Electronic Branched Flow in Graphene: Theory and Machine Learning Prediction

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Dirac Solids

Linear Energy Dispersion Relation of Dirac Solids

$$\epsilon_k = \pm v_F \hbar |k|$$

Quasi-classical dynamics: Ultra-relativistic Hamiltonian

$$\mathcal{H} = \pm v_F \sqrt{p_x^2 + p_y^2} + V(x, y)$$

Classical limit of Dirac equation

Small effective mass due to doping is neglected

$$m^* = \hbar (\pi n)^{1/2} / v_F$$
Motivation Experiments

Charged impurities form disordered potential. *Charge Puddles*

**hBN/graphene**

![Image of hBN/graphene](image1)


**Ir(111)/graphene**

![Image of Ir(111)/graphene](image2)

Samaddar *et al.*, *PRL* 116, 2016

Martin *et al.*, *PRB* 91, 2015

Branched Electronic Flow & Caustic Formation in parabolic 2DEGs

![Image of branched electronic flow](image3)


Disordered Potential

Random distributed charge puddles of radius $R = 4$ nm.

\[ V_r \ll \epsilon_k \]

A bias potential along the direction of motion $\mathcal{x}$

\[ V_d = -\alpha \mathcal{x} \]
Simulations

\[ \mathcal{H} = v_F \sqrt{p_x^2 + p_y^2} - \alpha x + V_r(x, y) \]

As bias increases:
(i) the passage to branched flow delays.
(ii) the caustics disperse slower.

Theoretical Model

Effective Hamiltonian for $p_x \gg p_y$

$$\mathcal{H} = p_x + \frac{p_y^2}{2p_x} - \alpha x + V_r(x,y)$$

Local curvature $u$ equation in the quasi-2D approach ($x = t$)

$$\frac{du}{dt} + \frac{u^2}{1 + \alpha t} + \frac{\partial^2}{\partial y^2} V_r(t,y) = 0$$

where $u(t, y) = \frac{\partial p_y}{\partial y}$

Caustic is an area with high intensity occurs when

$$|u(t_c)| \rightarrow \infty$$
Scaling of the First Caustic

The random potential acts as white noise with variance $\sigma^2$

$$\frac{\partial^2}{\partial y^2} V_r(t, y) = \sigma^2 \xi(t) \quad \text{where} \quad \langle \xi(t) \rangle = 0, \quad \langle \xi(t) \xi(t') \rangle = 2\delta(t - t')$$

Langevin equation for the local curvature

$$\dot{u} = -\frac{u^2}{1 + \alpha t} + \sigma^2 \xi(t)$$

Solve approximately the first passage problem for $|u| \to \infty$

$$\langle t_c \rangle \sim \sigma^{-2/3} \left(1 + 2\tilde{\alpha} + 3\tilde{\alpha}^2 + \frac{10}{3}\tilde{\alpha}^3\right), \quad \tilde{\alpha} = 1.11\alpha\sigma^{-2/3}$$

Conventional 2D metals:

In the presence of bias the quasi-2D approach fails.
First Caustic Time

Points & Dashed lines:
Simulations

Solid lines:
Theoretical Prediction
up to $\alpha \sigma^{-2/3} < 1$
Machine Learning Predicts Caustics

The Deep Learning method *Reservoir Computing* is utilized for accurate prediction of *Singular Events* in wave dynamics.

*Lu et al., Chaos 27, 2017*
Conclusion

- Branched electronic flow in Dirac Solids focusing on graphene.
- A Langevin Eq. for the local curvature of an ultra-relativistic biased electronic flow is derived.
- Scaling-type relationship between the first caustic location and the statistical properties of disordered potential.
- Machine Learning prediction of singular events in wave dynamics.

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