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# MONOPOLY WITH ASYMMETRIC INFORMATION ABOUT QUALITY

## Behavior and Regulation

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## 1. Introduction

It has long been conventional wisdom that a monopolist can be adequately controlled with taxes as long as distributional issues are ignored [Cournot (1938), Green (1962)]. Although Guesnerie and Laffont (1978) show that control with linear taxes is limited by the general nonconcavity of the monopolist's objective function, nonlinear taxes circumvent this problem.

Recently, considerable attention has been paid to the regulation of a monopolist with private information. Baron and Myerson (1982) and Guesnerie and Laffont (1984) examine the optimal control of a monopolist with superior knowledge about its costs. In such a setting, marginal cost pricing must be sacrificed to mitigate the informational rent of the monopolist.

Clearly, informational asymmetries in practice are not limited to costs. A more systematic study of the limitations on control imposed by these asymmetries is called for. In this paper we study a monopolist with private information about the quality of the good it sells. We use a simplified version of the model found in Laffont and Maskin (1986). The monopolist sells a good to a large number of consumers who cannot discern quality before purchase. Quality is observable after purchase but not verifiable by a third party so that warranties cannot be used. Buyers do not participate in the market frequently enough for a reputation effect to overcome the informational asymmetry.

Section 2 describes the model and our equilibrium concept. There are many equilibria because, in general, the monopolist conveys some of its information through the price it charges, i.e., there is a signaling effect.

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However, as explained in our earlier paper, it is natural to focus on the equilibrium most favorable to the seller. In such an equilibrium, the monopolist exercises market power not only in the conventional way, but through its choice of how much information to reveal through prices. Section 4 discusses the implications for regulation.

## 2. The model

We consider a single seller endowed with one unit of a consumption good. The good's quality, which is exogenous, is represented by the positive real number $\theta$ lying in the set $\Theta = \{\theta_1, \ldots, \theta_n\}$, where $1 \leqq \theta_1 < \cdots < \theta_n \leqq 2$. Higher values of $\theta$ denote higher quality levels. Ex ante, $\theta$ is a random variable for which the probability of $\theta_i$ is $f_i$. The seller, but not the buyers, observes the realization of $\theta$ before trade occurs.

The monopolist has a reservation value $v(\theta)$ for the commodity. If $p$ is the price at which the seller sells $q$ units, its payoff is therefore

$$pq + v(\theta)(1-q).$$

To avoid corner problems we assume here that $v(\theta)$ is increasing and that $v(\theta_n) < 1$. The seller faces a continuum of infinitesimal identical buyers, each with a von Neumann–Morgenstern utility function of the form

$$\theta q - \tfrac{1}{2}q^2 - pq.$$

Trade unfolds in two stages. After learning $\theta$, the seller sets a price $p$ (stage 1). In stage 2, buyers demand a certain quantity of the good $q(p)$, and exchange occurs. At that point buyers learn the value of $\theta$. For simplicity, we assume that the seller must meet demand at the price it sets.

A seller's strategy is a mapping $p\colon \Theta \to R_+$ that associates a price to each quality $\theta$. [In principle, $p(\theta)$ could be a random function, but, as shown in Laffont–Maskin (1986), such a strategy is never optimal for the seller. We thus confine attention to deterministic functions.] A buyer's strategy is a mapping $q\colon R_+ \to R_+$ that associates a quantity to each price.

We are interested in the perfect Bayesian equilibria (PBE) of this game. A PBE is a pair of strategies $(p(\theta), q(p))$ and a conditional probability function $g(\theta \mid p)$ (where $g(\theta \mid p)$ represents the probability that buyers attach to quality $\theta$ given that price is $p$) such that

(i) for all $p$ in the range of $p(\cdot)$, $g(\cdot \mid p)$ is the conditional probability of $\theta$ obtained by updating the prior $(f_i)$ in Bayesian fashion using $p(\cdot)$,
(ii) for all $p$, $q(p) \in \arg\max_q \sum_i (\theta_i q - \frac{1}{2}q^2 - pq) g(\theta_i \mid p)$,
(iii) for all $\theta$, $p(\theta) \in \arg\max_p [pq(p) + v(\theta)(1-q(p))]$.

Condition (i) requires that buyers have rational expectations. Condition (ii) describes buyers' optimal purchasing behavior given their expectations. Because buyers' utility functions are quadratic, (ii) simplifies to

$$p=\sum_i \theta_i g_i(\theta_i \mid p)-q(p). \tag{1}$$

Finally, (iii) describes the monopolist's best pricing strategy. It gives rise to a set of 'incentive' constraints:

$$p(\theta)q(p(\theta))+v(\theta)(1-q(p(\theta)))\geqq p(\theta^0)q(p(\theta^0))+v(\theta)(1-q(p(\theta^0)))$$
$$\text{for all} \quad \theta,\ \theta^0\in \textcircled{H}. \tag{2}$$

These constraints imply that the following standard monotonicity properties are satisfied by a PBE.

*Proposition 1. In any PBE, $p(\theta)$ is nondecreasing and $q(p)$ is nonincreasing.*

*Proof.* Standard [see Laffont and Maskin (1986)].

In view of Proposition 1, we need only impose the so-called 'adjacent upward' incentive constraint, as long as $p(\theta)$ and $q(p)$ are monotonic.

*Proposition 2. If $p(\theta)$ is nondecreasing, $q(p)$ is nonincreasing, and*

$$p(\theta_i)q(p(\theta_i))+v(\theta_i)(1-q(p(\theta_i)))$$
$$=p(\theta_{i+1})q(p(\theta_{i+1}))+v(\theta_i)(1-q(p(\theta_{i+1}))) \quad \textit{for all} \quad i, \tag{3}$$

*then all the constraints (2) are satisfied.*

*Proof.* Standard.

## 3. Transmission of information through prices

The behavior of the equilibrium function $p(\theta)$ dictates how much information about quality is revealed to buyers through prices. On the one hand, if $p(\theta)$ is constant, buyers learn nothing; on the other hand, if $p(\theta)$ is strictly increasing, buyers can infer quality precisely.

As in common in signaling models, there are many PBE's in our game. Indeed, there is one corresponding to each possible degree of revelation. At one extreme, there is a 'no revelation' or 'pooling' equilibrium in which

$$p(\theta)=\sum_i\left(\frac{\theta_i+v(\theta_i)}{2}\right)f_i \quad \text{for all} \quad \theta.$$

By contrast, there is a 'separating' equilibrium in which all information gets revealed

$$p(\theta_1)=\frac{\theta_1+v(\theta_1)}{2}$$

and, for all $i \geqq 2$,

$$(p(\theta_i)-v(\theta_{i-1}))(\theta_i-p(\theta_i))=(p(\theta_{i-1})-v(\theta_{i-1}))(\theta_{i-1}-p(\theta_{i-1})), \tag{4}$$

where in deriving (4) we have made use of (1) and (3). Similarly, there is a complete spectrum of intermediate equilibria.

In Laffont and Maskin (1986) we argue that, in a variety of circumstances, it is natural to concentrate on the PBE that maximizes the seller's ex ante payoff (before $\theta$ is realized). In particular, as long as the seller places sufficient weight on future profit, this is the equilibrium that would emerge as the long-run steady-state of a learning process in which buyers begin with prior beliefs about the statistical relationship between price and quality and naively update these over time in Bayesian fashion. Alternatively, the PBE is the unique stable equilibrium [in the sense of Kohlberg and Mertens (1982)] of a game where the seller can first commit itself to choose from a specified set of prices and then trade proceeds as above. Henceforth, we will focus on this most favorable PBE for the seller.

*Proposition 3. If $\theta_i-v(\theta_i)$ is nondecreasing in i (i.e., the buyers' marginal utility of consumption increases with quality at least as fast as that of the seller), then the PBE most favorable to the seller is a no revelation equilibrium.*

*Proof.* Let us illustrate the argument for the case of two quality levels $\theta_1<\theta_2$. Consider the program

$$\max_{q_1,q_2}\sum_i f_i(q_i(\theta_i-q_i)-v(\theta_i)q_i) \qquad \text{subject to}$$

$$q_1(\theta_1-q_1)-v(\theta_1)q_1 \geqq q_2(\theta_2-q_2)-v(\theta_1)q_2, \tag{5}$$

Because $\theta_i-v(\theta_i)$ is nondecreasing, the constraint (5) is binding, as one may readily verify. Moreover, the solution satisfies $q_1>q_2$. Hence, Proposition 2 implies that the solution of this program maximizes the seller's ex ante payoff. Because (5) holds with equality, the maximized value of the objective function is less than

$$\max_{q_1}(q_1(\theta_1-q_1)-v(\theta_1)q_1),$$

which in turn equals

$$\frac{(\theta_1-v(\theta_1))^2}{4}. \tag{6}$$

By contrast, the most favorable pooling equilibrium ex ante maximizes

$$\sum_i f_i(q(\theta_i-q)-v(\theta_i)q), \tag{7}$$

where we have again invoked (1). But the maximized value of (7) is

$$\frac{(\sum f_i(\theta_i-v(\theta_i)))^2}{4},$$

which is at least as big as (6). Q.E.D.

Proposition 3 has a simple intuitive explanation. When $\theta_i-v(\theta_i)$ is nondecreasing, it is (first-best) efficient for the monopolist to sell more when $\theta=\theta_2$ than when $\theta=\theta_1$. However, the incentive constraints interfere with efficiency; from Proposition 1 we know that $q(p(\theta_2))$ can be no greater than $q(p(\theta_1))$. Indeed, in a separating equilibrium, the former must be *strictly* less than the latter. A pooling equilibrium has the virtue of eliminating the incentive constraint, and, therefore, the requirement that $q(p(\theta_i))$ be less than $q(p(\theta_1))$. In fact, these two quantities must be equal, which implies that in the pooling equilibrium (relative to the separating equilibrium) $q(p(\theta_2))$ has shifted in the direction of first-best efficiency.

If $\theta_i-v(\theta_i)$ decreases rapidly enough, the conclusion of Proposition 3 may fail to hold, and a separating equilibrium becomes best. For example, if $\theta_1=1$, $\theta_2=5/4$, $v(\theta_1)=0$, and $v(\theta_2)=1$, the seller's incentive constraint (5) is not binding, and so the seller can do as well with a separating equilibrium as though there were perfect information about quality.

## 4. Regulation

Let us continue to assume that $\theta$ takes on only the values $\theta_1<\theta_2$. Suppose that a regulator wishes to impose a tax on the monopolist to maximize the expected sum of consumer and producer surplus. Like buyers, the government is assumed not to be able to observe quality directly. Thus, if the tax is to depend on quality, it must be imposed incentive compatibility. That is, if $T_i$ is the tax when quality equals $\theta=\theta_i$, we must have[1]

$$q(\theta_1-q_1)-v(\theta_1)q_1-T_1 \geqq q_2(\theta_2-q_2)-v(\theta_1)q_2-T_2. \tag{8}$$

The government chooses $q_1$, $q_2$, $T_1$ and $T_2$ to maximize

$$\sum f_i(\theta_i q_i-\tfrac{1}{2}q_i^2-v(\theta_i)q_i-T_i)$$

subject to (8) (see footnote 1 for an explanation of why the other incentive constraint is omitted),

$$\sum f_i T_i \geqq 0, \quad \text{and} \tag{9}$$

$$q_1 \geqq q_2. \tag{10}$$

Constraint (9) ensures a balanced budget and, in view of Proposition 1, constraint (10) is a necessary condition. Notice that the Lagrange multiplier of constraint (8) must be zero because, given a solution, we can reduce $T_1$ by $\Delta T_1$ and increase $T_2$ by $(f_1/f_2)\Delta T_1$, thereby relaxing (8) without affecting the maximand or the other constraints. Thus if constraint (10) is not binding, the solution to the program entails

$$q_i^*=\theta_i-v(\theta_i), \qquad i=1,2. \tag{11}$$

If $\theta_i-v(\theta_i)$ is nonincreasing in $i$, therefore, constraint (10) is satisfied automatically, and (11) constitutes the socially optimal sales levels of the monopolist. Notice that, in this case, the monopolist's private information entails no deviation from first-best efficiency. The monopolist is induced to produce according to (11) by being (i) given a lump sum subsidy for selling $q_1^*$, (ii) being assessed a lump sum tax for selling $q_2^*$, (iii) being assessed an 'infinite' lump sum tax for selling any other quantity.

If $\theta_i-v(\theta_i)$ is increasing in $i$, then (11) violates constraint (10). We conclude that the social optimum entails $q_1^*=q_2^*=q^*$, where

$$q^*=\sum f_i(\theta_i-v(\theta_i)). \tag{12}$$

This quantity is enforced by a zero tax if the monopolist sells $q^*$ and an infinite tax for a sale of any other level.

[1]There is, of course, another constraint as well, but thanks to Proposition 2, it is satisfied automatically.

In this second case – by contrast with that when $\theta_i - v(\theta_i)$ is nonincreasing – the monopolist's private information *does* imply a social loss. Interestingly, the social second best solution is to 'pool' the two types, just as pooling was the unregulated monopolist's most favorable equilibrium strategy.

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