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Laurence M. Ball
N. Gregory Mankiw

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ABSTRACT

This paper examines the optimal accumulation of capital and the effects of government debt in neoclassical growth models in which firms have market power and therefore charge prices above marginal cost. In this environment, the real interest rate earned by savers is less than the net marginal product of capital. We establish a new method for evaluating dynamic efficiency that can be applied in such economies. A plausible calibration suggests that the wedge between the real interest rate and the marginal product of capital is more than 4 percentage points and that the U.S. economy is dynamically efficient. In addition, government Ponzi schemes can have different implications for welfare than they do under competition. Even if the government can sustain a perpetual rollover of debt and accumulating interest, the policy may nonetheless reduce welfare by depressing steady-state capital and aggregate consumption. These findings suggest that even with low interest rates, as have been observed recently, fiscal policymakers should still be concerned about the crowding-out effects of government debt.

Laurence M. Ball
Department of Economics
Johns Hopkins University
Baltimore, MD 21218
and NBER
lball@jhu.edu

N. Gregory Mankiw
Department of Economics
Littauer 223
Harvard University
Cambridge, MA 02138
and NBER
ngmankiw@fas.harvard.edu

This paper explores the role of market power in neoclassical models of economic growth with a focus on the implications for the real rate of interest. Our analysis is based on a well-known principle: When firms with market power set their prices as a markup over marginal cost, they also choose a capital stock at which the marginal product of capital exceeds the cost of capital. Other things being equal, this wedge between the marginal product of capital and the cost of capital depresses the real interest rate.

The link between market power and the real interest may be especially important in the contemporary economy. Over the past few decades, the U.S. economy, along with many other economies around the world, has experienced a dramatic decline in real interest rates. Many observers have suggested that, during this period, markets have become less competitive, and markups have increased. [See, for example, Philippon (2020) and De Loecker, Eeckhout, and Unger (2020).] If they are correct, then increasing market power may be one reason for the declining real interest rate. In this environment, understanding the role of market power may be necessary to correctly interpret data on the real interest rate.

The connection between market power and the real interest rate provides a new lens through which to view fiscal policy and optimal national saving and investment. Discussions of fiscal policy often emphasize that government budget deficits can impede economic growth by crowding out capital. Recently, however, some observers have pointed to low real interest rates to suggest that crowding out may now be of little concern. [See, for example, Blanchard (2019) and Furman and Summers (2020).] The existence of market power calls this inference into question. If market power is depressing the real interest rate, then the real interest rate does not reflect the social value of additional capital.

To make these points as simply as possible, we start by building on Solow's (1956) model of economic growth and Phelps's (1961) analysis of optimal capital accumulation. The main change we make to this textbook model is to add firms with market power. We show that this amendment profoundly alters how the standard growth framework is applied to interpret the data, especially data on the real interest rate. Market power can push the real interest rate below the economy's growth rate. Under competition, this condition is associated with the overaccumulation of capital—a condition known as dynamic inefficiency. But that is not the case in an economy with market power.

We produce a new method for evaluating dynamic efficiency in the presence of market power. The method starts with measured capital income per unit of capital, which is commonly used to gauge rates of return in competitive economies. It then applies two corrections. An estimate of the typical markup in the economy corrects for the underpayment of capital that arises from market power. An estimate of Tobin's q corrects for the fact that the national accounts include the pure profits that firms receive in measured capital income.

To get a sense for the practical importance of these effects, we calibrate this expanded Solow model to roughly match data for the U.S. economy. If typical prices are marked up 20 percent over marginal cost, as some of the literature suggests, the real interest rate is more than 4 percentage points below the social return on capital. Correcting for the effects of market power yields an estimate of the social return of more than 8 percent. Because this estimated social return on capital is much higher than the economy's growth rate, it indicates that the U.S. economy is dynamically efficient.

After examining these effects in the Solow model, we consider the sustainability and welfare effects of government debt. To do so, we build on the classic work of Diamond (1965) on capital accumulation in an overlapping-generations model. Recall that, in competitive economies, there is a close connection between dynamic efficiency and government Ponzi schemes. [See, for example, O'Connell and Zeldes (1988).] If the real interest rate is below the growth rate, indicating dynamic inefficiency, the government can run a permanent primary deficit: it can give a transfer to every generation and roll over the resulting government debt forever. The crowding out that results from the debt issuance increases welfare because the economy has more capital than is optimal.

This close connection between dynamic efficiency and Ponzi schemes breaks down in an economy with market power. We show that if market power pushes the real interest rate below the growth rate, the government can again run a Ponzi scheme. However, such a Ponzi scheme, though feasible, may not be a free lunch. Even a successful Ponzi scheme may reduce welfare in the steady state.

The impact of a government Ponzi scheme on welfare depends on two effects. The first we call the *aggregate effect*. Because government debt crowds out capital, it depresses aggregate consumption if the net marginal product of capital exceeds the growth rate and increases aggregate consumption in the opposite case. [This is parallel to the classic Phelps (1961) result.] The second we call the *generational effect*: the introduction of government debt improves the allocation of aggregate consumption between the young and the old. This effect is similar to the Pareto improvement that can result from pay-as-you-go social security when the growth rate exceeds the interest rate (Samuelson, 1958; Weil, 2008). In a competitive economy, the aggregate and generational effects of debt work in the same direction. In an economy with market power, when the economy's growth rate is greater than the real interest rate but less than the net marginal product of capital, the two effects conflict.

The relative size of the two effects depends on parameters. For example, the magnitude of the generational effect depends on the intertemporal elasticity of substitution in consumption. If that elasticity is small, the generational effect is small, and the aggregate effect dominates. More generally, these findings show that, in the presence of market power, the real interest rate is a poor guide by itself for judging the impact of increased government indebtedness.

To be sure, while this paper studies the role of market power, the real interest rate and the marginal product of capital can differ for other reasons. Capital returns are risky, and other things being equal, the risk premium depresses the risk-free rate. [See, for example, Abel et al. (1989) and more recently Barro (2020).] This phenomenon does not arise here because we consider models with certainty. Of course, in the world, both uncertainty and market power influence the real interest rate. But we believe we can best clarify matters by considering only one of these forces. Just as much previous work has focused on the case of uncertainty without market power, here we focus on the case of market power without uncertainty.

Our analysis is related to several strands of the literature. Farhi and Gouria (2018) and Basu (2019) note that market power can drive a wedge between the marginal product of capital and the cost of capital, but they do not explore the implications for optimal capital accumulation or fiscal policy. Judd (1997) suggests that imperfect competition can imply that the optimal tax on capital is negative. He aims to use tax policy to correct the distortion that we take as given. Reis (2020) examines debt policy in an economy where credit-market imperfections cause the interest rate to fall below the marginal product of capital. Though the wedge in his model arises from a very different source, his analysis shares with ours the importance of distinguishing between the social return on physical capital and the private return on financial assets. Finally, much of the endogenous growth literature, such as Aghion and Howitt (1992), emphasizes the role of market power as a driver of technological innovation. By contrast, we follow in the neoclassical tradition of taking technological change as given to explore the implications of market power for optimal capital accumulation and fiscal policy.

The Solow Model with Market Power

We incorporate market power into the Solow model in four steps. First, we consider the behavior of individual firms. Second, we imbed these firms into the standard Solow framework. Third, we reconsider Phelps's conclusions about optimal capital accumulation in this model. Fourth, we provide a numerical calibration to roughly match the U.S. economy.

The Behavior of Individual Firms

The economy consists of N identical firms, each of which sells a differentiated product. Each firm produces its good using capital, labor, and intermediate goods purchased from other firms. The firm's output can be used as consumption, investment, or an intermediate good for other firms.

Following Rotemberg and Woodford (1995), we assume that producing one unit of the firm's output requires one unit of a composite of capital and labor and α units of intermediate goods. The composite is produced by the function

$$Q = F[K, A(L - X)]$$

where Q is the composite (which in turn equals the quantity of the firm's output), K is capital, L is labor, A is a measure of the state of labor-augmenting technology, X is a constant that represents fixed overhead labor, and $F()$ is homogenous of degree one. If $X = 0$, firms have constant returns to scale. If $X > 0$, then because of this fixed cost, firms exhibit increasing returns to scale.¹ Increasing returns to scale are not required for our analysis, but it is important to allow for this possibility. A sizable markup with constant returns would imply implausibly high pure profits as a share of national income. Similarly, intermediate goods are not crucial for the theory but are important when calibrating the model to actual economies.

Each firm has market power and charges a markup γ over marginal cost:

$$P = \gamma MC$$

where P is the price of the firm's output. In this part of the paper, we take the markup as exogenously given. The markup is presumably determined by the elasticity of demand and perhaps other aspects of market structure, which for now we do not need to specify. To ensure the solution to the model is well-behaved, we assume $\gamma\alpha < 1$.

Each firm hires labor at rate W and rents capital at rate R in competitive factor markets. It also buys goods from other firms, which it uses as its intermediate goods, at price P_i . An extra unit of output can be produced with either additional capital or additional labor, plus the necessary intermediate goods. Therefore, cost minimization implies that

$$MC = \frac{R}{F_1} + \alpha P_i = \frac{W}{AF_2} + \alpha P_i$$

where F_1 and F_2 are the partial derivatives of $F()$ with respect to the first and second arguments.

Because of the symmetry among the many identical firms, all firms' prices are the same, so

$$P_i = P.$$

As a result, the share of revenue each firm spends on intermediate goods equals the parameter α . Combining the last three equations, the price of output can be written as

¹ We use this formulation of fixed costs as overhead labor because it prevents fixed costs from vanishing in importance as the economy grows.

$$P = \left(\frac{\gamma}{1 - \gamma\alpha} \right) \frac{R}{F_1} = \left(\frac{\gamma}{1 - \gamma\alpha} \right) \frac{W}{AF_2}.$$

The price of the firm's output depends on its markup, the cost of capital and labor, its production function, and the intermediate-goods share.

To relate these firm-level variables to measures in the national income accounts, note that the firm's real value added is its gross output Q minus the value of intermediate goods αQ :

$$Y = (1 - \alpha)Q = (1 - \alpha)F[K, A(L - X)].$$

The marginal products of capital and labor in terms of value added are

$$\frac{\partial Y}{\partial K} = (1 - \alpha)F_1$$

$$\frac{\partial Y}{\partial L} = (1 - \alpha)AF_2.$$

From these equations and the preceding equation for P , we obtain

$$P = \mu \frac{R}{\partial Y / \partial K} = \mu \frac{W}{\partial Y / \partial L}$$

where

$$\mu = \frac{(1 - \alpha)\gamma}{1 - \alpha\gamma}.$$

Equivalently,

$$\frac{R}{P} = \frac{\partial Y / \partial K}{\mu}$$

$$\frac{W}{P} = \frac{\partial Y / \partial L}{\mu}.$$

This equation shows the wedge μ between the marginal products of capital and labor and their real factor prices. This wedge plays a crucial role in what follows.

The variable μ can be interpreted as the economy-wide markup: the ratio of the price to the social cost of producing a marginal unit of output. As Rotemberg and Woodford (1995) and Basu (2019) emphasize, when expenditure on intermediate goods is a substantial share of firm revenue, the economy-wide markup is much larger than the firm-level markup γ . This difference arises because of double marginalization: the cost of producing intermediate goods is marked up repeatedly along the chain of production. Hereafter, we work mostly in terms of the economy-

wide markup. The firm-level markup and the intermediate-goods share will return, however, when we calibrate the model.

The economy-wide markup determines the pure profits that accrue to firms. Each firm's share of profits in its value added:

$$\Pi = \frac{PY - RK - WL}{PY}.$$

Using the above relationships and Euler's theorem, we can express the profit share as

$$\Pi = 1 - \frac{1}{\mu} - \varphi \frac{X}{L}$$

where $\varphi = WL/(PY)$ is the labor share of value added. A firm's profit share depends on the economy-wide markup μ and the size of the fixed labor cost X relative to its labor input L . Though there may be sizable pure profits in this economy, that need not be the case, depending on the magnitude of the fixed cost.

In this classical model, only relative prices matter. Therefore, without loss of generality, we can hereafter normalize P , the price of output for these identical firms, to equal one.

The Aggregate Economy

Now let's embed these firms with market power in an otherwise standard Solow growth model as presented, for example, in the first chapter of Romer (2019). The economy's final output is the sum of the valued added of the N firms. The economy's aggregate production function is

$$Y = (1 - \alpha)F[K, A(L - NX)]$$

where Y , K , and L now represent aggregate output, capital, and labor. Because the firms are identical and the function $F()$ is homogenous of degree one, this aggregate production function follows from the individual firms' production functions for value added.

Final output can be used for consumption or investment. Labor is inelastically supplied and grows at rate n . Technological progress increases A at rate g . The economy saves an exogenous fraction s of output, and a fraction δ of capital depreciates each period. To ensure balanced growth, we assume the number of firms grows with the population:

$$N = \theta L$$

where θ is a constant. Its reciprocal is the number of workers in each firm. As a result, the aggregate production function becomes

$$Y = (1 - \alpha)F[K, AL(1 - \theta X)].$$

As is standard, we define AL to be the effective labor force, which grows at rate $n + g$. The economy's production function per effective worker is

$$y = f(k)$$

where $y = Y/(AL)$, $k = K/(AL)$, and $f(k) = (1 - \alpha)F(k, 1 - \theta X)$. Throughout the analysis, we hold α , θ , and X constant, so these arguments in the production function can be suppressed. Except for the fixed cost and the correction for intermediate goods, the production function is standard. We assume that $f'(k) > 0$ and $f''(k) < 0$.

The equation for capital accumulation is also standard:

$$\frac{dk}{dt} = sf(k) - (n + g + \delta)k.$$

Therefore, the steady-state capital stock k^* , defined by $dk/dt = 0$, is determined by

$$sf(k^*) = (n + g + \delta)k^*.$$

As is usual, output per effective worker is constant in steady state, and total output grows at rate $n + g$. Notice that the markup μ does not affect the steady-state capital stock and output. This result follows from the Solow model's assumption of an exogenously fixed saving rate. The markup does, however, affect the distribution of national income among capital, labor, and profit.

The markup also affects real interest rate r . In particular, recall that each unit of capital is paid the following:

$$R = \frac{f'(k^*)}{\mu}$$

where μ is the economy-wide markup. The return on holding capital is $R - \delta$. Arbitrage requires that this return equals the return on holding a financial asset r . In other words, as is standard, the rental price of capital R equals the cost of capital $r + \delta$. Therefore,

$$r + \delta = \frac{f'(k^*)}{\mu}$$

or

$$r = \frac{f'(k^*)}{\mu} - \delta.$$

This equation shows that, other things being equal, increased market power reduces the real interest rate.

In essence, market power reduces firms' demand for capital. Because the Solow model assumes a fixed saving rate, the supply of capital is inelastic, so the reduced demand is reflected in a lower cost of capital. Hence, the economy has a lower real interest rate. For example, suppose that the gross marginal product of capital f' is 13 percent and the intermediate goods share α is 0.5—numbers consistent with our calibration below. Then an increase in the firm-level markup γ from 1.0 to 1.2 increases the economy-wide markup μ from 1.0 to 1.5 and reduces the real interest rate by 4.3 percentage points.

Dynamic Efficiency

Let's now examine the optimal accumulation of capital. In this economy, steady-state consumption per person is

$$c^* = f(k^*) - (n + g + \delta)k^*.$$

As a result, the consumption-maximizing steady-state capital stock, which Phelps (1961) dubbed the Golden Rule capital stock, is determined by:

$$f'(k_{GR}^*) = n + g + \delta.$$

At the Golden Rule capital stock, the net marginal product of capital $f' - \delta$ equals the growth rate of the economy $n + g$. If the saving rate is larger than necessary to yield this capital stock, the economy is said to be dynamically inefficient because the economy can reduce saving and increase consumption at all points of time. The existence of market power does not alter these familiar results.

How can one judge whether an economy is dynamically efficient? The key is to compare the net marginal product of capital $f' - \delta$ with the economy's growth rate $n + g$. Because of diminishing marginal product, if $f' - \delta < n + g$, the economy has more capital than at the Golden Rule steady state, and if $f' - \delta > n + g$, it has less.

The real interest rate might seem useful here. Under competition, the markup μ equals one, and the real interest rate equals the net marginal product of capital $f' - \delta$. That is one reason why comparisons of the real interest rate with the growth rate appear so often in discussions of economic growth. This approach is not robust, however. With market power, the real interest rate equals $f'/\mu - \delta$. If $f' - \delta > n + g > f'/\mu - \delta$, an observer who assumed the economy is competitive would measure the net marginal product of capital with the interest rate and erroneously conclude that the economy is dynamically inefficient.

Another approach to judging dynamic efficiency, rather than relying on the real interest rate, is to measure the marginal product of capital with capital income per unit of capital. [For

example, see Abel et al. (1989)² and Reis (2020).] But this approach does not solve the problems introduced by market power. If capital income reported in the national accounts combines the net payments to capital $RK - \delta K$ and pure profits ΠY , then the measured net return per unit of capital, which we denote as m , is

$$m = \frac{RK - \delta K + \Pi Y}{K}.$$

This can be written as

$$m = \frac{f'}{\mu} - \delta + \frac{\Pi y}{k}.$$

As a gauge of the true marginal value of capital $f' - \delta$, this measure suffers from two problems. First, the markup reduces the payments to capital below its marginal product, which tends to decrease the measured return on capital. Second, pure profits appear to accrue to capital, which tends to increase the measured return on capital. These two biases push in opposite directions. Depending on the size of the markup and the size of the fixed costs, either bias could be larger.

We can, however, correct the measured return m for these biases. To do so, it is useful to define steady-state Tobin's q : the ratio of the market value of existing firms to the replacement cost of capital. If firms are infinitely lived,³ we can use the well-known Gordon growth formula to evaluate their market value:

$$M = \frac{RK - \delta K - gK + \Pi Y}{(r - g)}.$$

The numerator is the firms' dividends: their income $RK + \Pi Y$ minus their expenditure on capital investment $\delta K + gK$. (To maintain growth at rate g , firms must both replace depreciating capital and increase their capital stocks at rate g .) Dividends are discounted at $r - g$ because they grow at rate g in the steady state. Tobin's q is

$$q = \frac{M}{K}.$$

² In particular, Abel et al. (1989) recommend comparing the gross cash flow generated by capital [which in our notation equals $(m + \delta)K$] with gross investment [which equals $(n + g + \delta)K$ in the steady state]. In our framework, this is equivalent to comparing net capital income per unit of capital m with the growth rate $n + g$. Abel et al. show that this condition works under uncertainty and argue, therefore, that it is better than drawing inferences from the risk-free real interest rate. Throughout their analysis, they assume constant returns to scale and competitive markets.

³ If firms are not infinitely lived but instead die with a hazard rate of d , then future profits are discounted at rate $r + d - g$. All the expressions that follow remain the same, except g is replaced with $g - d$. Firm death would also allow for the possibility of $r < g$ without sending the market value of firms to infinity.

Using these expressions and the fact that $R = r + \delta$, Tobin's q can be written as

$$q = 1 + \frac{\Pi y}{(r - g)k}.$$

The excess of q over 1 equals the present value of pure profits from existing firms per unit of capital.

We can use now derive an expression for the marginal product of capital as a function of variables that can be observed or estimated:

$$f' = \mu \left(\frac{m - g}{q} + g + \delta \right).$$

This equation follows from the equation for q , the equation for m , and the equation for r . In the competitive case in which $\mu = 1$ and $q = 1$, it simplifies to the familiar $f' = m + \delta$. More generally, we can infer the true marginal product of capital from the measured net return m with two corrections. The multiplication by μ accounts for the underpayment of capital that arises from market power. The division by q accounts for the pure profits that firms receive.⁴

In short, evaluating dynamic efficiency in an economy with market power is tricky. An observer requires an estimate of the marginal product of capital. The real interest rate is an unreliable gauge because it is depressed by market power. Measuring the return on capital as firm income per unit of capital yields an estimate that can be either above or below the marginal product. Fortunately, our last equation provides a way to estimate the marginal product of capital, which we will apply shortly.

An Approximate Calibration to the U.S. Economy

Let's now calibrate this model with some numbers that roughly describe the U.S. economy today. We use the following figures as our baseline:

- The capital stock is three times annual output. That is, $K/Y = 3$.
- Depreciation is 15 percent of output. That is, $\delta K = .15Y$.
- Measured gross capital income is 33 percent of gross output. That is, $1 - \phi = 0.33$.
- Income per person grows at 2 percent per year. That is, $g = .02$.
- The share of intermediate goods in firm revenue is 50 percent. That is, $\alpha = 0.5$

⁴ Abel et al. (1989) note that q was averaging around 1 when they were writing, and they use that fact to justify their assumption of constant returns and competition. We can now see the flaw in that inference. A value of q equal to 1 could arise because fixed costs are large enough to eliminate pure profits, but that fact by itself does not render the measured capital return m a useful gauge for the net marginal product of capital $f' - \delta$. If q equals 1, the measured capital return understates the net marginal product of capital if $\mu > 1$.

- The market value of firms is twice the replacement cost of their capital. That is, $q = 2$.
- Firms mark up prices 20 percent over their marginal cost. That is, $\gamma = 1.2$.

The first five numbers are based on the national income accounts and related BEA data. The sixth is based on estimates of Tobin's q as we write this paper in the year 2021. This number is hard to pin down because stock market valuations vary substantially over time, so we will examine the sensitivity of our conclusions to plausible variations in it.

The most challenging number in the model to calibrate is the markup γ . Basu (2019) offers a good overview of the literature on estimating markups, which has not reached a consensus. Many studies find that markups have increased over the last several decades, though there is also not a consensus whether that is the case and, if so, by how much. We use an estimate of 1.2 as our baseline value of γ , but our sensitivity analysis allows γ to range from 1.05 to 1.4. Most of the markup estimates cited by Basu lie within this range.

Taking these seven numbers as our baseline and applying the above equations, we can infer the other key variables in the model. From the firm-level markup γ and the intermediate-goods share α , the economy-wide markup μ is computed using the equation derived earlier:

$$\mu = \frac{(1 - \alpha)\gamma}{1 - \alpha\gamma}.$$

We find that μ equals 1.5.

Our numbers for the depreciation share and the capital–output ratio imply that the depreciation rate δ is 5 percent. Using the definition of the labor share φ , we can then compute the measured capital return m as follows:

$$m = \frac{(1 - \varphi)}{k/y} - \delta.$$

From m , we then obtain an estimate of the marginal product of capital:

$$f' = \mu \left(\frac{m - g}{q} + g + \delta \right).$$

Next, we compute the implied real interest rate:

$$r = \frac{f'}{\mu} - \delta.$$

In the end, we find a real interest rate r of 4 percent, a measured net return on capital m of 6 percent, and a social return on capital $f' - \delta$ of 8.5 percent. This estimate of the social return on capital is well above the economy's average growth rate of about 3 percent, indicating that the U.S. economy is dynamically efficient.

The share of pure profits in gross national income is calculated as follows:

$$\Pi = (q - 1)(r - g)(k/y).$$

With the above numbers, pure profits are 6 percent of income. Given the intermediate-goods share of one half, pure profits are 3 percent of each firm's gross revenue.

Finally, we can also infer the degree of increasing returns, as measured here by the percentage of workers in the economy that represent a fixed cost:

$$\frac{NX}{L} = \frac{1 - \frac{1}{\mu} - \Pi}{\varphi}.$$

This equation follows from the earlier equation for an individual firm's profit share Π and the fact that there are N firms. For our calibration, NX/L is 41 percent. Such a large number is required to explain why a substantial economy-wide markup of 50 percent generates relatively small pure profits of only 6 percent of income. Given that the intermediate-goods share is one half and that labor represents 67 percent of value added, the cost of this overhead labor is 14 percent of firms' gross revenue.⁵

It is worth noting the real interest rate in our calibration of 4 percent is higher than the return on risk-free assets like Treasury bills, which in recent years have returned close to zero (and sometimes less) in real terms. No doubt, this discrepancy occurs because our model excludes uncertainty. As many authors have noted, other things being equal, uncertainty together with risk aversion drives down the risk-free rate. It might be best to interpret the real interest rate in our model not as the risk-free rate but as the average return on the financial assets that fund capital accumulation. From this perspective, the calibrated estimate of 4 percent seems reasonable.

Table 1 shows the sensitivity of our results to plausible changes in μ and q . For comparison, the top line shows the competitive benchmark ($\gamma = 1$, $q = 1$). In this case, all three measures of the rate of return (r , m , and $f' - \delta$) give an estimate of 6 percent, and there are no pure profits. The second panel shows the effect of varying μ around our baseline calibration ($\gamma = 1.2$, $q = 2$), and the third panel shows the effect of varying q . In all these cases, the real interest rate r is depressed below the social return on capital $f' - \delta$. Except in the case when the economy is close to competitive ($\gamma = 1.05$), the size of this wedge is at least 2 percentage points and often much larger.

⁵ The results change only slightly if firms are not infinitely lived (see footnote 3). Suppose that firms live on average for 50 years, so d is 2 percent. Then under our baseline, the real interest rate r is 3.0 percent, the measured net return on capital m is 6.0 percent, and the social return on capital $f' - \delta$ is 7.0 percent. Thus, in this case, market power depresses the real interest rate below the social return on capital by 4.0 percentage points. The profit share Π is 9.0 percent, and the fixed cost NX/L is 36 percent.

The social return on capital is always above the economy's average growth rate of about 3 percent, often by a large margin, indicating dynamic efficiency.

Table 2 shows what happens to the social return on capital $f' - \delta$ as Tobin's q and the markup γ vary together. Here we exclude entries that would imply negative fixed costs (which occurs when $\gamma = 1$ and $q > 0$). Note that for every plausible combination of these parameters, the estimated social return is substantial.

The bottom line is that, when firms have market power, the social return on capital necessary to gauge dynamic efficiency can be much larger than the real return earned by savers in the economy. In our main calibration, the gap is more than 4 percentage points. The next section examines what this conclusion implies for fiscal policy.

Table 1**Sensitivity check**

All numbers in the table are in percent.

	r	m	$f' - \delta$	Π	NX/L
Competitive benchmark					
$\gamma = 1.0, q = 1$	6.0	6.0	6.0	0	0
Effect of varying γ (with $q = 2$)					
$\gamma = 1.05$	4.0	6.0	4.9	6.0	5
$\gamma = 1.1$	4.0	6.0	6.0	6.0	18
$\gamma = 1.2$ (baseline)	4.0	6.0	8.5	6.0	41
$\gamma = 1.3$	4.0	6.0	11.7	6.0	60
$\gamma = 1.4$	4.0	6.0	16.0	6.0	76
Effect of varying q (with $\gamma = 1.5$)					
$q = 1$	6.0	6.0	11.5	0	50
$q = 1.5$	4.7	6.0	9.5	4.0	44
$q = 2$ (baseline)	4.0	6.0	8.5	6.0	41
$q = 2.5$	3.6	6.0	7.9	7.2	39
$q = 3.0$	3.3	6.0	7.5	8.0	38

Table 2**Sensitivity check, continued**

All numbers in the table represent the net marginal product of capital $f' - \delta$ and are in percent.

	$\gamma = 1.0$	$\gamma = 1.05$	$\gamma = 1.1$	$\gamma = 1.2$	$\gamma = 1.3$	$\gamma = 1.4$
$q = 1$	6.0	7.2	8.4	11.5	15.4	20.7
$q = 1.5$	---	5.7	6.8	9.5	13.0	17.6
$q = 2$	---	4.9	6.0	8.5	11.7	16.0
$q = 2.5$	---	4.5	5.5	7.9	11.0	15.1
$q = 3.0$	---	4.2	5.2	7.5	10.6	14.4

The Sustainability and Welfare Effects of Government Debt

We now consider the effects of government debt in an economy with market power. For this purpose, we move from the Solow model to Diamond's (1965) overlapping-generations (OLG) model, a fully specified general equilibrium model with optimizing agents. The Diamond model is a classic tool for analyzing fiscal policy.

We focus on the questions of whether the government can run a Ponzi scheme and, if so, to what effect. In other words, can the government run up a debt and then roll over the principal and the accumulating interest forever? And if it can avoid future taxes to service or repay the debt, how does such a policy affect welfare? This longstanding issue has received renewed interest recently, with prominent economists suggesting that low interest rates make such a policy feasible and perhaps desirable. [See, for example, Blanchard (2019).] As we will see, the answers to these questions are markedly different in economies with market power than they are under competition.

In competitive OLG models with certainty, the effects of fiscal policy are closely linked to whether the economy is dynamically efficient. In that setting, dynamic efficiency can be determined by comparing the growth rate of the economy with either the net marginal product of capital or the real interest rate (because the two are equal). Under dynamic efficiency, a Ponzi scheme is infeasible in the sense that the government must eventually raise future taxes to service any debt it issues. Moreover, because government debt crowds out capital, it reduces welfare for future generations. On the other hand, if the economy is dynamically inefficient, Ponzi schemes are feasible, and issuing debt can yield a Pareto improvement by reducing the overaccumulation of capital.

As in the Solow model, the key effect of market power is to introduce a wedge between the marginal product of capital and the real interest rate. This wedge raises the possibility that the net marginal product of capital exceeds the economy's growth rate while the real interest rate is less than the growth rate.⁶ In that case, the low interest rate allows the government to run a perpetual Ponzi scheme by making debt-financed transfers to every generation. The scheme is feasible because the debt-income ratio, rather than exploding, converges to a constant level. In contrast to the competitive case, however, the feasibility of this Ponzi scheme does not imply that it increases welfare. With the marginal product of capital above the growth rate, the crowding out of capital reduces aggregate consumption, and the welfare loss from this effect can exceed the welfare gains that individuals obtain from the transfers.

⁶ Reis (2020) focuses on this scenario as well, though the wedge in his model comes from a credit-market imperfection rather than from market power.

Assumptions

Aside from the introduction of market power, our essential assumptions follow the Diamond model. [See, for example, the second chapter of Romer (2019)]. A cohort of individuals is born each period and lives for two periods. For simplicity, we assume that the number of individuals in a cohort is constant over time and normalize it to one. An individual works when young, receives earnings, and divides the earnings between consumption and saving. Saving is used to buy capital and any bonds issued by the government. When old, the individual receives a transfer from the government (which can be positive or negative) and consumes this transfer plus the gross return on savings.⁷

To introduce market power, we assume that each young person is a yeoman farmer who produces and sells a differentiated good. A farmer produces her good using an endowment of one unit of her own labor along with capital that she rents from the old in a competitive factor market.⁸ We assume the same production function as in the Solow model, except that we exclude intermediate goods for simplicity. With one unit of labor, the farmer produces

$$Y = F[K, A(1 - X)],$$

where Y is output, K is the capital rented by the farmer, and the technology parameter A grows at the rate g each period. We again use the notation of $y = Y/A$ for output per unit of effective labor, $k = K/A$ for capital per unit of effective labor, and $y = f(k) = F(k, 1 - X)$ for the production function in terms of y and k . Again, we assume that $f'(k) > 0$ and $f''(k) < 0$.

Each farmer consumes the goods produced by all the farmers. We let C_i denote an individual's consumption of good i , with i distributed uniformly on $[0, 1]$. Following the canonical Dixit-Stiglitz (1977) model, a farmer's utility depends on a consumption aggregate C :

$$C = \left(\int_0^1 C_i^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

where ε is the elasticity of substitution between different goods. The lifetime welfare of a farmer born in period t is

$$V_t = U(C_{y,t}) + \beta U(C_{o,t+1}), \quad \beta > 0,$$

where C_y and C_o are the farmer's levels of consumption C when young and old. In addition, the

⁷ Our main results do not depend on whether the transfer is received when old or young. We assume it is received when old because doing so will make it easier to see the equivalence between government debt and unfunded social security.

⁸ In our choice of pronouns, we assume the economy is entirely female, as in Themyscira. Our results are robust if instead the farmers are male, non-binary, or a mixture of genders.

utility function is

$$U(C) = \frac{C^{1-\sigma}}{1-\sigma}, \quad \sigma > 0.$$

This functional form for $U(C)$ implies a constant elasticity of intertemporal substitution $1/\sigma$, which is necessary for a steady state with balanced growth.

The same aggregate of individual goods that produces one unit of consumption C can also produce one unit of capital K . Therefore, as in the original Diamond model, one unit of consumption is freely tradable for one unit of capital. Capital produced in period t is used in production in period $t + 1$ and then fully depreciates.

Equilibrium in the Goods and Capital Markets

In each period, a young farmer chooses a price for her good to maximize profit, which is the difference between revenue from selling the good and the cost of renting capital to produce the good. We can think of this profit as a mixture of wages for the farmer's labor and economic profits derived from her market power.

As is usual in the Dixit-Stiglitz model, the elasticity of demand for each farmer's product equals the substitution parameter ε , and the profit-maximizing price equals the farmer's marginal cost times a markup $\mu = \varepsilon/(\varepsilon - 1)$. (Here, because there are no intermediate goods, the economy-wide markup and the firm-level markup are the same.) In equilibrium, every farmer charges the same price, produces the same level of output, and rents the same amount of capital. We normalize the price of each farmer's good to one.

Each farmer divides her consumption expenditure equally among all the differentiated goods: C_i is the same for every good i . This fact and the definition of the consumption aggregate C imply that C equals the common C_i , and that the price of a unit of C equals one, the price of an individual good.

With the prices of all goods equal to one, the condition for profit-maximization becomes $1 = \mu(MC)$. Marginal cost MC is R/MPK , where R is the rental price of capital and MPK is the marginal product of capital $f'(k)$. Therefore, as in the Solow model, $R = MPK/\mu$. This condition defines a farmer's demand for capital. The supply of capital is determined by the saving decision made by the currently old generation in the previous period, as described below. The supply and demand for capital determine the equilibrium value of R .

Intertemporal Optimization

Given the period-by-period equilibrium in the goods and capital markets, an individual who is young in period t chooses a level of saving S_t to maximize welfare V_t . Saving is divided between capital accumulation and purchases of government debt. The gross return on capital equals the rental price of capital in the next period, R_{t+1} . In our non-stochastic model, the gross return on government debt must also equal R_{t+1} because capital and debt are perfect substitutes for savers.

The individual's budget constraint is

$$C_{y,t} = F[K_t, A_t(1 - X)] - R_t K_t - S_t \quad (1)$$

$$C_{o,t+1} = R_{t+1} S_t + \Omega_{t+1}$$

where Ω is the transfer from the government to the old (or tax if $\Omega < 0$). The first equation says that consumption when young equals earnings (the farmer's output minus the cost of renting capital) minus saving. The second equation says that consumption when old equals the return on saving plus the transfer. We assume that the government announces the transfer Ω_{t+1} in period t so the individual accounts for it in choosing S_t .

Given the utility function $U(C)$, the first-order condition for a person's optimal consumption path is

$$\frac{C_{o,t+1}}{C_{y,t}} = (\beta R_{t+1})^{1/\sigma} . \quad (2)$$

The ratio of a person's consumption when old and young is determined by βR_{t+1} with an elasticity equal to $1/\sigma$, the elasticity of substitution between consumption in the two periods. This equation and the budget constraint define $C_{y,t}$, S_t , and $C_{o,t+1}$ in terms of the interest rate in periods t and $t + 1$, the government transfer, and the level of K_t , which was determined in period $t - 1$.

Fiscal Policy

We focus on the following fiscal policy, which is always feasible. Initially, the transfer Ω is zero, and there is no government debt. Then, in some period t_0 , the government issues some bonds, which it sells to the young, and uses the proceeds to make a positive transfer to the old (an intervention announced at least one period prior to t_0).⁹ Let B be the level of government debt. At every $t > t_0$, the government keeps B_t/A_t constant at the level established in t_0 , which we call b , and sets the transfer Ω_t as needed to achieve that outcome. A constant B_t/A_t means that debt increases

⁹ If instead the policy is a surprise (a so-called MIT shock), some of the details that follow are different, but the main results regarding the feasibility and welfare effects of the policy are the same.

at the same rate as productivity, which in steady state implies a constant ratio of government debt to the economy's output.

The transfers or taxes implied by this policy are determined as follows. The level of debt evolves according to

$$B_t = R_t B_{t-1} + \Omega_t$$

where the first term on the right is the interest and principal on the debt rolled over from the previous period and the second is the current transfer. For $t > t_0$, the government's policy of holding b constant implies $B_t = A_t b$ and $B_{t-1} = A_{t-1} b = A_t b / (1 + g)$, which lead to

$$\Omega_t = A_t \left(1 - \frac{R_t}{1 + g} \right) b.$$

In a steady state with a constant R , there is a constant level of transfers per unit of effective labor, $\omega = \Omega/A$:

$$\omega = \left(\frac{g - r}{1 + g} \right) b$$

where $r = R - 1$ is the net return on capital and the interest rate on debt.

Notice that the sign of the steady-state ω depends on $g - r$. If $r > g$, then $\omega < 0$: in steady state, the government levies a perpetual tax to service the debt that it created through its transfer in period t_0 . If $r < g$, however, $\omega > 0$: the government provides a perpetual transfer that is never financed with taxes. In this case, the government runs a Ponzi scheme. This policy is feasible because the debt-income ratio tends to fall when the economy's growth rate exceeds the interest rate.

The feasibility of a Ponzi scheme when $r < g$ is a well-known result in OLG models. This economy, however, differs from those in previous work that assumes competitive markets. Because market power places a wedge between the marginal product of capital and the interest rate, a Ponzi scheme may be feasible even if the economy is dynamically efficient. As we will see, this difference has important implications for the welfare effects of government debt.

Debt and Steady-State Capital

Let's now consider the evolution of the capital stock after period t_0 when the government first issues debt. An individual's saving in all periods $t \geq t_0$ is divided between government bonds and investment in capital, which will be productive at $t + 1$. This implies that $K_{t+1} = S_t - B_t$. As described above, S_t is determined by an individual's optimal saving problem. The levels of R_t and R_{t+1} , which appear in the individual's optimization problem, are determined in equilibrium by the

condition $R = f'(k)/\mu$, and the transfer Ω_t is determined by the condition that the government maintains a constant level of b .

In the Appendix, we use these results to derive an equation that defines K_{t+1} as a function of K_t and B_t . As is common in work on the Diamond model, we assume that this equation determines a unique stable steady state for k , the level of capital per unit of effective labor, and that k approaches the steady state monotonically. (This assumption implicitly puts restrictions on the utility and production functions.) This steady state k^* depends on b , the level of government debt per unit of effective labor: $k^* = k^*(b)$.

While the condition defining k^* is complex, our main results follow from a simple property of k^* :

$$\frac{dk^*}{db} < 0.$$

That is, an increase in government debt reduces steady-state capital. This result is proved in the Appendix. It is a conventional crowding-out effect: an increase in debt diverts part of saving from capital accumulation to purchases of government bonds. We will see that all the steady-state effects of debt on welfare, whether positive or negative, flow from this crowding out.

Steady-State Welfare

To understand the effects of debt, it is useful to analyze the basic determinants of welfare as follows. Consider the lifetime welfare of a person born in period t :

$$V_t = U(C_{y,t}) + \beta U(C_{o,t+1}).$$

In steady state, $C_{y,t} = A_t c_y$ and $C_{o,t+1} = A_{t+1} c_o$, where c_y and c_o are the consumption of the young and old per unit of effective labor. We can rewrite $C_{o,t+1}$ as $A_t(1+g)c_o$. Aggregate consumption per unit of effective labor is $c = c_y + c_o$. Letting z equal c_o/c_y , the ratio of consumption of the old and young, we can write $c_y = c/(1+z)$ and $c_o = cz/(1+z)$. Combining these expressions, the steady-state welfare of a person born at t is

$$V_t^* = U\left(\frac{cA_t}{1+z}\right) + \beta U\left[\frac{cA_t(1+g)z}{1+z}\right]. \quad (3)$$

The last equation shows that steady-state welfare can be expressed in terms of two variables: c , aggregate consumption per unit of effective labor, and z , the old-to-young consumption ratio. This result will prove to be helpful because both c and z are determined in simple ways by k , the steady-state level of capital per unit of effective labor.

Specifically, the steady-state level of c is determined by:

$$\begin{aligned}
c &= y - i \\
&= f(k) - (1 + g)k
\end{aligned}
\tag{4}$$

where y and i are output and investment per unit of effective labor. The second line uses the fact that investment equals capital in the following period, which is $(1 + g)$ times capital in the current period. This equation is the same as the equation for consumption per unit of effective labor in the Solow model with the assumptions here that the depreciation rate is one and the population growth rate is zero.

An expression for $z = c_o/c_y$ follows from the first-order condition for an individual's optimization, which in steady state is:

$$\frac{C_{o,t+1}}{C_{y,t}} = (\beta R)^{1/\sigma}.$$

Using the facts that $C_{y,t} = A_t c_y$, $C_{o,t+1} = A_t(1 + g)c_o$, and $R = f'(k)/\mu$, we obtain:

$$z = \left[\frac{\beta f'(k)}{\mu} \right]^{1/\sigma} \left(\frac{1}{1 + g} \right).$$
(5)

The ratio z depends on the marginal product of capital $f'(k)$ because the marginal product influences the real interest rate and hence an individual's allocation of resources over time.

We now have a compact way to express steady-state welfare. Welfare is a function of aggregate consumption per unit of effective labor c and the ratio $z = c_o/c_y$, as shown by equation (3). Both c and z are determined by capital per unit of effective labor k , as shown by equations (4) and (5).

Welfare Effects of Debt

How does this fiscal policy of issuing government debt affect welfare? The answer is potentially different for different generations. At least one generation benefits: the generation that is old in period t_0 and therefore receives the first of the government transfers.

To see this result, recall that the transfer at t_0 is assumed to be positive, so the generation that receives it can use the transfer to increase consumption. We have assumed that the transfer is anticipated when that generation is young, so there is also a more subtle benefit: the extra resources when old lead the generation to reduce their saving, which reduces the capital stock when they are old and therefore increases the interest rate they receive on their saving. There is no adverse effect on the generation; in particular, no crowding out of capital has yet occurred when they are young,

so there is no loss of earnings when young.¹⁰

What are the effects of this debt on the welfare of future generations? We address this question by examining the steady-state effects of debt. If the policy lowers steady-state welfare, it obviously does not yield a Pareto improvement. In this case, the debt increases welfare in the short run but imposes a long-run burden.

Recall that steady-state welfare is determined by the variables c and $z = c_o/c_y$, and that both these variables are determined by k , the steady-state level of capital per unit of effective labor. Therefore, the effect of the debt level b on steady-state welfare operates through its effect on k . Differentiating the expression for welfare (3) yields:

$$\frac{dV_t^*}{db} = \frac{dV_t^*(c, z)}{dk} \frac{dk}{db}.$$

This expression can in turn be expanded:

$$\frac{dV_t^*}{db} = \left(\frac{\partial V_t^*}{\partial c} \frac{dc}{dk} + \frac{\partial V_t^*}{\partial z} \frac{dz}{dk} \right) \left(\frac{dk}{db} \right).$$

$$\begin{array}{ccccccc} (+) & (?) & & (?) & (-) & & (-) \\ \underbrace{\hspace{1.5cm}} & & & \underbrace{\hspace{1.5cm}} & & & \end{array}$$

AGGREGATE GENERATIONAL
EFFECT EFFECT

Under this equation, we have written the signs of the various derivatives that determine dV_t^*/db and labels to identify the two different effects of debt on steady-state welfare. To interpret this equation, recall first that dk/db is negative: debt crowds out capital. This fact implies that the effect of debt on welfare has the opposite sign of the effect of greater capital accumulation on welfare. The effect of capital on welfare consists of two effects operating through aggregate consumption c and the ratio $z = c_o/c_y$. In general, both these effects have ambiguous signs.

We call the effect operating through c the *aggregate effect*. It is clear from an individual's

¹⁰ Formally, the welfare of an individual born in period t can be written as a function of the variables in her budget constraint: $V_t(K_t, R_t, S_t, R_{t+1}, \Omega_{t+1})$. For the first generation to receive a transfer, $t = t_0 - 1$. The introduction of debt and transfers at t_0 does not affect K_t or R_t for this generation. It does affect an individual's choice of S_t , but there is no welfare effect through this channel because $\partial V_t/\partial S_t = 0$ (this is the first-order condition for optimal saving). The policy intervention increases Ω_{t+1} , and it increases R_{t+1} because it reduces K_{t+1} . Both the higher Ω_{t+1} and the higher R_{t+1} raise the individual's consumption when old and hence her welfare.

utility function that $\partial V_t^*/\partial c > 0$: holding constant the distribution of consumption between the old and young, higher aggregate consumption raises welfare. The sign of the aggregate effect is determined therefore by the sign of dc/dk . Differentiating equation (4) yields a familiar condition:

$$\frac{dc}{dk} > 0 \text{ iff } f'(k) - 1 > g.$$

This condition is the usual one for dynamic efficiency: more capital increases consumption if the net marginal product of capital [$f'(k)$ minus the depreciation rate of 1] exceeds the growth rate of the economy. When this condition holds, the aggregate effect of capital on welfare is positive; because debt crowds out capital, the welfare effect of debt through this channel is negative. When the condition does not hold, debt raises welfare by reducing the overaccumulation of capital.

We call the effect operating through z , the ratio of consumption when old and young, the *generational effect*. It is easy to show using equation (5) that $dz/dk < 0$: more capital shifts consumption away from the old. The reason is that more capital reduces the real interest rate and thereby causes individuals to reduce consumption when old relative to consumption when young.

We can derive $\partial V_t^*/\partial z$ using the expression for welfare, equation (3). Taking the partial derivative with respect to z leads after some algebra (see Appendix) to

$$\frac{\partial V_t^*}{\partial z} > 0 \text{ iff } g > r.$$

Recall that $dz/dk < 0$ and $dk/db < 0$, which together mean that an increase in debt leads to an increase in z . (The mechanism is that debt crowds out capital, which raises the interest rate, which in turn causes individuals to shift their consumption toward old age). Combining these results, we find that an increase in debt raises welfare through the generational channel if $g > r$ and reduces welfare through this channel if $g < r$.

What explains this result? An increase in z for a given c means that more aggregate consumption is shifted from the young to the old within each period. Over a person's life cycle, she loses consumption when young and gains consumption when old. Because c grows at rate g in steady state, the extra consumption that a person receives when old is $1 + g$ times the consumption she loses when young. From the first-order condition for optimal saving, the marginal utility of consumption when old equals $1/(1 + r)$ times the marginal utility of consumption when young. Combining these facts, the ratio of the utility gain when old to the loss when young is $(1 + g)/(1 + r)$. The net effect on welfare is positive if this ratio exceeds one, that is, if $g > r$.

We can build intuition for these results by recognizing that in this model, as in many OLG models, the steady-state effects of government debt are the same as those of a pay-as-you-go social security system. Specifically, in our model, the steady state with an amount b of debt per unit of effective labor is equivalent to a social security system that transfers bA_t from the young to the old in period t . We can see this from the cash flows between an individual born in period t and the

government. In our steady state with debt, a young individual gives the government bA_t to purchase its debt. When that person is old at $t + 1$, she receives $(1 + r)bA_t = [(1 + r)/(1 + g)]bA_{t+1}$ on the maturing debt and a transfer $\Omega_{t+1} = A_{t+1}b(g - r)/(1 + g)$. The total receipts when old sum to bA_{t+1} . The individual's payment to the government when young and total receipts when old are the same as in the social security system.

To focus on the generational effect of government debt or social security, it is useful to consider an endowment economy (Samuelson, 1958; Weil, 2008). In this setting, aggregate consumption is fixed, and social security shifts consumption toward the old: the policy affects welfare only through z . In an endowment economy, social security raises welfare if $g > r$, where r is the interest rate on consumption loans (which in equilibrium are in zero supply and demand). As in our model, $g > r$ implies that the utility gain from shifting consumption toward the old exceeds the loss when young. An endowment economy, however, does not have the aggregate effect because fiscal policy cannot alter total consumption c .

The Overall Welfare Effect of Debt

To summarize: Debt raises steady-state welfare through the aggregate effect if the economy's growth rate g exceeds the net marginal product of capital $f'(k) - 1$. Debt raises welfare through the generational effect if g exceeds the interest rate r . Because of market power, we know that $r < f'(k) - 1$, so our results imply the following about the total effect of debt:

1. If $g \leq r$, then $\frac{dV_t^*}{db} < 0$.
2. If $r < g < f'(k) - 1$, then $\frac{dV_t^*}{db}$ is ambiguous.
3. If $f'(k) - 1 \leq g$, then $\frac{dV_t^*}{db} > 0$.

These results arise because the two effects of debt have the same sign in the first and third cases and opposite signs in the second case.

Note that the competitive benchmark of $\mu = 1$ implies $r = f'(k) - 1$. In this case, the middle case of opposing aggregate and generational effects disappears, and the two effects always have the same sign. This result explains why previous research on debt in OLG models has not emphasized the difference between the two effects: this work has studied competitive models in which the two effects always operate in the same direction.

For the middle case of $r < g < f'(k) - 1$, the sign of the overall effect depends on how close g is to r and to $f'(k) - 1$. It also depends on the parameter σ in the utility function and the shape of the production function. Recall that $1/\sigma$ is the elasticity of intertemporal substitution in consumption. The key feature of the production function is the elasticity of substitution between

capital and labor, which we denote by v . In general, the condition determining the sign of the net welfare effect is complex, but we can gain some intuition from special cases:

$$\text{If } r < g < f'(k) - 1,$$

$$\frac{dV_t^*}{db} < 0 \text{ as } \sigma \rightarrow \infty \text{ or } v \rightarrow \infty,$$

$$\frac{dV_t^*}{db} > 0 \text{ as } \sigma \rightarrow 0 \text{ or } v \rightarrow 0.$$

That is, the net welfare effect of debt is negative for the middle range of g if the substitutability of consumption when young and old is very low (a high σ) and when the substitutability of labor and capital in production is very high (a high v). The net welfare effect is positive in the opposite cases.

These results, which are proved in the Appendix, arise because of how σ and v affect dz/dk and hence the size of the generational effect. Consider the limiting cases when $\sigma \rightarrow \infty$ or $v \rightarrow \infty$. In both these cases, $dz/dk \rightarrow 0$: the level of capital has no effect on the ratio of consumption when old and young. That is, the generational effect vanishes, and debt affects welfare only through the aggregate effect, which is negative because $f'(k) - 1 > g$.

To understand why $dz/dk \rightarrow 0$ for high values of σ and v , recall that an increase in k reduces r and the fall in r reduces z due to intertemporal substitution:

$$\frac{dz}{dk} = \frac{dz}{dr} \frac{dr}{dk}$$

When $\sigma \rightarrow \infty$, $dz/dk \rightarrow 0$ because $dz/dr \rightarrow 0$. Consumption when old and young are not substitutable, so individuals keep the ratio fixed when the return on saving changes. When $v \rightarrow \infty$, $dz/dk \rightarrow 0$ because $dr/dk \rightarrow 0$. In this case, the production function approaches linearity, which implies a constant $f'(k)$ and hence a constant r .

Conclusion

This paper has addressed a classic topic: the optimal accumulation of capital and the welfare effects of government debt in neoclassical growth models. Over the past several decades, the thinking of many economists has been shaped by the Solow (1956) growth model and the Diamond (1965) overlapping-generations model, which assume certainty and competitive markets. In these models, the real interest rate reflects the marginal product of capital. As a result, the low real interest rates experienced in recent years have tempted some economists to conclude that capital accumulation now has only small social value at the margin and, therefore, that the crowding out of capital by government debt is of little concern.

The innovation in this paper is to expand these models to include firms with market power. In this environment, the real interest rate earned by savers is below the net marginal product of capital. For a plausible calibration of the expanded Solow model, the wedge between them can be more than 4 percentage points. The calibration also suggests that the U.S. economy is dynamically efficient. When market power is introduced into the Diamond model, government Ponzi schemes can have different implications for welfare than they do under competition. Even if a perpetual rollover of government debt and accumulating interest is feasible, the crowding out of capital may still reduce steady-state welfare.

Previous work has established a close connection between the feasibility of government Ponzi schemes and the possibility of rational speculative bubbles. [See, for example, Tirole (1985), O'Connell and Zeldes (1988), and Martin and Ventura (2018).] This equivalence suggests that in an overlapping-generations model with market power, rational bubbles are possible whenever government Ponzi schemes are. But unlike under competition, these bubbles may reduce welfare because they could occur even in dynamically efficient economies. By diverting saving away from capital accumulation, bubbles may depress aggregate consumption. This topic could be addressed in future research.

Much previous work on dynamic efficiency and Ponzi schemes has stressed the role of uncertainty about capital returns. Under uncertainty, the risk-free real interest rate is below the expected marginal product of capital, and attempted Ponzi schemes by the government are risky (Abel et al., 1989; Ball, Elmendorf, and Mankiw, 1998; Blanchard 2019). In this paper, we abstract from these issues by considering models with certainty. A more realistic analysis of the U.S. economy would include both uncertainty and market power. We also leave that topic for future work.

APPENDIX

Determination of the Steady-State Capital Stock

Combining an individual's first-order condition for optimal saving, equation (2), and the budget constraint, equation (1), leads to an expression for saving:

$$S_t = \frac{F[K_t, A_t(1 - X)] - R_t K_t - \Omega_{t+1}(\beta R_{t+1})^{-\frac{1}{\sigma}}}{1 + \beta^{-\frac{1}{\sigma}}(R_{t+1})^{\frac{\sigma-1}{\sigma}}}.$$

To study the dynamics of the capital stock, we substitute $S_t = K_{t+1} + B_t$ and $\Omega_{t+1} = A_{t+1}[1 - R_{t+1}/(1 + g)]b$ into the last equation. We also use $R = f'(k)/\mu$ and divide by A_t to derive a relationship between k_t and k_{t+1} , the levels of capital per unit of effective labor in periods t and $t+1$:

$$\begin{aligned} [(1 + g)k_{t+1} + b] \left\{ 1 + \beta^{-\frac{1}{\sigma}} \left[\frac{f'(k_{t+1})}{\mu} \right]^{\frac{\sigma-1}{\sigma}} \right\} + \left[\frac{\beta f'(k_{t+1})}{\mu} \right]^{-\frac{1}{\sigma}} \left[1 + g - \frac{f'(k_{t+1})}{\mu} \right] b \\ = f(k_t) - \frac{f'(k_t)k_t}{\mu}. \end{aligned}$$

In this equation, notice that k_{t+1} and b appear only on the left side and k_t appears only on the right. We will therefore write the equation as $H(k_{t+1}, b) = J(k_t)$, where $H()$ and $J()$ are the two sides of the equation.

We assume that this equation defines a unique k_{t+1} for a given k_t . We also assume that the dynamics of the implied functional relationship $k_{t+1}(k_t)$ have a unique stable steady state, k^* , and that k converges monotonically to k^* . These conditions require $0 < dk_{t+1}/dk_t < 1$ at $k_t = k^*$, which holds if utility and production functions are sufficiently well-behaved. (One much-studied case that satisfies this condition is a Cobb-Douglas production function and $\sigma = 1$, that is, logarithmic utility.)

The Effect of Debt on Steady-State Capital

Using the notation developed above, the steady-state level of capital per unit of effective labor, k^* , is defined by $H(k^*, b) = J(k^*)$. The effect of debt on k^* is given by

$$\frac{dk^*}{db} = \frac{\frac{\partial H}{\partial b}}{\frac{dJ}{dk} - \frac{\partial H}{\partial k}}$$

with all derivatives evaluated at k^* . Using the formula for $H()$, the numerator of this expression is

$$\frac{\partial H}{\partial b} = 1 + (1 + g) \left[\frac{\beta f'(k)}{\mu} \right]^{-\frac{1}{\sigma}} > 0.$$

It is easy to see that $dJ/dk > 0$. Because $dk_{t+1}/dk_t = (dJ/dk_t)/(\partial H/\partial k_{t+1})$, our assumption that $0 < dk_{t+1}/dk_t < 1$ at the steady state implies $\partial H/\partial k > dJ/dk$. Therefore, the denominator of dk^*/db is negative. Together these results imply

$$\frac{dk^*}{db} < 0.$$

That is, a higher level of debt reduces steady-state capital.

The Sign of the Generational Effect

We show above that the sign of the generational effect of debt on welfare is the same as the sign of $\partial V_t^*/\partial z$, where $V_t^*(c, z)$ is given by equation (3). Differentiating equation (3) yields

$$\begin{aligned} \frac{\partial V_t^*}{\partial z} &= U'(C_{y,t})A_t c \left[-\frac{1}{(1+z)^2} \right] + \beta U'(C_{o,t+1})A_t(1+g)c \left[\frac{1}{(1+z)^2} \right] \\ &\propto -U'(C_{y,t}) + \beta U'(C_{o,t+1})(1+g). \end{aligned}$$

The first-order condition for optimal consumption implies $U'(C_{y,t}) = \beta R U'(C_{o,t+1})$. Substituting this fact into the previous equation and simplifying yields:

$$\frac{\partial V_t^*}{\partial z} \propto g - r.$$

As claimed in the text, $\partial V_t^*/\partial z$ has the same as $g - r$.

Special Cases for the Effect of Debt on Steady-State Welfare

As described in the text, if $r < g < f'(k) - 1$, the net effect of government debt on steady-state welfare is ambiguous because the aggregate effect is negative but the generational effect is positive. However, we can determine the net effect in the limiting cases in which $1/\sigma$, the intertemporal elasticity of substitution in consumption, or v , the elasticity of substitution between capital and labor in production, approach zero or infinity.

In each of these cases, the term dz/dk approaches zero or infinity. In the equation for dV_t^*/db , dz/dk affects the size of the generational effect. One can show that both dc/dk and

$(\partial V_i^*/\partial c)/(\partial V_i^*/\partial z)$, the ratio of the marginal effects of c and z on utility, remain bounded away from zero and infinity. Therefore, if dz/dk approaches zero, the generational effect vanishes in importance and the aggregate effect dominates, implying $dV_i^*/db < 0$. The opposite occurs if dz/dk approaches infinity.

To determine the behavior of dz/dk , we use the fact that k affects z through the interest rate r :

$$\frac{dz}{dk} = \frac{dz}{dr} \frac{dr}{dk}$$

To see the effect of the parameter σ , notice first that this utility function parameter does not affect dr/dk , which is determined by the production function and the markup μ . The expression for z , equation (5), can be written as

$$z = \frac{[\beta(1+r)]^{\frac{1}{\sigma}}}{1+g}$$

which implies

$$\frac{dz}{dr} = \frac{\beta^{\frac{1}{\sigma}}(1+r)^{\frac{1-\sigma}{\sigma}}}{\sigma(1+g)}$$

Combining the last two equations yields

$$\frac{dz}{dr} = \frac{z}{\sigma(1+r)}$$

We consider the effect of varying σ while holding constant the steady-state level of capital, and hence z . (Implicitly this requires an adjustment in β .) We can see that if $\sigma \rightarrow \infty$, $dz/dr \rightarrow 0$. In this case, $dz/dk \rightarrow 0$ and the generational effect vanishes. If $\sigma \rightarrow 0$, $dz/dr \rightarrow \infty$. In this case, $dz/dk \rightarrow \infty$ and the generational effect dominates the aggregate effect.

To see the effect of the elasticity of substitution in the production function, v , note again that

$$\frac{dz}{dk} = \frac{dz}{dr} \frac{dr}{dk}$$

The term dz/dr does not depend on the production function, so the effect of v on dz/dk depends on its effect on dr/dk . We know $r = f'(k)/\mu - 1$, so $dr/dk = f''(k)/\mu$. We consider the effect of changing v while holding constant the equilibrium y , k/y , and capital income, which means that $f''(k)$ changes as v changes. In essence, the elasticity of substitution determines how rapidly diminishing returns set in.

As $v \rightarrow \infty$, $f''(k) \rightarrow 0$ (the production function becomes linear), so $dr/dk \rightarrow 0$ and $dz/dk \rightarrow 0$. The generational effect vanishes. As $v \rightarrow 0$, $f''(k) \rightarrow \infty$ (the production function approaches Leontief), so the generational effect dominates the aggregate effect.

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