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The Implementation of Social Choice Rules: Some General Results on Incentive Compatibility

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1. INTRODUCTION

We shall assume that the objectives of a society are embodied in a certain social choice rule. A social choice rule (SCR) selects a set of feasible social states for each possible configuration of individual preferences and other characteristics. One interprets the choice set as the set of welfare optima. For example, given an Arrow social welfare function which embodies individual preferences in a social ordering, then a natural social choice rule is derived by maximizing this social ordering over the feasible set. Alternatively, the Pareto rule is the social choice rule which selects all Pareto efficient states, given individual preferences and the feasible set. These are two particular social choice rules which have received much attention, but our discussion will cover social choice rules in general.

If the relevant characteristics of individual agents, such as preferences, happen to be publicly known, then the social choice rule can be implemented trivially because the choice set itself is known. The problem of incentive compatibility arises precisely because these characteristics are not known by the planner *a priori*. The planner may attempt to learn characteristics directly by asking agents to reveal them. In general, however, if the agents realize how the information they reveal is to be used, they will have an incentive to misrepresent. Then the task of the planner in implementing the social choice rule is more difficult. Obviously, he must use a planning mechanism of some kind, whose outcomes are possible social states. We shall assume that, when he devises the mechanism, the planner knows what social states are feasible, so that he can ensure that the final outcome is feasible. (See, however, Hurwicz, Maskin and Postlewaite (1978), which considers the more general problem where feasibility itself depends on unknown characteristics.) The planner, however, relies on signals from the individual agents to help him implement the social choice rule. It is assumed that each individual agent sends his own signal. The planner's mechanism is then a rule which specifies a social state for each list of signals sent by the individual agents. It is assumed that each agent knows the precise form of the mechanism the planner is using. Then each agent realizes that he is involved in a game, because the outcome of the mechanism depends on the signals which he and all the other

agents send to the planner. More precisely, this is a " game form ", in which there is a fixed set of strategies, consisting of signals to the planner, and in which the outcomes of these strategies are known to all " players ". This is a " game form " rather than a " game ", however, because the players' preferences over outcomes have not been specified. It is then assumed that the players in this game form, who are the individuals in the society, reach some kind of equilibrium which depends on their true characteristics—in particular, their preferences. The mechanism generates a particular social state given these equilibrium signals. Presumably, one wants this social state to be in the social choice set given the agents' true characteristics—i.e. to be something the planner might have chosen had he known these characteristics right from the start.

The basic problem of the planner, then, is to devise a game form which always has at least one equilibrium, and whose possible outcomes in equilibrium all belong to the appropriate social choice set for the individuals' true characteristics. A mechanism (or game form) with this property is said to *implement* the social choice rule.

This task of implementing social choice rules is what has come to be called the problem of " incentive compatibility ", a term due to Hurwicz (1972) (and hinted at in Hurwicz (1960, p. 28)). The last few years have seen the publication of many papers in this topic, including this *Review of Economic Studies* symposium. Such an explosion of published work may seem bewildering, and some kind of classification overdue. We have therefore been moved to develop and collect together a number of general results which assist the classification, and which summarize or extend known results on the possibilities of implementation.

Let us restrict ourselves at first to decision mechanisms which are *individually* incentive compatible—that is, mechanisms with non-cooperative concepts of equilibrium. Even within this restricted class, one can distinguish between two quite different kinds of mechanism. The first we shall call " direct " mechanisms.[1] In these, each agent's signal to the planner is a characteristic—a set of preferences, endowments, production possibilities, and whatever else happens to be relevant. Notice, however, that agents need *not* report their true characteristics, even in equilibrium, and for this reason, it may be misleading to call even direct mechanisms " preference revelation mechanisms ".

Nonetheless, the natural reason for considering such direct mechanisms is, presumably, the appeal of an implementation that encourages each agent to reveal his true characteristic. Much work has accordingly been done on finding particular direct mechanisms which admit truthful equilibria. Indeed, where each agent has truthfulness as a dominant strategy for the game form, the mechanism is said to be *straightforward* (or " cheatproof "). There are also other solution concepts, such as maximin and expected utility equilibria, which may lead to truthful direct reporting of characteristics as an equilibrium strategy, as we shall explain below.

While straightforward mechanisms have an enormous superficial appeal, there are simply far too many economic environments for which such mechanisms cannot yield satisfactory outcomes. This has become clear from the papers of Gibbard and Hurwicz especially. In fact, the papers which find straightforward mechanisms restrict themselves to rather special economic environments. Either the preferences are special (as in Clarke (1971), Green and Laffont (1977), Groves and Loeb (1975)) or there is a large economy in which no one individual's lie can significantly affect the overall outcome (Hammond (1979), also Roberts and Postlewaite (1976)). We shall give a complete characterization (see Section 4) of those social choice rules which can be implemented by straightforward mechanisms.

As already suggested, it may be possible to construct direct mechanisms for which, even if truthfulness is not a dominant strategy, it is at least an expected utility maximizing or a maximin strategy for each agent. Then one does have a mechanism for " preference revelation ". But the maximin notion of equilibrium is not especially appealing, and an expected utility mechanism can be constructed only if the planner knows agents' subjective

probabilities concerning one another's characteristics, to which the mechanism will ordinarily be sensitive.

Thus, there are many economic environments for which there turns out to be no very satisfactory direct mechanisms.

For this reason, a number of other papers have developed rather a different kind of mechanism, in order to ensure satisfactory outcomes in a larger class of environments. Prominent examples are in Groves and Ledyard (1977), Hurwicz (1979), Schmeidler (1976), and Maskin (1977). In these mechanisms, agents do not necessarily reveal their characteristics at all; their messages may be quite arbitrary, without any obvious economic significance. For such indirect mechanisms, it is customary to assume that each agent behaves in a Nash-like manner. That is, each agent takes the messages of others as given and chooses his own message so that the resulting outcome is the one most desirable for him. As we do for straightforward mechanisms, we shall provide a comprehensive discussion (in Section 7) of when an SCR can be implemented by a mechanism with Nash equilibrium as the solution concept.

The organization of the paper is as follows. In Sections 2 and 3, we introduce definitions and notation as well as prove a number of useful technical results. In Section 4, we offer a detailed account of implementation by straightforward mechanisms—both individually and coalitionally incentive compatible. In particular, we give necessary and sufficient conditions for the existence of implementations and apply those conditions to several economic domains of interest. We also examine the relationship of implementability to the existence of social welfare functions and social aggregation functions. Sections 5 and 6 complete our discussion of direct mechanisms by taking up, in order, the Bayesian and maximin solution concepts. Section 7 discusses Nash and Stackelberg equilibrium. We first show why direct mechanisms will not suffice when Nash equilibrium is the solution concept. We then characteristize those SCR's which are Nash implementable, after which we discuss the connection between implementation in dominant strategies and that in Nash strategies. Finally we give a treatment of Stackelberg equilibrium, which brings us back full circle to dominant strategies. Section 8 summarizes our most important remarks and mentions some omitted topics.

2. IMPLEMENTATION BY GAME FORMS

It is assumed that there is a finite set $I = \{1, ..., n\}$ of agents. Choices are to be made from subsets of the underlying set X of social states or outcomes. Each agent $i \in I$ has a characteristic θ_i on which depends i's preference ordering $R(\theta_i)$ on the set X.

The planner's problem arises because nothing is publicly known about i's characteristic θ_i except that it is a member of the fixed set of possible characteristics Θ_i.

Let θ denote the list of characteristics $(\theta_i)_{i \in I}$. Then the planner knows only that $\theta \in \Theta := \Pi_{i \in I} \Theta_i$. θ should also determine a subset $A(\theta) \subseteq X$ of social states which are feasible. For example, states in X may be allocations of private goods, and each θ_i may determine i's initial endowment and consumption set. While for some solution concepts it seems possible to implement social choice rules when the feasible set is unknown (Hurwicz, Maskin and Postlewaite (1978), Postlewaite (1979)) we shall restrict our discussion to the simpler case when the feasible set of social states A is known to the planner and independent of θ. This simplification is in keeping with most work in this area. A *social choice rule* is a correspondence f which specifies, for each list of characteristics $\theta \in \Theta$ and for each possible feasible set $A \subseteq X$, a non-empty social choice set $f(\theta, A)$ of feasible social states. Thus, $f(\theta, A) \subseteq A$.

The planner's problem is to construct a mechanism or game form g which implements the social choice set $f(\theta)$ for every possible $\theta \in \Theta$, in a sense to be made precise below.

Strictly speaking, the planner should really construct a separate game form g_A for each feasible set A so that he can implement the social choice rule over its entire domain

of characteristics θ and feasible sets A. However, since under our assumptions the feasible A is known to the planner in advance, we need only ensure that for that *fixed A*, there exists a game form g implementing $f(\theta, A)$. In fact, we shall treat the feasible set A as fixed from now on, and so we shall write simply $f(\theta)$ instead of $f(\theta, A)$.

By a *game form g* we mean a mapping $g: S \rightarrow A$ from a product set $S := \Pi_{i \in I} S_i$ of individual strategy sets to outcomes $x = g(s) \in A$. Any $s_i \in S_i$ is to be interpreted as the signal sent by agent i to the planner.

In the later sections of this paper we shall consider a variety of solution concepts in turn—namely, dominant strategies (Section 4), expected utility and maximin equilibria (Sections 5 and 6) and Nash and Stackelberg equilibria (Section 7). For each of these solution concepts and each $\theta \in \Theta$ the game form g has a set of possible equilibria. Each equilibrium consists of a vector of strategies $s^* \in S$. The set of such equilibria, $E_g(\theta)$, depends on the agents' true characteristics θ; of course, $E_g(\theta)$ may be empty. Then, the set of equilibrium outcomes of the mechanism is $g(E_g(\theta)) := \{g(s^*) \mid s^* \in E_g(\theta)\}$.

Now we can define two different notions of implementation. The first of these is the weaker.

The mechanism g is said to *implement f* if, for every $\theta \in \Theta$:

(i) $E_g(\theta)$ is non-empty;

(ii) $g(E_g(\theta)) \subseteqq f(\theta)$.

Thus, under implementation, any equilibrium outcome is in the social choice set, and an equilibrium always exists.

Second, the mechanism g is said to *implement f fully* if, for every $\theta \in \Theta$,

$$g(E_g(\theta)) = f(\theta).$$

Thus, under full implementation, the set of possible equilibria is identical to the social choice set.

The above definitions have applied to general game forms. There is, however, a particular class of game forms which have a natural appeal and have received much attention in previous work. These are *direct* mechanisms, in which the strategy space S_i for each agent i is the set of possible characteristics Θ_i. In effect, then, each agent reports a possible characteristic but not necessarily his true one.

Nevertheless, the most appealing direct mechanisms are those in which truthful reporting of characteristics always turns out to be an equilibrium. It is the absence of such a mechanism which has been called the "free-rider" problem in the theory of public goods, and the problem of "strategic voting" arises because voting sincerely—according to one's true preferences—is not necessarily an equilibrium strategy. Perhaps the most appealing direct mechanisms of all are those for which each agent has truth as a dominant strategy—then we shall speak of a *straightforward mechanism*. Thus, a straightforward mechanism is a mechanism $g: \Theta \rightarrow A$ such that, for each $i \in I$, $\theta_i \in \Theta_i$ and each fixed $\bar{\theta}_{-i} \in \Theta_{-i}$:

$$g(\theta_i, \bar{\theta}_{-i}) R(\theta_i) g(\eta_i, \bar{\theta}_{-i}), \quad \text{for all } \eta_i \in \Theta_i$$

(where $\bar{\theta}_{-i}$ denotes the list $\bar{\theta}$ with the component $\bar{\theta}_i$ omitted).

There are some obvious advantages to straightforward mechanisms. They economize on information and on computation. The central planner need not understand the psychological motivations of the agents, beyond a basic self-interest. An additional advantage of straightforward mechanisms is that they avoid the undesirable outcomes which could occur if one or more agents were to tell the truth even though truth were not an equilibrium strategy for them. Finally, by contrast with the indirect mechanisms and the Nash equilibria we shall consider in Section 7, there are no problems with the stability of the mechanism.

A number of theorems in the sections below will demonstrate the existence of an

equivalent direct mechanism. It is worth explaining at some length what this means. Suppose we have a general game form or indirect mechanism $g: S \rightarrow A$. Suppose that this game form yields an equilibrium correspondence $E_g(\theta)$ which is defined and non-empty valued on the set Θ of possible profiles of characteristics. Then we can define an *equilibrium selection* as a mapping $s^*: \Theta \rightarrow S$ such that, for each profile $\theta \in \Theta$, $s^*(\theta)$ is a member of the equilibrium set $E_g(\theta)$. Given this particular equilibrium selection s^*, we can construct the composed mapping $h: \Theta \rightarrow A$ defined by:

$$h(\theta): = g(s^*(\theta)) \quad (\text{all } \theta \in \Theta).$$

Thus, h selects an outcome in A for every profile of characteristics θ, and is therefore a direct mechanism. As we shall see, there are a number of cases in which this direct mechanism itself yields equilibria $E_h(\theta)$ which have two properties:

(i) $\theta \in E_h(\theta)$ (so that truth is always a possible equilibrium);

(ii) For every $\theta \in \Theta$, $h(\theta) = g(s^*(\theta))$ (so that the outcome of the truthful equilibrium of the direct mechanism is always the same as the outcome of the equilibrium selection $s^*(\theta)$ for the indirect mechanism).

If the direct mechanism h has these two properties, we shall call it an *equivalent direct mechanism*.

Notice, however, that, if g is a mechanism which implements the social choice rule $f(\theta)$, and if h is an equivalent direct mechanism, then h need not implement f, according to the above definition. The reason is that h may well have other equilibria than the truthful equilibrium, and these equilibria may not correspond at all to equilibria of the game form g, nor need they yield outcomes in the social choice set $f(\theta)$. (See Example 4.1.2 below.) However, it has commonly been assumed that, if truthfulness is an equilibrium strategy for an agent in a direct mechanism, then that agent will choose to be truthful. This hypothesis motivates the following definiton.

Truthful Implementation. The direct mechanism $g: \Theta \rightrightarrows A$ is said to *implement the* SCR f *truthfully* (on the set A) if $\forall \theta \in \Theta$, $\theta \in E_g(\theta)$ and $g(\theta) \in f(\theta)$.

Clearly, if g implements f and g^* is an equivalent direct mechanism, then g^* truthfully implements f.

3. PROPERTIES OF SOCIAL CHOICE RULES

We shall be studying the implementation of SCR's in Sections 4–7. To prepare for this study, it will be necessary to collect a set of definitions and technical results. Since the feasible set A is fixed, we shall equate characteristics with preferences hereafter. That is, we shall write $\mathscr{R} = \Pi_{i \in I} \mathscr{R}_i$ and $f: \mathscr{R} \rightrightarrows A$, where $\mathscr{R}_i (\subseteq \mathscr{R}_A)$ is the preference domain of individual i and where $\mathscr{R}_A$ is the class of all logically possible orderings of A. R denotes a typical member of $\mathscr{R}$; that is, R is a profile of orderings $(R_1, R_2, ..., R_n)$.

3.1. Rich Domains

We shall first take up a property of preference domains of social choice rules which we will subsequently find useful.

First, define the *restriction* $R: B$ of the relation R to the set B by:

$$aR: Bb \Leftrightarrow a, b \in B \text{ and } aRb.$$

Rich Domain. For a set of alternatives, A, the domain $\mathscr{R} \subseteq \mathscr{R}_A$ is said to be *rich* iff $\forall \{R, R'\} \subseteq \mathscr{R}$ and $\forall \{a, b\} \subseteq A$ such that $aRb \Rightarrow aR'b$ and $aPb \Rightarrow aP'b$, then, there exists $R'' \in \mathscr{R}$ such that $\forall c \in A$, $aRc \Rightarrow aR''c$ and $bR'c \Rightarrow bR''c$. If $\mathscr{R}_i$ is rich for all $i \in I$, then $\mathscr{R} = \Pi \mathscr{R}_i$ will also be called *rich*.

The meaning of this definiton will become more transparent if we consider some examples of rich domains.

Example 3.1.1.   If $\mathscr{R} = \mathscr{R}_A$, the set of all logically possible orderings on A, then $\mathscr{R}$ is obviously rich.

Example 3.1.2.   Let $\mathscr{R}^E$ be the set of all continuous, strictly monotonic, strictly convex preference orderings over the m-good commodity space $\mathbb{R}^m_+$. We shall demonstrate that $\mathscr{R}^E$ is rich.

Consider R, $R' \in \mathscr{R}^E$ such that for some $\{a, b\} \subseteq \mathbb{R}^m_+$, $aRb \Rightarrow aR'b$ and $aPb \Rightarrow aP'b$, while not both bPa and $bP'a$.

Choose utility functions u and u' for R and R' respectively. There are then six possibilities: (i) $u(a) > u(b)$ and $u'(a) > u'(b)$; (ii) $u(a) = u(b)$ and $u'(a) > u'(b)$; (iii) $u(a) = u(b)$ and $u'(a) = u'(b)$; (iv) $u(b) > u(a)$ and $u'(a) > u'(b)$; (v) $u(b) > u(a)$ and $u'(a) = u'(b)$; (vi) $u(b) > u(a)$ and $u'(b) > u'(a)$. We consider them in turn.

(i) Normalize and set $u(a) = u'(a)$ and $u(b) = u'(b)$. Take R'' as the preference ordering represented by the utility function $u'' := \min(u, u')$. Figure 1 describes the situation for the case where $m = 2$. One notes that u'' is defined by the intersections of the upper contour sets at a and b, depicted by the shaded areas. One verifies readily that $\forall c \in A$, $u(a) \geqq u(c) \Rightarrow u''(a) \geqq u''(c)$ and $u(b) \geqq u(c) \Rightarrow u''(b) \geqq u''(c)$. Furthermore, u'' is clearly continuous, strictly monotonic and strictly convex, because u and u' are.

(ii) Normalize and set $u(a) = u'(a)$. Again take $u'' := \min(u, u')$. As Figure 2 shows u'' is defined by the intersections of the upper contour sets at a and the upper contour set for u' at b. It is readily checked that u'' represents the required R''.
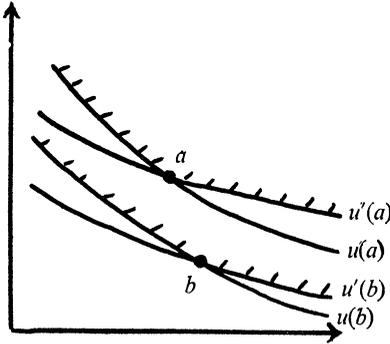

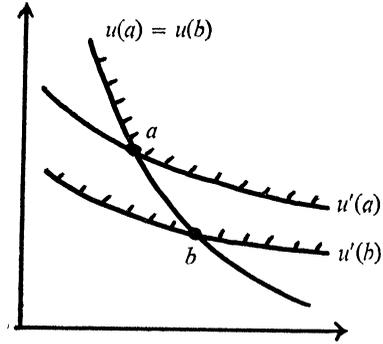FIGURE 1   FIGURE 2

(iii). Normalize and set $u(a) = u'(a)$. Take $u'' := \min(u, u')$. As Figure 3 shows u'' is defined by the intersections of the upper contour sets at points a and b. It is readily checked that u'' represents the required R''.
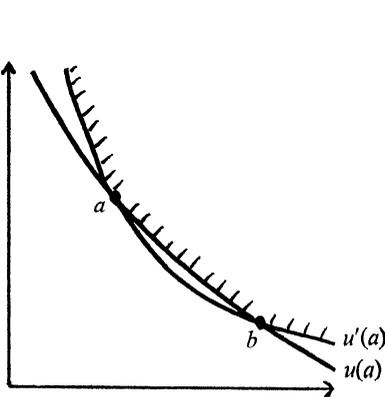

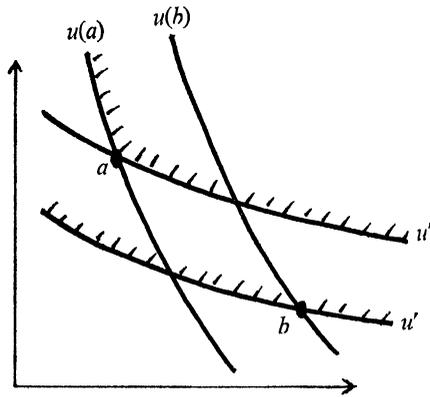FIGURE 3   FIGURE 4

(iv) Normalize and set $u(a) = u'(a)$. Then by hypothesis $u(b) > u(a) = u'(a) > u'(b)$. Take $u'' := \min(u, u')$. As Figure 4 shows u'' is defined by intersection of the upper contour sets at a and the upper contour set for u' at b. One verifies that u'' represents the required R''.

(v) Normalize and set $u(b) = u'(b)$. It follows that $u(b) = u'(b) = u'(a) > u(a)$. Take $u'' := \min(u, u')$. As Figure 5 shows u'' is defined by the intersection of the upper contour sets at b and the upper contour set for u at a. One verifies that u'' represents the required R''.
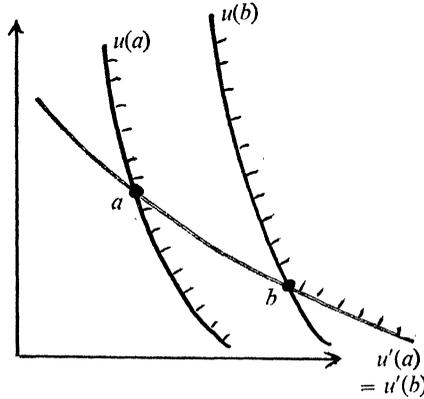

FIGURE 5

(vi) Normalize and set $u(b) = u'(b)$ and $u(a) = u'(a)$. Take $u'' := \min(u, u')$. This works as in case (i), except that a and b are interchanged in Figure 1.

We have therefore verified that $\mathcal{R}^E$ is rich.

Example 3.1.3. Our next example is that of a single peaked domain. Let $A = \{a_i, ..., a_n\}$, and arrange the alternatives along a line in ascending order. Let $\mathcal{R}_{SP}$ be the maximal class of single peaked preferences on A. That is,

$$\mathcal{R}_{SP} = \{R \in \mathcal{R}_A \mid \forall i < j < k, \ a_i P a_j \Rightarrow a_j R a_k\}.$$

Consider $\{a_i, a_j\} \subseteq A$ and $\{R, R'\} \subseteq \mathcal{R}_{SP}$ such that $(a_i R a_j \Rightarrow a_i R' a_j)$ and $(a_i P a_j \Rightarrow a_i P' a_j)$. Assume $i < j$ and construct R'' so that

   (i) $a_i P'' a_{i-1} P'' \ ... \ P'' a_1$

and

   (ii) $a_i P'' a_{i+1} P'' \ ... \ P'' a_m$, if $a_i P' a_j$

or

   (iii) $a_i I'' a_{i+1} I'' \ ... \ I'' a_j P'' a_{j+1} P'' a_{j+2} P'' \ ... \ P'' a_m$, if $a_i I' a_j$

or

   (iv) $a_j P'' a_{j-1} P'' \ ... \ P'' a_i P'' a_{j+1} P'' a_{j+2} P'' \ ... \ P'' a_m$, if $a_j P' a_i$.

It is immediately verified that R'' fulfils the hypotheses of the definition of rich domains.

Having presented several examples of rich domains, we shall next describe a non-rich domain of preferences which is commonly used in the literature on incentive compatibility.

Example 3.1.4. Let g denote the vector of public goods in the economy and let there be a single private good (call this money), denoted by x. Let U_Q be the set of quasi-linear utility functions defined in the space of goods in this economy; i.e.

$$u \in U_Q, \quad \text{iff } u(g, x) = v(g) + x$$

for some function v defined on the space of public goods. It will be instructive to see why the argument which we used in Example 3.1.2 to prove that $\mathcal{R}^E$ is rich will not work

here. Figure 6 portrays the situation where there is a single public good, and where u and u' at the chosen points a and b satisfy the conditions in case (i) of Example 3.1.2. Since preferences are quasi-linear the indifference curve $u(a)$ is a horizontal translation of $u(b)$ and $u'(a)$ is likewise a horizontal translation of $u'(b)$. As before, normalize and set $u(a) = u'(a)$ and $u(b) = u'(b)$. Let $u'' = \min(u, u')$. As Figure 6 shows, u'' is defined by the intersection of the upper contour sets at a and b. But the indifference curve $u''(a)$ is not a horizontal translation of $u''(b)$. Hence u'' is not quasi-linear.
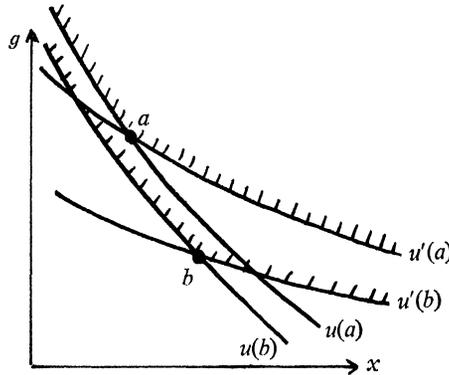

FIGURE 6

3.2. *Monotonicity*

We shall now define certain properties of SCR's which bear on their implementability.

Monotonicity. An SCR $f: \mathscr{R} \rightrightarrows A$ is *monotonic* iff $\forall\{R, R'\} \subseteq \mathscr{R}$, $\forall a \in A$, if $a \in f(R)$ and if $\forall i \in I$, $\forall b \in A$, $aR_ib \Rightarrow aR'_ib$, then $a \in f(R')$.

Monotonicity is a property satisfied, for example, by the majority rule choice rule (for strong orderings) and most of its variants. It is satisfied by the Pareto rule and, in market contexts, by the rule which selects all core allocations. Monotonicity does, however, impose some serious restrictions on an SCR. For one thing, suppose we have two societies in which the preference profiles are identical. Monotonicity then implies that the social choice sets must be the same. This means that we must exclude interpersonal comparisons of utility, and revert to a kind of Arrow social choice rule. Specifically, monotonicity excludes such rules as maximum and utilitarianism which do incorporate interpersonal comparisons. Finally, it is worth noting that monotonicity implies a limited degree of independence of irrelevant alternatives. For suppose we have two profiles R, R' and that $a \in f(R)$. Suppose too that the only difference between R and R' is a re-ordering for some individuals of alternatives in $A - \{a\}$. Then monotonicity requires that $a \in f(R')$. Thus the rank order (Borda) rule in particular violates monotonicity.

We shall also be concerned with other monotonicity properties. To define these properties, we shall work with SCR's whose choice sets are singletons (i.e. $f(R)$ is a singleton for all R). Such SCR's we shall call *singleton valued* SCR's or SSCR's.

(R'_i, R_{-i}) will denote the profile R with i's ordering R_i replaced by R'_i. (R'_C, R_{-C}) will denote the profile R with the orderings $R_i (i \in C)$ replaced by R'_i, but the orderings $R_i (i \notin C)$ unchanged.

Independent Person-by-Person Monotonicity (IPM). The SSCR $f: \mathscr{R} \rightrightarrows A$ satisfies IPM iff $\forall R \in \mathscr{R}$, $\forall i \in I$, $\forall R'_i \in \mathscr{R}_i$, $\forall\{a, b\} \subseteq A$, if $a \in f(R)$ and $aR_ib \Rightarrow aP'_ib$, then $b \notin f(R'_i, R_{-i})$.

Suppose that for a given profile of preferences, R, a is the chosen alternative. Consider now an alternative b and an individual i for whom aR_ib. What IPM says is that if

instead of R_i, individual i were to have an ordering R_i', with the single proviso that $aP_i'b$, then b would not be society's chosen element if R_i is replaced by R_i' in the original profile of preferences.

A stronger property than IPM is:

Independent Weak Monotonicity (IWM).   The SSCR $f\colon \mathscr{R} \rightrightarrows A$ satisfies IWM iff

$$\forall R \in \mathscr{R}, \ \forall C \subseteq I, \ \forall R_C' \in \prod_{i \in C} \mathscr{R}_i, \ \forall \{a, b\} \subseteq A, \ \text{if } a \in f(R) \text{ and } \forall i \in C, \ aR_ib \Rightarrow aP_i'b$$

then $b \notin f(R_C', R_{-C})$.

As we shall see in Section 7.2., IPM does not generally imply monotonicity nor does the converse hold.   The three properties IPM, IWM and monotonicity can be related, however, in some important special cases.

Theorem 3.2.1.   *If $\mathscr{R}$ consists of strong orderings and if the SSCR $f\colon \mathscr{R} \rightrightarrows A$ satisfies IPM, then it satisfies monotonicity.*

Proof.   Suppose that f satisfies the hypotheses of the theorem.

Consider $\{R, R'\} \subseteq \mathscr{R}$ and $a \in A$ such that $a \in f(R)$ and $\forall i \in I, \ \forall b \in A, \ aR_ib \Rightarrow aR_i'b$. Suppose $c \in f(R_1', R_{-1})$ for some $c \in A$.   If $c \neq a$, then $cP_1'a$ and aP_1c by IPM.   But $aP_1c \Rightarrow aP_1'c$ by hypothesis.   Therefore $a = c$.   Similarly $a \in f(R_{\{1, 2\}}', R_{-\{1, 2\}})$.   Continuing iteratively, $a \in f(R')$.   ‖

Theorem 3.2.2.   *If $\mathscr{R}$ is rich and if the SSCR $f\colon \mathscr{R} \rightrightarrows A$ is monotonic, then f satisfies IWM (and hence IPM).*

Proof.   Let $\mathscr{R}$ and f satisfy the hypotheses.   Consider

$$R \in \mathscr{R}, \ \{a, b\} \subseteq A, \ C \subseteq I, \ R_C' \in \prod_{i \in C} \mathscr{R}_i$$

such that $\forall i \in C, \ aR_ib \Rightarrow aP_i'b$, and $a \in f(R)$ but also $b \in f(R_C', R_{-C})$.   By the richness of $\mathscr{R}$, $\exists R_C'' \in \prod_{i \in C} \mathscr{R}_i$ such that

$$\forall c \in A, \ \forall i \in C, \ aR_ic \Rightarrow aR_i''c \ \text{and} \ bR_i'c \Rightarrow bR_i''c.$$

From monotonicity applied to R_C and R_C'', $a \in f(R_C'', R_{-C})$.   From monotonicity applied to R_C' and R_C'', $b \in f(R_C'', R_{-C})$.   But f is an SSCR, a contradiction.   Therefore $b \notin f(R_C', R_{-C})$. ‖

Corollary 3.2.3.   *If $\mathscr{R}$ is a rich domain consisting of strong orderings, then IPM, IWM and monotonicity are equivalent in an SSCR $f\colon \mathscr{R} \rightrightarrows A$.*

3.3.   *Other Properties.*   Other properties of SCR's which will concern us below include:

No Veto Power: $f\colon \mathscr{R} \rightrightarrows A$ satisfies no veto power iff $\forall a \in A, \ \forall i \in I$, if $\exists R \in \mathscr{R}$ such that $\forall j \neq i, \ \forall b \in A, \ aR_jb$, then $a \in f(R)$.

In words, suppose that for a given profile of preferences, R, all individuals other than i find alternative a one of the most desirable in the set A.   The SCR f satisfies the no veto power property if a is in the choice set for this profile.   Thus i does not have a power of veto in this case.

General No Veto Power: $f\colon \mathscr{R} \rightrightarrows A$ satisfies general no veto power iff

$$\forall a \in A, \ \forall i \in I, \ \forall R_i \in \mathscr{R}_i, \ \exists R_{-i} \in R_{-i} \text{ such that } a \in f(R_i, R_{-i}).$$

In words, what this means is that given any alternative a, and any preference ordering R_i for a given individual i, there exists a profile of preferences for the remaining members of society which ensures that a is in the choice set when the SCR is applied.   Notice that no veto power implies general no veto power.

Dictatorship: $f\colon \mathscr{R} \rightrightarrows A$ is dictatorial if there exists $i \in I$ such that

$$\forall R \in \mathscr{R}, \ \forall a \in A: a \in f(R) \text{ only if } \forall b \in A, \ aR_ib.$$

Pareto Optimality: $f: \mathscr{R} \rightrightarrows A$ is Pareto optimal iff $\forall R \in \mathscr{R}$, $\forall \{a, b\} \subseteq A$, if $\forall i \in I$, aP_ib, then $b \notin f(R)$.

Citizen Sovereignty (CS): $f: \mathscr{R} \rightrightarrows A$ satisfies CS if $\forall a \in A$, $\exists R \in \mathscr{R}$ such that $a \in f(R)$.

Triple Restriction: $f: \mathscr{R} \rightrightarrows A$ satisfies triple restriction iff $\forall R \in \mathscr{R}$, $\forall \{a, b, c\} \subseteq A$, $\exists R' \in \mathscr{R}$ such that $R': \{a, b, c\} = R: \{a, b, c\}$ and $f(R') \subseteq \{a, b, c\}$.

The significance of CS and triple restriction may be made clearer by the following two results, which will be useful in Section 4.

Theorem 3.3.1. *Let $\mathscr{R}$ be a rich domain consisting of strong orderings. If $f: \mathscr{R} \rightrightarrows A$ is an SSCR satisfying CS and IPM, then f is Pareto optimal.*

Proof. Suppose that f and $\mathscr{R}$ satisfy the hypotheses of the theorem but that f is not Pareto optimal. Then there exists $R' \in \mathscr{R}$ and $\{a, b\} \subseteq A$ such that $\forall i \in I$, $aP_i'b$ but $b \in f(R')$. By CS, there exists $R \in \mathscr{R}$ such that $a \in f(R)$. By Corollary 3.2.3, f satisfies IWM. Since $\forall i \in I$, $aR_ib \Rightarrow aP_i'b$ and $a \in f(R)$, IWM implies that $a \in f(R')$, a contradiction. ‖

Theorem 3.3.2. *If $f: \mathscr{R}_A \rightrightarrows A$ is an SCR satisfying Pareto optimality, then f satisfies triple restriction.*

Proof. Suppose f is Pareto optimal. Let $R \in \mathscr{R}_A$. For some $\{a, b, c\} \subseteq A$ let $R' \in \mathscr{R}_A$ be a profile such that $R': \{a, b, c\} = R\{a, b, c\}$ and $[x \in \{a, b, c\}, y \in \{a, b, c\} \Rightarrow xP_iy$ for all $i]$. By Pareto optimality, $f(R') \subseteq \{a, b, c\}$. ‖

4. DIRECT MECHANISMS: DOMINANT STRATEGIES AND STRAIGHTFORWARD MECHANISMS

4.1. *Equivalent Direct Mechanisms.* Let $g: S \to X$ be a fixed game form or mechanism.

Dominant Strategy: s_i^* is a *dominant strategy for i given θ_i* if: $\forall s_i \in S_i$, and $\forall \bar{s}_{-i} \in S_{-i}$:

$$g(s_i^*, \bar{s}_{-i})R_i(\theta_i)g(s_i, \bar{s}_{-i}).$$

Dominant Strategy Mechanism: The mechanism g is a *dominant strategy mechanism* for Θ if, for all $i \in I$ and for each $\theta_i \in \Theta_i$, there exists a dominant strategy for i given θ_i.

There may, of course, be more than one dominant strategy for i given θ_i. But, for a dominant strategy mechanism, there exists at least one *dominant strategy selection* $s^*: \Theta \to S$, such that $s^*(\theta) = (s_i^*(\theta_i))_{i \in I}$ and, for each $i \in I$ and each $\theta_i \in \Theta_i$, $s_i^*(\theta_i)$ is a dominant strategy for i given θ_i.

Theorem 4.1.1.[2] *Let $g: S \to X$ be a dominant strategy mechanism. For each dominant strategy selection $s^*: \Theta \to S$ (with $s^*(\theta) = (s_i^*(\theta_i))_{i \in I}$) there exists a straightforward mechanism which is equivalent to g.*

Proof. Define the direct mechanism $h: \Theta \to X$ by $h(\theta) := g(s^*(\theta))$ (for all $\theta \in \Theta$). Now, for every fixed $\bar{\eta}_{-i} \in \Theta_{-i}$, and every $\eta_i \in \Theta_i$, we know that:

$$s^*(\eta_i, \bar{\eta}_{-i}) = (s_i^*(\eta_i), \ s_{-i}^*(\bar{\eta}_{-i})).$$

So, for every $\theta_i \in \Theta_i$, because $s_i^*(\theta_i)$ is a dominant strategy for i given θ_i:

$$g(s_i^*(\theta_i), s_{-i}^*(\bar{\eta}_{-i}))R_i(\theta_i)g(s_i^*(\eta_i), s_{-i}^*(\bar{\eta}_{-i}))$$

i.e.

$$h(\theta_i, \bar{\eta}_{-i})R_i(\theta_i)h(\eta_i, \bar{\eta}_{-i}).$$

So, in the mechanism h, truthfulness is always a dominant strategy—i.e. h is straightforward. ‖

A consequence of this theorem is that if g implements an SCR f in dominant strategies, then there exists a direct mechanism which implements it truthfully. Therefore, if truthful

implementation rather than implementation is all that we require, we need never consider indirect mechanisms. On the other hand, notice that the notions of an equivalent direct mechanism and truthful implementation are rather weak. One requires only that true revelation in the direct mechanism be *a* dominant strategy, not that it be uniquely dominant. This laxness is for good reason. In moving from indirect dominant strategy mechanisms to direct ones, as in the preceding theorem, one may introduce dominant strategies which are *not* truthful. More troubling, these additional strategies may create a situation where the indirect mechanism is an implementation of a given SCR, while the equivalent direct mechanism is not. The following example illustrates this possibility.

Example 4.1.2. Suppose that the set of feasible social alternatives is given by $A = \{a, b, c, d, e, p, q, r\}$ and that characteristics can be identified with preferences. Suppose that the domains of possible preferences for individuals 1 and 2 are, respectively, $\mathscr{R}_1 = \{R_1, R_1'\}$ and $\mathscr{R}_2 = \{R_2, R_2'\}$, where

$$R_1 = \begin{bmatrix} q \\ a-c-e \\ d-b-p \\ r \end{bmatrix}, \quad R_1' = \begin{bmatrix} c-b-p \\ a-d-e \\ q-r \end{bmatrix}, \quad R_2 = \begin{bmatrix} r \\ d-a-e \\ b-c-p \\ q \end{bmatrix}, \quad R_2' = \begin{bmatrix} d \\ b-c \\ a \\ e-p-q-r \end{bmatrix}$$

Consider the SCR f defined so that:

$$f(R_1, R_2) = \{a, e\}$$

$$f(R_1', R_2) = \{c, p, b\}$$

$$f(R_1, R_2') = \{d\}$$

$$f(R_1', R_2') = \{b\}.$$

f is not a badly behaved SCR; indeed, it satisfies monotonicity and the strong Pareto criterion. Moreover, it is implemented in dominant strategies by the following game form:

$$g_1 \quad \begin{array}{|c|c|c|} \hline a & d & e \\ \hline c & b & p \\ \hline a & b & e \\ \hline \end{array}$$

In g_1, player I chooses rows as strategies, and player II, columns. Player I's dominant strategies are the first and third rows if his preference ordering is R_1 and row two if R_1'. Player II's dominant strategies are the first and third columns if his preference ordering is R_2 and the second column if R_2'. Consider the equivalent direct mechanism g_2 obtained from g_1 by associating rows 1 and 2 of g_1 with R_1 and R_1', respectively, and columns 1 and 2 with R_2 and R_2', respectively. Hence,

$$g_2 \quad \begin{array}{c c c} & R_2 & R_2' \\ R_1 & \boxed{a} & \boxed{d} \\ R_1' & \boxed{c} & \boxed{b} \end{array}$$

g_2, however, is not an implementation of f. The second row is dominant for player I with preferences R_1; the second column is dominant for player II with preferences R_2. The outcome when players choose the second row and column is, however, b, which is not in the choice set $f(R_1, R_2)$, nor is it Pareto optimal.

There are at least two reasons why one need not find this example disturbing. First, it remains true that while g_2 does not strictly speaking implement f, all equilibrium outcomes lie in the choice set of f when players use their *truthful* dominant strategies. That is, g_2 *truthfully implements* f in dominant strategies. In direct mechanisms where telling the truth is one of several dominant strategies, it may be reasonable to suppose that players will in fact tell the truth. Second, the construction of Example 4.1.2 depends on indifference between two or more alternatives in players' preference orderings. When preferences are strong the following results obtain.

Theorem 4.1.3. *If $\mathscr{R}$ contains only strong orderings, then if f is fully implementable in dominant strategies, f is singleton-valued.*

Proof. For $R \in \mathscr{R}$, consider $a, b \in f(R)$. If $g: S \to A$ fully implements f in dominant strategies, there exist dominant strategy equilibria $s, s' \in S$ for R such that $g(s) = a$ and $g(s') = b$. Because s_1 and s_1' are both dominant strategies for R_1,

$$g(s_1', s_2, ..., s_n) = g(s_1, s_2, ..., s_n) = a.$$

Similarly,

$$g(s_1', s_2', s_3, ..., s_n) = g(s_1', s_2, s_3, ..., s_n).$$

Continuing iteratively, $g(s') = a$. ∥

Corollary 4.1.4. *Suppose $\mathscr{R}$ contains only strong orderings. If $f: \mathscr{R} \to A$ is truthfully implemented in dominant strategies by $g^*: \mathscr{R} \to A$, then g^* implements f.*

Proof. Define $f^*: \mathscr{R} \to A$ so that $\forall R \in \mathscr{R}, f^*(R) = g^*(E_{g^*}(R))$, where $E_{g^*}(R)$ is the set of dominant strategy equilibria for preference profile R in game form g^*. Now by construction g^* fully implements f^*. Therefore $f^*(R)$ must be a singleton for all R. But $g^*(R) \in f(R)$ by Theorem 4.1.1. Therefore $f^*(R) \subseteq f(R)$ for all R, so g^* implements f. ∥

Corollary 4.1.4 demonstrates that implementation and truthful implementation are effectively identical concepts when working with strong orderings. It thus shows that the problem exhibited by Example 4.1.2 depends essentially on the possibility of individual indifference. Because of this and other pathologies (to be discussed below) introduced by indifference, we shall be particularly concerned with the case of strong orderings.

4.2. Social Aggregation Functions and Implementation

So far we have said little about the existence of implementations. We shall now proceed to remedy that state of affairs.

Consider a set A of social alternatives and classes $\mathscr{R}_1, ..., \mathscr{R}_n$ of preference orderings over A for individuals $1, ..., n$, respectively. Take $\mathscr{R} = \prod_{j=1}^{n} \mathscr{R}_j$. Let $\mathscr{B}_A$ be the class of all complete, reflexive, binary relations over A. A social aggregation function (SAF) is a mapping

$$F: \mathscr{R} \to \mathscr{B}_A.$$

If the range of F consists of acyclic relations, F is called a social decision function (see Sen (1970)) and if these relations are also transitive, F is a social welfare function (SWF) (see Arrow (1963)). The following are familiar properties of SAF's.

Pareto Property (PP): $\forall \{a, b\} \subseteq A, \forall R \in \mathscr{R}$ if $aP_i b$ for all i, then $aP(F(R))b$. (The notation " $aP(F(R))b$ " is equivalent to " $aF(R)b$ " and " $\sim bF(R)a$ ".)

Non-negative Response (NNR): $\forall \{a, b\}, \forall \{R, R'\} \subseteq \mathscr{R}$ if $\forall i[(aP_i b$ implies $aP_i'b)$ and $(aR_i b$ implies $aR_i'b)]$, then $(aP(F(R))b$ implies $aP(F(R'))b)$ and $(aF(R)b$ implies $aF(R')b)$.

Notice that NNR in fact implies independence of irrelevant alternatives (see Sen (1970)). We shall first be interested in SCR's which maximize SAF's.

Maximization of an SAF: An SCR f: $\mathscr{R} \rightrightarrows A$ *maximizes* the SAF F: $\mathscr{R} \rightarrow B_A$ iff $\forall R \in \mathscr{R}$, $a \in f(R)$ implies $[\forall b \in A, aF(R)b,$ and $\forall b \notin f(R), aP(F(R))b]$.

Maskin (1976b) presents a basic result relating SWF's to dominant strategy implementations. The following theorem is related to that result.

Theorem 4.2.1. *If* f: $\mathscr{R} \rightrightarrows A$ *maximizes the* SAF F: $\mathscr{R} \rightarrow B_A$, *where F satisfies* NNR, *then f can be truthfully implemented in dominant strategies.*

Proof. Suppose that f maximizes the SAF F, where F satisfies NNR. To get a direct mechanism, we constructed a tie-breaking rule as follows: Choose a strong ordering T which *well-orders* A (see Halmos (1960) for a discussion of well-ordering). For any $R \in \mathscr{R}$ let $T(f(R))$ be the " first " element in $f(\mathscr{R})$ according to the well-ordering. Consider the direct mechanism g: $\mathscr{R} \rightarrow A$ defined so that $\forall R \in \mathscr{R}$:

$$g(R) = T(f(R)).$$

We claim that g implements f truthfully. If not, then for some i and some $R_i \in \mathscr{R}_i$, there exist $R_{-i} \in \Pi_{j \neq i} \mathscr{R}_j$ and $R'_i \in \mathscr{R}_i$ such that $g(R'_i, R_{-i})P_i g(R_i, R_{-i})$. Let $a = g(R'_i, R_{-i})$ and $b = g(R_i, R_{-i})$. Because $b = T(f(R_i, R_{-i}))$, we have $b \in f(R_i, R_{-i})$ and so $bF(R_i, R_{-i})a$. Now, if $bP(F(R_i, R_{-i}))a$, then, because $aP_i b$ and by NNR, we have $bP(F(R'_i, R_{-i}))a$. But this contradicts $a = T(f(R'_i, R_{-i})) \in f(R'_i, R_{-i})$. If $bI(F(R_i, R_{-i}))a$, then $a \in f(R_i, R_{-i})$, and so bTa. But because $aP_i b$, we have, by NNR $bF(R'_i, R_{-i})a$, and so $b \in f(R'_i, R_{-i})$, a contradiction of $a = T(f(R'_i, R_{-i}))$ and bTa. Therefore g implements f truthfully. ‖

Using the methods of Theorem 4.2.1's proof, we can actually prove a considerably stronger theorem. A dominant strategy equilibrium, as we have defined it, is an entirely non-cooperative notion. We may well be interested in a strengthening of the concept to take into account the possibility of coalition formation. For the game form g: $\prod_{i=1}^{n} S_j \rightarrow A$, the strategy vector $\bar{s}_C \in \Pi_{j \in C} S_j$ is *dominant* for the coalition $C \subseteqq \{1, ..., n\}$ with preferences R_C if $\forall s_C \in \Pi_{j \in C} S_j$, $\forall s_{-C} \in \Pi_{j \notin C} S_j$, $g(\bar{s}_C, s_{-C})R_i g(s_C, s_{-C})$ for some $i \in C$. The mechanism g is a *coalitionally dominant strategy mechanism* with respect to $\mathscr{R}$ iff $\forall R \in \mathscr{R}$, $\exists \bar{s} \in S$ such that $\forall C \subseteqq \{1, ..., n\}$, $\bar{s}_C$ is dominant for C with the preferences R_C. It is of interest to note that Theorem 4.2.1 holds for coalitional dominance as well.

Theorem 4.2.2. *Under the hypotheses of Theorem* 4.2.1 f *can be truthfully implemented in coalitionally dominant strategies.*

Proof. Merely a repetition of the proof of Theorem 4.2.1 replacing individuals by coalitions. ‖

Although we are primarily concerned with non-cooperative mechanisms in this paper, we shall have more to say about coalitions below (see Theorem 4.5.1).

4.3. *The Necessary and Sufficient Conditions for Implementation*

Theorem 4.2.1, though helpful, is only a sufficient condition for the existence of an implementation. It may be difficult to tell how far from being necessary it is. Therefore we shall now develop conditions which are both necessary and sufficient. To do this, we shall first confine our attention to SSCR's. We shall, however, consider general SCR's as well (see Theorem 4.3.2). Note that, in view of Theorem 4.1.3, SSCR's are the only fully implementable SCR's when the domain contains only strong orderings.

Theorem 4.3.1. *An* SSCR f: $\mathscr{R} \rightrightarrows A$ *is truthfully implementable in dominant strategies iff it is independently person-by-person monotonic.*

Proof.

Sufficiency: Suppose that f satisfies IPM. Define g: $\mathscr{R} \rightarrow A$ so that $\forall R \in \mathscr{R}$, $g(R) \in f(R)$, ($g(R)$ is uniquely defined since f is singleton-valued). If g is not straightforward, then

there exists $R \in \mathscr{R}$ such that $a = g(R)$, $b = g(R_i', R_{-i})$, and bP_ia. Now $b = g(R_i', R_{-i})$ implies $b \in f(R_i', R_{-i})$. Since bP_ia, we conclude, by IPM, $a \notin f(R)$, contradicting $a = g(R)$. Therefore g is straightforward.

Necessity: For SSCR f, consider $R \in \mathscr{R}$ and, for some i, $R_i' \in \mathscr{R}_i$ such that

$$aR_ib \Rightarrow aP_i'b, \quad a \in f(R), \text{ and } b \in f(R_i', R_{-i}).$$

If f can be truthfully implemented we must have aR_ib; otherwise, individual i with preferences R_i would not tell the truth. Similarly, we must have $bR_i'a$. But by choice of R and R_i', we know that $aP_i'b$, a contradiction. Therefore $b \notin f(R_i', R_{-i})$, establishing IPM. ‖

There are two immediate inferences one can draw from Theorem 4.3.1. The first is the general existence theorem for SCR's.

Theorem 4.3.2. *An SCR f is truthfully implementable in dominant strategies iff there exists an SSCR f^*, which satisfies IPM, such that for all R, $f^*(R) \subseteq f(R)$.*

Proof. Let $g: \mathscr{R} \to A$ be a truthful implementation of f. Define $f^*(R) = \{g(R)\}$ for all $R \to \mathscr{R}$. Then g implements f^* truthfully. Therefore, by Theorem 4.3.1 f^* satisfies IPM. The converse also follows immediately from Theorem 4.3.1. ‖

Remark 4.3.3. NNR is a property of SAF's which guarantees that the corresponding SCR satisfies IPM, provided that it is single-valued. So our earlier Theorem 4.2.1 is not quite a corollary of Theorem 4.3.2.

Theorem 4.3.4. *If $\mathscr{R}$ contains only strong orderings, an SCR $f: \mathscr{R} \rightrightarrows A$ is fully implementable in dominant strategies if and only if f is an SSCR and satisfies IPM.*

Proof. From Theorem 4.3.1., IPM is necessary and sufficient for truthful implementation. From Corollary 4.1.4, truthful implementation is equivalent to implementation for strong orderings. From Theorem 4.1.3, an implementable SCR is fully implementable iff it is singleton-valued. ‖

4.4. *Implementation in Economic Environments*

One inference we can draw from Theorem 4.3.4 is a strengthening of a theorem due to Hurwicz (1972). Consider an economy with m goods ($m \geq 2$) and n consumers ($n \geq 2$). Suppose that there are fixed positive stocks $w_1, \ldots, w_m$ of each of the m goods. Let A^E consist of all allocations of these stocks among the consumers. That is,

$$A^E = \{(x(1), \ldots, x(n)) \mid x(i) \in R_+^m, \sum_{i=1}^{n} x(i) = (w_1, \ldots, w_m)\}.$$

Let each individual i's preference domain $\mathscr{R}_i^E$ consist of all individualistic (i.e. selfish), strictly convex and strictly monotonic preference orderings over A^E. Take

$$\mathscr{R}^E = \prod_{i=1}^{n} \mathscr{R}_i^E.$$

Theorem 4.4.1. *If $f: \mathscr{R}^E \rightrightarrows A^E$ is a Pareto optimal SSCR which can be truthfully implemented in dominant strategies, then f is dictatorial.*

Remark. This result strengthens that of Hurwicz by dropping the requirement of individual rationality. It strengthens that of Satterthwaite (1976) by dropping the requirement that f be differentiable.

Proof. We shall give a proof for the case $n = 2$, $m = 2$, which can be depicted in an Edgeworth box diagram. The extension to more goods is straightforward. If $n > 2$, we can apply a result in Maskin (1976b) to the effect that there exists a straightforward

mechanism for just two people, if and only if there exists a straightforward mechanism for any $n > 2$.

Without loss of generality, take aggregate endowments $w_1 = w_2 = 1$. For $i = 1, 2$ consider $R_i \in \mathcal{R}_i^E$. Since R_i is individualistic, we can describe R_i by giving i's preferences for his own consumption bundles. For $0 < t < 1$, choose $R_i^t \in \mathcal{R}_i$ so that:

$$(x_1(i), x_2(i))R_i^t(y_1(i), y_2(i))$$

iff

$$[x_1(i)]^t[x_2(i)]^{1-t} \geqq [y_1(i)]^t[y_2(i)]^{1-t}.$$

So that we can apply Corollary 3.2.3, we shall work with the domain $\mathcal{R}^{E*}$ of strong orderings in $\mathcal{R}^E$, however. So, corresponding to R_i^t, define a strong ordering P_i^{*t} so that:

$$(x_1(i), x_2(i))P_i^{*t}(y_1(i), y_2(i))$$

iff either

(a) $(x_1(i), x_2(i))P_i^t(y_1(i), y_2(i))$

or

(b) $(x_1(i), x_2(i))I_i^t(y_1(i), y_2(i))$ and $x_1(i) > y_1(i)$.

Thus, R_i^t is just an ordinary Cobb–Douglas preference ordering with parameter t. R_i^{*t}, however, is a lexical extension of R_i^t, with alternatives ordered on an I_i^t-indifference curve according to the quantity of good 1.

Suppose now that $f: \mathcal{R}^E \rightrightarrows A^E$ is a Pareto optimal and non-dictatorial SSCR which is truthfully implementable in dominant strategies. By Theorem 4.3.1, f restricted to $\mathcal{R}^{E*}$ satisfies IPM. But, as in Example 3.1.2, the domain $\mathcal{R}^{E*}$ is rich. Therefore, by Corollary 3.2.3, f restricted to $\mathcal{R}^{E*}$ must satisfy IWM, and also monotonicity.

Consider the profile $(R_1^{*\frac{1}{2}}, R_2^{*\frac{1}{2}})$. In an Edgeworth box, the contract curve for this profile is the diagonal joining the corners $((1, 1), (0, 1)$ and $((0, 0), (1, 1))$.

Suppose that $a = ((1, 1), (0, 0)) \in f(R_1^{*\frac{1}{2}}, R_2^{*\frac{1}{2}})$. Then, for any other $b \in A^E$, and any $R \in \mathcal{R}^{E*}$, we have aP_1b, bP_2a. It follows from IWM applied to the coalition $C = \{1, 2\}$ that $b \notin f(R_1, R_2)$. So $a \in f(R_1, R_2)$ for all $R \in \mathcal{R}^{E*}$, which means that consumer 1 is a dictator. Similarly, $((0, 0), (1, 1)) \in f(R_1^{*\frac{1}{2}}, R_2^{*\frac{1}{2}})$ would imply consumer 2 being a dictator.

Therefore, we assume that for some c satisfying $0 < c < 1$:

$$w = (w(1), w(2)) = ((c, c), (1-c, 1-c)) \in f(R_1^{*\frac{1}{2}}, R_2^{*\frac{1}{2}}).$$

Consider now, for some $t \neq \frac{1}{2}$:

$$x = (x(1), x(2)) \in f(R_1^t, R_2^{\frac{1}{2}}).$$

Since x is Pareto optimal, it must lie on the contract curve $C(R_1^{*t}, R_2^{*\frac{1}{2}})$ (see Figure 7). But, for consumer 1 with preferences $R_1^{\frac{1}{2}}$ to tell the truth, it must be true that x lies on or below the $I_1^{\frac{1}{2}}$-indifference curve $I_1^{\frac{1}{2}}(w)$ which passes through w.

Suppose it is true that x lies above the line separating the indifference curves $I_1^{\frac{1}{2}}(w)$, $I_2^{\frac{1}{2}}(w)$. Then there exists a preference ordering $\bar{R}_1 \in \mathcal{R}_1^E$ such that each $\bar{I}_1$-indifference curve touches a corresponding $I_2^{\frac{1}{2}}$-indifference curve on the diagonal, the $\bar{I}_1$-indifference curves are all flatter than their $I_1^{\frac{1}{2}}$-counterparts, and such that $\bar{I}_1(w)$ is sufficiently flat to pass below x. Let $\bar{R}_1^*$ be the strong ordering derived from $\bar{R}_1$, in the same way that R_1^{*t} is derived from R_1^t.

Let $a \in f(\bar{R}_1^*, R_2^{*\frac{1}{2}})$. Then, for all $b \in A^E$ we have $a\bar{R}_1b \Rightarrow aR_1^{*\frac{1}{2}}b$. So, by monotonicity of f, $a \in f(R_1^{*\frac{1}{2}}, R_2^{*\frac{1}{2}})$. But then $a = w$, and so $w \in f(\bar{R}_1^*, R_2^{*\frac{1}{2}})$. However, $x \in f(R_1^t, R_2^{\frac{1}{2}*})$ and x lies above the indifference curve $\bar{I}_1(w)$, which is a contradiction, because consumer 1 could announce R_1^{*t} when his true preference ordering is $\bar{R}_1^*$. Therefore x must lie on or below the line separating $I_1^{\frac{1}{2}}(w)$ and $I_2^{\frac{1}{2}}(w)$ after all.
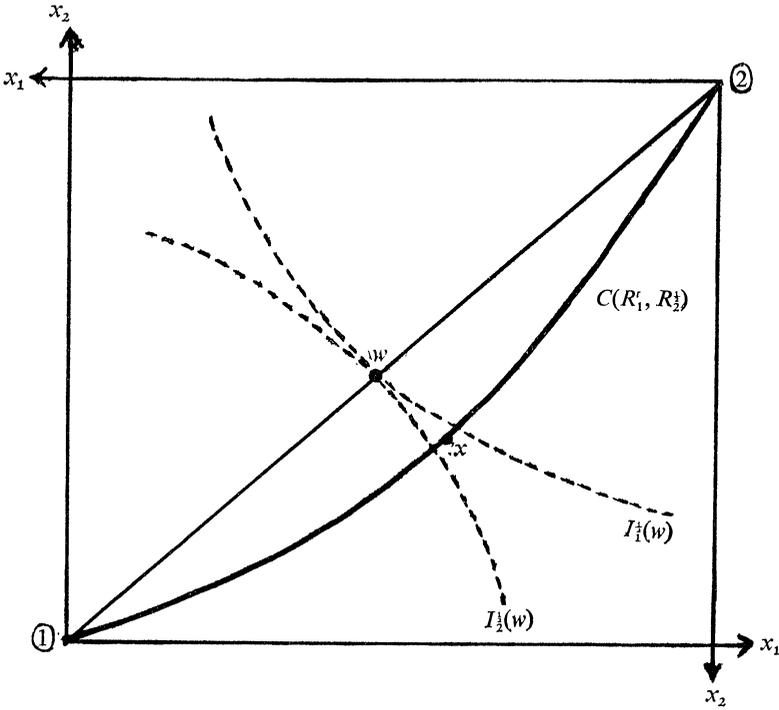
FIGURE 7

Next, let $y \in f(R_1^{*\frac{1}{2}}, R_2^{*1-t})$. Notice that $C(R_1^{*t}, R_2^{*\frac{1}{2}}) = C(R_1^{*\frac{1}{2}}, R_2^{*1-t})$. So x and y lie on the same contract curve. Furthermore, by an argument symmetric to the one given in the last paragraph, y must lie on or above the line separating $I_1^{\frac{1}{2}}(w)$ and $I_2^{\frac{1}{2}}(w)$.

Suppose it were true that $x \neq y$. Then, since

$$(R_1^{*t}, R_2^{*\frac{1}{2}}): \{x, y\} = (R_1^{*\frac{1}{2}}, R_2^{*1-t}): \{x, y\},$$

since all are strong orderings, and since $x \in f(R_1^{*t}, R_2^{*\frac{1}{2}})$, IWM implies $y \notin f(R_1^{*\frac{1}{2}}, R_2^{*1-t})$, and we have a contradiction. Therefore $x = y$, and x lies on the line separating $I_1^{\frac{1}{2}}(w)$ and $I_2^{\frac{1}{2}}(w)$.

Now consider $z \in f(R_1^{*t}, R_2^{*t})$. Notice that $C(R_1^{*t}, R_2^{*t}) = C(R_1^{*\frac{1}{2}}, R_2^{*\frac{1}{2}})$ and so, since z lies on this common contract curve, $(R_1^{*t}, R_2^{*t}): \{w, z\} = (R_1^{*\frac{1}{2}}, R_2^{*\frac{1}{2}}): \{w, z\}$. Therefore, applying IWM once again, it must be true that $w = z$. But $wP_2^{*\frac{1}{2}}x = y \in f(R_1^{*t}, R_2^{*t})$. So, with true preferences $R_2^{*\frac{1}{2}}$, consumer 2 will prefer to announce R_2^{*t} when consumer 1's preferences are R_1^{*t}.

This is a contradiction, so either consumer 1 or consumer 2 must be a dictator after all. $\|$

4.5. Implementation and Coalitions

IPM needs only to be strengthened to IWM to obtain results for coalitions.

Theorem 4.5.1. *An SSCR $f: \mathscr{R} \Rightarrow A$ is truthfully implementable in coalitionally dominant strategies if it satisfies* IWM.

Proof. An obvious modification of the proof of the sufficiency half of Theorem 4.3.1. $\|$

Notice that Theorem 4.5.1 provides only a sufficient condition for implementability in coalitionally dominant strategies. The following example shows that IWM is not necessary.

Example 4.5.2. Take $A = \{a, b, c, d\}$. Let $\mathscr{R}_1 = \{R_1, R'_1\}$ and $\mathscr{R}_2 = \{R_2, R'_2\}$, where

$$R_1 = \begin{bmatrix} a \\ c \\ d \\ b \end{bmatrix} \quad R'_1 = \begin{bmatrix} c \\ a \\ b \\ d \end{bmatrix} \quad R_2 = \begin{bmatrix} c \\ b \\ a \\ d \end{bmatrix} \quad R'_2 = \begin{bmatrix} b \\ d \\ c \\ a \end{bmatrix}.$$

Define $f: \mathscr{R}_1 \times \mathscr{R}_2 \to A$ so that

$$f(R_1, R_2) = \{a\}, f(R_1, R'_2) = \{d\},$$
$$f(R'_1, R_2) = \{c\}, f(R'_1, R'_2) = \{b\}.$$

It is easy to verify that f is implementable in coalitionally dominant strategies. It does not satisfy IWM, however, because

$$(R_1, R_2):\{a, b\} = (R'_1, R'_2):\{a, b\}$$

and $a \in f(R_1, R_2)$ but $b \in f(R'_1, R'_2)$ in violation of IWM.

We shall defer giving general conditions which are both necessary and sufficient for coalitional implementability to a future paper. For the time being, we shall make do with the following observations.

Corollary 4.5.3. *An* SCR *is truthfully implementable in coalitionally dominant strategies if there exists an* SSCR f^*, *satisfying* IWM, *such that, for all* $R \in \mathscr{R}, f^*(R) \subseteqq f(R)$.

A necessary and sufficient condition in the case of rich domains of strong orderings is provided by

Corollary 4.5.4. *If* $\mathscr{R}$ *is a rich domain consisting of strong orderings, then an* SCR $f: \mathscr{R} \rightrightarrows A$ *is fully implementable in coalitionally dominant strategies if and only if f is an* SSCR *and satisfies* IPM.

Proof. For sufficiency, observe that if f is an SSCR satisfying IPM, then by Corollary 3.2.3, f satisfies IWM. Therefore, we obtain implementability by Theorem 4.5.1. For necessity, note that coalitional implementability implies ordinary implementability, which in turn implies IPM, by Theorem 4.3.4. $\|$

We should note that if, in Corollary 4.5.4 the assumption of richness is dropped, the necessity half of the theorem fails.

Example 4.5.5. Let $A = \{a, b, c, d\}$, and $\mathscr{R}_1 = \{R_1, R'_1\}$, $\mathscr{R}_2 = \{R_2, R'_2\}$, where

$$R_1 = \begin{bmatrix} a \\ c \\ d \\ b \end{bmatrix} \quad R'_1 = \begin{bmatrix} c \\ a \\ b \\ d \end{bmatrix} \quad R_2 = \begin{bmatrix} a \\ d \\ c \\ b \end{bmatrix} \quad R'_2 = \begin{bmatrix} d \\ a \\ b \\ c \end{bmatrix}.$$

Define $f^*: \mathscr{R}_1 \times \mathscr{R}_2 \rightrightarrows A$ so that:

$$f^*(R_1, R_2) = \{a\}, f^*(R_1, R'_2) = \{d\},$$
$$f^*(R'_1, R_2) = \{c\}, f^*(R'_1, R'_2) = \{b\}.$$

f^* satisfies IPM (and so is implementable in dominant strategies). f^* is not implementable in coalitionally dominant strategies (and hence does not satisfy IWM), however, because $f^*(R'_1, R'_2) = b$ so that if the true preference profile were (R'_1, R'_2), both players would prefer the joint announcement (R_1, R_2) to the truth. Notice that $\mathscr{R}_1$ and $\mathscr{R}_2$ are not rich domains.

Example 4.5.6. Let $A = \{a, b, c, d, e\}$, and $\mathscr{R}_1 = (R_1, R_1')$, $\mathscr{R}_2 = (R_2, R_2')$, where

$$R_1 = \begin{bmatrix} a-c \\ b-d-e \end{bmatrix} \quad R_1' = \begin{bmatrix} a-c \\ b-d \\ e \end{bmatrix} \quad R_2 = \begin{bmatrix} a-d \\ b-c-e \end{bmatrix} \quad R_2' = \begin{bmatrix} a-d \\ b-c \\ e \end{bmatrix}.$$

Define $f^{**}: \mathscr{R}_1 \times \mathscr{R}_2 \rightrightarrows A$ so that

$$f^{**}(R_1, R_2) = \{a\}, f^{**}(R_1, R_2') = \{d\},$$

$$f^{**}(R_1', R_2) = \{c\}, f^{**}(R_1', R_2') = \{b\}.$$

f^{**} satisfies IPM and can therefore be truthfully implemented in dominant strategies. It is clearly not truthfully implementable coalitionally, however, because the players 1 and 2 with preferences (R_1', R_2') would, as a coalition, pretend to have the preference profile (R_1, R_2).

In view of Theorems 4.3.1 and 4.5.1, it is clear that only the difference between IPM and IWM prevents coalitional dominance in those SSCR's which are truthfully implementable in dominant strategies. The difference between IWM and IPM is, in fact, nonexistent in the case of rich domains of strong orderings. Notice that, in the hypotheses of Corollary 4.5.4 we could have stipulated " IWM " instead of " IPM ", by virtue of Corollary 3.2.3. The equivalence of IPM and IWM on rich domains of strong orderings has, then, the implication that, in this case, implementation in dominant strategies implies coalitional implementation. This implication does not hold in general when preference orderings are weak, or the domain is not rich, as the so-called Clarke–Groves–Vickrey mechanisms well illustrate.

4.6. *The Clarke–Groves–Vickrey Mechanisms*

Some of the power of our framework can be seen from the extra ease with which the following results can be proved.

Following Green and Laffont (1977), consider a public project space $\{0, 1\}$ where " 1 " denotes the adoption of a public project and " 0 " denotes its rejection. Let A be the set of vectors $(x, t_1, ..., t_n)$, where $x \in \{0, 1\}$ and denotes adoption or rejection of the project and t_i represents a transfer to individual i. For each individual i in the society, let the space of characteristics be $\Theta_i = \mathbb{R}$ (the real numbers). Let preferences be defined so that:

$$(x, t_1, ..., t_n) R(\theta_i)(y, t_1', ..., t_n') \Leftrightarrow x\theta_i + t_i \geq y\theta_i + t_i'.$$

Take $\Theta = \prod_{i=1}^{n} \Theta_i$. Consider $f: \Theta \rightrightarrows A$ such that $\forall \theta \in \Theta$:

$$f(\theta) = \{(x(\theta), t_1(\theta), ..., t_n(\theta))\} \quad \text{where } x(\theta) = \begin{cases} 1, & \text{if } \Sigma\theta_i \geq 0 \\ 0, & \text{otherwise.} \end{cases} \quad ...(4.6.1)$$

and $t_i: \Theta \to \mathbb{R}$ $(i = 1$ to $n)$ are well-defined transfer functions.

The basic theorem which characterizes those f's satisfying (4.6.1) which are implementable by dominant strategies is:

Theorem 4.6.1. (Green and Laffont (1977)).[3] *f is implementable in dominant strategies iff there exist functions $h_i(\theta_{-i})$ such that*

$$t_i(\theta) = \begin{cases} \sum_{j \neq i} \theta_j + h_i(\theta_{-i}), & \text{if } \Sigma\theta_j \geq 0 \\ h_i(\theta_{-i}), & \text{otherwise.} \end{cases} \quad ...(4.6.2)$$

Proof. Let f be an SSCR satisfying (4.6.1). Without loss of generality, we may write

$$t_i(\theta) = \begin{cases} \sum_{j \neq i} \theta_j + h_i(\theta), & \text{if } \Sigma\theta_j \geq 0 \\ h_i(\theta), & \text{otherwise.} \end{cases}$$

f is implementable iff there exists a mechanism $g: \Theta \to A$ which has truth as a dominant strategy such that $g(\theta) \in f(\theta)$. Consider $\theta_i, \bar{\theta}_i \in \Theta_i, \theta_{-i} \in \Theta_{-i}$ such that $\theta_i, \bar{\theta}_i \geqq -\sum_{j \neq i} \theta_j$ and

$$h_i(\theta_i, \boldsymbol{\theta}_{-i}) \geqq h_i(\bar{\theta}_i, \boldsymbol{\theta}_{-i}). \qquad \qquad ...(4.6.3)$$

If (4.6.3) holds with strict inequality, then

$$g(\theta_i, \boldsymbol{\theta}_{-i}) P_i(\theta_i) g(\bar{\theta}_i, \boldsymbol{\theta}_{-i})$$

$$g(\theta_i, \boldsymbol{\theta}_{-i}) P_i(\bar{\theta}_i) g(\bar{\theta}_i, \boldsymbol{\theta}_{-i})$$

which is a contradiction of IPM, since $f(\theta_i, \boldsymbol{\theta}_{-i}) \neq f(\bar{\theta}_i, \boldsymbol{\theta}_{-i})$. Therefore, if f satisfies IPM, $h_i(\theta_i, \boldsymbol{\theta}_{-i})$ is a constant for $\theta_i \geqq -\sum_{j \neq i} \theta_j$. Similarly, $h_i(\theta_i, \boldsymbol{\theta}_{-i})$ is constant for $\theta_i < \sum_{j \neq i} \theta_j$. It remains only to show that $h_i(\theta_i, \boldsymbol{\theta}_{-i})$ is constant over the whole domain Θ_i. Consider $\bar{\bar{\theta}}_i \geqq -\sum_{j \neq i} \theta_j > \bar{\theta}_i$. Because truth dominates:

$$\bar{\bar{\theta}}_i + \sum_{j \neq i} \theta_j + h_i(\bar{\bar{\theta}}_i, \boldsymbol{\theta}_{-i}) \geqq h_i(\bar{\theta}_i, \boldsymbol{\theta}_{-i}) \qquad \qquad ...(4.6.4)$$

and

$$\bar{\bar{\theta}}_i + \sum_{j \neq i} \theta_j + h_i(\bar{\theta}_i, \boldsymbol{\theta}_{-i}) \leqq h_i(\bar{\bar{\theta}}_i, \boldsymbol{\theta}_{-i}). \qquad \qquad ...(4.6.5)$$

Taking $\bar{\bar{\theta}}_i = -\sum_{j \neq i} \theta_j$ in (4.6.4) we obtain $h_i(\theta_i, \boldsymbol{\theta}_{-i}) \geqq h(\theta_i', \boldsymbol{\theta}_{-i})$ for all

$$\theta_i \geqq -\sum_{j \neq i} \theta_j > \theta_i'.$$

But by considering in (4.6.5) a sequence $\bar{\bar{\theta}}_i^n$ converging from below to $-\sum_{j \neq i} \theta_j$, we see that $h_i(\theta_i, \boldsymbol{\theta}_{-i}) \leqq h(\theta_i', \boldsymbol{\theta}_{-i})$ for all $\theta_i \geqq -\sum_{j \neq i} \theta_j > \theta_i'$. Therefore $h_i(\theta_i, \boldsymbol{\theta}_{-i})$ remains constant for all θ_i and may, therefore, be written as $h_i(\boldsymbol{\theta}_{-i})$. ‖

The proof of Theorem 4.6.1 is essentially a demonstration that the only SSCR's of the form (4.6.1) which satisfy IPM are those which are of the form (4.6.2). Interestingly, none of these SSCR's is implementable in coalitionally dominant strategies.

Theorem 4.6.2. (Green and Laffont (1979)). *For $n \geqq 2$, there exists no SSCR of the form (4.6.1) which is implementable in coalitionally dominant strategies.*

Proof. If there were such an SSCR of the form (4.6.1), it would have to satisfy (4.6.2). It suffices to show, therefore, that SSCR's of the form (4.6.2) do not satisfy IWM. If such an SSCR f did satisfy IWM, then for $\bar{\theta}_1, \bar{\bar{\theta}}_1 \in \Theta_1, \bar{\theta}_2, \bar{\bar{\theta}}_2 \in \Theta_2, \boldsymbol{\theta}_{-1-2} \in \Theta_{-1-2}$ such that $\bar{\theta}_1 + \bar{\theta}_2 + \sum_{j \neq 1, 2} \theta_j \geqq 0, \bar{\bar{\theta}}_1 + \bar{\bar{\theta}}_2 + \sum_{j \neq 1, 2} \theta_j \geqq 0$, we have

$$\bar{\theta}_1 + \bar{\theta}_2 + \sum_{j \neq 1, 2} \theta_j + h_1(\bar{\theta}_2, \boldsymbol{\theta}_{-1-2}) + h_2(\bar{\theta}_1, \boldsymbol{\theta}_{-1-2})$$
$$= \bar{\bar{\theta}}_1 + \bar{\bar{\theta}}_2 + \sum_{j \neq 1, 2} \theta_j + h_1(\bar{\bar{\theta}}_2, \boldsymbol{\theta}_{-1-2}) + h_2(\bar{\bar{\theta}}_1, \boldsymbol{\theta}_{-1-2}). \quad ...(4.6.6)$$

Simplifying (4.6.6), recalling that h_1 does not depend on θ_1 nor h_2 on θ_2, we obtain

$$h_1(\bar{\theta}_2, \boldsymbol{\theta}_{-1-2}) - h_1(\bar{\bar{\theta}}_2, \boldsymbol{\theta}_{-1-2}) = \bar{\bar{\theta}}_2 - \bar{\theta}_2$$

and

$$h_2(\bar{\theta}_1, \boldsymbol{\theta}_{-1-2}) - h_2(\bar{\bar{\theta}}_1, \boldsymbol{\theta}_{-1-2}) = \bar{\bar{\theta}}_1 - \bar{\theta}_1. \qquad ...(4.6.7)$$

Note that (4.6.7) holds for *all* $\bar{\theta}_1, \bar{\bar{\theta}}_1, \bar{\theta}_2, \bar{\bar{\theta}}_2, \boldsymbol{\theta}_{-1-2}$. By symmetry, we may rewrite (4.6.7) as

$$h_1(\boldsymbol{\theta}_{-1}) = -\sum_{j \neq 1} \theta_j + h_1(\mathbf{0}),$$
$$h_2(\boldsymbol{\theta}_{-2}) = -\sum_{j \neq 2} \theta_j + h_2(\mathbf{0}). \qquad ...(4.6.8)$$

Now choose $\bar{\theta}_1, \bar{\bar{\theta}}_1 \in \Theta_1, \bar{\theta}_2, \bar{\bar{\theta}}_2 \in \Theta_2$, and $\boldsymbol{\theta}_{-1-2} \in \Theta_{-1-2}$ such that

$$\bar{\theta}_1 + \bar{\theta}_2 + \sum_{j \neq 1, 2} \theta_j < 0, \quad \bar{\bar{\theta}}_1 + \bar{\bar{\theta}}_2 + \sum_{j \neq 1, 2} \theta_j < 0, \quad \text{and} \quad \bar{\theta}_1 + \bar{\theta}_2 \neq \bar{\bar{\theta}}_1 + \bar{\bar{\theta}}_2.$$

From IWM, we have

$$h_1(\bar{\theta}_2, \boldsymbol{\theta}_{-1-2}) + h_2(\bar{\theta}_1, \boldsymbol{\theta}_{-1-2}) = h_1(\bar{\bar{\theta}}_2, \boldsymbol{\theta}_{-1-2}) + h_2(\bar{\bar{\theta}}_1, \boldsymbol{\theta}_{-1-2}) \qquad ...(4.6.9)$$

Expanding (4.6.9), using (4.6.8), we have:

$$-\bar{\theta}_2 - \sum_{j \neq 1,\,2} \theta_j + h_1(0) - \bar{\theta}_1 - \sum_{j \neq 1,\,2} \theta_j + h_2(0)$$
$$= -\bar{\theta}_2 - \sum_{j \neq 1,\,2} \theta_j + h_1(0) - \bar{\theta}_1 - \sum_{j \neq 1,\,2} \theta_j + h_2(0).$$

Or, simplifying, $\bar{\theta}_1 + \bar{\theta}_2 = \bar{\theta}_1 + \bar{\theta}_2$, which contradicts our choice of $\bar{\theta}_1$, $\bar{\theta}_2$, $\bar{\theta}_1$, $\bar{\theta}_2$. ‖

4.7. Independent Monotonicity and Social Aggregation Functions

We shall now draw a connection between the sufficient conditions for implementation (Theorems 4.2.1 and 4.2.2) and the necessary and sufficient conditions (Corollary 4.5.4).

Theorem 4.7.1. *If $\mathcal{R}$ is rich and consists of strong orderings, the SSCR f can be (fully) implemented in (coalitionally) dominant strategies iff f maximizes a SAF $F: \mathcal{R} \to \mathcal{B}_A$, where F satisfies* NNR. *If, furthermore, f satisfies citizen sovereignty then F can be taken as Pareto optimal.*

Remark. Either or both the parenthetic words can be deleted from the statement of the theorem without altering its truth.

Proof.

Sufficiency: From Theorem 4.2.1, if f maximizes F, then it is truthfully implementable and so, by Theorem 4.3.1, f satisfies IPM. By Corollary 4.5.4, it is fully implementable in coalitionally dominant strategies.

Necessity: If f is implementable, then f satisfies IWM. Construct a SAF as follows. For each $R \in \mathcal{R}$ and $\{a, b\} \subseteq A$, define the binary relation $F(R)$ so that: $aF(R)b$ iff either

    (i) $\forall R^* \in \mathcal{R}$ such that $\forall i$, $aP_i b \Rightarrow aP_i^* b$, $b \notin f(R^*)$

or

    (ii) $a = b$.

From (ii), $F(R)$ is a reflexive relation. To see that it is complete, suppose that, for some $R \in \mathcal{R}$ and $\{a, b\} \subseteq A$, $\sim aF(R)b$. Then from (i), $\exists R^* \in \mathcal{R}$ such that $\forall i$, $aP_i b \Rightarrow aP_i^* b$ and $b \in f(R^*)$. By IWM, there does not exist $R^{**} \in \mathcal{R}$ such that $\forall i$, $bP_i^* a \Rightarrow bP_i^{**} a$ and $a \in f(R^{**})$. Therefore, by choice of R^*, there does not exist $R^{**} \in \mathcal{R}$ such that $\forall i$, $bP_i a \Rightarrow bP_i^{**} a$ and $a \in f(R^{**})$. Thus $bF(R)a$. $F(R)$ is, therefore, complete, and so F is an SAF. It is equally straightforward to show that F satisfies NNR and that f maximizes F.

Now suppose that f satisfies CS. Choose $R \in \mathcal{R}$ such that for some $\{a, b\} \subseteq A$, $aP_i b$ for all i. By CS, there exists $R^* \in \mathcal{R}$ such that $a \in f(R^*)$. Since $\forall i$, $bP_i a$ implies $bP_i^* a$ vacuously, but $a \in f(R^*)$, we conclude from (i), that $\sim bF(R)a$. Hence $aP(F(R))b$, establishing Pareto optimality. ‖

One might conjecture that the statement "f is truthfully implementable iff f maximizes an SAF satisfying NNR " is true for weak orderings. Obviously the sufficiency half holds, but the following example illustrates that the necessity half is false.

Example 4.7.2. Let $A = \{a, b, c\}$ and $\mathcal{R}_1 = (R_1, R_1')$, $\mathcal{R}_2 = (R_2, R_2')$, where

$$R_1 = R_2 = \begin{bmatrix} a-b \\ c \end{bmatrix} \quad R_1' = R_2' = \begin{bmatrix} c \\ a-b \end{bmatrix}.$$

Define $f: \mathcal{R}_1 \times \mathcal{R}_2 \rightrightarrows A$ so that

$$f(R_1, R_2) = \{a\}$$
$$f(R_1, R_2') = f(R_1', R_2) = \{b\}$$
$$f(R_1', R_2') = \{c\}.$$

One may easily verify that f satisfies IPM and so is truthfully implementable. If f maximizes an SAF $F: \mathscr{R}_1 \times \mathscr{R}_2 \to \mathscr{B}_A$, then because $f(R_1, R_2) = \{a\}$, we must have $aP(F(R_1, R_2))b$. On the other hand, because $f(R_2, R_2') = \{b\}$ we must have $bP(F(R_1, R_2'))a$. Therefore F cannot satisfy NNR.

There has been some interest in the connection between dominant strategy mechanisms and social welfare functions satisfying the Arrow axioms (see Maskin (1976b), Kalai and Muller (1977), Satterthwaite (1975)). Theorem 4.7.1 demonstrates that a dominant strategy mechanism is an essentially weaker concept, corresponding to an SAF rather than an SWF. That is, dominant strategy mechanisms do not necessarily generate transitive social preferences. There are at least two alternative natural conditions one can impose on game forms to get an exact correspondence between dominance and SWF's. One is a consistency condition described in Maskin (1976b). The other is the so-called triple restriction property presented in Section 3.3. We have, in fact, the following result.

Theorem 4.7.3. *If $\mathscr{R}$ is rich and consists of strong orderings and the SSCR $f: \mathscr{R} \rightrightarrows A$ satisfies triple restriction, then f can be (fully) implemented in (coalitionally) dominant strategies iff f maximizes an SWF $F: \mathscr{R} \to \mathscr{B}_A$, where F satisfies NNR. If, furthermore, f satisfies CS, then F can be chosen to satisfy PO (Pareto optimality).*

Remark. A social welfare function $F: \mathscr{R} \to \mathscr{R}_A$ satisfying PO and NNR has been called (see, for example, Maskin (1976a)) an *Arrow* SWF, because NNR is basically the intersection of " positive association " (see Arrow (1963)) and independence of irrelevant alternatives, and because the three properties—PO, positive association, and independence, are the three properties that Arrow demands of SWF's. (We ignore here, for obvious reasons, the unrestricted domain and dictatorship conditions.) Theorem 4.7.2 then does provide equivalence between the Arrow problem and the Gibbard–Satterthwaite problem for SSCR's satisfying triple restriction.

Proof. Sufficiency is established by the proof of Theorem 4.7.1. To establish necessity, assume that $f: \mathscr{R} \rightrightarrows A$ can be implemented in dominant strategies and satisfies triple restriction. Construct an SAF $F: \mathscr{R} \to \mathscr{B}_A$ as in the proof of Theorem 4.7.1. Consider $\{a, b, c\} \subseteq A$ and $R \in \mathscr{R}$ such that $aF(R)bF(R)c$. Suppose $\sim aF(R)c$. Then there exists $R^* \in \mathscr{R}$ such that for all $i(aP_ic$ implies $aP_i^*c)$ and $c \in f(R^*)$. By triple restriction, there exists $R' \in \mathscr{R}$ with $R': \{a, b, c\} = R: \{a, b, c\}$ and $f(R') \subseteq \{a, b, c\}$. If $a \in f(R')$, then by IWM, $c \notin f(R^*)$, a contradiction of our above conclusion. But, if $b \in f(R')$ or $c \in f(R')$, we infer $\sim aF(R)b$ or $\sim bF(R)c$, respectively—also contradictions. Therefore, $\sim aF(R)c$ is impossible after all, and we have $aF(R)c$, establishing the transitivity of $F(R)$. ‖

The Gibbard–Satterthwaite theorem is one corollary that we can quickly derive from Theorem 4.7.3.

Corollary 4.7.4. (Gibbard (1973), Satterthwaite (1975)). *Let A contain at least three elements and let $f: \mathscr{R}_A^n \rightrightarrows A$ be an n-person SSCR which is truthfully implementable in dominant strategies and satisfies consumer sovereignty. Then f is dictatorial.*

Proof. Consider the restriction (call it f^*) of f to $(\mathscr{R}_A^*)^n$, where $\mathscr{R}_A^*$ consists of all strong orderings in $\mathscr{R}_A$. By Theorem 4.3.1, f^* satisfies IPM. By Theorem 4.3.4, f^* is fully implementable. By Theorem 3.3.1, f is Pareto optimal. By Theorem 3.3.2 f therefore satisfies triple restriction. By Theorem 4.7.3, f^* maximizes a SWF $F: (\mathscr{R}_A^*)^n \to \mathscr{R}_A$ which satisfies NNR and PO. By the Arrow Impossibility Theorem, F is a dictatorial SWF. Therefore f^* is dictatorial and so is f. ‖

5. DIRECT MECHANISMS: BAYESIAN EQUILIBRIUM

In this section, we briefly treat the so-called Bayesian mechanisms, previously studied by Ledyard, d'Aspremont and Gérard-Varet (1977a), Arrow (1977), Laffont and Maskin

(1978), Rosenthal (1978) and Myerson (1979). We shall not attempt here to characterize implementable SCR's as we have done for dominant-strategy and Nash equilibrium.

Suppose that we have a game form or mechanism $g: S \rightarrow X$. We assume that each agent has incomplete information, in the sense of Harsanyi (1967).

Thus, each agent i knows his own characteristic θ_i but does not know the characteristics θ_{-i} of the other agents. Instead, he assesses the probabilities that these other agents have various configurations of characteristics; these assessments are summarized by a probability measure Π_i on the set Θ_{-i}. In principle, this probability measure should depend on the agent's own characteristic θ_i. We write, therefore, $\Pi_i(\cdot \mid \theta_i)$. We shall assume that the planner knows a priori the functional form Π_i, though not, of course, the true value of θ_i.

In this game form, a " Bayesian equilibrium " takes the form of a vector of strategy rules $(\sigma_1(\cdot), \ldots, \sigma_n(\cdot))$, where the strategy s_i^* that player i chooses is a well-defined, measurable function of his true characteristic θ_i. What player i does is to choose his s_i^*, given θ_i, in order to maximize expected utility, given his probability measure Π_i on Θ_{-i}. Formally, then, a Bayesian equilibrium is a vector $(\sigma_1(\cdot), \ldots, \sigma_n(\cdot))$, where $\sigma_i: \Theta_i \rightarrow S_i$ for each i, such that $\forall i \in I, \forall \theta_i \in \Theta_i, s_i = \sigma_i(\theta_i)$ maximizes

$$\int_{\Theta_{-i}} u_i(g(s_i, \sigma_{-i}(\theta_{-i})); \theta_i) d\Pi_i(\theta_{-i} \mid \theta_i).$$

Here, of course, $u_i(. ; \theta_i)$ denotes player i's von Neumann–Morgenstern utility function, which is determined by i's characteristic θ_i.

As formulated, since each agent must choose his strategy before observing the others' strategies, he must understand how other agents' strategies s_{-i} depend on their unknown characteristics θ_{-i}. To work out this dependence, he really needs to know the probability measures Π_{-i} of the other agents. Thus, in this framework, one ordinarily assumes that the entire list of probability measures $\Pi = (\Pi_1, \ldots, \Pi_n)$ is common knowledge to all players, as well as to the planner. One plausible way that this common knowledge could arise is by the planner's publishing a historical frequency distribution.

A simple result relating Bayesian to dominant strategy implementation is

Theorem 5.1. *Let $f: \Theta \rightrightarrows A$ be an* SCR. *Then f is truthfully implementable in Bayesian equilibrium strategies for all possible probability measures Π if and only if f is truthfully implementable in dominant strategies.*

Proof. Sufficiency is immediate.

Necessity: Suppose f is not truthfully implementable in dominant strategies. There exists $i \in I$ and $\bar{\theta} \in \Theta$ such that truthfulness for i is not a Nash equilibrium strategy given $\bar{\theta}_{-i}$. Now choose Π so that $\Pi_i(\{\bar{\theta}_{-i}\}) = 1$. Then truthfulness is not a Bayesian strategy either—a contradiction. ‖

We previously observed (Theorem 4.1.1) that, if a planner were content with truthful implementation rather than implementation, there would be no need ever to consider any but *direct* mechanisms, if dominant strategies were the solution concept. We will now show that the same principle applies to Bayesian equilibrium.

Theorem 5.2. *If the SCR $f: \Theta \rightrightarrows X$ is implemented in Bayesian strategies by the game form $g: S \rightarrow X$, given the agents' probability measures Π_i on Θ_{-i}, then there exists an equivalent direct mechanism $h: \Theta \rightarrow X$ for which truthfulness is always a Bayesian equilibrium.*

Proof. Let $(\sigma_1, \ldots, \sigma_n)$ be a Bayesian equilibrium for the game form $g: S \rightarrow X$, given the agents' probability assessments Π_i. Define $h: \Theta \rightarrow X$ so that

$$\forall \theta \in \Theta, \quad h(\theta) = g(\sigma(\theta)).$$

Because $\sigma_i(\cdot)$ is an equilibrium strategy rule for agent $i \in I$, $s_i = \sigma_i(\theta_i)$ maximizes

$$\int_{\Theta_{-i}} u_i(g(s_i, \sigma_{-i}(\theta_{-i})); \theta_i) d\Pi_i.$$

It follows that $\eta_i = \theta_i$ maximizes

$$\int_{\Theta_{-i}} u_i(g(\sigma_i(\eta_i), \sigma_{-i}(\theta_{-i})); \theta_i) d\Pi_i.$$

That is, $\eta_i = \theta_i$ maximizes

$$\int_{\Theta_{-i}} u_i(h(\eta_i, \theta_{-i}); \theta_i) d\Pi_i.$$

So $(\sigma_1^*, ..., \sigma_n^*)$, where, for all i, $\sigma_i^*(\theta_i) = \theta_i$, is a Bayesian equilibrium for h.   ‖

As with dominant strategies, the equivalent direct Bayesian mechanism h may not actually implement f, although h truthfully implements it. This, again, is because there may be untruthful equilibria whose associated outcomes are not in the choice set of f.

We should note that, although we have taken agents' beliefs to be probability assessments of other agents' characteristics, we could have made the *signals* s_{-i} of the other agents the objects of player i's uncertainty. Formally this would be captured by a probability measure μ_i over S_{-i}. This second approach has the advantage of describing explicitly an agent's beliefs about the *behaviour* of other agents. Unfortunately, it means that the informational burden on the planner may be considerable; in general, there will be a different Π for each game form g, because each game form has a different S. This means too that Theorem 5.2 no longer holds; it may be impossible to reduce a game form to a direct mechanism.

6. DIRECT MECHANISMS: MAXIMIN EQUILIBRIUM

We next turn to the concept of maximin equilibrium. Implementation in maximin strategies has been studied by Thomson (1978). As with Bayesian behaviour, maximin attitudes can take either of two forms: either agent i believes the other agents will *act* (i.e. choose those feasible strategies) so as to yield the worst possible outcome from i's standpoint, or he believes that others' *characteristics* are so distributed that when they choose maximin strategies the outcome is, in his eyes, the worst possible. The first formulation is probably more pervasive in game theory. The second approach, however, has the merit of endowing the agent with a bit more sophistication than maximin behaviour usually entails. It has the further advantage of enabling us, under certain circumstances (see Theorem 6.1) to restrict our concern to direct mechanisms as in Theorems 4.1.1 and 5.2. The second approach is a restricted version of maximin behaviour in that the agent determines the consequences of each of his possible strategy choices by minimizing only with respect to others' possible maximin strategies. We shall therefore call it *restricted maximin*. Formally, a *restricted maximin equilibrium* for a game form $g: S \to X$ with respect to the characteristic space Θ is a vector of correspondences, $(\sigma_1, \sigma_2, ..., \sigma_n)$, where $\forall i$, $\sigma_i: \Theta_i \rightrightarrows S_i$, such that $\forall i \in I$, $\forall \theta_i \in \Theta_i$, $s_i^* \in \sigma_i(\theta_i)$ iff

$$\min \{u_i(g(s_i^*, s_{-i}); \theta_i) \mid s_{-i} \in \sigma_{-i}(\theta_{-i}), \theta_{-i} \in \Theta_{-i}\}$$
$$= \max_{s_i \in S_i} \min \{u_i(g(s_i, s_{-i}); \theta_i) \mid s_{-i} \in \sigma_{-i}(\theta_{-i}), \theta_{-i} \in \Theta_{-i}\}.$$

With restricted maximin, we shall be able to prove the existence of an equivalent direct mechanism only in a special case which includes, in particular, the case where each agent's restricted maximin strategy is always unique.

Theorem 6.1. *Let $g\colon S \to A$ be a game form with a restricted maximin equilibrium correspondence $\sigma\colon \Theta \rightrightarrows S$ which admits a maximin selection $s^*\colon \Theta \to S$ such that $s_i = s_i^*(\theta_i)$ maximizes $\min_{\theta_{-i} \in \Theta_{-i}} u_i(g(s_i, s_{-i}^*(\theta_{-i})); \theta_i)$. Then there exists an equivalent direct mechanism $h\colon \Theta \to A$ for which truthfulness is always a maximin strategy.*

Remark. This theorem implies that if g implements an SCR f, then g truthfully implements f. The same *caveat* applies here, however, as to all other equivalent direct mechanisms: they merely implement the SCR *truthfully*; there may be non-truthful equilibria which yield outcomes not in the choice set of the SCR.

Proof. Define $h\colon \Theta \to A$ by $h(\theta) = g(s^*(\theta))$. Because of the definitions of $s_i^*(\theta_i)$ and because $s_i^*(\eta_i) \in S_i$ for all $\eta_i \in \Theta_i$, it follows that θ_i is a solution to

$$\max_{\eta_i \in \Theta_i} \min_{\theta_{-i} \in \Theta_{-i}} u_i(h(\eta_i, \theta_{-i}); \theta_i). \quad \|$$

The postulation of a maximin selection s^* as in Theorem 6.1 is quite stringent. The following example demonstrates how there may be no equivalent direct mechanism when maximin strategies are not unique.

Example 6.2. Let $A = \{a, b, c, d, e, x, y\}$, $\mathcal{R}_1 = \{R_1, R_1'\}$, $\mathcal{R}_2 = \{R_2, R_2'\}$, $\mathcal{R}_3 = \{R_3\}$, where

$$
R_1 = \begin{pmatrix} a \\ d \\ b \\ x \\ y \\ e \\ c \end{pmatrix}, \quad
R_1' = \begin{pmatrix} a \\ c \\ x \\ e \\ y \\ d \\ b \end{pmatrix}, \quad
R_2 = \begin{pmatrix} b \\ a \\ d \\ c \\ y \\ x \\ e \end{pmatrix}, \quad
R_2' = \begin{pmatrix} b \\ a \\ d \\ c \\ e \\ y \\ x \end{pmatrix}, \quad \text{and } R_3 = \begin{pmatrix} b \\ e \\ y \\ d \\ c \\ x \\ a \end{pmatrix}.
$$

Define the SCR $f\colon \mathcal{R}_1 \times \mathcal{R}_2 \times \mathcal{R}_3 \rightrightarrows A$ by the following matrix:

	R_2	R_2'
R_1	$\{a\}$	$\{b, e\}$
R_1'	$\{c, x\}$	$\{d, y\}$

Notice that we need not include individual 3's preferences in our table. f is implemented in maximin strategies by the following game form

g:

s_3

	s_2	s_2'
s_1	a	b
s_1'	c	d

s_3'

	s_2	s_2'
s_1	a	e
s_1'	x	y

Observe that s_1 and s_1' are maximin strategies for player 1 with preferences R_1 and R_1', respectively. s_2 and s_2' are maximin strategies for player 2 with preferences R_2 and R_2', respectively. s_3 and s_3' are maximin strategies for player 3. It can immediately be verified that g implements f. Also, the maximin strategies are actually restricted maximin strategies too. However, no direct mechanism truthfully implements f, as we shall now see.

A direct mechanism, g^*, which truthfully implemented f would have to be of the form

	R_2	R_2'
R_1	a	b or e
R_1'	c or x	d or y

If (R_1', R_2) resulted in outcome c, then $g^*(R_1, R_2', R_3) = b$ and $g^*(R_1', R_2', R_3) = d$, otherwise player 2 with preferences R_2' would not tell the truth. But, in such a case, player 2 with preferences R_2 would not tell the truth because dP_2c. Therefore, we must have $g(R_1', R_2, R_3) = x$. But x is preferred only to e by player 2 with preferences R_2. Therefore, to induce truthtelling in player 2, we must have $g(R_1, R_2', R_3) = e$. But e is preferred only to c by player 1 with preferences R_1. Therefore to obtain truthtelling in player 1, we must have $g(R_1', R_2, R_3) = c$, which contradicts the above. Therefore, there exists no direct mechanism which implements f.

Example 6.2 demonstrates that it will not always suffice to confine our attention to direct mechanisms when working with maximin as a solution concept. Nevertheless, there is a large class of SCR's which can be truthfully implemented in maximin strategies as the following theorems show. We shall postpone to the future a complete characterization of all such SCR's.

Theorem 6.3. *If* SSCR $f: \mathscr{R} \rightrightarrows A$ *satisfies general no veto power, then it can be truthfully implemented in maximin strategies—in particular, all strategies are maximin, and hence so is the truth.*

Proof. The result is immediate, since, if we define $g: \mathscr{R} \rightrightarrows A$ so that $g(R) \in f(R)$, a player's changing strategies does not affect the scope of possible alternatives which could arise as others' strategies vary. In particular, the worst outcome remains the same. Therefore, all strategies are maximin. ‖

Theorem 6.4. *If* SSCR $f: \mathscr{R} \rightrightarrows A$ *satisfies* IPM, *then it can be truthfully implemented in maximin strategies.*

Proof. Immediate from Theorem 4.3.1, because dominant strategies are maximin strategies. ‖

7. INDIRECT MECHANISMS: NASH AND CONJECTURAL STRATEGIES

7.1. *Direct versus Indirect Mechanisms*

In the previous three sections we were concerned, for the most part, with direct mechanisms. Our attention could be safely confined to such mechanisms because of theorems which asserted that, with some qualifications in the case of Bayesian or maximin equilibrium, any implementation of an SCR can be transformed into an equivalent direct mechanism which truthfully implements the SCR.

This section represents a departure from direct mechanisms. When Nash equilibrium is the solution concept, demanding the existence of a truthful implementation requires, in fact, the existence of a *dominant-strategy* mechanism, as the following almost trivial theorem shows.

Theorem 7.1.1. *The SCR* $f: \mathscr{R} \rightrightarrows A$ *can be truthfully implemented in Nash strategies if and only if it can be truthfully implemented in dominant strategies.*

Proof. If f can be truthfully implemented in dominant strategies, it obviously is implementable in Nash strategies since a dominant strategy equilibrium is a Nash equilibrium. Suppose that $g: \mathscr{R} \rightarrow A$ truthfully implements f in Nash strategies. Then

$\forall R \in \mathscr{R}$, truthful revelation constitutes a Nash equilibrium. But this means that $\forall i$, $\forall R \in \mathscr{R}$, $\forall R'_i \in \mathscr{R}_i$, $g(R)R_i g(R'_i, R_{-i})$. In other words, the truth is a dominant strategy. ‖

This simple result already suggests that, when looking at Nash equilibrium, there is no compelling reason to focus on direct mechanisms. If one demands truthful revelation, one is back in the dominant strategy case, whereas, if one tries to go farther with Nash than with dominant strategy equilibrium, (i.e. tries to expand the class of implementable SCR's by weakening dominant strategies to Nash strategies) one must expect individuals, in general, to be lying in equilibrium. To the extent that individuals lie, we might argue, the case for *direct* mechanisms is weakened, for why should we be especially interested in preference announcements if they are false? Yet there is an even more compelling reason for turning our attention to indirect mechanisms, which is that there are some important SCR's which cannot be implemented in Nash strategies by direct mechanisms but which can be implemented by appealing to indirect mechanisms.

Example 7.1.2. Let $A = \{a, b, c, d, e, x\}$, $\mathscr{R}_1 = \{R_1, R'_1\}$, $\mathscr{R}_2 = \{R_2, R'_2\}$, and $\mathscr{R}_3 = \{R_3\}$, where

$$R_1 = \begin{pmatrix} c \\ a \\ e \\ x \\ b \\ d \end{pmatrix}, \quad R'_1 = \begin{pmatrix} d \\ a \\ e \\ x \\ b \\ c \end{pmatrix}, \quad R_2 = \begin{pmatrix} e \\ b \\ c \\ d \\ a \\ x \end{pmatrix}, \quad R'_2 = \begin{pmatrix} x \\ b \\ c \\ d \\ a \\ e \end{pmatrix}, \quad \text{and } R_3 = \begin{pmatrix} a \\ b \\ c \\ d \\ e \\ x \end{pmatrix}$$

Define the SSCR f^*: $\mathscr{R}_1 \times \mathscr{R}_2 \times \mathscr{R}_3 \rightrightarrows A$ so that

$$f^*(R_1, R_2, R_3) = f^*(R'_1, R'_2, R_3) = \{a\}.$$

and

$$f^*(R_1, R'_2, R_3) = f^*(R'_1, R_2, R_3) = \{b\}.$$

Now if f^* could be implemented in Nash strategies by a direct mechanism g, g would be a game form with two strategies for player 1, two for player 2 and one for player 3. Such a game form can be represented by a 2×2 matrix, where player 1 chooses rows as strategies and player 2, columns. Because a must be a Nash equilibrium outcome for the preference profiles (R_1, R_2, R_3), a must be an entry somewhere in the matrix. Without loss of generality, assume it falls in the upper left-hand box. If this entry is to be a Nash equilibrium for (R_1, R_2, R_3), but not for (R_1, R'_2, R_3) or (R'_1, R_2, R_3), then the matrix must take the form:

a	x
d	

.

Now a must be a Nash equilibrium outcome for the profile (R'_1, R'_2, R_3), but the upper left-hand corner is not an equilibrium outcome for this profile. Therefore, a must occur in the lower right-hand corner. But, this precludes b being an equilibrium outcome for $R_1, R'_2, R_3)$ or (R'_1, R_2, R_3). Therefore, there is no direct mechanism which implements f^* in Nash strategies. On the other hand, it is a simple matter to verify that f^* is monotonic and satisfies no veto power. Thus, by Theorem 7.1.3 below, f^* *can* be implemented in Nash strategies. f^* is, therefore, an example of an SSCR which can be implemented only by appealing to indirect mechanisms.

The general theorem on the existence of implementation in Nash strategies is the following:

Theorem 7.1.3.   *Consider the (n-person) SCR f: $\mathscr{R} \rightrightarrows A$, where $n \geq 3$. If f is fully implementable in Nash strategies, then it is monotonic. Furthermore, if it is monotonic and satisfies no veto power, then it is fully implementable.*

Proof. See Theorems 2 and 5 of Maskin (1977). ‖

As Theorem 7.1.3 asserts, monotonicity is necessary but not quite sufficient for full implementation. Actually, in a large class of cases—including most economic environments —monotonicity is both necessary and sufficient. Suppose there exists a fixed stock of some desirable private commodity, the distribution of which among individuals is part of the description of a social alternative. Suppose furthermore, that for each possible distribution of this commodity there corresponds at least one social alternative and that preferences of each individual are such that he always prefers an increase in his own consumption of the good to that of anyone else. Then if there are at least three individuals, no two individuals can agree that a given allocation is best because not both individuals can receive all of the private commodity in that allocation. Therefore, the NVP condition is satisfied vacuously.

7.2. *Monotonicity and* IPM: *Nash versus Dominance*

In those environments where NVP holds vacuously we can make a useful comparison between the conditions for Nash and dominant strategy implementability.

First, it is of some interest to note that IPM does *not* imply monotonicity. That is. dominant strategy implementability does not imply Nash implementability.

Example 7.2.1.[4] Let $A = \{a, b\}$, and $I = \{1, 2, 3\}$. Define f: $\mathscr{R}_A \rightrightarrows A$ so that

$$f(R_1, R_2, R_3) = \begin{cases} \{b\}, & \text{if for two or more individuals, } bP_i a \\ \{a\}, & \text{otherwise} \end{cases}$$

This f satisfies IWM, as can easily be checked, and so is fully implementable in coalitionally dominant strategies. f is not monotonic, however, because if $bP_1'a$, $bP_2'a$, and $bP_1''a$, $aI_2''b$, $aP_3''b$ then $bR_i'a \Rightarrow bR_i''a$ (all $i \in I$) and $b \in f(R')$ but $b \notin f(R'')$. So f is not implementable in Nash strategies. This observation may seem somewhat startling because the concept of a dominant strategy equilibrium is very much more demanding than that of a Nash equilibrium. The apparent paradox is resolved by noticing that the difficulty with implementation (as opposed to truthful implementation) may be not so much ensuring the existence of equilibrium outcomes in the choice as the *non-existence* of equilibrium outcomes *outside* the choice set. For instance, in Example 7.2.1, if preferences satisfy $bP_i a$ ($i = 1, 2, 3$) and f itself is used as a direct mechanism, then the Nash equilibrium $aP_i b$ ($i = 1, 2, 3$) gives rise to the non-f-optimal outcome a. In overcoming the problem of non-optimal equilibria, dominant strategies have an advantage over Nash dominant strategy equilibria by their very stringency, making non-optimal equilibria less likely.

A dominant strategy implementable SSCR which is not Nash implementable is impossible when one considers domains consisting only of strong orderings.

Theorem 7.2.2.   *If $\mathscr{R}$ consists of strong orderings and if f: $\mathscr{R} \rightrightarrows A$ is an n-person ($n \geq 3$) SSCR satisfying NVP which is implementable in dominant strategies, then f is implementable in Nash strategies.*

Proof. If f and $\mathscr{R}$ satisfies the hypotheses of the theorem, then f satisfies IPM by Theorem 4.3.1. By Theorem 3.2.1, f satisfies monotonicity. By Theorem 7.1.3, f is implementable in Nash strategies. ‖

Not surprisingly, monotonicity does not in general imply IPM. Nevertheless there is an important class of cases where IPM does follow from monotonicity.

Theorem 7.2.3. *If $\mathcal{R}$ is a rich domain, then if the SSCR $f: \mathcal{R} \rightrightarrows A$ is implementable in Nash strategies, it is truthfully implementable in dominant strategies.*

Proof. If f and $\mathcal{R}$ satisfy the hypotheses of the theorem, then f is monotonic. By Theorem 3.2.2, f satisfies IPM. The result follows from Theorem 4.3.1.   ‖

Theorem 7.2.3 which is a generalization of a theorem due to Roberts (1977), should not be misinterpreted; it applies only to singleton-valued SCR's—that is, SSCR's. There are many non-singleton-valued SCR's—e.g. the Pareto rule—which are monotonic, hence Nash implementable (if they also satisfy NVP, as does the Pareto rule) but certainly not dominant strategy-implementable. Nevertheless, Theorem 7.2.3 is a highly negative result for planners who insist on always associating a unique social optimum with any given preference profile. The theorem implies that, at least for rich domains, there is nothing to be gained from looking for Nash- rather than dominant strategy-implementations. The set of dominant strategy-implementable SSCR's subsumes those implementable by Nash strategies and, in view of Example 7.2.1, may *properly* subsume them. In the case of a multi-valued SCR f, like the Pareto rule, which is Nash—but not dominant strategy—implementable, Theorem 7.2.3 tells us that there does not exist a monotonic singleton-valued selection. That is, there does *not* exist an SSCR $f^*: \mathcal{R} \rightrightarrows A$, with $f^*(R) \subseteq f(R)$ for all R, such that f^* is monotonic.

It should be noted that Theorem 7.2.3 depends on the hypothesis of richness as Example 7.1.2 shows. Recall that the SSCR in Example 7.1.2 is Nash implementable. Observe, however, that it does not satisfy IPM because

$$(R_1, R_2, R_3): \{a, b\} = (R_1', R_2, R_3): \{a, b\}$$

and $a \in f^*(R_1, R_2, R_3)$, yet $b \in f^*(R_1', R_2, R_3)$. Therefore f^* is not truthfully implementable in dominant strategies. Notice that f^* is not a contradiction of Theorem 7.2.3 because its domain is not rich.

Example 7.1.2 should not be construed as a particularly persuasive argument in favour of Nash implementation over dominant strategy implementation. It is probably true that most domains of economic interest are rich and therefore fall within the province of Theorem 7.2.3. In particular we have the following corollaries.

Corollary 7.2.4. *Let A^E be the set of allocations of m goods ($m \geq 2$) among n consumers ($n \geq 2$) and let $\mathcal{R}^E$ be the class of profiles of " economic " preferences, as in Example 3.1.2. If $f: \mathcal{R}^E \rightrightarrows A^E$ is an SSCR satisfying the Pareto property and which is implementable in Nash strategies, then f is dictatorial.*

Proof. If f satisfies the hypotheses of the corollary, then, since the domain is rich, f is truthfully implementable in dominant strategies, by Theorem 7.2.3. By Theorem 4.4.1, f is dictatorial.   ‖

Corollary 7.2.5. *If A contains at least three alternatives, if $\mathcal{R}_A$ is the class of all preference orderings over A, and if $f: \mathcal{R}_A^n \rightrightarrows A$ is an n-person SSCR which satisfies citizen sovereignty and which is implementable in Nash strategies, then f is dictatorial.*

Proof. If f satisfies the hypotheses of the corollary it is implementable in dominant strategies by Theorem 7.2.3 since $\mathcal{R}_A$ is rich. Therefore, by Corollary 4.7.4, f is dictatorial.   ‖

A game form $g: S \rightarrow A$ has *interchangeable Nash equilibria* (with respect to the domain $\mathcal{R}$) if, $\forall R \in \mathcal{R}$, $\forall s', s'' \in S$, whenever s' and s'' are Nash equilibria of g for profile R, then any s such that $\forall i$, $s_i \in \{s_i', s_i''\}$ is also a Nash equilibrium. An SCR is *implementable in interchangeable Nash strategies* if it has an implementation with interchangeable Nash equilibria.

Using Corollary 7.2.5, one can easily establish that if we require that the Nash equilibria of an implementation be interchangeable, then when the domain of preferences is unrestricted, the only SCR's satisfying citizen sovereignty are dictatorial. This result was first proved by Sussangkarn (1978).

Corollary 7.2.6. *If A contains at least three alternatives and if $f: \mathscr{R}_A^n \rightrightarrows A$ is an n-person SCR which satisfies citizen sovereignty and which is implementable in interchangeable Nash strategies, then f is dictatorial.*

Proof. Let $\mathscr{R}_A^*$ be that subset of $\mathscr{R}_A$ consisting of strong orderings, and let f^* be the restriction of f to $(\mathscr{R}_A^*)^n$. Suppose that $g: S \to A$ implements f in interchangeable Nash strategies. g clearly implements f^*. Suppose that for some $R \in (\mathscr{R}_A^*)^n$ and $a \neq b$, $\{a, b\} \subseteq f^*(R)$. Let s^a and s^b be Nash equilibria for R such that $g(s^a) = a$ and $g(s^b) = b$. From interchangeability, (s_1^b, s_{-1}^a) is also a Nash equilibrium, and because individual 1 is not indifferent between distinct alternatives, $g(s_1^b, s_{-1}^a) = a$. Continuing iteratively, $g(s_1^b, \ldots, s_i^b, s_{i+2}^a, \ldots, s_n^a) = a$ for any i, and, hence, $g(s^b) = a$, a contradiction. Therefore, f^* must be singleton-valued. From Corollary 7.2.5, f^* is dictatorial, and hence so is f. ‖

7.3. *Generalized Stackelberg Equilibrium*

While Nash equilibrium is a very compelling solution concept for non-cooperative games of complete information, it is often considered to imply rather naive behaviour on the part of agents in games of incomplete information. This attitude is taken because the story that we tell about how Nash equilibrium arises with incomplete information is one of iterative adjustment, and in any adjustment procedure, a " sophisticated " agent can manipulate the direction in which adjustment occurs to his advantage by behaving in a non-Nash-like manner. In view of this possibility, Hurwicz (1975) examines the notion of " manipulable Nash equilibrium ", where sophisticated agents play a higher order game. Instead of studying this possibility, we will look into the notion of a generalized joint Stackelberg equilibrium.

A sophisticated agent will presumably believe that his own choice of strategy will influence the choice of strategies by others. This belief can be captured by the notion of a conjecture. Given a game form $g: S \to A$ and a space of characteristics Θ a conjecture for agent i is a mapping $\psi_i: S_i \times S \times \Theta \to S_{-i}$. Given the current vector of strategies $s \in S$, his own contemplated deviation s_i', and the characteristics of agents $\theta \in \Theta$, $\psi_i(s_i', s, \theta)$ is agent i's conjecture about other players' responses to his deviation. Nash behaviour is captured by the conjecture $\psi_i(s_i, \bar{s}, \theta) = \bar{s}_{-i}$. What we shall call *generalized Stackelberg conjectures* are those of the form

$$\psi_i(s_i, \bar{s}, \theta) \equiv \psi_i^*(s_i, \theta_{-i}).$$

That is, the agent believes that the response by other agents to his own deviation s_i depends only on that deviation and their characteristics. Current strategies are irrelevant. A special case of a generalized Stackelberg conjecture is the *ordinary Stackelberg conjecture* $\psi_i(s_i, \bar{s}, \theta) \equiv \psi_i^{**}(s_i, \theta_{-i})$, where $(s_i, \psi_i^{**}(s_i, \theta_{-i}))$ constitutes a Nash equilibrium for the players other than i. That is, if $\bar{s}_{-i} = \psi_i^{**}(s_i, \theta_{-i})$,

$$g(s_i, \bar{s}_{-i})R(\theta_j)g(s_i, s_j, \bar{s}_{-i-j}) \quad \text{for all } j \neq i \text{ and all } s_j \in S_j.$$

For generalized Stackelberg conjectures $\psi_1^*, \ldots, \psi_n^*$, *joint generalized Stackelberg equilibrium* (JGSE) (with respect to a profile θ of characteristics) is a strategy vector $s^* \in S$ such that:

$$\forall i \in I, \forall s_i \in S_i, \ g(s^*)R(\theta_i)g(s_i, \psi_i^*(s_i, \theta_{-i}))$$

and

$$s_{-i}^* = \psi_i^*(s_i^*, \theta_{-i}).$$

We might hope, *a priori*, that the notion of a joint generalized Stackelberg equilibrium might provide an intermediate concept between Nash and dominant strategy equilibrium. Ostensibly, it appears to dictate behaviour which is not so naive as that of Nash but yet does not make as severe demands on game form construction as do dominant strategies. That this hope is not well-founded is demonstrated by our final result, related to a set of results by d'Aspremont and Gérard-Varet (1977*b*), which shows that JGSE is essentially equivalent to dominant strategy equilibrium.

Theorem 7.3. *An SSCR $f: \Theta \rightrightarrows A$ is implementable in generalized Stackelberg strategies iff it is truthfully implementable by dominant strategies.*

Proof. First observe that a dominant strategy equilibrium is trivially a joint ordinary Stackelberg equilibrium, so half of the theorem is immediate. Suppose that $g: S \rightarrow A$ implements f in generalized Stackelberg strategies. From the equilibrium correspondence, choose any selection $s^*: \Theta \rightarrow S$ and construct the equivalent direct mechanism

$$h(\theta) = g(s^*(\theta)).$$

Consider any i and any $\eta_i \in \Theta_i$. Because $s^*(\theta)$ is a JGSE

$$g(s^*(\theta))R(\theta_i)g(s_i^*(\eta_i, \theta_{-i}), \psi_i^*(s_i^*(\eta_i, \theta_{-i}), \theta_{-i})).$$

Because $s^*(\eta_i, \theta_{-i})$ is a JGSE,

$$\psi_i^*(s_i^*(\eta_i, \theta_{-i}), \theta_{-i}) = s_{-i}^*(\eta_i, \theta_{-i}).$$

Therefore, $g(s^*(\theta))R(\theta_i)g(s^*(\eta_i, \theta_{-i}))$

i.e. $h(\theta)R(\theta_i)h(\eta_i, \theta_{-i})$ for all $\eta_i \in \Theta_i, \forall \theta_{-i} \in \Theta_{-i}$.

So h is straightforward and obviously truthfully implements f. $\|$

8. CONCLUDING REMARKS

This paper does not purport to be a survey of what is known about incentive compatibility. Rather than summarizing results already in the literature, we have been more interested in developing general characterization theorems about implementability. Among these results, we feel the most important are:

(i) Theorem 4.4.1, which demonstrates that in any rich economic environment, any Pareto optimal SCR which can be implemented in dominant strategies must be dictatorial;

(ii) Theorem 4.7.1, which relates dominant strategy implementable SSCR's to social aggregation functions satisfying non-negative response;

(iii) Theorem 4.7.3, which relates dominant strategy implementable SSCR's to social welfare functions satisfying the Arrow conditions;

(iv) Theorem 7.2.3, which relates implementability of SSCR's in Nash strategies to implementability in dominant strategies;

(v) Corollary 7.2.4, which extends Theorem 4.4.1 for economic environments to SSCR's which can be implemented in Nash strategies.

(vi) Corollary 7.2.5, which extends the Gibbard–Satterthwaite Theorem (Corollary 4.7.4) to SSCR's which can be implemented in Nash strategies.

(vii) Theorem 7.3, which shows that implementability in generalized Stackleberg strategies is equivalent to truthful implementability in dominant strategies.

There are at least three general issues we have not discussed.

First, except for a short section on coalitional dominant strategies, we have confined ourselves to purely non-cooperative behaviour. Nonetheless, a considerable amount is

known about implementation in Nash strategies when coalitions can form; i.e. for implementation in strong equilibrium (see Kalai, Postlewaite and Roberts (1977a) and (1977b)). Indeed, characterization theorems in the same spirit as those provided for Nash equilibrium can be stated (see Maskin (1978a) and (1978b)).

Second, we have had nothing to say about the minimal message space size required for implementation. This is an issue treated by Hurwicz, Reiter and Saari (1978).

Finally, we have not discussed the stability of equilibrium. Stability is, of course, not an issue when the solution concept is dominant, Bayesian, or maximin strategies because then a player's best strategy does not depend on the strategies played by others, and so equilibrium can be attained in a one-shot procedure, where everyone simply plays his best strategy. The situation is markedly different, however, for Nash equilibrium, where whether an adjustment procedure convergences may be a legitimate source of concern to the planner. Unfortunately, little is known about the possibility of designing stable Nash implementations.

NOTES

1. " Direct " mechanisms are represented in the work of Vickrey (1961), Clarke (1971), Gibbard (1973), Groves (1973), Groves and Loeb (1975), Green and Laffont (1977), Hammond (1979), and Postlewaite (1979) amongst others. It is true that Gibbard (1973) and Groves (1973) allow arbitrary messages, but they look for dominant strategies and so, by Theorem 4.1.1, they are effectively looking for particular direct mechanisms.

2. A special case of Theorem 4.1.1 occurs in Green and Laffont (1977).

3. Sufficiency in Theorem 4.6.1 was proved by Groves (1973) who also showed that only " truthful " strategies, essentially, are dominant.

4. See Groves (1976) for another example, beside 7.2.1, in which not all Nash equilibria are optimal but all dominant strategy equilibria are.

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