JOURNAL OF ECONOMIC THEORY 32, 317–327 (1984)

Notes, Comments, and Letters to the Editor

On The Efficiency of Fixed Price Equilibrium*

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Received September 3, 1980; revised December 12, 1982

We study the efficiency properties of K-equilibrium, a species of fixed price equilibrium. In particular, we examine the interrelations among K-equilibria and two of its properties: order and voluntariness. We also consider several alternative concepts of optimality. *Journal of Economic Literature* Classification Numbers: 021, 024.

INTRODUCTION

This paper is an attempt to tie together some loose ends in the theory of fixed price equilibrium. In particular, we are concerned with the efficiency properties of K-equilibrium.[1]

K-equilibria possess two important properties: order (the requirement that at most one side of the market be quantity-constrained) and voluntariness (the stipulation that no one trade more of any good than he wants to). We examine the interrelations among K-equilibria, order, and voluntariness and their connection with the two most natural concepts of optimality in a fixed price economy: constrained Pareto optimality (optimality relative to trades

* This research was supported by the National Science Foundation and the British Social Science Research Council. We thank J. P. Benassy, J. M. Grandmont, R. Guesnerie, F. Hahn, and Y. Younès for helpful comments.

[1] The concept of K-equilibrium is due to Grandmont [3]. It embraces both the Drèze [2] and Benassy [1] equilibrium concepts if preferences are convex.

0022-0531/84 $3.00

that are feasible at the fixed prices[2]) and voluntary Pareto optimality (optimality relative to feasible trades that satisfy voluntariness). We show first (Proposition 1) that a common definition of order (cf. Grandmont *et al.* [3]), in fact, *implies* that exchange is voluntary (assuming that preferences are convex and differentiable and there are at least three goods). In particular, the orderly and K-equilibrium allocations are the same. We, therefore, consider a less demanding notion of order, weak order,[3] which is distinct from voluntariness. By analogy with weak order, we introduce a weaker form of voluntariness.[4] We observe (Proposition 2) that, with convexity, differentiability, and at least three goods, order is equivalent to the conjunction of weak voluntary exchange and weak order. We then demonstrate (Proposition 3) that constrained Pareto optima, although weakly orderly, are, except by accident, non-voluntary and, hence, non-orderly. Furthermore, (Proposition 4) voluntary Pareto optima need not be weakly orderly. These last two results mean that whether the economy is centralized or decentralized (i.e., whether traders are compelled to make trades or are free to make them), it may be efficient to "constrain" both sides of the market.

K-equilibria are not optimal in the conventional sense. In particular, it is quite possible for one K-equilibrium to Pareto-dominate another. We conclude by showing (Proposition 5) that even non-dominated K-equilibria (i.e., K-Pareto optima) need not be voluntary Pareto optima.

1. Notation and Definitions

Consider an economy of $m+1$ goods indexed by $h(h=0, 1,..., m)$, whose price vector p is fixed ($p_0 = 1$), and n traders indexed by i ($i = 1,..., n$) where trader i has a feasible net trade set $X^i \subseteq R^{m+1}$. We assume that X^i is convex and contains the origin (so that trading nothing is possible) and that trader i's preferences (denoted by $\succsim^i$) are continuous and strictly convex on this set. We will at times require preferences to be differentiable as well. Following Grandmont *et al.* [3], we define an equilibrium for such an economy as follows:

Definition 1. A *K-equilibrium* is a vector of net trades $(t^1,..., t^n)$ associated with the vector of quantity constraints $((\underline{Z}^1, \bar{Z}^1),..., (\underline{Z}^n, \bar{Z}^n))$ (with $\underline{Z}^i \leqslant 0$, $\bar{Z}^i \geqslant 0$, $\underline{Z}^i_0 = -\infty$ and $\bar{Z}^i_0 = +\infty$) such that, for all i,

[2] Younès [9] calls this concept p-optimality.

[3] The concept of weak order is due to Younès [9] and Malivaud–Younès [6].

[4] This concept was suggested to us by J.-P. Benassy.

(B) Budget feasibility (t^i is feasible at prices p): $t^i \in \tilde{X}^i = X^i \cap \{t^i \mid p \cdot t^i = 0\}$;

(R) Ration feasibility (quantity constraints are observed): $\underline{Z}^i \leqslant t^i \leqslant \bar{Z}^i$;

(V) Voluntariness (exchange is voluntary): t^i is the $\succsim^i$-maximal element among net trades satisfying (B) and (R);

(O) Order (exchange is orderly): if, for some commodity h, some agent i and j, $(\tilde{t}^i, \tilde{t}^j) \in \gamma_h^i(\underline{Z}, \bar{Z}) \times \gamma_h^j(\underline{Z}, \bar{Z}), \tilde{t}^i \succ^i t^i$ and $\tilde{t}^j \succ^j t^j$, then $(\tilde{t}_h^i - \bar{Z}_h^i)(\tilde{t}_h^j - \underline{Z}_h^j) \geqslant 0$, where $\gamma_h^i(\underline{Z}, \bar{Z}) = \{\tilde{t}^i \in \tilde{X}^i \mid \underline{Z}_k^i \leqslant t_k^i \leqslant \bar{Z}_k^i \ \forall k \neq 0, h\}$;

(F) Aggregate feasibility (net trades sum to zero): $\sum_i t^i = 0$.

If trader i's net trade on market h is t_h^i, then he acts as though his ration were equal to t_h^i. Hence, for vector t^i, define the "canonical" rations $\underline{Z}(t^i)$ and $\bar{Z}(t^i)$ so that, for $h \neq 0$,

$$\bar{Z}_h(t^i) = \begin{cases} t_h^i, & \text{if } t_h^i \geqslant 0 \\ 0, & \text{if } t_h^i \leqslant 0 \end{cases}$$

$$\underline{Z}_h(t^i) = \begin{cases} 0, & \text{if } t_h^i \geqslant 0 \\ t_h^i, & \text{if } t_h^i \leqslant 0. \end{cases}$$

Voluntariness implies that agents are not forced to trade more of any good than they want to. An allocation characterized by voluntariness is said to be voluntary. Formally, we have

DEFINITION 2. A *voluntary allocation*[5] is a vector of net trades $(t^1, \ldots, t^n)$ satisfying conditions (B), (R), (V), and (F) for the canonical rations association with these trades.

A market is orderly if buyers and sellers are not both constrained on that market. The next two definitions represent alternative attempts to capture the idea of order. First we introduce property (O′), which is equivalent to (see Grandmont *et al.* [4]) but somewhat easier to work with than (O).

(O′) A vector of net trades $(t^1, \ldots, t^n)$ satisfies property (O′) if, for all markets h, there exists no alternative vector $(\tilde{t}^1, \ldots, \tilde{t}^n) \in \prod_i \gamma_h^i(\underline{Z}(t^i), \bar{Z}(t^i))$ such that $\tilde{t}^i \succsim_i t^i$ (with at least one strict preference) and $\sum_i \tilde{t}_h^i = 0$.[6]

The following definition is equivalent to that in Grandmont [3].

DEFINITION 3. An *orderly allocation* is a vector of net trades $(t^1, \ldots, t^n)$ satisfying (B), (R), (O′), and (F) for the canonical rations associated with those trades.

[5] Younès [9] calls this concept a p-equilibrium.

[6] Grandmont *et al.* [4] call property (O′) market-by-market efficiency.

The problem with the above definition of an orderly allocation, if one is attempting to distinguish between the notions of order and voluntariness, is that it itself embodies elements of voluntariness. Indeed, we will show below (Proposition 1) that, with differentiability, the above concept of order *implies* voluntariness. Roughly speaking, this is because in the above definition of order (O′), the trade $\tilde{t}^i$ in $\gamma_h^i(\underline{Z}(t^i), \bar{Z}(t^i))$ could be preferred to t^i either because (a) $\tilde{t}^i$ relaxes a constraint on market h or (b) t^i entails forced trading on a market $k \neq h$, whereas $\tilde{t}^i$ does not. The non-existence of $\tilde{t}^i$'s of type (a) is what we intuitively mean by order, whereas (b) pertains to voluntary exchange. But (O′) does not distinguish between the two. Therefore, we define an alternative notion of order due to Younès [9]) that is free from the taint of voluntariness. We first define property (O″).

(O″) A vector of net trades $(t^1,..., t^n)$ satisfies property (O″) if, for all markets h, there exists no alternative vector $(\tilde{t}^1,..., \tilde{t}^n) \in \prod_{i=1}^n \bar{\gamma}_h^i(t^i)$ such that, for each i, $\tilde{t}^i \succsim^i t^i$ (with at least one strict preference) and $\sum_i \tilde{t}_h^i = 0$, where $\bar{\gamma}_h^i(t^i) = \{\tilde{t}^i \in \tilde{X}^i \mid \tilde{t}_k^i = t_k^i,\ k \neq 0, h\}$.

Notice that properties (O′) and (O″) are identical except that the latter requires that alternative net trade vectors be indentical to the original trades in all markets other than h and 0.

DEFINITION 4. A *weakly orderly allocation* is a vector of net trades $(t^1,..., t^n)$ satisfying properties (B), (R), (O″), and (F) for the canonical rations associated with the trades.

An orderly allocation is obviously weakly orderly. By analogy with weak order, we may define a concept of *weak voluntariness.* We first introduce a weaker version of property (V):

(V′) A vector of net trades $(t^1,..., t^n)$ satisfies property (V′) if, for all markets h, there do not exist i and $\tilde{t}^i \in \bar{\gamma}_h^i(t^i)$ such that $\tilde{t}^i \succsim^i t^i$ (with at least one strict preference) and $\underline{Z}_h(t^i) \leqslant \tilde{t}_h^i \leqslant \bar{Z}_h(t^i)$.

We now have

DEFINITION 5. A *weakly voluntary allocation* is a vector of net trades $(t^1,..., t^n)$ satisfying conditions (B), (R), (V′), and (F) for the canonical rations associated with these trades.

Below we shall be interested in the Pareto-maximal elements in the sets of K-equilibria, voluntary allocations, orderly allocations, and weakly orderly allocations, which will be called K-Pareto optima (KPO), voluntary Pareto optima (VPO), orderly Pareto optima (OPO), and weakly orderly Pareto optima (WPO), respectively. An ostensibly still stronger notion of optimality, selecting Pareto-maximal elements in the set of *all* budget and aggregately feasible allocations, is constrained Pareto optimality.

DEFINITION 6. A constrained Pareto optimum (CPO) is a Pareto optimum of the economy for feasible consumption sets $\tilde{X}^i = X^i \cap \{t^i \mid p \cdot t^i = 0\}$. That is, it solves the program

$$\max \sum_{i=1}^{n} \lambda^i u^i(t^i) \text{ subject to } t^i \in \tilde{X}^i \text{ and } \sum t^i = 0, \tag{*}$$

for some choice of non-negative λ^i's, where the u^i's are utility functions representing preferences over net trades.

2. ORDER AND VOLUNTARINESS

Let us first state several equivalence results that are either well known or simple to confirm.

Fact 1 (Grandmont *et al.* [4]). The definitions (O) and (O′) are equivalent.

It follows immeediately that

$\{K$-equilibria (BROFV)$\}$

$= \{$Voluntary Allocations (BRFV)$\} \cap \{$Orderly Allocations (BRO′F)$\}$.

Fact 2. If preferences are differentiable, a weakly voluntary allocation is voluntary. (This holds since, with differentiability, coordinate-wise maximization is equivalent to full maximization.) That is, $\{$BRFV$\} = \{$BRFV′$\}$.

Fact 3 (Younès [9] and Silvestre [7]). With differentiability, K-equilibria (voluntary and orderly allocations) and voluntary and weakly orderly allocations ("Younès equilibria") are equivalent. That is, $\{$BROFV$\} = \{$BRO″FV$\}$.

We can now demonstrate that if preferences are differentiable, and there are at least three markets, order implies voluntariness.

PROPOSITION 1. *If preferences are differentiable and $m \geqslant 2$, an orderly allocation is implementable. That is,* $\{$BRO′F$\} \subseteq \{$BRVF$\}$.

Proof. Consider an orderly allocation $(\bar{t}^1,..., \bar{t}^n)$. If this allocation is not weakly voluntary, then there exist i, h, and $\tilde{t}^i \in \bar{\gamma}_h(\bar{t}^i)$ such that $\tilde{t}^i >^i \bar{t}^i$ and $\underline{Z}(\bar{t}^i) \leqslant \tilde{t}^i \leqslant \bar{Z}(\bar{t}^i)$. Consider h' different from h and 0. Then $\tilde{t}^i \in \gamma^i_{h'}(\underline{Z}(\bar{t}^i), \bar{Z}(\bar{t}^i))$, and, because $\tilde{t}^i_{h'} = \bar{t}^i_{h'}$, $\sum_{j \neq i} \bar{t}^j_{h'} + \tilde{t}^i_{h'} = 0$. Thus, $(\bar{t}^1,..., \tilde{t}^i,..., \bar{t}^n)$ contradicts the order of $(\bar{t}^1,..., \bar{t}^n)$, and so we conclude that $(\bar{t}^1,..., \bar{t}^n)$ must be weakly voluntary after all. From Fact 2 $(\bar{t}^1,..., \bar{t}^n)$ is thus voluntary. Q.E.D.

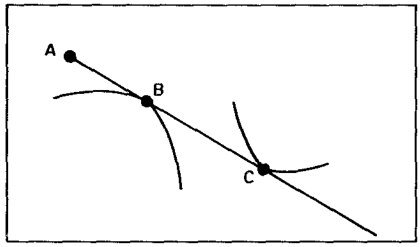

FIGURE 1

That there be at least three markets and that preferences be differentiable are hypotheses essential for the validity of the preceding proposition. Consider, for example, a two-market economy as represented in the Edgeworth box in Fig. 1.

Point A represents the initial endowment; the line through A, prices; and the curves tangent to the line, indifference curves. Any allocation between B and C is clearly orderly but not voluntary since it involves forced trading by the agent whose indifference curve is tangent at B. To see that differentiability is crucial, consider a two-person three-good economy where agents have preferences of the form $\{\alpha \log \min\{x_1, x_2\} + \log x_0\}$. Given these preferences, we can treat goods 1 and 2 together as a composite commodity, since traders will always hold goods 1 and 2 in equal amounts. Thus the economy is, in effect, reduced to two goods, and so Fig. 1 again becomes applicable.

We can combine Proposition 1 and Facts 2 and 3 to obtain:

PROPOSITION 2. *If preferences are differentiable and $m \geqslant 2$, order is equivalent to the conjunction of weak voluntariness and weak order, and thus completely characterizes K-equilibria. That is,*

$$
\begin{aligned}
&K\text{-equilibrium allocations (BROFV)} \\
&\quad = \{\text{Weakly Voluntary Allocations (BRV}'\text{F)}\} \\
&\qquad\qquad \cap \{\text{Weakly Order Allocations (BRO}''\text{F)}\} \\
&\quad = \{\text{Orderly Allocations (BROF)}\}.
\end{aligned}
$$

3. OPTIMALITY

We next turn to constrained Pareto optimality. We show that although a constrained Pareto optimal allocation is weakly orderly, it is virtually never voluntary or orderly when preferences are differentiable.

PROPOSITION 3. *A constrained Pareto optimum* (CPO) *is weakly orderly* (*implying that* {*Constrained Pareto optima*} = {*Weakly orderly Pareto optima*}). *Further, given differentiability, a non-Walrasian*[7] CPO *is neither voluntary nor* (*when* $m \geqslant 2$) *orderly if it lies in the interior of each trader's feasible consumption set, there is some* (*i.e.*, *nonzero*) *trade on every market, and every trader is assigned a strictly positive weight in the program* (*).[8]

Proof. Let $(t^1,..., t^n)$ be a CPO. If it were not weakly orderly, then trades could be altered on some market h, leaving trades on markets other than 0 undisturbed, in a Pareto-improving way, a contradiction of optimality. Therefore, the first part of the Proposition is established.

Suppose that the hypotheses of the second part are satisfied. We will establish that the CPO $(t^1,..., t^n)$ is not voluntary. Because it is not Walrasian, there exist a market h and an agent i who would prefer a trade different from t_h^i, given his trades on markets $k \neq 0, h$. If trader i is, say, a net buyer of h (the argument is symmetric if he is a net seller), he would like to buy either more or less of good h. If less, the non-voluntariness of $(t^1,..., t^n)$ follows immediately. Assume, therefore, that he would like to buy more.

Because, by assumption, there is non-zero trade on market h, there are traders who sell positive quantities of good h. If among these traders there exists an agent j who would like to sell less of good h, the proof is, again, complete. If there exists j who would like to sell more of good h (given his trades on markets other than 0 and h), i and j can arrange a mutually beneficial trade at prices p, contradicting constrained Pareto optimality. Therefore, assume that all sellers on market h are unconstrained. From differentiability, forcing them to sell a bit more of good h does not change their utility to the first order but does increase i's utility. Since $(t^1,..., t^n)$ is interior, furthermore, they are able to sell more. Therefore, if the allocation assigns positive weight to i in (*), it involves forced trading. Thus $(t^1,..., t^n)$ is not voluntary. If $m \geqslant 2$, Proposition 1 implies it is not orderly. Q.E.D.

The hypothesis of differentiability in Proposition 3 is, as in previous results, essential. Crucial too is the assumption that all traders have positive weight in the program (*). To see this, refer again to Fig. 1. Point B isboth constrained Pareto optimal and voluntary. However, the trader whose indif-

[7] With differentiable preferences, a Walrasian allocation is simply an allocation such that for each agent i and each good h, i's marginal rate of substitution between h and the numéraire is equal to p_h (for details on the definitions of a Walrasian allocation under non-differentiability see Silvestre [8]).

[8] A condition that is sufficient (but far from necessary) for all traders to have positive weight is that each trader be constrained on some market.

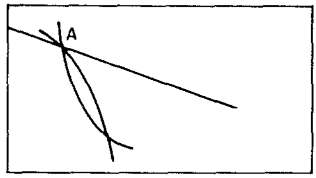

FIGURE 2

ference curve is tangent at C has zero weight. (Note, indicentally, that all the other CPO's—which constitute the line segment between B and C—are non-voluntary.) Finally, the hypothesis of non-zero trade on each market is necessary. Refer, for example, to the Edgeworth box economy of Fig. 2. Initial endowments are given by A, which is also a constrained Pareto optimum relative to the price line drawn. Although A does not involve forced trading, it does not violate the Proposition, as it involves no trade at all.

Althogh differentiability is a restrictive assumption, the non-zero weight and trade assumptions rule out only negligibly many CPO's. On the basis of Proposition 3, we may conclude that, with differentiability, CPO's are generically non-voluntary and non-orderly.

We now consider the set of Pareto optima among voluntary allocations: the voluntary Pareto optima. A VPO is an allocation that would arise if, given prices, a Paretian planner chose rations for all traders, who then optimised. Obvious questions are whether VPO's are necessarily orderly or even weakly orderly. The following proposition demonstrates that this is not the case.

PROPOSITION 4. *Voluntary Pareto optima need not be weakly orderly (nor, a fortiori, orderly).*

Proof. The proof takes the form of an example. Consider a two-trader, three-good economy in which trader A derives utility only from good 0 and has an endowment of one unit each of goods 1 and 2. Trader B has the utility function

$$U(x_0, x_1, x_2) = \frac{15}{8} x_1 + \frac{3}{2} x_2 - 3x_1 x_2 - 3x_1^2 - x_2^2 + x_0,$$

where x_i is *consumption* of good i, and an endownment of one unit of good 0. All prices are fixed at 1. It can verified that trader B's unconstrained demands for goods 1 and 2 at these prices are $1/12$ and $1/8$, respectively. This is a VPO in which all the weight is assigned to trader B. In this VPO, trader A is constrained on both markets, and buys $1/12 + 1/8 = 5/24$ units of good 0. If a non-weakly orderly VPO exists, trader B must be constrained either on market 1 or 2. If the constraint is on market 2, we have

$$x_1^{\mathrm{B}} + x_2^{\mathrm{B}} = 11/48 \qquad \text{(because trader A buys 11/48 units of 0)} \tag{1}$$

and

$$6x_1^{\mathrm{B}} + 3x_2^{\mathrm{B}} = 7/8$$

$$\text{(from maximization of utility with respect to good 1).} \tag{2}$$

Solving Eqs. (1) and (2), we find $x_2^{\mathrm{B}} = 1/6$, which is greater than B's unconstrained demand, 1/8. Thus, if the VPO exists, trader B must be constrained on market 1. Now, if trader B is constrained from buying more than 1/24 units of good 1, demand for good 2 is 3/16. Notice that $3/16 + 1/24 = 11/48$. Thus, if trade B is so constrained and trader A is constrained from selling more than 1/24 units of good 1 and 3/16 units of good 2, the resulting allocation *is* a VPO. However, it is not weakly orderly, because, given a purchase of 3/16 units of good 2, trader B would like to buy 5/96 units of good 1. Since $5/96 > 1/24$, both traders A and B are constrained on market 1. Q.E.D.

K-equilibria do not have the welfare properties associated with Walrasian equilibria. In particular, it is possible for one K-equilibrium to Pareto-dominate another.[9] Nonetheless, one might expect the Keynesian Pareto optima—the Pareto maximal allocations within the class of K-equilibria— to have "good" welfare properties. For instance, one might conjecture that they are VPO's. That this need not be so is demonstrated by the following:

PROPOSITION 5. *A* KPO *need not be a* VPO.

Proof. The proof is again by example. Consider an economy similar to that of the proof of Proposition 4 but with two additional goods. Specifically, take

$$U^{\mathrm{A}} = x_0 + \frac{15}{8}x_3 + \frac{3}{2}x_4 - 3xx_4 - 3x_3^2 - x_4^2$$

$$U^{\mathrm{B}} = x_0 + \frac{15}{8}x_1 + \frac{3}{2}x_2 - 3x_1x_2 - 3x_1^2 - x_2^2.$$

Suppose that trader A has endowments of 19/24, 1/3, and 1/3 units of goods

[9] To see that this is so, recall that Hahn [5] showed that at Walrasian prices, a non-Walrasian K-equilibrium can exist. But this equilibrium must be Pareto-dominated by the Walrasian equilibrium.

0, 1, and 2, respectively, whereas B's endowments consists of 19/24, 1/3, and 1/3 units of goods 0, 3, and 4, respectively. All prices are fixed at 1. It can be verified that if unconstrained on markets 3 and 4, trader A demands 1/12 and 1/8 units, respectively, independent of constraints he faces on other markets. Similarly, trader B demands 1/12 and 1/8 units, respectively, of goods 1 and 2 if unconstrained on those markets. Thus, the unconstrained demands on all four markets are less than the unconstrained supplies: 1/3 units in each case. Consequently, from order, the only possible K-equilibrium is one in which demand is unconstrained on every market. Trader A's equilibrium net trade vector is therefore $(0, -1/12, -1/8, 1/12, 1/8)$. The two traders enjoy utilities of 205/192 each. Because this is the unique K-equilibrium it is a KPO. Now suppose that trader B is constrained from buying more than 1/24 units of good 1 and that A is constrained from buying more than 1/24 units of good 3. It is easily checked that B will then demand 3/16 units of good 3 and A 3/16 units of good 4. Thus, we obtain a voluntary allocation in which trader A's net trade vector is $(0, -1/24, -3/16, 1/24, 3/16)$ and B's is $(0, 1/24, 3/16, -1/24, -3/16)$. But these net trades generate utilities of 835/768 for each trader. Because $835/768 > 205/192$, this implies that the KPO is not a VPO. Q.E.D.

We can summarize the results (with differentiability) in a schematic diagram (Fig. 3).

The no-spillover case. One "unappealing" feature of Fig. 3 is that the set of VPO's is neither completely within nor without the set of weakly orderly allocations, and, more specifically, the set of KPO's. However, with an additional strong hypothesis, this unaesthetic property disappears.

By the absence of spillovers we mean that a change in a constraint on a market does not alter net trades in any of the other markets, except the

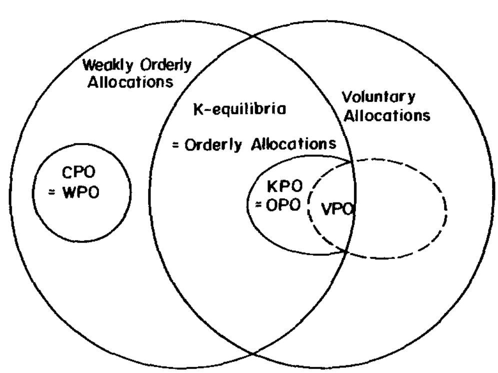

FIGURE 3

unconstrained market. A sufficient condition to obtain no spillovers in that traders' utility functions take the form $u^i = t_0^i + \sum_{h=1}^{n} \Phi_h^i(t_h^i)$. In the no-spillover case, the only change in Fig. 3 is that the VPO set shrinks to coincide with the KPO and OPO sets. We have

PROPOSITION 6. *In the case of no spillovers*, $\{\text{VPO}\} = \{\text{KPO}\} = \{\text{OPO}\}$.

Proof. A VPO must be orderly. Otherwise, slightly relaxing the constraints in market h for one demand-constrained and one supply-constrained agent would be voluntary (since it would not disturb the other markets) and Pareto improving. Q.E.D.

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