Electromagnetic wave propagation in gradient index metamaterials, plasmonic systems and optical fiber networks

Marios Mattheakis

Department of Physics, University of Crete, Heraklion

31 October 2014
Outline

Introduction

Methods for light propagation

Networks of Luneburg Lenses

Caustic formation

Rogue waves formation

Optical fiber lattices

Active plasmonic systems
Electromagnetic wave propagation in gradient index metamaterials, plasmonic systems and optical fiber networks

Introduction

Outline

Introduction

Metamaterials (MMs)
Gradient Index Refractive index (GRIN) lenses

Methods for light propagation

Networks of Luneburg Lenses

Caustic formation

Rogue waves formation

Optical fiber lattices
Metamaterials (MMs)

- Artificial subwavelength structures
- Macroscopic properties obtained by the microscopic structure and properties of the compositional materials
- Provide properties that have not been found in nature, such as
  - Negative refractive index
  - Cloaking
  - Flat slab perfect imaging
  - Gradient refractive index (GRIN)
Metamaterials (MMs)

- Negative Index
- Cloaking
- Perfect Imaging
Gradient Refractive Index (GRIN) metamaterials

- Formed via spatial variation of the refractive index
- Lead to enhanced light manipulation in a variety of circumstances

Luneburg Lens

\[ n(r) = \sqrt{2 - \left(\frac{r}{R}\right)^2} \]

Maxwell fisheye

\[ n(r) = \frac{n_0}{1 + \left(\frac{r}{R}\right)^2} \]
Outline

Introduction

Methods for light propagation
- Quasi 2D ray tracing
- Parametric 2D ray tracing
- Helmholtz wave 2D approach
- Numeric solution of Maxwell equations

Networks of Luneburg Lenses

Caustic formation

Rogue waves formation
Quasi 2D ray tracing

Polar coordinates

- Fermat Principle of least time for a refractive index $n(r)$

$$ S = \int_{A}^{B} n(r) \, ds = \int_{A}^{B} n(r) \sqrt{1 + r^2 \dot{\phi}^2} \, dr , \quad \delta S = 0 $$

- Optical Lagrangian $\mathcal{L}$ and Hamiltonian $\mathcal{H}$

$$ \mathcal{L}(\phi, \dot{\phi}, r) = n(r) \sqrt{1 + r^2 \dot{\phi}^2} , \quad \mathcal{H} = - \frac{\sqrt{n^2 r^2 - p^2_\phi}}{r} $$

- First integral of motion

$$ \int d\phi = \int \frac{p_\phi}{r \sqrt{n^2 r^2 - p^2_\phi}} \, dr \quad \text{where} \quad p_\phi = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = \text{constant} $$
Ray tracing solution for a Luneburg Lens (LL)

- Complete ray solution for an LL for rays with initial angle $\theta$.

$$y(x) = \frac{(2x_0y_0 + R^2 \sin(2\theta)) x}{2x_0^2 + (1 + \cos(2\theta)) R^2} + \frac{\sqrt{2} R y_0 \cos(\theta) \sqrt{(1 + \cos(2\theta)) R^2 + 2x_0^2 - 2x^2}}{2x_0^2 + (1 + \cos(2\theta)) R^2} - \frac{x_0 \sin(\theta) \sqrt{(1 + \cos(2\theta)) R^2 + 2x_0^2 - 2x^2}}{2x_0^2 + (1 + \cos(2\theta)) R^2}$$

- For rays parallel to the propagation axis $x$ ($\theta = 0$)

$$y(x) = \frac{y_0}{x_0^2 + R^2} \left( x_0 x + R \sqrt{R^2 + x_0^2 - x^2} \right)$$

Break down of quasi 2D approach

- Quasi 2D method failures to describe backscattered rays
- This failure is due to the assumption that the radial coordinate plays the role of time
Parametric 2D ray tracing

In Cartesian coordinates

- Fermat Principle of least time for a refractive index \( n(x, y) \), for \( x(\tau) \) and \( y(\tau) \) where \( \tau \) the generalized time parameter

\[
S = \int_{A}^{B} n(x, y) \sqrt{\dot{x}^2 + \dot{y}^2} \, d\tau , \quad \delta S = 0
\]

- Optical Lagrangian \( \mathcal{L} \) and Hamiltonian \( \mathcal{H} \)

\[
\mathcal{L} = n \sqrt{\dot{x}^2 + \dot{y}^2} , \quad \mathcal{H} = \frac{k_x^2}{2} + \frac{k_y^2}{2} - \frac{n^2}{2} , \quad \left( k_q = \frac{\partial \mathcal{L}}{\partial \dot{q}} \right)
\]

- Ray tracing equation

\[
\dddot{\vec{r}} = \frac{1}{2} \nabla n(\vec{r})^2 \quad \text{(where } \vec{r} = (x, y)\text{)}
\]
Helmholtz wave equation approach

In Cartesian coordinates

- Helmholtz wave equation and standard assumption

\[
\left[ \nabla^2 + (nk_0)^2 \right] u(x, y) = 0, \quad u(x, y) = A(x, y)e^{i\phi(x, y)}
\]

- Resulting equations and derivation of Hamiltonian $\mathcal{H}$

\[
(\nabla \phi)^2 - (nk_0)^2 = \frac{\nabla^2 A}{A} \quad \Rightarrow \quad \mathcal{H} = \frac{\kappa^2}{2k_0} - \frac{k_0}{2} n(\vec{r})^2
\]

The term $\frac{\nabla^2 A}{A}$ is called Helmholtz potential and preserves the wave behavior in the ray tracing equation. In geometric optic approach can be neglected
Application of 2D ray solution to Luneburg index

\[ \vec{r} = \frac{1}{2} \nabla n(\vec{r})^2 \]

\[ n(\vec{r}) = \sqrt{2 - \left( \frac{\vec{r}}{R} \right)^2} \]

\[ \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \cos \left( \frac{t}{R} \right) + \begin{pmatrix} k_{x0} \\ k_{y0} \end{pmatrix} R \sin \left( \frac{t}{R} \right) \]

Finite Difference in Time Domain (FDTD)

- FDTD solves numerically the time dependent Maxwell’s equations
- Discretization both in space and time with grid unit cells $(\Delta x, \Delta y)$ and $\Delta t$ respectively
- Stability criterion
  \[ \Delta x \ll \lambda_{\text{min}} \quad \text{and} \quad \Delta y \ll \lambda_{\text{min}} \]
- Courant limit
  \[ \Delta t < \frac{\sqrt{(\Delta x)^2 + (\Delta y)^2}}{c_{\text{max}}} \]
Application of FDTD to a Luneburg lens

- FDTD is applied for a monochromatic EM plane wave source with wavelength $\lambda$
- Assumed the vacuum as bulk material ($\varepsilon = 1$)
- An LL is used with $\varepsilon = 2 - (r/R)^2$ with $R = 10\lambda$
- The steady state of the electric intensity is plotted

Transverse Magnetic waves (TM polarization)

\[
\frac{\partial H_x}{\partial t} = -\frac{1}{\mu} \frac{\partial E_z}{\partial y},
\]

\[
\frac{\partial H_y}{\partial t} = \frac{1}{\mu} \frac{\partial E_z}{\partial x},
\]

\[
\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon} \left( \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right)
\]
Outline

Introduction

Methods for light propagation

Networks of Luneburg Lenses
  Waveguides formed by GRIN lenses
  Beam splitter formed by GRIN lenses

Caustic formation

Rogue waves formation

Optical fiber lattices
Luneburg lens waveguides

Ray tracing solutions

Electromagnetic waveguides can be formed by LLs.

Luneburg lens waveguides

FDTD wave simulations

Electromagnetic waveguides can be formed by LLs

Beam splitter formed by Luneburg lens

A beam splitter can be formed by LLs

The losses are $\sim 10\%$. 90% of the incoming rays are split and guided through LLs configuration.
Outline

Introduction

Methods for light propagation

Networks of Luneburg Lenses

Caustic formation

Statistics of caustics

Simulations and Results

Rogue waves formation

Optical fiber lattices
Caustics

- Caustics are areas with high intensity
- Caustics and branched flow can arise in light propagation through random fluctuated refractive index
Statistics of caustics

- Derivation of a Hamilton-Jakobi equation by an ordinary Hamiltonian with unit mass \((m = 1)\)

\[
\mathcal{H} = \frac{p^2}{2} + V(t, y) \\
\tilde{p} = \frac{\partial S}{\partial y}
\]

\[
\Rightarrow \frac{\partial}{\partial t} S(t, y) + \frac{1}{2} \left( \frac{\partial S}{\partial y} \right)^2 + V(t, y) = 0
\]

- Definition of curvature of classical action \(S\)

\[
u \equiv \frac{\partial p}{\partial y} = \frac{\partial^2 S}{\partial y^2}
\]

- The singularities of curvature \((\nu \to \infty)\) denote caustics
Statistics of caustics

- Ordinary Differential Equations for curvature $u$ derived by HJE
  
  \[ \frac{du}{dt} + u^2 + \frac{\partial^2}{\partial y^2} V(t, y) = 0 \]

- Potential acts as delta correlated noise $\Gamma(t)$
  
  \[ \frac{\partial^2}{\partial y^2} V(t, y) = \Gamma(t) \quad \langle \Gamma(t)\Gamma(t') \rangle = 2\sigma\delta(t - t') \]

- Relation between standard deviation $\sigma$ and Diffusion coefficient $D$
  
  \[ D = 2\sigma^2 \]

- Ordinary Stochastic Differential Equation for curvature $u$
  
  \[ \frac{du}{dt} = -u^2 - \sigma \Gamma(t) \]
Statistics of caustics

Scaling law

- Fokker-Plank equation (FPE) for probability density $P$

$$\frac{\partial}{\partial t} P(u, t) = \left[ \frac{\partial}{\partial u} u^2 + \frac{\partial^2}{\partial u^2} \frac{D}{2} \right] P(u, t)$$

- Backward Fokker-Plank equation (BFPE) for probability density $p_f$

$$\frac{\partial}{\partial t} p_f(u, t) = \left[ -u_0^2 \frac{\partial}{\partial u_0} + \frac{D}{2} \frac{\partial^2}{\partial u_0^2} \right] p_f(u, t)$$

- Looking for the average first time that $u \to \infty$ for arbitrary initial curvature $u_0$, obtaining a scaling law for the mean time of the first caustic onset

$$\langle t_c \rangle \sim \sigma^{-2/3}$$
Simulations setup

- Monochromatic electromagnetic waves propagate through disordered Luneburg networks
- 150 randomly located generalized LLs each radius with $R = 10\lambda$ in lattice with dimensions $460\lambda \times 360\lambda$
- Generalized LL refraction index

$$n(r) = \sqrt{\alpha (n_L^2 - 1)} + 1$$

- $\alpha$ is called “strength” parameter and it is proportional to random potential standard deviation $\sigma$

$$\sigma \simeq 0.1\alpha$$
FDTD simulations

Simulation for different values of strength parameter $\alpha$

- Left figure: $\alpha = 0.07$
- Right figure: $\alpha = 0.1$

Mattheakis et. al., “Branched flow through optical complex systems” (working paper)
Scintillation index

- The scintillation index $\sigma_I$ shows the deviation of the Intensity $I$ of the mean value of intensity $\langle I \rangle$

$$\sigma_I^2 = \frac{\langle I(x)^2 \rangle}{\langle I(x) \rangle^2} - 1$$

- Since caustics are high intensity areas, a maximum of $\sigma_I$ shows caustic formation.
Numerical results

- Simulation for several values of strength parameter $\alpha$ are taken place, resulting to the correspond scintillation indexes plots.

(a) $\sigma_1^2(x)$ for different values of $\sigma$
(b) maximum position of the $\sigma_1^2(x)$
(c) $\sigma_1^2(x)$ are plotted in rescaled $x$ axis

M. Mattheakis et. al., “Branched flow through optical complex systems” (working paper)
Outline

Introduction

Methods for light propagation

Networks of Luneburg Lenses

Caustic formation

Rogue waves formation
  Definition of Rogue Waves (RWs)
  Rogue waves arise in Luneburg hole lens networks

Optical fiber lattices
Definition of Rogue Waves (RWs)

- RWs are relatively large and spontaneous surface waves
- Waves with height at least two times greater than Significant Wave Height (SWH)
- SWH is the mean wave height of the highest (statistical) third of the waves
- Long tailed height distribution instead of Rayleigh distribution
- RWs have been found in
  - Ocean water surface waves
  - Microwaves propagation
  - Financial systems
Electromagnetic wave propagation in gradient index metamaterials, plasmonic systems and optical fiber networks

Rogue waves formation

Rogue waves arise in Luneburg hole lens networks

Luneburg Hole (LH) lens consists random photonic networks

- Luneburg Hole (LH) is a new GRIN lens with index

\[ n(r) = \sqrt{1 + \left( \frac{r}{R} \right)^2} \]

- LH has purely defocussing properties
FDTD Results

- 400 randomly located LH lenses, each radius with $R = 3.5\lambda$, consist a photonic disordered lattice
- Filling factor of the arrangement $f = 0.17$

M. Mattheakis et al., “Linear and nonlinear photonic rogue waves in complex transparent media” (working paper)
Experimental Results

- Photonic disordered lattices \((250 \times 250) \mu m^2\) consist of five superposed layers of 400 LHs for each layer

M. Mattheakis et al., “Linear and nonlinear photonic rogue waves in complex transparent media” (working paper)
Nonlinear permittivity function

- Introduce focusing nonlinearity (Kerr effect) in the permittivity function

\[ \varepsilon = n^2 = \varepsilon_L + \chi |E|^2 \quad (\chi \text{ varying from } 10^{-7} \text{ to } 10^{-6}) \]

- Linear RW position and statistics are not affected by the presence of relatively small nonlinearity

M. Mattheakis et. al., “Linear and nonlinear photonic rogue waves in complex transparent media” (working paper)
Outline

Introduction

Methods for light propagation

Networks of Luneburg Lenses

Caustic formation

Rogue waves formation

Optical fiber lattices

Optical fiber lattices

Transport properties
Optical fiber lattices

- Fiber networks consist of 400 fibers in a square lattice
- Fibers extend in $z$ axis, which in paraxial approximation plays the role of time
- Fibers are coupled through interfiber interaction (evanescent coupling)
- A disorder parameter $\rho$ is introduced to control the randomness level

Wave-packet propagation

- Discrete Nonlinear Schrödinger Equation (DNLS) is chosen for investigating transport properties

\[ i \frac{d\psi_n}{dt} = \sum_m V_{n,m} \psi_m - \chi |\psi_n|^2 \psi_n \]

- DNLS allows to explore both the tunneling effect and the influence of nonlinearity

- A wave-packet is placed in the central fiber (excitation of a single fiber)

- Investigate the diffusion exponent by plotting the Mean Square Displacement (MSD) as function of time \( t \)

- No Anderson localization takes place since the excitation is initially localized
Electromagnetic wave propagation in gradient index metamaterials, plasmonic systems and optical fiber networks

Optical fiber lattices

Transport properties

Diffusion exponent

- Passage from ballistic to sub-diffusion due to structural disordered
- Transition from ballistic to sub-diffusion due to nonlinearity
- The combination of structural disordered with nonlinearity leads to almost complete localization

Electromagnetic wave propagation in gradient index metamaterials, plasmonic systems and optical fiber networks

Active plasmonic systems

Outline

Introduction

Methods for light propagation

Networks of Luneburg Lenses

Caustic formation

Rogue waves formation

Optical fiber lattices

Active plasmonic systems

Surface plasmon characteristics
Surface Plasmon Polaritons SPPs

- Quasi particles formed by coupling of an EM wave with metal’s free electron-oscillation field.
- Surface waves with evanescent decaying EM field in transverse axis.
- Due to ohmic loss the SPPs decays exponentially also in propagation axis.
- Maxwell equations support SPPs solutions for
  - Transverse Magnetic Polarization (TM)
  - Interface between metal ($\text{Re}[\varepsilon_m] < 0$) and dielectric ($\text{Re}[\varepsilon_d] > 0$)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_1$</td>
<td>2.25</td>
</tr>
<tr>
<td>$\varepsilon_2$</td>
<td>$-15.13 - 0.93i$</td>
</tr>
<tr>
<td>$\varepsilon_3$</td>
<td>1.69</td>
</tr>
<tr>
<td>d</td>
<td>50 nm</td>
</tr>
<tr>
<td>$\theta$</td>
<td>$66.74^\circ$</td>
</tr>
</tbody>
</table>
SPPs excitation (COMSOL simulation)

- A monochromatic EM wave is used for SPPs excitation in a dielectric-metal-dielectric configuration.

SPP characteristics

- Dispersion relation $\beta$
  
  $\beta = k_0 \sqrt{\frac{\varepsilon_d \varepsilon_m}{\varepsilon_d + \varepsilon_m}}$

- Propagation length $L$
  
  $L = \frac{1}{2\text{Im}[\beta]}$

- Penetration length $t_{d(m)}$
  
  $t_{d(m)} = \frac{1}{k_0} \text{Re} \left[ \sqrt{\frac{\varepsilon_d + \varepsilon_m}{-\varepsilon_d^2(m)}} \right]$
Active dielectrics

- Dielectrics with complex permittivity and refractive index
  \[ \varepsilon = \varepsilon' + i\varepsilon'' = (n_R + i\kappa)^2 \quad (\varepsilon'' \text{ and } \kappa \text{ account for gain}) \]

- Gain counterbalance the ohmic loss of metal
- Enhanced propagation and penetration length
- Enhanced the SPP intensity
Infinite propagation Length

There is a critical value of gain for which $\text{Im}[\beta] = 0$ resulting to lossless SSPs propagation ($L \rightarrow \infty$)

---

Outline

Introduction

Methods for light propagation

Networks of Luneburg Lenses

Caustic formation

Rogue waves formation

Optical fiber lattices

Active plasmonic systems
Conclusion

- Four methods for studying light propagation are developed
- GRIN lenses can form waveguides and beamsplitters
- Caustic and rogue waves formation arise in EM propagation through random GRIN networks
- Wavepacket sub-diffuses in fiber lattices due to randomness and nonlinearity
- Active (gain) dielectrics enhance SPP propagation and penetration length
Acknowledgements

Thanks for your attention

I would like to deeply thank ...

- Prof. Giorgos Tsironis
- Prof. Stelios Tzortzakis
- Dr. Giorgos Neofotistos
- Dr. Nikos Lazaridis
- Dr. Thomas Oikonomou
- Dr. Patrick Navez
Publications I

M. Mattheakis, G. P. Tsironis
*Extreme waves and branching flow in optical media.*
Chapter in a volume of Springer Series on Material Science (2014)

M. Mattheakis, G. P. Tsironis, V. I. Kovanis
Luneburg lens waveguide networks

F. Perakis, M. Mattheakis, G. P. Tsironis
Small-world networks of optical fiber lattices
Publications II

C. Athanasopoulos, M. Mattheakis, G.P. Tsironis
Enhanced surface plasmon polariton propagation induced by active dielectrics
(Excerpt from the Proceedings of the 2014 COMSOL Conference in Cambridge (2014))

M. Mattheakis, J.J. Metzger, G.P. Tsironis, R. Fleischmann
Branched flow through optical complex systems
(working paper)

M. Mattheakis, I. J. Pitsios, G. P. Tsironis, S. Tzortzakis
Linear and nonlinear photonic rogue waves in complex transparent media
(working paper)