

THE GAINS TO MAKING LOSERS PAY

IN HIGH BID AUCTIONS

by

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A major theme of the recent theoretical advances in the theory of auctions is that auction rules which maximize expected revenue are not efficient, ex-post. That is, a seller exploiting his monopoly power to the maximum will design a scheme in which there is a finite probability that the agent with the highest valuation will not end up with the object for sale.

Discussion of the issue to date has focussed on the simplest auction in which each buyer has a valuation which is perceived by others to be an independent random draw from some known distribution  $F(\cdot)$ . In his seminal paper Vickrey (1961) established that, with risk neutral buyers, expected seller revenue is the same for high and second bid auctions. While this conclusion holds for any announced minimum price Vickrey focussed on the case in which the minimum price is equal to the seller's own use value of the object. Since the winner is the buyer with the highest valuation in excess of the minimum price both auctions are efficient.

Butters (1975) first posed the question as to the optimal design of an auction from the seller's viewpoint. He showed that expected revenue was necessarily increased by announcing a minimum price in excess of the seller's use value.

Independently Harris and Raviv (1978), Riley and Samuelson (1981), and Myerson (1981) have all considered optimal auction design from the seller's viewpoint under successively weaker assumptions about the distribution  $F(v)$ . From these papers we know that when buyers are risk neutral and a mild restriction on  $F(v)$  is satisfied, there is no auction which yields greater expected revenue than the high bid (or second bid) auction with the appropriately selected minimum price.<sup>1</sup> Then, for the revenue maximizing auction, the

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<sup>1</sup>For the most complete discussion of the revenue maximizing auction when  $F(v)$  does not satisfy the restriction see Maskin and Riley (1980).

inefficiency is associated with the possibility that one or more buyers have valuations between the seller's use value and his announced minimum price.

Various authors including Holt (1979) and Matthews (1979) have also shown that, when buyers are risk averse, the high and second bid auctions no longer generate the same expected revenue. In the second bid auction the payment by the winning buyer is independent of his bid. Therefore buyers continue to bid their reservation values. However, in the high bid auction, risk averse buyers place a lower marginal valuation on larger gains. To understand the implications of this it is easiest to consider the open auction equivalent of the high bid auction. In this "Dutch" auction the auctioneer calls out successively lower prices. Loosely speaking, a risk averse buyer has a greater fear of losing and so signals to stop the auction more quickly than if he were risk neutral. The resulting bids are therefore higher on average. This conclusion holds regardless of the preannounced price at which the seller will withdraw the object. Thus, for any minimum price, the high bid auction yields greater expected revenue than the second bid auction.

It is then natural to inquire as to whether the high bid auction can itself be improved upon. The answer turns out to be in the affirmative. For a broad class of auctions which include the independent valuations auction and the "mineral rights" auction (with mild additional restrictions) Maskin and Riley (1980) establish that a seller can increase expected revenue if losers share the burden of payment with the winner. Making losers pay lowers the equilibrium bids and thus raises the expected gains of the winner. The outcome of the auction is thus more risky than with payment only by the winner. This exacerbates each buyer's fear of loss. As a result

bids are not lowered by so much that the decline in expected revenue from the winner completely offsets the revenue received from losers.

Since the analysis of the general case is complicated the goal of this paper is to illustrate our conclusion by means of an example. In the following section we shall show, for a simple two buyer auction with risk averse buyers, that the seller increases expected revenue by employing both a minimum price and an entry fee. General results are summarized in section 2.

### 1. An Example

Consider two buyers, each with a valuation of some object  $v_i \in [0,1]$  where  $v_i$  is an independent random draw from the uniform distribution:  $F(v) = v$ . Each buyer has the piecewise linear utility function

$$(1) \quad U(x) = \begin{cases} x & , \quad x \leq w \\ x + \alpha(x-w) & , \quad x > w \end{cases} \quad 0 < \alpha \leq 1$$

The smaller is  $\alpha$  the greater the kink at  $x = w$  and hence the more risk averse are the buyers.

The seller announces that the object will be sold to the highest bidder submitting a bid above some minimum price  $m$ . Moreover each bidder must also submit an entry fee of  $c$  along with the bid.

We wish to establish that the seller can raise more expected revenue by utilizing some pair  $\langle m, c \rangle$  which is strictly positive than if he employs only a minimum price  $m$ . For any pair  $\langle m, c \rangle$  there is some valuation  $\underline{v}$  below which there is no incentive to enter the auction. A buyer with the "entry valuation"  $\underline{v}$  bids the minimum,  $m$ , and wins if the other buyer has

a lower valuation, that is, with probability  $F(\underline{v}) = \underline{v}$ . With no entry fee  $\underline{v} = m$ . Therefore, with  $c$  sufficiently small, the wealth of the buyer, if he wins, is less than  $w$  and the entry value satisfies

$$(2) \quad EU = \underline{v}(\underline{v}-m) - c = 0.$$

To determine the symmetric Bayesian equilibrium bid function,  $b(v)$ , we begin by assuming that buyer 2 bids according to a bid function with the general characteristics depicted in Figure 1. We then show that there is some such function with the property that buyer 1's best response is to employ the same bid function. That is, with buyer 2 making a bid  $b_2 = b(v_2)$  buyer 1's best response  $b_1 = b(v^*)$  is to choose  $v^* = v_1$ .

At this point it is useful to note that expected utility is a function of buyer 1's true valuation, his bid  $b$  and his probability of winning  $p$ . That is, we may write

$$EU = U(-b, p, v_1).$$

In making a bid buyer 1 considers the trade off between a higher bid, and hence a smaller increase in wealth if he wins, and a higher probability of winning.

Consider first the case in which buyer 1's valuation is close to the entry value  $\underline{v}$ . Then, win or lose, his wealth is less than  $w$ . Expected utility is therefore

$$(3) \quad U(-b, p, v_1) = p[v_1 - b - c] + (1-p)[-c].$$

But with a bid of  $b_1 = b(v)$  buyer 1 wins with probability  $v$ . Therefore

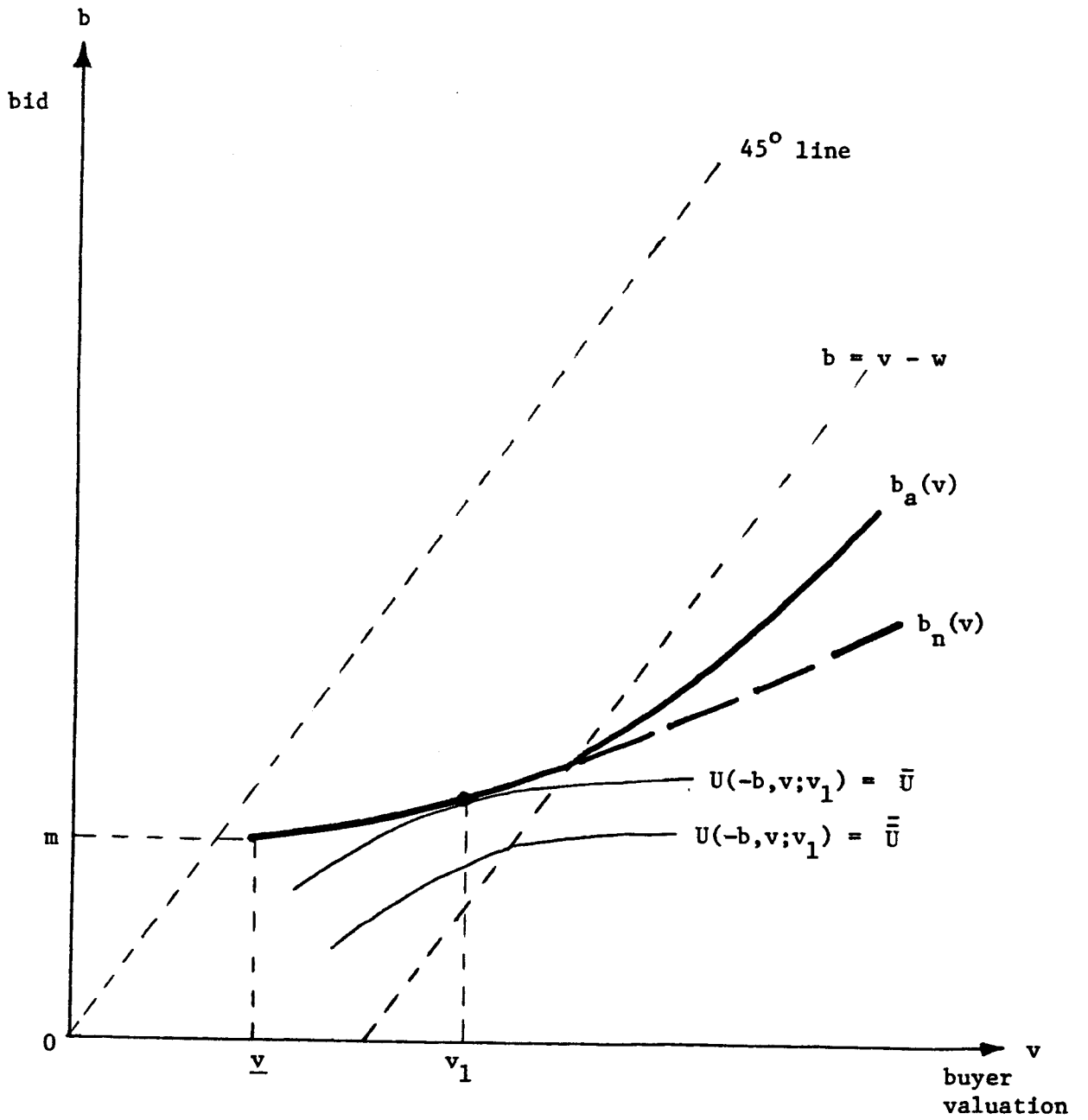


Figure 1: Equilibrium bid function

expected utility can be rewritten as

$$U(-b(v), v; v_1) = v(v_1 - b(v) - c) - (1-v)c$$

The optimization problem of buyer 1 can then be thought of as choosing a point on the curve  $b = b(v)$  to maximize  $U(-b, v; v_1)$ . This is illustrated in Figure 1. For  $b(\cdot)$  to be the equilibrium bidding strategy expected utility must take on its maximum at  $v = v_1$ , that is,

$$\frac{dU}{dv} = v_1 - \frac{d}{dv}(vb(v)) = 0, \quad \text{at } v = v_1 .$$

Integrating and making use of the boundary condition,  $b_1(\underline{v}) = m$  we have

$$(4) \quad b_n(v) = \frac{1}{2}v + \frac{mv - \frac{1}{2}v^2}{v} .$$

The subscript n denotes the fact that this is the equilibrium bid function for a buyer remaining always in his risk neutral range. From (4), the wealth of buyer 1, if he is the high bidder, is

$$v_1 - b(v_1) - c = \frac{1}{2}v_1 - \frac{mv - \frac{1}{2}v^2}{v_1} - c .$$

At some valuation,  $v'$ , this wealth is equal to  $w$ . Substituting for  $c$  from

(2) we then have

$$(5) \quad \frac{1}{2}v' - (mv - \frac{1}{2}v^2)/v' - \underline{v}(\underline{v}-m) = w .$$

For all higher valuations the wealth of the winner is at least  $w$ . Therefore expected utility is no longer given by (2). Instead, if buyer 1 wins, his wealth is beyond the kink and we have

$$\begin{aligned} U(-b(v), v; v_1) &= v[w + \alpha(v_1 - b(v) - c - w)] - (1-v)c \\ &= \alpha v \left[ \left( \frac{1-\alpha}{\alpha} \right) (w+c) + v_1 - b(v) \right] - c \end{aligned}$$

Arguing exactly as above,  $b(v)$  is the Bayesian equilibrium bidding strategy if  $U$  has its maximum at  $v = v_1$ . Then

$$\frac{dU}{dv} = \alpha \left[ \left( \frac{1-\alpha}{\alpha} \right) (w+c) + v_1 - \frac{d}{dv} (vb(v)) \right] = 0 \quad \text{at } v = v_1.$$

Reintegrating we have

$$(6) \quad b_a(v) = \frac{1}{2}v + \left( \frac{1-\alpha}{\alpha} \right) (w+c) + \frac{k}{v}, \quad v > v'.$$

The subscript  $a$  denotes the fact that for  $v > v'$  the bidder is risk averse. The constant of integration is determined by the requirement that the bid function be continuous. Utilizing (4) we therefore have

$$(7) \quad b_a(v) = \frac{1}{2}v + \frac{mv - \frac{1}{2}v^2}{v} + \left( \frac{1-\alpha}{\alpha} \right) \left( 1 - \frac{v'}{v} \right) (w+c), \quad v > v'$$

$$= b_n(v) + \left( \frac{1-\alpha}{\alpha} \right) \left( 1 - \frac{v'}{v} \right) (w+c), \quad v > v'.$$

The curves  $b_n(v)$  and  $b_a(v)$  are depicted in Figure 1. For  $c \geq 0$  and sufficiently small it can be confirmed that  $b_n(v)$  is an increasing function with slope less than  $1/2$ . Thus it intersects the curve  $b = v - w$  only once at  $v'$ . Moreover, from (7), it follows that, for  $\alpha$  sufficiently close to 1,  $b_a(v)$  must also lie below the curve  $b = v - w$ . Thus  $b_a(v)$  is indeed the equilibrium bid function over the interval  $[v', 1]$ <sup>2</sup>.

Summarizing, we have established that the equilibrium bid function is<sup>3</sup>

$$(8) \quad b(v) = \begin{cases} 0 & , v < \underline{v} \\ b_n(v) & , \underline{v} < v \leq v' \\ b_a(v) & , v' < v \leq 1 \end{cases}$$

<sup>2</sup>For  $\alpha$  sufficiently small the equilibrium bid function has a second kink at some point  $v'' > v'$ . For  $v > v''$  the equilibrium bid function is  $b(v) = b_a(v'') + v - v''$ .

<sup>3</sup>To be precise we have only considered necessary conditions for  $b_1 = b(v_1)$  to be buyer 1's optimal response. Using arguments similar to those in Riley and Samuelson (1981) it is not difficult to confirm that this is buyer 1's globally optimal response.



Since  $b_a(v) > b_n(v)$ , for all  $v > v'$ , it follows immediately that the sellers expected revenue is higher when buyers are risk averse. Moreover, as the kink becomes larger ( $\alpha$  declines) expected revenue increases. Thus the more risk averse are the buyers the greater is the seller's expected revenue. These results illustrate more general conclusions obtained by Riley and Samuelson (1981).

To determine the effects of introducing an entry fee we are interested in perturbing the equilibrium from an initial situation in which there is no entry fee. The expected revenue of the seller is the sum of the expected entry fees plus the expected high bid. Since the high bid is made by the individual with the high valuation and  $\text{Prob}\{\text{high value is less than } v\} = F^2(v)$ , we may write expected revenue as

$$(9) \quad R_\alpha(m, \underline{v}) = 2 \int_{\underline{v}}^1 c dF(v) + \int_{\underline{v}}^1 b(v) dF^2(v)$$

Substituting for  $b(v)$  from (4) and (7) we have

$$R_\alpha(m, \underline{v}) = 2 \int_{\underline{v}}^1 c dF(v) + \int_{\underline{v}}^1 b_n(v) dF^2(v) + \int_{v'}^1 (b_a(v) - b_n(v)) dF^2(v)$$

If  $\alpha = 1$  (no kink so that buyers are everywhere risk neutral) the third term is zero. Moreover the first two terms are independent of  $\alpha$ . Therefore we may write

$$(10) \quad R_\alpha(m, \underline{v}) = R_1(m, \underline{v}) + \int_{v'}^1 (b_a(v) - b_n(v)) dF^2(v)$$

Substituting from (2) and (4)

$$(11) \quad R_1(m, \underline{v}) = \int_{\underline{v}}^1 (\underline{v}^2 + v^2) dv$$

Note that  $R_1$  is independent of  $m$ . That is, when buyers are risk neutral, any pair  $\langle m, c \rangle$  yielding the same entry value,  $\underline{v}$ , yields the same expected revenue. Moreover, it is readily confirmed that  $R_1$  takes on its maximum at  $\underline{v} = \frac{1}{2}$ . Then, from (2), any pair  $\langle m, c \rangle$  satisfying

$$c = \frac{1}{4} - \frac{1}{2} m$$

maximizes expected revenue. In particular expected revenue is maximized by the pair  $\langle m, c \rangle = \langle \frac{1}{2}, 0 \rangle$ . Returning to expression (10) and substituting from (2) and (7) we have

$$(12) \quad R_\alpha(m, \underline{v}) = R^1(\underline{v}) + \left(\frac{1-\alpha}{\alpha}\right)(1-v')^2(w + \underline{v}^2 - \underline{v}m).$$

Differentiating (12) with respect to  $m$  we have

$$(13) \quad \frac{\partial R_\alpha}{\partial m} = \left(\frac{1-\alpha}{\alpha}\right)[- \underline{v}(1-v')^2 - 2(1-v') \frac{\partial v'}{\partial m}].$$

From (2)  $m = \underline{v} - \frac{c}{v}$ . We wish to establish that at  $c = 0$ , so that  $m = \underline{v}$ ,

$\frac{\partial R_\alpha}{\partial m}$  is negative. This will be the case if  $\frac{\partial v'}{\partial m}$  is positive. From (5)

$$(14) \quad \left[\frac{1}{2} + \frac{m\underline{v} - \frac{1}{2}\underline{v}^2}{(v')^2}\right] \frac{\partial v'}{\partial m} - \underline{v}\left(\frac{1}{v'} - 1\right) = 0$$

At  $m = \underline{v}$  the bracket is positive. Then  $\frac{\partial v'}{\partial m}$  is positive and the proof is complete.

We conclude this section by comparing expected revenue with and without the entry fee. For concreteness we consider the special case

$$w = 1/8, \alpha = 1/2.$$

In Samuelson and Riley (1981) it is established that, for any entry fee and minimum price, the expected revenue from a second bid auction is equal to

the expected revenue from a high bid auction when buyers are risk neutral. We have already seen that with risk neutral buyers the seller can do no better than announce a minimum price of one-half and a zero entry fee. Moreover, in the second bid auction risk aversion has no effect on buyers' strategies. Then the second row of Table 1 indicates the greatest expected revenue obtainable using a second bid auction. As a standard of comparison row 1 indicates expected revenue in the absence of a minimum price.

| auction    | minimum price | entry fee | expected revenue | % increase over row 1 |
|------------|---------------|-----------|------------------|-----------------------|
| second bid | 0             | 0         | .333             | -                     |
| " "        | .5            | 0         | .416             | 25                    |
| high bid   | .5            | 0         | .432             | 30                    |
| " "        | .445          | 0         | .435             | 31                    |
| " "        | .209          | .093      | .483             | 45                    |

Table 1: Expected Revenue from Alternative Auctions

(two buyers,  $w = 1/8$ ,  $\alpha = 1/2$ )

Turning to the high bid auction, the equilibrium bid function has a kink at  $v'$  which is implicitly defined by equation (5). Setting the entry fee equal to zero so that  $m = \underline{v}$  we can solve for  $v'$  obtaining

$$(15) \quad v' = w + (w^2 + \underline{v}^2)^{1/2}$$

Combining (11), (12), and (15) we then have

$$(16) \quad R = \frac{1}{3} + \underline{v}^2 - \frac{4}{3}\underline{v}^3 + \left(\frac{1-\alpha}{\alpha}\right)(1-w-(w^2+\underline{v}^2)^{\frac{1}{2}})^2 w$$

Row 3 of Table 1 indicates expected revenue when the minimum price  $m(= \underline{v})$  is set at the level which is optimal for the second bid auction. Row 4 indicates the outcome when the minimum price is chosen to maximize expected revenue, as given by (16).

Finally, we consider the gain to using an entry fee as well as a minimum price. Given the piecewise linearity of the objective function it is perhaps not surprising that, holding  $\underline{v}$  constant, the seller's optimal strategy is to raise the entry fee until the winner is always in the risk averse range.

In terms of Figure 1, as the entry fee is raised and the minimum bid lowered the kink moves to the left. Expected revenue continues to rise until  $v'$ , the point at which the bid function is kinked, approaches the entry value  $\underline{v}$ . From (5) this occurs when

$$(17) \quad c = \frac{wv}{1-v} ,$$

and hence, from (2)

$$(18) \quad m = \underline{v} - \frac{w}{\underline{v}(1-\underline{v})} .$$

Substituting (17) and (18) into (12) and making use of (11) we have

$$(19) \quad R_{\alpha} = \frac{1}{3} + \underline{v}^2 - \frac{4}{3}\underline{v}^3 + \left(\frac{1-\alpha}{\alpha}\right)(1-\underline{v})w$$

Setting  $\alpha = 1/2$  and  $w = 1/8$  it is readily confirmed that  $R_{\alpha}$  takes on its maximum at

$$\underline{v} = (2 + 2^{\frac{1}{2}})/8 \approx .427$$

In reaching this result we have assumed that for all  $v > \underline{v}$  the wealth of the winner,  $v - b_a(v)$ , exceeds  $w$ . To confirm this we note that, substituting (17) and (18) into (7), we have

$$b_a(v) = \frac{1}{2}v + \left[ \frac{v^2}{2} - \frac{wv}{(1-v)} \right] / v + \left( \frac{1-\alpha}{\alpha} \right) \left( 1 - \frac{v}{\underline{v}} \right) \frac{wv}{(1-v)}$$

For the parameter values used in Table 1 the bracket is negative so that  $b_a(v)$  is a concave function. But, by construction  $\underline{v} - b_a(\underline{v}) = w$ . Then, for all  $v > \underline{v}$ ,  $v - b_a(v)$  does in fact exceed  $w$ .

The final row of Table 1 is then computed using (17), (18) and (19).

## 2. General Results

In Maskin and Riley (1980) a general auction model is developed which encompasses a broad class of "one-shot" auctions. Rather than elaborate here on this general model, we shall summarize the main conclusions for the two most commonly studied models: (i) the "independent values auction" and (ii) the "common value" or "mineral rights" auction.

The former is precisely the model examined in the previous section. Each agent has a valuation  $v_i$  which is an independent draw from some known distribution  $F(v)$ . As in our illustration, it is natural to introduce risk aversion by making each buyer's return a concave function of the difference between the gross gain ( $v_i$  for the agent with the highest valuation and zero for the others) and any payments due. For such a model it can be shown that making some losers pay always raises expected revenue.

Formally, we have

Proposition 1: Making some losers pay in the Independent Values Auction.

Suppose each of  $n$  buyers has a valuation  $v_i$ ,  $i = 1, \dots, n$ , an independent draw from the distribution  $F(v)$ . Suppose also that each buyer's return can be expressed as

$$U(v_i - w), \text{ if the object is awarded to agent } i$$

$$U(-w), \text{ otherwise}$$

where  $w$  is wealth and  $U(\cdot)$  is a concave function.

Then it is always possible to raise expected revenue by giving buyers a choice as to whether or not to pay an entry fee. "Free bids" are considered only if no entry fee is received.

The first step in the proof of Proposition 1 involves establishing that, for any  $v^*$ , there is an equilibrium bid strategy in which only those with valuations exceeding  $v^*$  have an incentive to pay the entry fee. The second step involves demonstrating that, for sufficiently high  $v^*$ , introduction of a small entry fee raises expected revenue.

From the same general theorem in Maskin and Riley (1980) we also have:

Proposition 2: Making all losers pay in the Independent Values Auction.

Under the assumptions of Proposition 1 it is always possible to raise expected revenue from a high bid auction *cum* positive reserve price by lowering the latter and introducing a required entry fee.

As the example in section 1 makes clear, expected revenue is generally maximized by establishing auction rules such that those with sufficiently low valuations in excess of the seller choose not to participate. The resulting auction is therefore inefficient, ex-post, because there is a

chance that some buyer with a valuation in excess of the seller remains out of the auction.

It might then be argued that in the sale of some right to access by a governmental unit, which has no alternative governmental use, an auction should be designed so that any buyer with a positive valuation has an incentive to bid. Accepting this argument, Proposition 1 then implies that the governmental unit can do better than utilize a high bid auction with zero minimum price. Expected revenue is revised by introducing a voluntary entry fee and giving priority to those submitting the fee with their bids. \*

Finally we turn to the "common value" auction. Each of  $n$  buyers is assumed to observe a signal,  $x_i$ , of which is jointly distributed with the true value,  $s$ , according to the continuous density function  $g(s, x_i)$ .

A buyer's signal provides information about the true value in the following sense. For any  $s, s', x_i, x'_i$

$$(s-s')(x_i-x'_i) > 0 \leftrightarrow g(s, x_i)g(s', x'_i) \geq g(s, x'_i)g(s', x_i)$$

That is, for any pair of draws,  $(s, x_i), (s', x'_i)$ , it is more likely that  $s-s'$  and  $x_i-x'_i$  have the same rather than opposite signs. Following Milgrom and Weber (1980) we shall say that  $x_i$  and  $s$  are positively related.

Suppose that buyer 1 has a signal or "estimate"  $x_1$  and that the highest of the other  $n-1$  signals is  $y_1$ . If buyer 1 pays an amount  $b$  and wins the auction when the true value is  $s$ , his utility is assumed to be  $u(s-b)$  where  $u$  is a concave function. Knowing only  $x_1$  and  $y_1$ , buyer 1's expected utility

\*Note that we cannot appeal to Proposition 2 for this result. A buyer with a zero valuation will never pay an entry fee since his probability of winning is zero.

is therefore

$$v(x_1, y_1, -b) = E\{U(s-b) | x_1, y_1\}$$

Under our assumptions the joint distribution of  $s$  and  $x_1, \dots, x_n$  is

$$(20) \quad f(s, x_1, \dots, x_n) = \prod_{i=1}^n g(s, x_i) / \left( \int_{-\infty}^{\infty} g(x, s) dx \right)^{n-1}$$

From (20) the conditional density  $f(s, x_3, \dots, x_n | x_1, x_2)$  can be calculated.

Then

$$v(x_1, y_1, -b) = (n-1) \int_{x_3=-\infty}^{y_1} \dots \int_{x_n=-\infty}^{y_1} \int_{s=-\infty}^{\infty} U(s-b) f(s, x_3, \dots, x_n | x_1, y_1) dx_3, \dots, dx_n ds$$

Knowing (20) buyer 1 can also compute the conditional density function

$h(y_1 | x_1)$ . In the high bid auction buyer 1 wins if and only if he outbids all the others. Then if all but buyer 1 are utilizing the bid function  $b(x)$  and buyer 1 bids  $\hat{b}_1 = b(\hat{x}_1)$ , his expected utility is

$$\phi(x_1, \hat{x}_1) = \int_{-\infty}^{x_1} v(x_1, y_1, -b(x_1)) h(y_1 | x_1) dy_1$$

The equilibrium bid function is then described by the requirement that, for all  $x_1$ ,  $\phi(x_1, \hat{x}_1)$  takes on its maximum at  $\hat{x}_1 = x_1$ . That is, when others are bidding according to  $b(x)$  buyer 1's best response is to bid  $b_1 = b(x_1)$ .

In Maskin and Riley (1980) the following result is derived.

**Proposition 3:** Making some losers pay in the common value auction.

Suppose that the utility function  $U(\cdot)$  satisfies conditions such that each buyer exhibits non-increasing absolute risk aversion. Then if buyers are sufficiently risk averse and the rate of absolute risk aversion



does not decrease too quickly with wealth it is possible to raise expected revenue by giving buyers a choice as to whether or not to pay an entry fee and looking at bids without a fee only if there is no fee paying buyer.

We also have the following counterpart to Proposition 2.

Proposition 4: Making all losers pay in the common value auction.

If buyers exhibit sufficiently large and constant absolute risk aversion it is always possible to raise expected revenue from a high bid auction *cum* positive reserve price by lowering the latter and introducing a required entry fee.

A few concluding remarks are in order concerning the role of our assumptions about risk aversion. First of all it is critical that any buyer with a very favorable signal should have a lower marginal utility of income if he submits the high bid than if he does not. That is, a buyer with a favorable signal would like to purchase fair insurance against losing out in the bidding. In the absence of such insurance the seller is able to exploit buyers' fear of loss by introducing payments for losers.

With constant absolute risk aversion all buyers satisfy this "insurance condition." With decreasing absolute risk aversion it is not satisfied by buyers with sufficiently unfavorable signals. However, unless absolute risk aversion decreases rapidly the insurance condition must be satisfied by buyers with highly favorable signals.

To see how the degree of absolute risk aversion plays a role we first define

$$\bar{v}_3(x_1) = \int_{-\infty}^{x_1} v_3(x_1, y, -b(x_1)) h(y | x_1, y \leq y_1) dy$$

and

$$\bar{v}_{33}(x_1) = \int_{-\infty}^{x_1} v_{33}(x_1, y, -b(x_1)) h(y | x_1, y \leq y_1) dy$$

In proving Propositions 3 and 4 it is assumed that the following inequality holds for all those buyers satisfying the insurance condition.

$$\frac{\bar{v}_{33}(x_1)}{\bar{v}_3(x_1)} < \frac{[v_3(x_1, x_1, -b(x_1)) - \bar{v}_3(x_1)]}{v(x_1, x_1, -b(x_1)) - U(0)}$$

With buyers' signals positively correlated it can be shown that the right hand side is negative. Thus the information effect, absent in the independent valuations model, makes it necessary to introduce the assumption that buyers are sufficiently risk averse.

## REFERENCES

- Gerard R. Butters, "Equilibrium Price Distributions and the Economics of Information," Ph.D. Dissertation, Department of Economics, University of Chicago, Chicago, Illinois, 1975.
- M. Harris and A. Raviv, "Allocation Mechanisms and the Design of Auctions," Working Paper, Graduate School of Industrial Administration, Carnegie-Mellon University, Pittsburgh, Pa., 1978.
- Charles A. Holt, Jr., "Competitive Bidding for Contracts under Alternative Auction Procedures," 88, (1980), 433-445.
- Eric S. Maskin, John G. Riley, "Auctioning an Indivisible Object," Discussion Paper, Kennedy School of Government, Harvard University, 1980.
- Stephen Matthews, "Risk Aversion and the Efficiency of First and Second Price Auctions," Working Paper, Department of Economics, University of Illinois, 1979.
- P. Milgrom and R. Weber, "A Theory of Auctions and Competitive Bidding," Working Paper, Kellogg Graduate School of Management, Northwestern University, 1980.
- R. B. Myerson, "Optimal Auction Design," Mathematics of Operations Research, forthcoming, 1980.
- J. G. Riley and W. F. Samuelson, "Optimal Auctions," American Economic Review, 71, June 1981.
- William Vickrey, "Counterspeculation, Auctions and Competitive Sealed Tenders," J. Finance, 16, 8-37, 1961.