SFI TRANSMISSION
COMPLEXITY SCIENCE FOR COVID-19

STRATEGIC INSIGHT: **Mechanism design can aid the market in meeting extraordinary needs under unusual circumstances.**[1]

FROM: **Eric Maskin, Harvard University**

DATE: 4 May 2020 **NO:** 026.1

Under normal circumstances, most goods and services are produced, bought, and sold through free markets. But in an emergency like a pandemic, markets may not suffice. Imagine, for example, that society suddenly needs to undertake tens or even hundreds of millions of virus tests a week (so that employers can put their employees back to work safely). To whom can we turn to produce the testing equipment? There may be many potential manufacturers, and how can we know who they all are? Even if we know their identities, how do we decide which ones should actually do the producing? How much should each produce? And what price should a producer receive to cover its costs?

If we had the luxury of time, the market might resolve all these questions: prices and quantities would adjust until supply and demand were brought into balance. But getting a new market of this size to equilibrate quickly is unrealistic. Furthermore, markets don't work well when there are concentrations of power on either the buying or selling side, as there might well be here. Fortunately, mechanism design can be enlisted to help.

1. Markets

Before getting to mechanism design, let's review why markets normally work so well. Suppose that there are many buyers and producers for some good. Suppose that buyer i enjoys (gross) benefit $b_i(x_i)$ from quantity x_i. Similarly, each producer j incurs cost $c_j(y_j)$ to produce y_j. Hence, society's net social benefit is:

$$\sum_i b_i(x_i) - \sum_j c_j(y_j). \tag{1}$$

[1] This piece is based on a Santa Fe Institute webinar talk on the Complexity of COVID-19, April 14, 2020.

At a social optimum, (1) is maximized subject to the constraint that supply equals demand:

$$\sum x_i = \sum y_j. \tag{2}$$

The solution to this constrained maximization is optimal in several senses:

(i) total production $\sum y_j$ and total consumption $\sum x_i$ are optimal;

(ii) y_j is optimal for each producer j; and

(iii) x_i is optimal for each buyer i.

Achieving all three optimalities may seem complicated, but the market provides a simple solution. If p is the price at which the good can be bought and sold, then each buyer i maximizes

$$b_i(x_i) - px_i \text{ (net benefit)} \tag{3}$$

and the first-order condition for this maximization is

$$b_i'(x_i) = p \ \ (b' \text{ denotes the derivative of } b). \tag{4}$$

Similarly, each producer j maximizes

$$py_j - c_j(y_j) \text{ (profit)} \tag{5}$$

with first-order condition

$$p = c_j'(y_j). \tag{6}$$

But notice that (4) and (6) are also the first-order conditions for the problem of maximizing (1) subject to (2). And so the market outcome attains the social optimum as long as p is chosen so that (2) holds. (Mathematically, p is the Lagrange multiplier for (2).)

But how do we get the right choice of p? In a free market, p falls if supply exceeds demand and rises if demand exceeds supply. Eventually, the equilibrating price is found. But this process takes time. In the meantime, the price may be way too high, in which case, buyers who need tests are being "gouged," or too low, in which case there may be a serious shortage of tests.

There is an additional problem with the market solution: it relies on producers and buyers being "small" so that they can't individually affect the price. If some of these agents are big (e.g., if one of the equipment-producers supplies a significant fraction of demand), then the optimizations in (3) and (5) have to be modified and a social optimum no longer obtains. Moreover, by withholding supply, a big producer can distort the price-adjustment procedure and generate an outcome in which the price is too high and market supply is too low relative to the optimum. (A big buyer can do just the opposite.)

2. Mechanism Design to the Rescue[2]

For both reasons, we now turn to mechanism design.[3] For now, let us assume that the government attaches (gross) benefit $b(\sum y_j)$ to total production $\sum y_j$. (In the next section we decompose $b(\sum y_j)$ into the underlying benefits $\{b_i(y_j)\}$ of test-equipment users.)

The government is interested in maximizing the net social benefit

$$b(\sum_j y_j) - \sum_j c_j(y_j)$$

but it doesn't know the cost functions $\{c_j\}$ (and may not even know the full set of potential producers). We solve this difficulty using a variant of the Vickrey–Clarke–Groves mechanism (Vickrey (1960), Clarke (1971), Groves (1973)). Specifically, the government announces a call for test-equipment production and has each potential producer j submit a cost function $\hat{c}_j$. It then computes the production levels $\{\hat{y}_j\}$ that maximize the *apparent* net social benefit

$$b(\sum_j y_j) - \sum_j \hat{c}_j(y_j) \tag{7}$$

and has producer k produce $\hat{y}_k$ and gives producer k a payment:

$$\left[b(\sum_j \hat{y}_j) - \sum_{j\neq k} \hat{c}_j(\hat{y}_j)\right] - \left[b(\sum_{j\neq k} \hat{y}_j^*) - \sum_{j\neq k} \hat{c}_j(\hat{y}_j^*)\right], \tag{8}$$

[2] This section and the next are a bit math-heavy. For a simple explanation, see section 4.

[3] An alternative to markets or mechanism design would be for government to simply order some company or companies to produce all the equipment. But this might be an extraordinarily inefficient outcome if these companies aren't up to the task or if there are other companies who could produce it much more cheaply (which the government is not likely to know in advance). Moreover, how does the government know equipment level is "right"?

where the levels $\{\hat{y}_j^*\}_{j\neq k}$ maximize $b(\sum_{j\neq k} y_j) - \sum_{j\neq k} \hat{c}_j(y_j)$.

Claim: Given that the government chooses $\{\hat{y}_j\}$ to maximize (7) and pays producer k the amount (8), it is optimal for producer k to report its costs *truthfully*, i.e., it will take $\hat{c}_k = c_k$.

Proof: The second expression in square brackets in (8) doesn't depend on $\hat{c}_k$ and so doesn't affect producer k's maximization. In effect, producer k maximizes

$$b(\sum_j \hat{y}_j) - \sum_{j\neq k} \hat{c}_j(\hat{y}_j) - c_k(\hat{y}_k). \tag{9}$$

But (9) is just net social benefit with cost functions c_k and $\{\hat{c}_j\}_{j\neq k}$, i.e., producer k's objective is the same as society's. Thus, the optimal choice of $\hat{c}_k$ is indeed c_k. Q.E.D.

3. Buyers' Benefits

Let us now decompose $b(\cdot)$ into $\sum_i b_i(\cdot)$.

Because government doesn't know the benefit functions $\{b_i\}$, it will have buyers report $\{\hat{b}_i\}$ (as well as having producers report $\{\hat{c}_j\}$) and, instead of maximizing (7) it will choose $\{\hat{x}_i\}$ and $\{\hat{y}_j\}$ to maximize

$$\sum_i \hat{b}_i(\hat{x}_i) - \sum j\hat{c}_j(\hat{y}_j) \text{ subject to } \sum_i \hat{x}_i - \sum_j \hat{y}_j \tag{10}$$

Buyer h then receives $\hat{x}_h$ and pays

$$\left[\sum_j \hat{c}_j(\hat{y}_j) - \sum_{i\neq h} \hat{b}_i(\hat{x}_i)\right] - \left[\sum_j \hat{c}_j(\hat{y}_j^*) - \sum_{i\neq h} \hat{b}_i(\hat{x}_i^*)\right],$$

where $\{\hat{x}_i^*\}$ and $\{\hat{y}_j^*\}$ maximize

$$\sum_{i\neq h} \hat{b}_i(x_i) - \sum_j \hat{c}_j(y_j). \tag{11}$$

By analogy with producer k's problem in section 2, it is optimal for buyer h in these circumstances to set $\hat{b}_h = b_h$.

4. Simple Example

Imagine that there is just a single buyer with benefit function $b(\cdot)$ and a single producer with cost function $c(\cdot)$. In that case, the government

(i) has the buyer report $\hat{b}(\cdot)$ and the producer report $\hat{c}(\cdot)$;

(ii) calculates z^* to maximize $\hat{b}(z) - \hat{c}(z)$;

(iii) has the producer produce z^* and deliver this to the buyer; and

(iv) pays the producer $\hat{b}(z^*)$ and taxes the buyer $\hat{c}(z^*)$.

Notice that the buyer's objective function is

$$b(z) - \hat{c}(z)$$

and the producer's is

$$\hat{b}(z) - c(z)$$

and so it is optimal for the buyer to report $\hat{b} = b$ and for the producer to report $\hat{c} = c$.

As usual in the mechanism design literature, the way to align social and individual goals is to give individual producers and buyers monetary transfers (either positive or negative) that transform their personal objective functions into the social objective function.

REFERENCES

1 Clarke, E. 1971. "Multipart Pricing of Public Goods." *Public Choice*. 11: 17-33.

2 Groves, T. 1973. "Incentives in Teams." *Econometrica*. 41(4): 617-631.

3 Vickrey, W. 1961. "Counterspeculation, Auctions, and Competitive Sealed Tenders." *Journal of Finance*. 16 (1): 8-37.

Read more posts in the Transmission series, dedicated to sharing SFI insights on the coronavirus pandemic: santafe.edu/covid19