Pervasive Stickiness – Appendix

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This appendix accompanies the paper "Pervasive Stickiness." It contains a description of the model used in the paper and the algorithm that solves it.

I. The economic environment

Households. There is a continuum of households distributed in the unit interval and indexed by j. They live forever discounting future utility by a factor $\xi \in (0,1)$ and obtaining utility each period according to:

$$U(C_{t,j}, L_{t,j}) = \frac{C_{t,j}^{1-1/\theta} - 1}{1 - 1/\theta} - \frac{\varkappa L_{t,j}^{1+1/\psi}}{1 + 1/\psi}, \tag{1}$$

where: θ is the intertemporal elasticity of substitution, ψ is the Frisch elasticity of labor supply, $\varkappa$ captures relative preferences for consuming goods or leisure, $C_{t,j}$ is the consumption by household j at date t, and $L_{t,j}$ is the labor supplied by household j at date t. Consumption $C_{t,j}$ is a Dixit-Stiglitz aggregator of the consumption of varieties indexed by i, $C_{t,j}(i)$, with an elasticity of substitution ν:

$$C_{t,j} = \left(\int_0^1 C_{t,j}(i)^{\frac{\nu}{\nu-1}} di \right)^{\frac{\nu-1}{\nu}} \tag{2}$$

At each date t, the household faces a budget constraint:

$$P_t C_{t,j} + B_{t,j} = W_{t,j} L_{t,j} + (1 + i_{t-1}) B_{t-1,j} + T_{t,j}. \tag{3}$$

The new notation stands for: P_t is the dollar price of goods at date t, $B_{t,j}$ are holdings of nominal bonds, $W_{t,j}$ is the nominal wage paid to household j, and i_{t-1} is the nominal net return at t on a bond purchased at $t-1$. Finally $T_{t,j}$ are lump-sum nominal transfers received by the the household from two sources. First, they come from receiving profits from firms, which are equally owned by all households. Second, we assume that consumers signed an insurance contract at the beginning of time so that they all start each period with with the same wealth. This way, we do do not have to track the wealth distribution. The payments from this contract are in $T_{t,j}$. The Dixit-Stiglitz aggregator has an associated static price index:

$$P_t = \left(\int_0^1 P_t(i)^{1-\nu} di \right)^{\frac{1}{1-\nu}}. \tag{4}$$

Technologies. Households own a continuum of firms in the unit interval indexed by j. Firm j operates a technology that combines the labor supplied by each household i, $N_{t,j}(i)$, into the output of a particular variety of good $Y_{t,j}$. The production function is:

$$Y_{t,j} = A_t N_{t,j}^{\beta}, \tag{5}$$

$$N_{t,j} = \left(\int_0^1 N_{t,j}(i)^{\frac{\gamma}{\gamma-1}} di \right)^{\frac{\gamma-1}{\gamma}}. \tag{6}$$

A_t stands for exogenous aggregate productivity, which follows a random walk in logs with standard deviation of shocks σ_a. The parameter $\beta \in (0,1)$ is the labor share of income and measures the degree of diminishing returns to scale. The composite of inputs used by firm j, $N_{t,j}$, is a Dixit-Stiglitz aggregator of the different varieties of labor hired with an elasticity of substitution γ. It implies a dual minimum-expenditure static wage index:

$$W_t = \left(\int_0^1 W_t(i)^{1-\gamma} di \right)^{\frac{1}{1-\gamma}}. \tag{7}$$

Markets. In financial assets, there is an anonymous market for nominal bonds. They are in zero net supply so for the market to clear:

$$\int_0^1 B_{t,j} dj = 0, \tag{8}$$

There is a goods market for each variety i, in which all consumers are buyers and the sole seller is firm i that has a monopoly over its variety. Market clearing requires:

$$\int_0^1 C_{t,j}(i) dj = Y_{t,i}. \tag{9}$$

Across all varieties, total output is:

$$Y_t = \left(\int_0^1 Y_{t,i}^{\frac{\nu}{\nu-1}} di \right)^{\frac{\nu-1}{\nu}}. \tag{10}$$

Finally, there is a labor market for each variety of labor i. The buyers are all firms and the seller is the household that has the monopoly over labor services i. In equilibrium:

$$\int_0^1 N_{t,j}(i) dj = L_{t,i}. \tag{11}$$

Total labor is:

$$L_t = \left(\int_0^1 L_{t,i}^{\frac{\gamma}{\gamma-1}} di \right)^{\frac{\gamma-1}{\gamma}}. \tag{12}$$

Decision-makers and information. Consumers wish to maximize the expected discounted sum of utility at each date (1) given the preferences in (2) subject to the sequence of budget constraints (3) from t into infinity and a no-Ponzi scheme condition.

There are two decision-makers within a consumer, who cannot exchange information. One is a shopper whose job is to pick at each date the consumption of each variety taking total expenditure at that date as given. The shopper has full information and searches the markets for all varieties for the best bundle of goods at lower cost.

The other decision-maker is a planner, whose job is to choose the total amount of expenditure at each date and how much to save. The planners are inattentive only sporadically updating their information. When a planner updates her information, she obtains full information, but in between updates she obtains no new information. Reis (2004) presents a model in which costs of acquiring, absorbing, and processing information lead planners to optimally choose how often to update their information. Here, we take this behavior as given. Moreover, following Mankiw and Reis (2002), we assume that there is sticky information, understood as a constant probability δ at each date that any planner receives new information. Our assumptions imply that planners differ only with regards to their information. To continue using j to index planners, we change its meaning. From now on, j denotes how long ago a planner last updated her information so $C_{t,j}$ is the expenditure of a planner who last updated her information j periods ago. Thus, the unit mass of planners is divided into a countable number of groups of consumers, each with mass $\delta(1-\delta)^j$.

A second set of agents includes workers. They have the same objective and face the same constraints as consumers—they share a household. Their choice is what wage to charge for their labor services. They post a wage monopolistically taking into account the demand for their labor and commit to supplying the labor necessary to ensure that the market clears at that wage. They are also inattentive with sticky information, as in Mankiw and Reis (2003). The wage $W_{t,j}$ is set by a worker that last updated her information j periods ago and sticky information implies that at every date a fraction ω of workers update their information.

The final set of private agents are firms. Within the firm, there are two departments making decisions. The hiring department takes as given the choice of how much to produce and hires the combination of labor inputs that minimizes costs using full information. The sales department sets a price that takes into account its monopoly power and the demand for its product and commits to producing as much of the good as necessary to clear the market. They are inattentive as modelled explicitly in Reis (2005), who provides also a set of conditions under which information is sticky as in Mankiw and Reis (2002). Each date, a fraction λ of sales departments in firms obtain new information.

Monetary Policy. In this cashless economy, we assume that the government can enforce the use of a unit of account and issue nominal bonds. This gives it the power to set the nominal interest rate. (See Woodford, 2003, for an exposition of how interest rates are set in cashless economies.) We assume that policy mechanically follows a Taylor rule:

$$i_t = \phi_y \log\left(\frac{Y_t}{Y_t^n}\right) + \phi_\pi \log\left(\frac{P_t}{P_{t-1}}\right) + \varepsilon_t. \tag{13}$$

Y_t^n will be defined later and ε_t are policy shocks that follow the process: $\varepsilon_t = \rho\varepsilon_{t-1} + e_t$ where e_t is white noise with mean zero and standard deviation σ_e. The parameter ϕ_π is larger than one, respecting the Taylor principle, while the parameter ϕ_y is non-negative reflecting a desire for stabilization. Finally, note that the interest rate rule does not ensure determinacy of the price level. This indeterminacy is well-known and there are many slight modifications of the model that eliminate it (see, for instance, Woodford, 2003, chapter 2). We do not wish to complicate the model further by addressing this issue directly. Instead, we peg the initial price level at an initial condition: $P_{-1} = 1$.

II. Equilibrium of the economy

To solve the model, we must first describe optimal behavior. We start with consumers and their two choices. Optimal behavior by shoppers implies that the demand for each variety by consumer j:

$$C_{t,j}(i) = C_{t,j}\left(P_t(i)/P_t\right)^{-\nu}. \tag{14}$$

Summing over all consumers and using the market clearing condition for variety i in (9) implies:

$$Y_{t,i} = \left(P_t(i)/P_t\right)^{-\nu}\left(\int_0^1 C_{t,j}dj\right). \tag{15}$$

Moving next to planners, recall that they obtain new information with probability δ every period. Recall also that all planners are identical aside from how long they last planned j. Letting $A_{t,j} \equiv [(1+i_{t-1})B_{t-1,j} + W_{t,j}L_{t,j} + T_{t,j}]/P_t$ denote the real resources with which planner j enters period t, the assumption of perfect insurance implies that $A_{t,j} = A_t$, the same for all planners. We denote by $V(A_t, .)$ the value function for planners that plan at period t. The second argument in the value function includes other state variables that may be useful at forecasting the future—we will omit it from now onwards.

The planner solves the dynamic program:

$$V(A_t) = \max_{\{C_{t+i,i}\}}\left\{\sum_{i=0}^{\infty}\xi^i(1-\delta)^i U(C_{t+i,i},.) + \xi\delta\sum_{i=0}^{\infty}\xi^i(1-\delta)^i E_t\left[V\left(A_{t+1+i}\right)\right]\right\}, \tag{16}$$

$$s.t. \quad : \quad A_{t+1+i} = R_{t+1+i}\left(A_{t+i} - C_{t+i,.}\right) + \frac{W_{t+1+i,.}L_{t+1+i,.} + T_{t+1+i,.}}{P_{t+1}}. \tag{17}$$

The first term in the Bellman equation equals the expected discounted utility if the planner never updates her information again. The second term includes the sum of the continuation values over all of the possible future dates at which the agent may plan again, each occurring with a probability $\delta(1-\delta)^i$. The constraint comes from re-writing of the budget constraint, where

$$R_{t+1} \equiv \left(1+i_t\right)P_t/P_{t+1}, \tag{18}$$

the real return on bonds. Note that the consumer takes work choices as given.

The first-order conditions for optimality are:

$$\xi^i(1-\delta)^i C_{t+i,i}^{-1/\theta} = \xi\delta\sum_{k=i}^{\infty}\xi^k(1-\delta)^k E_t\left[V'\left(A_{t+1+k}\right)\bar{R}_{t+i,t+1+k}\right], \tag{19}$$

for all $i = 0, 1, ...,$ where $\bar{R}_{t+i,t+1+k} = \prod_{z=t+i}^{t+k} R_{z+1}$, the compound return between two dates.

The envelope theorem condition is:

$$V'(A_t) = \xi\delta \sum_{k=0}^{\infty} \xi^k (1-\delta)^k E_t \left[V'(A_{t+1+k}) \bar{R}_{t,t+1+k} \right]. \tag{20}$$

Combining condition (19) at $i = 0$ and (20) shows that $V'(A_t) = C_{t,0}^{-1/\theta}$. The marginal value of an extra unit of resources equals the marginal utility of using them immediately for consumption. Using this result to replace for the marginal value terms in the optimality conditions, they simplify to:

$$C_{t,0}^{-1/\theta} = \xi E_t \left[R_{t+1} C_{t+1,0}^{-1/\theta} \right], \tag{21}$$

$$C_{t+j,j}^{-1/\theta} = E_t \left[C_{t+j,0}^{-1/\theta} \right], \tag{22}$$

holding for all t and all j. The first condition is the usual Euler equation for an attentive consumer. The second condition shows that an inattentive consumer sets the marginal utility of her consumption equal to her expectation of the marginal utility of the attentive consumer.

Moving next to firms, the hiring department minimizes costs with full information. The optimal demand by firm j for labor services of variety i is:

$$N_{t,j}(i) = N_{t,j} \left(W_t(i)/W_t \right)^{-\gamma}. \tag{23}$$

Summing over all firms and using the market clearing condition in labor variety i, we obtain:

$$L_t(i) = \left(W_t(i)/W_t \right)^{-\gamma} \left(\int_0^1 N_{t,j} dj \right). \tag{24}$$

The sales department maximizes profits subject to the production function (5) and the iso-elastic demand for its product in (15). For a firm that last updated its information j periods ago, the first-order condition is:

$$P_{t,j} = \frac{\nu}{\nu - 1} \times \frac{E_{t-j} \left(W_t Y_{t,j}^{1/\beta} A_t^{-1/\beta} \right)}{E_{t-j} \left(\beta Y_{t,j} \right)}. \tag{25}$$

This states the usual result that with iso-elastic preferences, nominal prices are a fixed markup over nominal marginal costs. The markup equals $\nu/(\nu - 1)$. The other fraction in the expression, for a firm that is planning, equals the nominal marginal cost—the nominal wage divided by the marginal product of the composite labor input.

Finally, we move to workers. Their problem is similar to that faced by the consumption planners. Jumping to the optimality conditions:

$$W_{t,0} = \frac{\gamma}{\gamma-1} \times \frac{P_t \varkappa L_{t,j}^{1/\psi}}{\hat{V}_t'(.)}, \tag{26}$$

$$\frac{L_{t,0}^{1/\psi} P_t}{W_{t,0}} = E_t \left(\frac{\xi R_{t+1} L_{t+1,0}^{1/\psi} P_{t+1}}{W_{t+1,0}} \right), \tag{27}$$

$$W_{t+j,j} = \frac{E_t \left(L_{t+j,j}^{1+1/\psi} \right)}{E_t \left(L_{t+j,0}^{1/\psi} L_{t+j,j} / W_{t+j,0} \right)}. \tag{28}$$

The first condition states that wages equal a constant markup $\gamma/(\gamma-1)$ over the marginal opportunity cost of labor. For the worker that is planning, this equals the marginal disutility of labor $\varkappa L_{t,j}^{1/\psi}$ divided by the marginal utility of an extra dollar for the worker $\hat{V}_t'$. The second equation is a standard Euler equation for an attentive worker. Supplying an extra unit of labor today leads to a fall in utility of $L_{t,0}^{1/\psi}$. In return, the worker receives $W_{t,0}/P_t$, which after invested in bonds returns R_{t+1} per unit the next period. The worker can then work less $(R_{t+1}W_{t,0}/P_t)P_{t+1}/W_{t+1,0}$ units tomorrow which raise utility by this amount times the marginal utility of labor tomorrow $L_{t+1,0}^{1/\psi}$ discounted by the factor ξ. At an optimum, the cost of anticipating work must equal its expected benefit, so the equality must hold. Finally, the third condition states that an inattentive worker sets wages so that her expected disutility from working mirrors the expected disutility from working of an attentive worker.

We can now define a competitive equilibrium of this economy: it is an allocation of total expenditures and savings, consumption of varieties, labor supplied of the different varieties, and output produced of each variety such that consumers, workers and firms all behave optimally, monetary policy follows the Taylor rule, and all markets clear.

The equations above characterize this equilibrium. However, they are difficult to handle. We proceed by log-linearizing around the stationary point where $\sigma_a = \sigma_e = 0$ so all variables are constant. Small letters denote the log-linear deviation of the respective capital variables from the steady state, with the exception of r_t, which denotes the log-linear deviation of

$E_t[R_{t+1}]$. The set of log-linearized optimality conditions is:

$$
\begin{aligned}
r_t &= i_t - E_t\left(\Delta p_{t+1}\right), && (29)\\
y_{t,j} &= y_t - \nu\left(p_{t,j} - p_t\right), && (30)\\
l_{t,j} &= l_t - \gamma(w_{t,j} - w_t), && (31)\\
c_{t,j} &= E_{t-j}\left(c_{t+1,0} - \theta r_t\right), && (32)\\
p_{t,j} &= E_{t-j}\left[w_t + (1/\beta - 1)y_{t,j} - a_t/\beta\right], && (33)\\
w_{t,j} &= E_{t-j}[p_t + l_{t,j}/\psi - r_t + w_{t+1,0} - p_{t+1} - l_{t+1,0}/\psi]. && (34)
\end{aligned}
$$

The log-linearized definitions of the aggregate production function and the price, wage, and output indices are:

$$
\begin{aligned}
y_t &= a_t + \beta l_t && (35)\\
p_t &= \lambda\sum_{j=0}^{\infty}(1-\lambda)^j p_{t,j}, && (36)\\
w_t &= \omega\sum_{j=0}^{\infty}(1-\omega)^j w_{t,j} && (37)\\
y_t &= \delta\sum_{j=0}^{\infty}(1-\delta)^j c_{t,j}, && (38)
\end{aligned}
$$

Finally the log-linear Taylor rule is:

$$i_t = \phi_y(y_t - y_t^n) + \phi_\pi \Delta p_t + \varepsilon_t. \qquad (39)$$

This set of 11 equations over time provide the competitive equilibrium solution for the set of 11 variables (y_t, $y_{t,j}$, $c_{t,j}$, $l_{t,j}$, l_t, w_t, $w_{t,j}$, p_t, $p_{t,j}$, i_t, r_t) as a function of the exogenous processes a_t, ε_t and y_t^n.

III. Reduced-form representations of the model

The fully attentive economy. In the classical economy, $\lambda = \delta = \omega = 1$ so all are attentive. Following convention, we label the equilibrium in this economy "natural." Equations (36)-(38) imply that $p_{t,j}^n = p_t^n$, $w_{t,j}^n = w_t^n$ and $c_{t,j}^n = y_t^n$, while (30)-(31) imply that $l_{t,j}^n = l_t^n$ and $y_{t,j}^n = y_t^n$. This reflects the fact that all agents are identical. A few steps of algebra using equations (32)-(35) shows that: $l_t^n = (y_t^n - a_t)/\beta$, $w_t^n - p_t^n = y_t^n - l_t^n$, $r_t^n = 0$ and that $y_t^n = \Xi a_t$, where $\Xi = (1 + 1/\psi)/(1 + 1/\psi + \beta/\theta - \beta)$. Note that all real variables are determined as a function of only the exogenous technology shock a_t, independently of monetary policy. The classical dichotomy holds in this economy. Monetary policy shocks determine the nominal interest rate and inflation through the Taylor rule and the Fisher equation. The solutions are: $\Delta p_t^n = -\varepsilon_t/(\phi_\pi - \rho)$ and $i_t^n = r_t^n + E_t\left(\Delta p_{t+1}^n\right)$.

We have solved for the natural equilibrium. Note that for preferences consistent with a balanced growth path ($\theta = 1$) the parameter $\Xi = 1$. Therefore, output and real wages are proportional to productivity and labor supplied is constant. Since output is always at its natural level that follows a stochastic trend, there is no output gap and so the acceleration correlation is zero. Moreover, as a_t and ε_t are independent, output and inflation in the attentive economy are statistically independent. Since real wages are proportional to output per hour, the two are equally variable. And since output follows a random walk, the standard deviation of its quarterly changes equals one-half of the standard deviation of its annual changes.

The reduced-form sticky information economy. Combining equations (30) and (33) to replace for $p_{t,j}$ and $y_{t,j}$ in equation (36) gives the aggregate supply relation:

$$p_t = \lambda \sum_{j=0}^{\infty} (1-\lambda)^j E_{t-j}\left[p_t + \frac{\beta(w_t - p_t) + (1-\beta)y_t - a_t}{\beta + \nu(1-\beta)}\right]. \tag{40}$$

Denoting by mc_t (real marginal costs) the fraction on the right-hand side, we can re-arrange this equation to obtain a sticky-information Phillips curve:

$$\Delta p_t = \frac{\lambda mc_t}{1-\lambda} + \lambda \sum_{j=0}^{\infty} (1-\lambda)^j E_{t-1-j}\left(\Delta p_t + \Delta mc_t\right). \tag{41}$$

Iterating forward on equation (32), we get:

$$c_{t,j} = -\theta \sum_{i=0}^{T} E_{t-j}\left(r_{t+i}\right) + E_{t-j}\left(c_{t+T+1,0}\right). \tag{42}$$

Next, take the limit as $T \to \infty$. As time elapses to infinity all become aware of past news so $\lim_{i\to\infty} E_t\left(r_{t+i}\right) = \lim_{i\to\infty} E_t\left(r_{t+i}^n\right) = 0$. Moreover, since the probability of remaining inattentive falls exponentially with the length of the horizon, we approach this limit fast enough to ensure that the sum in the first term converges. As for the second term,

$\lim_{i\to\infty} E_t\left(c_{t+i,0}\right) = \lim_{i\to\infty} E_t\left[y^n_{t+i}\right] = y^n_t$. The first equality holds because consumers are fully insured every period and in the limit all are informed. The second equality holds because y^n_t follows a random walk. The expression above therefore becomes:

$$c_{t,j} = -\theta E_{t-j}\left(R_t\right) + y^n_{t-j}, \tag{43}$$

where $R_t = E_t\left(\sum_{i=0}^{\infty} r_{t+i}\right)$, the long real interest rate. Replacing for $c_{t,j}$ in (38) gives the IS curve:

$$y_t = \delta \sum_{j=0}^{\infty}(1-\delta)^j E_{t-j}\left(y^n_t - \theta R_t\right). \tag{44}$$

We can first-difference this equation to obtain an alternative representation of the IS:

$$\begin{aligned} y_t &= E_t\left(y_{t+1}\right) - \delta^2 \sum_{j=0}^{\infty}(1-\delta)^j \left[y^n_t - E_{t-j}\left(y^n_t\right)\right] \\ &\quad -\theta\delta(r_t - R_t) - \theta\delta \sum_{j=0}^{\infty}(1-\delta)^j E_{t-j}\left[(1-\delta)r_t + \delta R_t\right]. \end{aligned}$$

Similar steps, iterating forward on (34) and using the fact that $\psi(w^n_t - p^n_t) - l^n_t = \psi y^n_t/\theta$ show that:

$$\psi w_{t,j} = E_{t-j}(l_{t,j} + \psi p_t - R_t) + \psi y^n_{t-j}/\theta. \tag{45}$$

Using this result as well as (33) and (35) to replace for $l_{t,j}$, $w_{t,j}$ and l_t in (37), gives a wage curve:

$$w_t = \omega \sum_{j=0}^{\infty}(1-\omega)^j E_{t-j}\left[p_t + \frac{\gamma(w_t - p_t)}{\gamma+\psi} + \frac{y_t - a_t}{\beta(\gamma+\psi)} + \frac{\psi\left(y^n_t - \theta R_t\right)}{\theta(\gamma+\psi)}\right]. \tag{46}$$

The AS, the IS and the wage curve, together with the Fisher equation and the Taylor rule characterize the equilibrium for $(y_t,\, p_t,\, w_t,\, r_t,\, i_t)$ given exogenous shocks to (a_t, ε_t) in the sticky information economy.

IV. Properties of the sticky information equilibrium

Finding the sticky information equilibrium. We find the sticky information equilibrium using a method of undetermined coefficients. That is we guess that:

$$y_t = \sum_{n=0}^{\infty} \left[\hat{y}_n e_{t-n} + \tilde{y}_n (a_{t-n} - a_{t-1-n})\right], \tag{47}$$

$$p_t = \sum_{n=0}^{\infty} \left[\hat{p}_n e_{t-n} + \tilde{p}_n (a_{t-n} - a_{t-1-n})\right], \tag{48}$$

$$w_t = \sum_{n=0}^{\infty} \left[\hat{w}_n e_{t-n} + \tilde{w}_n (a_{t-n} - a_{t-1-n})\right], \tag{49}$$

$$r_t = \sum_{n=0}^{\infty} \left[\hat{r}_n e_{t-n} + \tilde{r}_n (a_{t-n} - a_{t-1-n})\right], \tag{50}$$

$$i_t = \sum_{n=0}^{\infty} \left[\hat{\imath}_n e_{t-n} + \tilde{\imath}_n (a_{t-n} - a_{t-1-n})\right], \tag{51}$$

and look for the coefficients with hats and tildes.

The AS in equation (40) implies that:

$$\left[\frac{\beta + \nu(1-\beta)}{\Lambda_n} - \nu(1-\beta)\right] \hat{p}_n = (1-\beta)\hat{y}_n + \beta \hat{w}_n, \tag{52}$$

$$\left[\frac{\beta + \nu(1-\beta)}{\Lambda_n} - \nu(1-\beta)\right] \tilde{p}_n = (1-\beta)\tilde{y}_n + \beta \tilde{w}_n - 1, \tag{53}$$

for all n, where $\Lambda_n = \lambda \sum_{i=0}^{n} (1-\lambda)^i$, the share of firms that have learned about a shock n periods after it occurs. The IS in equation (44) implies that:

$$\hat{y}_n = -\Delta_n \theta \hat{R}_n, \tag{54}$$

$$\tilde{y}_n = \Delta_n \left(\Xi - \theta \tilde{R}_n\right), \tag{55}$$

for all n, where $\Delta_n = \delta \sum_{i=0}^{n} (1-\delta)^i$, the share of consumers that have learned about a shock n periods after it occurs. The definition of the long rate implies that $\hat{R}_n = \sum_{i=0}^{\infty} \hat{r}_{n+i}$ and $\tilde{R}_n = \sum_{i=0}^{\infty} \tilde{r}_{n+i}$. The wage curve in (46) in turn implies that:

$$(\gamma + \psi - \Omega_n \gamma)\, \hat{w}_n = \Omega_n \psi \hat{p}_n + \Omega_n \left(\hat{y}_n/\beta - \psi \hat{R}_n\right), \tag{56}$$

$$(\gamma + \psi - \Omega_n \gamma)\, \tilde{w}_n = \Omega_n \psi \tilde{p}_n + \Omega_n \left[(\hat{y}_n - 1)/\beta + \psi(\Xi - \theta \tilde{R}_n)/\theta\right], \tag{57}$$

for all n with $\Omega_n = \omega \sum_{i=0}^{n} (1-\omega)^i$.

Using (52) to replace for $\hat{w}_n$ and (54) to replace for $\hat{R}_n$ in (56), and first-differencing

(54) allows us to drop wages and the long rate and reduce the problem to finding prices, output and the short rate with two equations:

$$
\begin{aligned}
\hat{y}_n &= \Psi_n \hat{p}_n, && (58)\\
\theta \hat{r}_n &= \frac{\hat{y}_{n+1}}{\Delta_{n+1}} - \frac{\hat{y}_n}{\Delta_n}. && (59)
\end{aligned}
$$

where we defined a new parameter:

$$
\Psi_n = \frac{\theta\Delta_n \left\{ [\psi + \gamma(1-\Omega_n)] \left[\frac{\beta+\nu(1-\beta)}{\Lambda_n} - \nu(1-\beta) \right] - \Omega_n \beta\psi \right\}}{(1-\beta)(\gamma+\psi)\theta\Delta_n + \Omega_n \left\{ \theta\Delta_n \left[1-\gamma(1-\beta)\right] + \psi\beta \right\}} \tag{60}
$$

For the coefficients involving productivity shocks, we have instead:

$$
\begin{aligned}
\tilde{y}_n &= \Psi_n \tilde{p}_n + \Upsilon_n, && (61)\\
\theta \tilde{r}_n &= \frac{\tilde{y}_{n+1}}{\Delta_{n+1}} - \frac{\tilde{y}_n}{\Delta_n}. && (62)
\end{aligned}
$$

where the new parameter is:

$$
\Upsilon_n = \frac{\theta\Delta_n \left[\gamma + \psi + \Omega_n(1-\gamma)\right]}{(1-\beta)(\gamma+\psi)\theta\Delta_n + \Omega_n \left\{ \theta\Delta_n \left[1-\gamma(1-\beta)\right] + \psi\beta \right\}}. \tag{63}
$$

Finally, using the Fisher equation to substitute nominal interest rates out of the Taylor rule and rearranging leads to:

$$
\begin{aligned}
\phi_\pi \hat{p}_n &= \phi_y \hat{y}_{n+1} + (1+\phi_\pi)\hat{p}_{n+1} - \hat{p}_{n+2} - \hat{r}_{n+1} + \rho^{n+1}, && (64)\\
\phi_\pi \tilde{p}_n &= \phi_y \tilde{y}_{n+1} + (1+\phi_\pi)\tilde{p}_{n+1} - \tilde{p}_{n+2} - \tilde{r}_{n+1} - \phi_y \Xi, && (65)
\end{aligned}
$$

for $n = 0, 1, 2...$ There is also an initial condition from the Taylor rule at date 0:

$$
\begin{aligned}
\phi_y \hat{y}_0 + (1+\phi_\pi)\hat{p}_0 - \hat{p}_1 - \hat{r}_0 + 1 &= 0, && (66)\\
\phi_y \tilde{y}_0 + (1+\phi_\pi)\tilde{p}_0 - \tilde{p}_1 - \tilde{r}_0 &= \phi_y \Xi. && (67)
\end{aligned}
$$

We have all the conditions we need to solve for the undetermined coefficients on the impact of monetary and productivity shocks. Our algorithm that finds the impact of monetary shocks starts by choosing a very large number T and setting $\hat{y}_n = \hat{r}_n = 0$ and $\hat{p}_n = \bar{p}$

for $n \geq T$. We know that for $T = +\infty$ this guess is correct for some unknown positive value of $\bar{p}$. Starting with a guess for $\bar{p}$, the system made by equations (58), (59) and (64) recursively gives the solution for $\hat{y}_n$, $\hat{r}_n$, and $\hat{p}_n$ for $n = T-1, T-2, ...0$. The final step is to check the initial condition (66). One can then search for the $\bar{p}$ that ensures that (66) holds, which concludes the algorithm.

For productivity shocks, the algorithm is similar. For large T, $\tilde{y}_n = \Xi$, $\tilde{r}_n = 0$ and $\tilde{p}_n = \bar{p}$ for $n \geq T$, and the system of equations (61), (62) and (65) recursively gives the solution for $\tilde{y}_n$, $\tilde{r}_n$, and $\tilde{p}_n$ for $n = T-1, T-2, ..., 0$, while (67) is the initial condition used to pin down $\bar{p}$.

Finally, (52) and (53) give the solution for $\hat{w}_n$ and $\tilde{w}_n$. Using the production function: $\hat{y}_t - \hat{l}_t = (1 - 1/\beta)\hat{y}_t$ and $\tilde{y}_t - \tilde{l}_t = (1 - 1/\beta)\tilde{y}_t + 1/\beta$. Note also that

$$\pi_t = \sum_{n=0}^{\infty} \left[\hat{\pi}_n e_{t-n} + \tilde{\pi}_n (a_{t-n} - a_{t-1-n})\right], \tag{68}$$

with $\hat{\pi}_0 = \hat{p}_0$ and $\hat{\pi}_n = \hat{p}_n - \hat{p}_{n-1}$ for $n \geq 1$, and the same for $\tilde{\pi}_n$.

Calculating the predicted moments. We have characterized the representation of the economy in (47)-(51) and for $y_t - l_t$ in the previous paragraph. The population moment: $\rho(\pi_{t+2} - \pi_{t-2}, y_t - y_t^n) =$

$$\frac{\sum_{n=0}^{\infty} \left(\hat{\pi}_{n+2} - \hat{\pi}_{n-2}\right) \hat{y}_n \sigma_e^2 + \sum_{n=0}^{\infty} \left(\tilde{\pi}_{n+2} - \tilde{\pi}_{n-2}\right) (\tilde{y}_n - \Xi) \sigma_a^2}{\sqrt{\left[\sum_{n=0}^{\infty} \left(\hat{\pi}_{n+4} - \hat{\pi}_n\right)^2 \sigma_e^2 + \sum_{n=0}^{\infty} \left(\tilde{\pi}_{n+4} - \tilde{\pi}_n\right)^2 \sigma_a^2\right] \left[\sum_{n=0}^{\infty} \hat{y}_n^2 \sigma_e^2 + \sum_{n=0}^{\infty} (\tilde{y}_n - \Xi) \sigma_a^2\right]}}, \tag{69}$$

where $\hat{\pi}_{-i} = \tilde{\pi}_{-i} = 0$ for any positive i. Turning next to fact 2:

$$\frac{\sigma\left[\Delta(w-p)\right]}{\sigma\left[\Delta(y-l)\right]} = \sqrt{\frac{\sum_{n=0}^{\infty} \left[\hat{w}_{n+1} - \hat{p}_{n+1} - (\hat{w}_n - \hat{p}_n)\right]^2 \sigma_e^2 + \sum_{n=0}^{\infty} \left[\tilde{w}_{n+1} - \tilde{p}_{n+1} - (\tilde{w}_n - \tilde{p}_n)\right]^2 \sigma_a^2}{\sum_{n=0}^{\infty} \left[\hat{y}_{n+1} - \hat{l}_{n+1} - (\hat{y}_n - \hat{l}_n)\right]^2 \sigma_e^2 + \sum_{n=0}^{\infty} \left[\tilde{y}_{n+1} - \tilde{l}_{n+1} - (\tilde{y}_n - \tilde{l}_n)\right]^2 \sigma_a^2}}. \tag{70}$$

Finally, to assess fact 3 we can calculate:

$$\frac{2\sigma(y_t - y_{t-1})}{\sigma(y_t - y_{t-4})} = 2\sqrt{\frac{\sum_{n=0}^{\infty} \left(\hat{y}_{n+1} - \hat{y}_n\right)^2 \sigma_e^2 + \sum_{n=0}^{\infty} \left(\tilde{y}_{n+1} - \tilde{y}_n\right)^2 \sigma_a^2}{\sum_{n=0}^{\infty} \left(\hat{y}_{n+4} - \hat{y}_n\right)^2 \sigma_e^2 + \sum_{n=0}^{\infty} \left(\tilde{y}_{n+4} - \tilde{y}_n\right)^2 \sigma_a^2}}. \tag{71}$$

Fitting the model to the data. We solve the problem:

$$\min_{\lambda,\delta,\omega}\left\{(\rho(\pi_{t+2}-\pi_{t-2},y_t-y_t^n)-0.47)^2+\left(\frac{\sigma(\Delta(w-p))}{\sigma(\Delta(y-l))}-0.69\right)^2+\left(\frac{2\sigma(y_t-y_{t-1})}{\sigma(y_t-y_{t-4})}-0.79\right)^2\right\}, \tag{72}$$

using the formulae in (69)-(71) and the coefficients provided by the algorithm above as a function of the parameters. Since we know little about the function that we are minimizing, we tried different non-linear minimization algorithms to search for the optimal $\{\lambda,\delta,\omega\}$ in the space $[0,1]^3$.

References not in the main text of the paper

Woodford, Michael. *Interest and Prices*, Princeton: Princeton University Press, 2003.