CHAPTER 9

A SECOND-BEST APPROACH TO INCENTIVE COMPATIBILITY

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1. Introduction

Public economics has recently witnessed major advances in the theory of incentives. Procedures that enable decision makers to take correct public decisions despite imperfect information have been designed and their properties studied.[1] These procedures are typically games in which decisions are taken (in game theoretical language, outcomes result) on the basis of information revealed (the strategies selected) by private individuals. One class of procedures, moreover, has the property that revelation of the truth constitutes a dominant strategy; that is, an individual does best by revealing the truth regardless of the behavior of others.[2]

The limitations of these procedures are numerous and will not be reviewed here. One major drawback of the dominant strategy mechanisms, however, is that they work only for a limited class of social welfare functions. Indeed, Roberts (1979) and Laffont and Maskin (1979a) showed that, essentially, only the linear social welfare functions can, in general, be optimized incentive compatibly. This limitation forces us either to weaken the strong incentive requirement of dominant strategies (see, for example, Laffont and Maskin, 1979b) or to lower our sights to second-best optimization in the design of incentive compatible mechanisms.

Any second-best problem requires a detailed specification of the constraints and, in particular, the available instruments. In incentives theory, the description of the instruments is the difficult part, and we do not yet have a characterization of the various

[1] See, for example, Green and Laffont (1979) and Laffont (1979).
[2] See Laffont and Maskin (1979a).

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approaches to set up appropriate incentives. For example, one may design pure dominant strategy mechanisms which use no information besides that which individuals reveal, or one may, as in the optimal tax literature, make use of additional data, such as incomes. If certain information is available only *ex post*, one may introduce *ex post* penalty functions.

At this stage of our study of incentives, the generality of a second-best study can only be very limited and based on a rather restrictive description of available instruments. In this chapter we explore a very special type of second-best problem where a second-best optimum is obtained by approximating the social welfare function of interest with one that can be implemented by a dominant strategy mechanism.

In section 2 we describe the model and summarize the major theorems of incentive theory that we use. Section 3 solves the second-best problem in a simple case. Section 4 states the general nature of the problem. The appendix analyzes implementation of the "maximin" social welfare function.

2. The model

We consider a society with $n (n \geqslant 2)$ agents, indexed by $i = 1, ..., n$, and one private good, say income. The utility function of agent i depends on his consumption of private good x_i, $i = 1, ..., n$, and on a public decision, K. The cost (in private good) of the public decision for agent i, $c_i(K)$, is defined *ex ante*. Social cost is $c(K) = \Sigma c_i(K)$. Agent i's initial endowment of private good is $\bar{x}_i$. We assume that his utility function is quasi-linear, i.e.

$$\begin{aligned} u_i(K, x_i - c_i(K)) &= w_i(K) + x_i - c_i(K) \\ &= v_i(K) + x_i. \end{aligned}$$

Thus, $w_i(K)$ is the gross and $v_i(K)$ the net willingness to pay for the public decision, K. For mathematical convenience, we parameterize the net willingness to pay functions. For $i = 1, ..., n$, let Θ_i be an open interval of $\mathbb{R}$ and let v_i: $\mathbb{R}_+ \times \Theta_i \to \mathbb{R}$ be a continuously differentiable function.

A social welfare function is a mapping F: $\mathbb{R}^{2n} \to \mathbb{R}$, where $F(v_1, ..., v_n, x_1, ..., x_n)$ is the social welfare level attached to the net willingness to pay vector $(v_1, ..., v_n)$ and the private consumption levels $(x_1, ..., x_n)$. We shall suppose that F is increasing and twice continuously differentiable and that it satisfies the following two assumptions.

Assumption 1: The matrix of second-order partials of F with respect to the v_j's is negative semi-definite.

Assumption 2: There exist functions $K^*(\cdot), x_1(\cdot), ..., x_n(\cdot)$, from $\Theta = \Pi_{i=1}^n \Theta_i$ to $\mathbb{R}_+$ such that $K^*(\cdot), x_1(\cdot), ..., x_n(\cdot)$ are continuously differentiable and, for all $\theta \in \Theta$, $K^*(\theta), x_1(\theta), ..., x_n(\theta)$ solve the problem:

$$\max F(v_1(K,\theta_1), \ldots, v_n(K,\theta_n), x_1, \ldots, x_n), \qquad (*)$$

such that

$$\sum_{i=1}^{n} x_i \leq \sum_{i=1}^{n} \bar{x}_i - c(K), \quad x_i \geq 0, \quad i = 1, \ldots, n$$

The functional forms $v_i(\cdot, \cdot)$ are assumed to be known publicly, but the true value, $\hat{\theta}_i$, of the parameter θ_i is known only to agent i, *a priori*. We can represent the family of possible vectors of valuations by

$$\mathcal{V} = \{v_1(\cdot, \theta_1), \ldots, v_n(\cdot, \theta_n) \mid \theta_1, \ldots, \theta_n \in \Theta_1 \times \ldots \times \Theta_n\}.$$

The problem of the decision-maker is to choose a public decision which maximizes social welfare, even though he does not know the vector $\hat{\theta} = (\hat{\theta}_1, \ldots, \hat{\theta}_n)$. One approach the decision-maker could follow is to behave as a Bayesian statistician and solve the program:

$$\max_{K, x_1, \ldots, x_n \geq 0} \int_{\theta \in \Theta} F(v_1(K,\theta_1), \ldots, v_n(K,\theta_n), x_1, \ldots, x_n) \, \mathrm{d}\psi(\theta_1, \ldots, \theta_n)$$

s.t.

$$\sum_{i=1}^{n} x_i \leq \sum_{i=1}^{n} \bar{x}_i - c(K),$$

where $\psi(\theta_1, \ldots, \theta_n)$ is his subjective prior distribution on $\hat{\theta}$. Let $K_B, x_{1B}, \ldots, x_{nB}$ be the solution and let V_B be the optimal value of the maximand:

$$V_B = E_\psi F(v_1(K_B, \theta_1), \ldots, v_n(K_B, \theta_n), x_{1B}, \ldots, x_{nB}).$$

In contrast with this Bayesian optimum is the "first best" optimum, given by:

$$V_{FB} = E_\psi F(v_1(K^*(\theta), \theta_1), \ldots, V_n(K^*(\theta), \theta_n), x_1(\theta), \ldots, x_n(\theta)),$$

where the functions $K^*(\cdot)$, $x_1(\cdot)$, ..., $x_n(\cdot)$ are as in assumption 2.

Is it possible for the decision-maker to improve on V_B and, perhaps, even to attain V_{FB}? To do better than V_B, the decision-maker must make the private agents participate in the decision process. The most straightforward way of doing so is simply to ask them to reveal their true characteristics. Unfortunately, agents will, in general, find it in their interest to misrepresent these characteristics. That is to say, they will view themselves as playing a game where the outcome (a vector $(K, x_1, \ldots, x_n)$) depends on their strategy (professing a characteristic). The difficulty this strategic behavior creates for public decision-making is sometimes called the free rider problem.

Recognizing that agents will behave strategically, the decision-maker can consider his problem as that of designing a game or procedure, the equilibria of which come as close as possible to solving the first-best optimization. Formally, the decision-maker must choose strategy spaces $S_1, \ldots, S_n$ and functions

$$K, x_1, \ldots, x_n\colon\ S_1 \times \ldots \times S_n \to \mathbb{R},$$

where $K(s_1, \ldots, s_n)$ is interpreted by the public decision-taker as the consequence of strategies $s_1, \ldots, s_n$ and $x_i(s_1, \ldots, s_n)$ as agent i's consequent private consumption level. To be able to speak of the equilibrium outcomes of such a procedure, one must first state the solution concept. The strongest solution concept is that of dominant strategies. In a dominant strategy equilibrium each player employs a strategy that he would be willing to use regardless of the strategies chosen by other players.

It turns out that for dominant strategy equilibrium, there is no loss of generality in assuming that an agent's strategy space is his space of characteristics (i.e. $S_i = \Theta_i$) with the truth as a dominant strategy (see Green and Laffont, 1979, theorem 4-8). Consequently, we say that the free rider problem is *solvable for $\mathscr{V}$ and F* if there exists a procedure $g(\cdot) = (K(\cdot), x_1(\cdot) \ldots, x_n(\cdot))$ form Θ into $\mathbb{R}_+^{n+1}$ such that for any $\hat{\theta} \in \Theta$:

$$\begin{cases} \forall i \forall \theta_i \in \Theta, v_i(K(\hat{\theta}_i, \theta_{-i}), \hat{\theta}_i) + x_i(\hat{\theta}_i, \theta_{-i}) \geqslant v_i(K(\theta_i, \theta_{-i}), \hat{\theta}_i) + x_i(\theta_i, \theta_{-i}) \\ \text{and } g(\hat{\theta}) \text{ solves the first-best optimization } (*).^3 \end{cases} \qquad (**)$$

We now review some results on the free rider problem's solvability. Let $\mathscr{F}_L$ be the class of "linear" social welfare functions:

$$F(v_1, \ldots, v_n, x_1, \ldots, x_n) = \sum_{i=1}^{n} \lambda_i (v_i + x_i), \quad \lambda_i > 0, \quad i = 1, \ldots, n.$$

Let $\mathscr{V}_Q$ be the family of quadratic net willingness-to-pay functions:

$$\mathscr{V}_Q = \{\theta_1 K - \left(\frac{K^2}{2}\right), \ldots, \theta_n K - \left(\frac{K^2}{2}\right) \mid \theta_i \in \Theta_i \text{ for all } i\}.$$

Theorem 1 The Impossibility Theorem (Green and Laffont, 1979; Hurwicz, 1975).
For all $F \in \mathscr{F}_L$ there exists a family $\mathscr{V}$ such that the free rider problem is not solvable for $\mathscr{V}$ and F.

Theorem 2 The Quadratic Theorem (Groves and Loeb, 1975).
The free rider problem is solvable for $\mathscr{V}_Q$ and F if $F \in \mathscr{F}_L$.

We say that the free rider problem is *weakly solvable for $\mathscr{V}$ and F* if there exists a procedure $g(\cdot) = (K(\cdot), x_1(\cdot), \ldots, x_n(\cdot))$, satisfying the constraints of $(*)$ such that $K(\cdot) = K^*(\cdot)$, where K^* is as in assumption 2 and $(**)$ is satisfied. When $F = \Sigma_{i=1}^n (v_i + x_i)$, weak solvability implies that the public decision is Pareto efficient and that

[3] $\theta_{-i} = (\theta_1, \ldots, \theta_{i-1}, \theta_{i+1}, \ldots, \theta_n)$ $(\theta_i, \theta_{-i}) = (\theta_1, \ldots, \theta_{i-1}, \theta_i, \theta_{i+1}, \ldots, \theta_n)$

private consumption is feasible. However, typically $\Sigma_i x_i(\theta) < \Sigma_i \bar{x}_i - c(K)$, so that use of the procedure g entails some loss of private good.

Theorem 3 The Possibility Theorem (Clarke, 1971; Groves 1973; Vickrey, 1961).
The free rider problem is weakly solvable for any $\mathscr{V} \times F$ with $F \in \mathscr{F}_L$.

Theorem 4 The Generalized Impossibility Theorem (Laffont and Maskin, 1979a; Roberts, 1979).[4]
Suppose the matrix of second partials of F with respect to the v_i's is strictly negative definite and that $\mathscr{V}$ contains $\mathscr{V}_Q$. Then the free rider problem is not weakly solvable for $\mathscr{V}$ and F.

As is clear from above, a weak solution to the free rider problem is already second-best, and yet theorem 4 asserts the impossibility of such a solution for all but a narrow class of cases.

The purpose of this chapter is to avoid the negative conclusions of theorem 4 by relaxing the optimality requirement still further. From theorem 4 we know that it is impossible to elicit truthful responses if a concave and nonlinear social welfare function $F(\cdot)$ is to be maximized by the decision-maker. The decision-maker can maximize only those social welfare functions which are implementable by dominant strategy mechanisms. The second-best problem is, therefore, to choose the best such function given that $F(\cdot)$ is the actual social welfare function.

3. A solution to the second-best problem

We shall confine our study to a society with two agents and to the family of valuation functions:

$$\mathscr{V}_{LQ} = \left\{\eta_1 + \theta_1 K - \frac{K^2}{2}, \eta_2 + \theta_2 K - \frac{K^2}{2}\right\}, \quad \eta_i \in [-1, +1], \quad \theta_i \in [-1, +1].$$

From theorem 4 above we know that, other than the linear functions, there exists no social welfare function (satisfying assumption 1) for which the free rider problem is weakly solvable. Suppose nonetheless that the social preferences of society are represented by a function $F(v_1(K, \theta_1), v_2(K, \theta_2))$ which is symmetric and strictly concave in the v_i's in the relevant range. The only way to elicit the true characteristics $(\hat{\theta}_1, \hat{\theta}_2)$ is to replace F by a linear social welfare function:

$$\lambda_1 v_1(K, \theta_1) + \lambda_2 v_2(K, \theta_2), \quad \lambda_1 + \lambda_2 = 1, \quad \lambda_1 \geqslant 0, \quad \lambda_2 \geqslant 0.$$

[4] See also Laffont and Maskin (1980).

This "surrogate" SWF can then be maximized with appropriate Clarke–Groves transfers.
The solution of the program

$$\max_{K \geq 0} \lambda_1 \left[\eta_1 + \theta_1 K - \frac{K^2}{2}\right] + \lambda_2 \left[\eta_2 + \theta_2 K - \frac{K^2}{2}\right]$$

is

$$K^*(\theta, \lambda) = \lambda_1 \theta_1 + \lambda_2 \theta_2 .$$

The second-best problem reduces here to the choice of the best vector (λ_1, λ_2) given that one wishes to optimize $F(\cdot)$ but that one is, for incentive reasons, constrained to the $K^*(\cdot)$ family of public decision functions.

Proposition 1. Under assumption 1, if $F(\cdot)$ and the prior distribution of the decision maker on θ are both symmetric, then the optimal public decision function in the class

$$\{K(\theta) = \lambda_1 \theta_1 + \lambda_2 \theta_2, \quad \lambda_1 + \lambda_2 = 1\}$$

is

$$K^*(\theta) = \frac{\theta_1 + \theta_2}{2} .$$

Proof. Suppose, to the contrary, that the optimal public decision function were $K^1(\theta) = \lambda \theta_1 + (1-\lambda)\theta_2$, $\lambda \in [0,1]$, $\lambda \neq \frac{1}{2}$. Then, by symmetry of the problem, $K^2(\theta) = (1-\lambda)\theta_1 + \lambda\theta_2$ is also a solution. But $K^*(\theta) = (\theta_1 + \theta_2)/2 = \frac{1}{2} K^1(\theta) + \frac{1}{2} K^2(\theta)$. For each value of (θ_1, θ_2):

$$F(v_1(K^*(\theta), \theta_1), v_2(K^*(\theta), \theta_2))$$
$$> \tfrac{1}{2} F(v_1(K^1(\theta), \theta_1), v_2(K^1(\theta), \theta_2)) + \tfrac{1}{2} F(v_1(K^2(\theta), \theta_1), v_2(K^2(\theta), \theta_2)),$$

since by assumption 1 F is strictly concave in the v_i's and the v_i's are concave in K. Taking expectation on both sides with respect to the prior distribution, we obtain:

$$E_\psi F(v_1(K^*(\theta), \theta_1), v_2(K^*(\theta), \theta_2))$$
$$> \tfrac{1}{2} E_\psi F(v_1(K^1(\theta), \theta_1), v_2(K^1(\theta), \theta_2)) + \tfrac{1}{2} E_\psi F(v_1(K^2(\theta), \theta_1), v_2(K^2(\theta), \theta_2)),$$

contradicting the assumed optimality of $K^1(\cdot)$ and $K^2(\cdot)$. ■

The constraint of incentive compatibility on public decision functions leads to an expected value of social welfare which may not actually be larger than the expected value when the decision is taken solely on the basis of prior expectations by a Bayesian statistician. With a symmetric social welfare function and a symmetric prior, a Bayesian statistician can do no better than to set $K = 0$. We might ask, then, when a revelation procedure can do better than simply fixing K at zero. We explore this question with several examples. We shall compare the fixed K rule with the revelation scheme that chooses $K^*(\theta) = \frac{1}{2}\theta_1 + \frac{1}{2}\theta_2$.

When the prior distributions on $\theta = (\theta_1, \theta_2)$ and $\eta = (\eta_1, \eta_2)$ are independent, one may as well assume that $\eta_1 = \eta_2 = 0$ (see below). We shall, therefore, do so.

Example 1. Perfect negative correlation (fig. 9.1).
Distribution of θ:

$\theta_1 = 1, \theta_2 = -1$ with probability 1/2,

$\theta_1 = -1, \theta_2 = 1$ with probability 1/2.

In both cases $K^*(\theta) = \frac{1}{2}\theta_1 + \frac{1}{2}\theta_2$ is identically zero. That is, under perfect negative correlation the decision rule is the same as the *a priori decision*.

Therefore, under perfect negative correlation, there is no gain to using a revelation procedure. Indeed, there is a loss if it is costly to operate.

Example 2. Perfect positive correlation (fig. 9.2).
Distribution on θ:

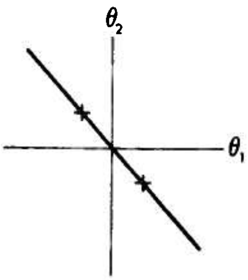

Figure 9.1

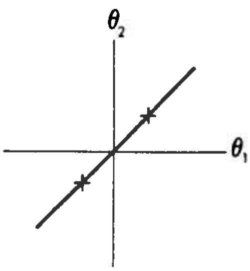

Figure 9.2

$$\theta_1 = 1, \theta_2 = 1 \quad \text{with probability } 1/2,$$

$$\theta_1 = -1, \theta_2 = -1 \quad \text{with probability } 1/2.$$

When

$$\theta_1 = \theta_2 = 1, \quad K^* = \tfrac{1}{2}\theta_1 + \tfrac{1}{2}\theta_2 = 1, \quad v_1 = v_2 = \tfrac{1}{2},$$

$$\theta_1 = \theta_2 = -1, \quad K^* = \tfrac{1}{2}\theta_1 + \tfrac{1}{2}\theta_2 = -1, \quad v_1 = v_2 = \tfrac{1}{2}.$$

The expected value of the procedure is then

$$V_P = \tfrac{1}{2}F(\tfrac{1}{2},\tfrac{1}{2}) + (\tfrac{1}{2})F(\tfrac{1}{2},\tfrac{1}{2}) = F(\tfrac{1}{2},\tfrac{1}{2}).$$

Since $F(\cdot,\cdot)$ is increasing, V_P is larger than

$$V_B = (\tfrac{1}{2})F(0,0) + (\tfrac{1}{2})F(0,0) = F(0,0).$$

When $\eta_1 = \eta_2 = 0$, the public decision K^* is the first-best decision.

The reason why it is possible to neglect (η_1, η_2) can now be made clear.

For each value (η_1, η_2) we have:

$$V_{P/\eta_1,\eta_2} = F(\tfrac{1}{2} + \eta_1, \tfrac{1}{2} + \eta_2) > V_{B/\eta_1,\eta_2} = F(0 + \eta_1, 0 + \eta_2).$$

Taking expectations on both sides with respect to the distribution of η (assumed independent of the distribution of θ), the inequality is preserved.

Summing up, we conclude that under perfect positive correlation, the revelation procedure is valuable.

This result is really just a special case of the proposition that when the distribution is concentrated in orthants 1 and 3 the revelation procedure is worthwhile. Since θ_1 and θ_2 have the same sign, $K^*(\theta) = (\theta_1 + \theta_2)/2$ shares this sign and, therefore, $\theta_i K^* - K^{*2}/2 > 0, i = 1, 2$. Therefore, revelation procedures fail to be valuable only if the distribution of θ is concentrated in orthants 2 and 4. That this, however, is not a sufficient condition for lack of value is illustrated by the next example.

Example 3. Imperfect negative correlation (fig. 9.3).

Distribution of θ:

$$\begin{aligned} \theta_1 &= -1, \theta_2 = \tfrac{1}{2} \quad \text{with probability } \tfrac{1}{4} \\ &= \tfrac{1}{2} \qquad = -1 \quad \text{with probability } \tfrac{1}{4} \\ &= -\tfrac{1}{2} \qquad = 1 \quad \text{with probability } \tfrac{1}{4} \\ &= 1 \qquad = -\tfrac{1}{2} \quad \text{with probability } \tfrac{1}{4}. \end{aligned}$$

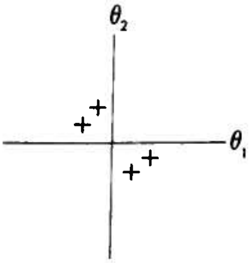

Figure 9.3

When

$$\begin{array}{lllll}
\theta_1 = -1, & \theta_2 = \frac{1}{2}, & K^* = -\frac{1}{4}, & v_1 = \frac{7}{32}, & v_2 = -\frac{5}{32}, \\
\theta_1 = \frac{1}{2}, & \theta_2 = -1, & K^* = -\frac{1}{4}, & v_1 = -\frac{5}{32}, & v_2 = \frac{7}{32}, \\
\theta_1 = -\frac{1}{2}, & \theta_2 = 1, & K^* = \frac{1}{4}, & v_1 = -\frac{5}{32}, & v_2 = \frac{7}{32}, \\
\theta_1 = 1, & \theta_2 = -\frac{1}{2}, & K^* = \frac{1}{4}, & v_1 = \frac{7}{32}, & v_2 = -\frac{5}{32}.
\end{array}$$

The expected value of this decision rule is

$$(\tfrac{1}{2})F(\tfrac{7}{32}, -\tfrac{5}{32}) + (\tfrac{1}{2})F(-\tfrac{5}{32}, \tfrac{7}{32}),$$

which is higher or less than $F(0,0)$ depending on the curvature of F, i.e. on the sensitivity of the social welfare function to inequality (see fig. 9.4).

For the social welfare function F_1, with high curvature, an *a priori* choice of $K = 0$

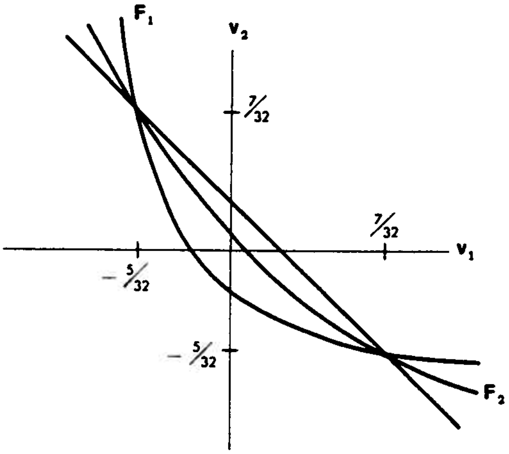

Figure 9.4

is better than the use of the procedure because of social aversion to inequality. On the other hand, for F_2, with low curvature, the use of the revelation procedure is advantageous. Indeed, if F is linear, the revelation scheme attains the first-best.

4. The general problem

The problem can be stated in its general form as follows:

$K^*(\theta, \lambda_1, ..., \lambda_n)$ solves the problem

$$\max_K \sum_{i=1}^{n} \lambda_i v_i(K, \theta_i),$$

as $(\lambda_1, ..., \lambda_n)$ varies in $L = \{\lambda_1, ..., \lambda_n \mid \lambda_1 + ... + \lambda_n = 1, \lambda_i \geqslant 0 \quad i = 1, ..., n\}$

and θ varies in Θ.

$K^*(\theta, \lambda_1, ..., \lambda_n)$ describes the public decision functions which are implementable by the Clarke–Groves mechanisms. From theorem 3 we know that they are essentially the only ones implementable by dominant strategy mechanisms; therefore, we have the complete description of the instruments.

The second-best problem is:

$$\max_{\lambda} \int_{\Theta} F(v_1(K^*(\theta, \lambda), \theta_1), ..., v_n(K^*(\theta, \lambda), \theta_n)) \, \mathrm{d}\, \psi(\theta_1, ..., \theta_n)$$

s.t.

$$(\lambda_1, ..., \lambda_n) \in L.$$

Let $(\phi_i(\theta, \lambda))$ denote the Clarke transfers[5] associated with the maximization of $\Sigma_{i=1}^{n} \lambda_i v_i(K, \theta_i)$. If $F(\cdot)$ depends on the levels of private goods, the problem takes the form:

$$\max_{\lambda \in L} \int_{\Theta} F(v_1(K^*(\theta, \lambda), \theta_1), ..., v_n(K^*(\theta, \lambda), \theta_n), \bar{x}_1 + \phi_1(\theta, \lambda), ..., \bar{x}_n + \phi_n(\theta, \lambda)).$$

If the $v_i(\cdot, \theta_i)$ functions are concave in $K, i = 1, ..., n$, the K^*'s can be characterized by the first-order condition:

$$\sum_{i=1}^{n} \lambda_i \frac{\partial v_i}{\partial K}(K^*, \theta_i) = 0, \quad \forall \theta \in \Theta.$$

[5] These transfers in private good ensure that a truthful answer is a dominant strategy for each agent (see Green and Laffont, 1979).

The second-best problem then takes the form:

$$\max_{\lambda} \int_{\Theta} F(v_1(K,\theta_1), ..., v_n(K,\theta_n), \bar{x}_1 + \phi_1(\theta,\lambda), ..., \bar{x}_n + \phi_n(\theta,\lambda))\, \mathrm{d}\,\psi(\theta_1, ..., \theta_n)$$

s.t.

$$\sum_{i=1}^{n} \lambda_i \frac{\partial v_i}{\partial K}(K,\theta_i) = 0, \quad \forall\theta\in\Theta \quad (\lambda_1, ..., \lambda_n)\in L.$$

Appendix

The approach developed in the text of this chapter requires differentiability of the decision functions. In this appendix we discuss the implementation of the maximin social welfare function by nondifferentiable decision functions.

Example A1.

$$V_1 = \{\theta_1 K - (K^2/2), \theta_2 K - (K^2/2), \theta_1 \in \mathbb{R}, \theta_2 \in \mathbb{R}\}, \quad K^*(\theta) = \min(\theta_1, \theta_2),$$

with zero transfers is a solution to the free rider problem for V_Q and $F = \min_K(v_1(K,\theta_1), v_2(K,\theta_2))$. Revelation of the truth is clearly a dominant strategy.

Example A2.

$$V_2 = \{-(K-\theta_1)^2, -(K-\theta_2)^2, \theta_1 \in \mathbb{R}, \theta_2 \in \mathbb{R}\}, \quad K^*(\theta) = \frac{\theta_1+\theta_2}{2},$$

i.e. the decision function which implements the maxmin criterion ($\min_i v_i(K,\theta_i)$) coincides with the one which maximizes the utilitarian criterion ($\Sigma_i v_i(K,\theta_i)$) so that we know that the Clarke–Groves mechanisms provide a weak solution to the free rider problem (theorem 3).

Example A3.

$$V_3 = \{\eta_1 + \theta_1 K - (K^2/2), \eta_2 + \theta_2 K - (K^2/2), \theta_1 > \theta_2, \eta_1 < \eta_2,$$
$$\theta_1 \in \mathbb{R}, \theta_2 \in \mathbb{R}, \eta_1 \in \mathbb{R}, \eta_2 \in \mathbb{R}\}.$$

The implementation of maxmin requires

$$v(\theta_1, K) = v(\theta_2, K) \quad \text{or} \quad K^* = \frac{\eta_2 - \eta_1}{\theta_1 - \theta_2}.$$

However, we observe that for η_1, η_2 fixed, we are back to the V_Q family and we know then that only the decision functions increasing in θ are implementable (Laffont and

Maskin, 1979b). Therefore the free rider problem is not weakly solvable for V since $K^*(\cdot)$ is not increasing in (θ_1, θ_2).

References

Clarke, E.H. (1971), "Multipart Pricing of Public Goods", *Public Choice* 2, 19–33.

Green, J. and J.J. Laffont (1977), "Characterization of Satisfactory Mechanisms for the Revelation of Preferences for Public Goods", *Econometrica* 45, 427–438.

Green, J. and J.J. Laffont (1976), "Revelation des Préférences pour les Biens Publics", *Cahiers du Séminaire d'Econométrie* (C.N.R.S. Paris).

Green, J. and J.J. Laffont (1979), *Incentives in Public Decision Making*, Studies in Public Economics, vol. 1 (Amsterdam: North-Holland Publishing Company).

Groves, T. (1973), "Incentives in Teams", *Econometrica* 41, 617–631.

Groves, T. and M. Loeb (1975), "Incentives and Public Inputs", *Journal of Public Economics* 4, 211–226.

Hurwicz, L. (1975), "On the Existence of Allocation Systems Whose Manipulative Nash Equilibria are Pareto Optimal", unpublished paper presented at the 3rd World Congress of the Econometric Society.

Laffont, J.J., ed. (1979), *Aggregation and Revelation of Preferences*, Studies in Public Economics, vol. 2 (Amsterdam: North-Holland Publishing Company).

Laffont, J.J. and E. Maskin (1979a), "On the Difficulty of Attaining Distributional Goals with Imperfect Information about Consumers", *Scandinavian Journal of Economics* 81, 227–237.

Laffont, J.J. and E. Maskin (1979b), "Nash and Dominant Strategy Implementation in Economic Environments", mimeo.

Laffont, J.J. and E. Maskin (1980), "A Differentiable Approach to Dominant Strategy Mechanisms", *Econometrica*.

Roberts, K. (1979), "The Characterization of Implementable Choice Rules", in: J.J. Laffont, ed., *Aggregation and Revelation of Preferences*, Studies in Public Economics, Vol. 2 (Amsterdam: North-Holland Publishing Company) ch. 18.

Vickrey, W. (1961), "Counterspeculation, Auctions and Competitive Sealed Tenders", *Journal of Finance* 16, 8–37.