Optimal Non-linear Taxation and the Design of Education Policy*

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Abstract

This paper studies the design of education policy in an optimal non-linear tax model with asymmetric information. It shows that both heterogeneity in ability and risky human capital investment (or the combination of the two) can provide a theoretical justification for government intervention in education. The sign of the optimal policy is exclusively determined by the Hicksian coefficient of complementarity. Specifically, when education increases (decreases) exposure to risk, or equivalently, when the wage elasticity of education is increasing (decreasing) in ability, the optimal policy is to tax (subsidise) education. But when heterogeneity and risk are combined, the sign of the optimal policy is indeterminate. Numerical results suggest that the magnitude of the optimal policy will depend on the strength of the insurance and redistributive motives. Income-contingent loans or education-dependent taxes and subsidies can implement the optimum.

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1 Introduction

1.1 Motivation

Government intervention in higher education, such as public subsidisation or outright provision, can be justified on many grounds. Some of these include providing equality of opportunity; alleviating credit constraints which can arise from the difficulty in posting collateral against risky human capital investments; the positive externalities of education and research; and remedying the (downward) distortionary impact of redistributive income taxes on human capital investments. However, irrespective of the motivation, (non-neutral) government policies will either incentivise or disincentivise human capital investment at the margin. So for ease of modelling, this paper will focus on the two simplest forms of government intervention: education taxes and subsidies.[1]

Education taxes and subsidies can impact individual decisions and outcomes through four channels. First, education taxes distort the labour supply of graduates, who effectively face a higher marginal tax rate. Second, subsidies are regressive in that they involve income transfers from non-graduates to graduates, with the latter typically having higher lifetime expected earnings (Autor et al., 1998).[2] Thirdly, by narrowing the gap between after-tax incomes for graduates, education taxes provide insurance against income risk. And fourthly, both education taxes and subsidies will distort individual human capital investment decisions at the margin. Therefore, there is a trade-off between equity and efficiency in designing education policies.

In this context, a growing number of countries[3] are combining redistributive income taxes with income-contingent loans (ICLs) to finance higher education — effectively a form of 'education tax'.[4] These loans typically offer cheap credit to students subject to the following conditions: repayment begins when post-graduation income exceeds a pre-determined threshold; annual repayments (as a proportion of income) are capped; and repayment ceases once the loan (plus interest)[5] has been repaid. While the intuitive benefits of these schemes — providing insurance against default and enabling consumption

[1]For a more general treatment of government intervention in higher education, including public provision, please see Schindler and Weigert (2008).

[2]Subsidies can also be regressive if the returns to education are biased towards higher ability individuals. This will be explored in Section 2.

[3]Countries that currently use ICLs to finance education include the United Kingdom, Australia and Zealand. See Chapman (2006) for a survey of the schemes employed in these and other countries.

[4]Of course, loan repayments are distinct from taxes. But they have a similar effect; both taxes and repayments distort decisions at the margin and reduce the returns to labour supply. Section 5 further clarifies the link between education dependent taxes and ICLs.

[5]In Australia the debt is indexed at the rate of inflation and there is no additional interest charged, delivering an implicit subsidy to graduates. By contrast, in the U.K., a non-zero real rate of interest is charged on student loans (Dearden et al., 2008).

smoothing — are well-known and understood, they have received only limited attention in the theoretical literature. To address this gap, one goal of this paper is to provide a theoretical foundation for the use of these schemes in an optimal non-linear tax setting.

Two candidate justifications for the use of ICLs and government intervention in higher education more generally will be considered in this paper: heterogeneity in the ability and the risky nature of human capital investments. To determine the validity of these justifications, this paper addresses the following three questions. First, can heterogeneity or uncertainty provide a theoretical justification for government intervention in education in an optimal non-linear tax setting? Second, what parameters are important for designing optimal human capital policies? And third, can ICLs or other policy instruments implement the optimum?

This paper will not consider the externalities or consumption benefits of education. Instead, the focus is purely on the observable private benefits of human capital accumulation: namely, increases in future earnings. This narrower focus allows for the impact of education policy on verifiable outcomes to be more precisely identified. However, it is important to acknowledge the role that externalities do play in shaping government policy in this area, particularly regarding the use of education subsidies.

1.2 Literature review

This paper builds on the optimal non-linear taxation literature pioneered by Mirrlees (1971) and Stiglitz (1982). This literature emphasises the role of informational asymmetries on the design of optimal tax schedules[6] but typically assumes exogenous human capital.

In a static model with heterogeneous agents who differ in innate ability, Bovenberg and Jacobs (2005) consider the role of education policies in an optimal non-linear tax setting. They model a homothetic human capital accumulation function and continuum of exogenous abilities in a one-period framework. They find that education subsidies should be set to alleviate the distortions of redistributive income taxes on human capital accumulation — a relationship they described to be akin to 'Siamese twins'. This result is an application of the efficiency theorem of Diamond and Mirrlees (1971) to a model with endogenous human capital accumulation. Maldonado (2008) and Bovenberg and Jacobs (2011) generalise this model to allow for more general earnings functions. In the former, the education elasticity of earnings depends on ability, while in the latter, it depends on both ability and labour supply. Importantly, both contributions highlight the importance of the complementarity between education and ability in determining the sign of the optimal policy.

One of the first papers to explore the interaction between optimal taxation and risky human capital investments was Eaton and Rosen (1980).[7] They prove that a non-zero income tax can be welfare improving because it provides wage insurance which is not present in lump-sum taxes and that without government intervention, there may be under-investment in education relative to the social optimum. More recently, Costa and Maestri (2007) and Anderberg (2009) have studied the role of uncertainty on the design of education policy in a optimal non-linear tax model with homogeneous agents.

[6]The presence of private information (e.g. ability types) motivates the use of non-linear income taxes to satisfy incentive compatibility.

[7]Hamilton (1987) was an important extension of Eaton and Rosen (1980) which considers the impact of allowing labour supply responses after the realisation of uncertainty.

However, their conclusions differ. The former, using binary productivity shocks, finds that the optimal policy always requires encouraging human capital investment at the margin. By contrast, the latter, using a continuum of productivity shocks, finds that the optimal policy should depend on the impact of education on wage risk. We affirm the latter contribution, finding that the interaction between human capital and risk will determine the sign of the optimal policy.

Findeisen and Sachs (2016) consider a two-period model with heterogeneous agents who differ in ability and face a binary education decision which is subject to uncertainty. Individual productivities are drawn from a continuous conditional cumulative distribution function which depends on ability and education. They find that ICLs can implement the optimum and use simulations to show that the welfare gain from using ICLs is increasing in the level of risk aversion in the agents' utility functions. Stantcheva (2017) derives optimal income tax and human capital policies in a life-cycle model with risky human capital. The government faces asymmetric information regarding agents ability, its evolution, and labour supply. Uncertainty arises from a per-period productivity shock which is incorporated in the earnings function. She finds that the sign of the optimal education subsidy is determined by the Hicksian coefficient of complementarity between education and ability, and that ICLs can again implement the second best optimum. Using simulations, she also shows that simple age-dependent linear taxes and subsidies can achieve most of the welfare gain from second best education taxes and subsidies.

While the two papers discussed immediately above are comprehensive, they do not address two important issues. First, it is not clear from their contributions how the impact of heterogeneity on the design of education policy differs from uncertainty (if at all). And second, they ignore they fail to consider (potential) interaction effects effects between the the two dimensions of private information.[8]

1.3 Summary of results

This paper fills these gaps in the literature. On the first point, we show that irrespective of whether there is heterogeneity or uncertainty, the sign of the optimal policy is determined by the same parameter — the Hicksian coefficient of complementarity between education and ability.[9] When this coefficient is greater than one, or equivalently, when the elasticity of wages with respect to ability is increasing in human capital, the second best policy is an education tax. And on the second point, we show that introducing uncertainty unambiguously increases the size of education taxes when agents differ in ability.

The rest of this paper is organised as follows. Section 2 analyses heterogeneity absent uncertainty using a one-period model. Section 3 extends this to a two-period model to explore the impact of risky human capital investments with homogeneous agents. Section 4 combines both agent heterogeneity and uncertainty and compares the second best allocation to the preceding chapters. Section 5 considers what decentralised policy instruments can implement the second best allocations and presents numerical simulations to shed more light on the design of an optimal ICL scheme. Finally, Section 6 concludes.

[8] We will characterise uncertainty by introducing idiosyncratic productivity shocks which are second-period private information in the two-period models considered in Sections 3 and 4.

[9] In the case of homogeneous ability and uncertainty, the appropriate Hicksian complementarity is between education and the productivity shock. For conciseness, we will hereafter refer to the 'Hicksian complementarity' in both cases.

2 Heterogeneity

This section analyses the impact of heterogeneity in innate ability on the design of education policy. The model we present is similar to Maldonado (2008) which builds on the original Stiglitz (1982) optimal taxation model with exogenous abilities (types).

The novel contribution of this section is the link we establish between the sign of the optimal policy and the Hicksian complementarity. We will also differ from the aforementioned papers by characterising the optimal policy in terms of 'wedges' — locally linear subsidies and taxes which would all be zero in the absence of government intervention — as is standard in the New Dynamic Public Finance (NDPF)[10] literature. The characterisation of the optimal policy in terms of wedges and the Hicksian complementarity enables easier comparison between the second best allocation in this section and the extensions presented in the sections that follow.

2.1 Set up

There is a measure one population consisting of heterogeneous agents who live for one period and differ in exogenous, innate ability θ^i. For simplicity,[11] we will consider the case of two ability levels θ^i, for $i \in \{H, L\}$ such that $\theta^H > \theta^L$, with the (exogenous) population proportions given by $\pi(\theta^i) = \pi^i$, for $i \in \{H, L\}$. We will refer to θ^H as the high-ability type and θ^L as the low-ability type.

Individuals derive utility from consumption $u(c)$ and disutility from labour supply $v(l)$. Preferences are additively separable,[12] $u(c) - v(l)$, where $u(.)$ is increasing, concave and continuously differentiable and $v(.)$ is decreasing, convex and continuously differentiable.

Productivity w depends on both exogenous ability θ^i and endogenous human capital e^i such that $w = w(e^i, \theta^i)$. Gross labour income is then given by $Y^i = w(e^i, \theta^i)l^i$. The wage (hereafter w) is strictly increasing in both education and ability $w_e > 0$, $w_\theta > 0$ and also strictly concave $w_{ee} < 0$, $w_{\theta\theta} < 0$ in both arguments.[13] Importantly, the sign of the cross-derivative $w_{e\theta}$ is not specified. As long as $w_{e\theta} \neq 0$, the marginal impact of education on wages will differ by ability.

[10] This literature considers policy instruments that are endogenously determined rather than restricting attention to pre-determined, implementable instruments.

[11] This assumption is not crucial and the results from the model will generalise to a continuum of abilities, as considered in Bovenberg and Jacobs (2011).

[12] Again, this assumption is not critical, but it ensures that the single-crossing property is always satisfied in this model.

[13] Unless otherwise indicated, this paper uses subscripts to denote partial derivatives. While θ is a binary variable and therefore w_θ is not well-defined, this notation is more general and implies the following ordering: $w(e^*, \theta^H) > w(e^*, \theta^L)$ for some level of education e^*.

Again for simplicity and to focus on heterogeneity as the justification for government intervention, we consider a binary education decision similar to Findeisen and Sachs (2016), e^i for $i \in \{H, L\}$, where e^i denotes the resource cost of education.[14] One possible interpretation is that e^H is the cost of graduating from university, while e^L is the cost of entering the labour market directly after high-school. Lastly, for ease of notation, prices of consumption and education are normalised to one without loss of generality, and we assume that there is no time-cost of education.

2.1.1 Hicksian complementarity

Following Stantcheva (2017), we will make use of the Hicksian coefficient of complementarity (Hicks, 1970), $\rho_{e\theta}$ (hereafter, the Hicksian complementarity).

$$\rho_{e\theta} = \frac{w w_{e\theta}}{w_e w_\theta} \tag{2.1}$$

In this heterogeneity model, the Hicksian complementarity is positive if and only if higher ability agents have higher marginal returns to education $w_{e\theta} > 0$. A Hicksian complementarity greater than one implies that higher ability agents have a higher proportional benefit from human capital, i.e. the wage elasticity with respect to ability is increasing in education $\frac{\partial}{\partial e}\left(\frac{\partial w}{\partial \theta}\frac{\theta}{w}\right) \geq 0$. The specification of the wage function is therefore critical for determining the value of this parameter. A separable wage function $w = e^i + \theta^i$ implies that $\rho_{e\theta} = 0$, while a multiplicative function, $w = e^i\theta^i$, as is typically found in the literature, means $\rho_{e\theta} = 1$. Finally, with a CES wage function of the form

$$w = \left[\alpha(\theta^i)^{1-\xi} + \alpha(e^i)^{1-\xi}\right]^{\frac{1}{1-\xi}} \tag{2.2}$$

ability and education can be substituted at fixed rate, $\rho_{e\theta} = \xi$. Throughout this paper however, we will not make any assumption on the functional form of the wage.[15]

2.2 Laissez faire

In the absence of government intervention, type i individuals will solve the following optimisation problem:

$$\max_{c^i, e^i} \quad u(c^i) - v\left(\frac{Y^i}{w(e^i, \theta^i)}\right)$$
$$\text{s.t.} \quad c^i \leq Y^i - e^i$$

This gives the usual first-order conditions (FOCs) determining labour supply and education at the optimum: the marginal rate of substitution between labour[16] and leisure

[14] In this model and the model in Section 4, the only cost of education is the resource cost. But in Section 3, we consider a model in which e^i incorporates forgone earnings.

[15] This generalisation is important because, as will be shown, the direction of optimal education policy in the second best allocation depends on the value of $\rho_{e\theta}$. The simulations presented in Section 5 use the CES wage function to isolate the impact of changing $\rho_{e\theta}$ on the optimal policy.

[16] Let $l^i = \frac{Y^i}{w(e, \theta^i)}$

is equated to the wage in Eq (2.3), and the marginal rate of substitution between education and consumption equals one[17] in Eq (2.4).

$$\frac{v'(l^i)}{u'(c^i)} = w(e^i, \theta^i) \tag{2.3}$$

$$\frac{v'(l^i)}{w(e^i, \theta^i)u'(c^i)} l^i w_e(e^i, \theta^i) = 1 \tag{2.4}$$

2.2.1 Wedges

Using Eq (2.3) and Eq (2.4) we can define the following wedges[18] (which are all zero in the *laissez faire* allocation). We will use these wedges — implicit[19] taxes or subsidies — to determine the amount and direction of distortion at an allocation relative to the *laissez faire* allocation. The labour wedge is defined as the gap between the marginal rate of substitution and the marginal rate of transformation between consumption and labour, while the education wedge is equal to the gap between the marginal cost and marginal benefit of human capital.

Labour

$$\tau_l(\theta^i) \equiv 1 - \frac{v'(l^i)}{u'(c^i)w(e^i, \theta^i)} \tag{2.5}$$

Gross education

$$\tau_e(\theta^i) \equiv 1 - (1 - \tau_l(\theta^i))l^i w_e(e^i, \theta^i) \tag{2.6}$$

Net education

$$t_e(\theta^i) \equiv \frac{\tau_e - \tau_l}{(1 - \tau_e)(1 - \tau_l)} \tag{2.7}$$

The distinction between the gross and net education wedges arises from the fact that there are two simultaneous distortions on human capital. A positive labour wedge will distort human capital investments downwards since part of the returns to education are taxed away. So even if the gross education wedge was zero, individual education decisions would still be distorted by a labour wedge (if present).[20] Therefore to account for the overall distortion on education, the net wedge is determined such that it equals zero when the tax system is neutral with respect to human capital.[21] In other words, when the net

[17] This follows from the normalisation of prices of education and consumption to one.

[18] We can interpret these wedges as follows: the labour wedge is positive (negative) if an individual works less (more) than they would in the *laissez faire* allocation.

[19] The wedges also have natural interpretations as marginal taxes or combinations of marginal taxes in a non-linear tax schedule. For example, the labour wedge would be the gradient of the tax function with respect to income.

[20] If there is no labour wedge at the optimum, it is sufficient to look at the gross education wedge.

[21] As shown by Stantcheva (2017), in this one period model, full-deductibility of human capital (education subsidy equal to the marginal cost of education times the labour tax rate) $\tau_e = \tau_l$ will ensure a zero net wedge.

wedge is zero, the tax scheme implements the *laissez faire* education allocation conditional on labour supply (i.e. Eq (2.4) holds).

2.3 First best

In the first best with complete information, the utilitarian social planner would solve the following problem:

$$\begin{aligned} \max_{c^i, Y^i, e^i} \quad & \sum_{i \in \{H,L\}} \pi^i \left[u(c^i) - v\left(\frac{Y^i}{w(e^i, \theta^i)} \right) \right] \\ \text{s.t.}(\lambda) \quad & \sum_{i \in \{H,L\}} \pi^i \left[Y^i - c^i - e^i \right] \geq 0 \end{aligned}$$

It can be shown that, at the optimum, labour and education wedges are equal to zero for both types, $\tau_l(\theta^i) = \tau_e(\theta^i) = 0$ for $i = \{H, L\}$, and the high-ability type works more $v'(l^H) > v'(l^L)$.[22]

2.4 Second best

The first best solution is not feasible when there is asymmetric information stemming from the inability to observe innate ability and labour supply.[23] This means that when observing a low gross income produced by an agent, the planner cannot tell whether this is due to low labour supply or low ability. As a result, the government must rely on indirect mechanisms (or equivalent direct mechanisms as per the revelation principle) which induce individuals to self-select themselves through the choice of observable allocation (c^i, Y^i, e^i) designed for their type. And following the usual practice in the optimal tax literature, we will assume that only the downward incentive compatibility constraint (μ) — the constraint preventing the high-ability type from mimicking the low-ability type — is binding at the optimum.[24]

Therefore the social planner will solve:[25]

[22]The planner also equalises (marginal utility of) consumption across both types in the first best. Derivation of the first best wedges is provided in Appendix A.1.

[23]We assume that human capital e^i, consumption c^i and gross labour income Y^i are observable.

[24]This assumption is valid due to the single-crossing property and from observing that in the first best allocation, both types receive the same level of consumption but the high-ability type works more. Therefore it is only the high-ability type that has an incentive to mimic.

[25]An equivalent approach from the mechanism design literature would be for the planner to design the mechanism (R^i, Y^i, e^i) – where $R^i = Y^i - T^i$ is disposable income and T^i is the non-linear income tax – which induces individuals to self-select themselves to the correct allocation for their type. This approach was used in Maldonado (2008).

$$
\begin{aligned}
\max_{c^i,Y^i,e^i} \quad & \sum_{i\in\{H,L\}} \pi^i \left[u(c^i) - v\left(\frac{Y^i}{w(e^i,\theta^i)}\right)\right] \\
\text{s.t.}(\lambda) \quad & \sum_{i\in\{H,L\}} \pi^i \left[Y^i - c^i - e^i\right] \geq 0 \\
(\mu) \quad & u(c^H) - v\left(\frac{Y^H}{w(e^H,\theta^H)}\right) \geq u(c^L) - v\left(\frac{Y^L}{w(e^L,\theta^H)}\right)
\end{aligned}
$$

2.4.1 Optimal labour wedge

Solving the FOCs with λ and μ as the Lagrange multipliers on the resource and incentive compatibility constraints respectively, the labour supply of the high-ability type is not distorted, while the labour supply of the low-ability type is distorted downwards:[26]

$$\tau_l(\theta^H) = 0 \tag{2.8}$$

$$\tau_l(\theta^L) = 1 - \frac{1 - \frac{\mu}{\pi^L}}{1 - \frac{\mu v'(l^{LH}) w(e^L,\theta^L)}{\pi^L v'(l^L) w(e^L,\theta^H)}} > 0 \tag{2.9}$$

The zero-marginal tax rate on the high-ability type and positive marginal tax rate on the low-ability type in the second best allocation is the standard result in optimal tax problems with two types (Mirrlees, 1971; Stiglitz, 1982; Guesnerie and Seade, 1982).

2.4.2 Optimal net education wedge

Similarly, the education decision of the high-ability type is not distorted,[27] while the net distortion on the low-ability type depends on how the education elasticity of the wage $\eta(e^i,\theta^i)$ varies with ability.[28]

$$\eta(e^i,\theta^i) = \frac{e^i w_e(e^i,\theta^i)}{w(e^i,\theta^i)} \tag{2.10}$$

If $\eta(e^i,\theta^i)$ is positive (negative), the proportional benefit of an additional unit of human capital is greater (smaller) for high-ability types relative to low-ability types.

$$\tau_e(\theta^H) = 0 \tag{2.11}$$

$$t_e(\theta^L) = \frac{1}{1-\tau_l(\theta^L)}\left[1 - \frac{\eta(e^L,\theta^H)}{\eta(e^L,\theta^L)}\right] \lesseqgtr 0 \tag{2.12}$$

There are then three possible cases for the net education distortion for the low-ability type, depending on the value of $\rho_{e\theta}$.[29]

[26] This follows from the convexity of $v(.)$ and that $l^{LH} = \frac{Y^L}{w(e^L,\theta^H)} < l^L$. Derivation of the second best labour wedges are provided in Appendix A.1.

[27] Since $\tau_l(\theta^H) = 0$, we can focus on the gross education wedge $t_e(\theta^H)$ for high ability.

[28] $\eta_\theta = \frac{e w w_{e\theta} - e w_e w_\theta}{w^2} = \frac{e w_e w_\theta}{w^2}(\rho_{e\theta} - 1)$. Therefore $\eta_\theta > 0 \Longleftrightarrow \rho_{e\theta} < 1$.

[29] Maldonado (2008) derives a similar result, but does not make the link between education elasticities and the Hicksian complementarity.

Case I: If $\rho_{e\theta} < 1$, the net education wedge for the low-ability type is positive, $t_e(\theta^L) > 0$.

Case II: If $\rho_{e\theta} = 1$, the net education wedge for the low-ability type is zero, $t_e(\theta^L) = 0$.

This is the 'Siamese twins' result first derived in Bovenberg and Jacobs (2005). The education subsidy is designed to make education expenses deductible against the labour income tax rate, creating a zero net wedge at the optimum.

Case III: Finally, if $\rho_{e\theta} > 1$, the net education wedge for the low-ability type is negative, $t_e(\theta^L) < 0$.

2.5 Discussion

The above analysis of the three possible cases gives rise to the following proposition which summarises the key finding from this section:

Proposition 1 *When individuals differ in innate ability and there is no uncertainty in returns to human capital investment, there is a net upward (downward) distortion on education at the optimum when the wage elasticity of education decreases (increases) in ability.*

This proposition suggests that whenever $\rho_{e\theta} \neq 0$, there will be a role for specific education policy, $\tau_e \neq 0$. Empirical work has confirmed this premise, finding that innate ability and other factors independent of education are important wage determinants (Taber, 2001). This implies that heterogeneity in ability can justify government intervention in education in an optimal non-linear tax system.

However, the model considered in this section abstracts away from one important characteristic of human capital investment: risk. Intuitively, uncertainty in the returns to education will distort human capital investments downwards and creates a role for insurance. Both of these aspects have implications for policy design. Therefore, we model the impact of uncertainty on the design of education policy in the next section.

3 Uncertainty

This section analyzes the impact of uncertainty on the design of education policy using a model with homogeneous agents. Uncertainty will be introduced through productivity shocks which affect the return to human capital investment. The model we present is similar to the model with binary productivity shocks considered in Costa and Maestri (2007) and with a continuum of productivity shocks considered in Anderberg (2009).

This section makes two contributions. First, we confirm the claim made in Anderberg (2009) that Costa and Maestri (2007) erred in their characterisation of the optimal policy. And second, we derive a new expression for the second best distortion which depends on the Hicksian complementarity.

3.1 Set up

To focus solely on the role of uncertainty, let us assume all agents are ex-ante identical (i.e. one representative agent). Unlike the model in Section 2, there are now two periods $t = 1, 2$ and we assume that all individuals have the same discount rate $\beta \in (0, 1]$. Individuals can save s in the first period and earn a return of R, and utility is assumed to be additive across periods.

Next assume that individuals allocate time between education e and labour l^1 in the first period and between labour l^2 and leisure $1 - l^2$ in the second period. This means, in contrast to the model in Section 2, the only cost of education is forgone labour income (i.e. a time cost). However, adding a resource cost of education would not change our results as long as both costs (resource and forgone labour income) are verifiable by the government and the investments can be subsidised (or taxed) (Jacobs and Yang, 2016; Costa and Maestri, 2007; Bovenberg and Jacobs, 2005). So to focus on the role of uncertainty, we do not include a resource cost of education in this section.

Productivity at $t = 1$ is w^1 for all individuals, normalised such that $w^1 = 1$. Therefore gross labour income in the first period is $1 - e$, the difference between the time endowment and time spent in human capital acquisition. At $t = 2$, productivity w^2 (hereafter w) depends on first-period education e and and a parameter θ^i, according to the wage function from Section 2, $w(e, \theta^i)$.

Following Stantcheva (2017), we will now interpret this parameter (θ^i) as an idiosyncratic productivity shock. Uncertainty arises from the fact that the productivity shock is second-period information. Therefore, individual human capital investments undertaken in the first period are subject to uncertainty. With homogeneous ability and uncertainty, $\rho_{e\theta}$ now represents the Hicksian complementarity between education and the productivity shock. It is positive (negative) when education increases (decreases) exposure to wage risk.

Importantly, the nature of the risk considered here is such that education may increase or decrease the wage at high shocks relative to low shocks, $w_{e\theta} \lesseqgtr 0$.[30] Following Costa and Maestri (2007), we assume that individuals cannot work for the same firm in both periods. This assumption precludes insurance contracts between firms and workers and introduces a potential role for government intervention.

As before, we will analyse the case of two discrete productivity shocks $\theta^i, i \in \{H, L\}$ with $\theta^H > \theta^L$. We will refer to θ^H as the high-shock and θ^L as the low-shock. While the shock the individual receives is second-period private information, we will assume the distribution of shocks is common knowledge with a PMF given by $\pi(\theta^i) = \pi^i, i \in \{H, L\}$.[31] Finally, because there is no heterogeneity at $t = 1$ all individuals consume a common amount c^1. By contrast, consumption at $t = 2$ will generally vary with the productivity shock θ^i and will be denoted by c^{2i}.

The representative agent's lifetime utility is then given by:[32]

$$u(c^1) + \beta \sum_{i=H,L} \pi^i \left[u'(c^{2i}) - v\left(\frac{Y^i}{w(e, \theta^i)}\right)\right] \tag{3.1}$$

3.2 Laissez faire

Absent government intervention, individuals who receive shock θ^i must solve the following optimisation problem in $t = 2$.

$$\begin{aligned} V^i(\theta^i, e, s) \equiv \quad & \max_{c^{2i}, Y^i} \quad u(c^{2i}) - v\left(\frac{Y^i}{w(e, \theta^i)}\right) \\ & \text{s.t.} \quad c^{2i} \leq Y^i + Rs \end{aligned}$$

Taking the first-order conditions yield the usual labour supply efficiency condition in the second period.

$$\frac{v'(l^i)}{u'(c^{2i})} = w(e, \theta^i) \tag{3.2}$$

And in the first period, individuals optimally allocate time between labour supply and education according to:

$$\begin{aligned} & \max_{c^1, e} \quad u(c^1) + \beta \sum_{i \in \{H,L\}} \pi^i V^i(\theta^i, e, s) \\ & \text{s.t.} \quad c^1 \leq 1 - e - s \end{aligned}$$

[30] While the impact of considering uncertainty over ex-post incomes when undertaking human capital investments is well-established in the empirical literature (e.g. Carneiro et al. (2003) and Chen (2008)), there is no consensus on whether wage uncertainty increases or decreases with the level of education.

[31] By the 'law of large numbers', we assume that the ex-post distribution of shocks is identical to the ex-ante probability distribution of shocks faced by the individuals.

[32] For simplicity we also assume that leisure is not valued in the first period.

Taking the first-order conditions with respect to consumption and education gives the following:

$$u'(c^1) = \beta R \sum_{i=H,L} \pi^i u'(c^{2i}) \tag{3.3}$$

$$u'(c^1) = \beta \sum_{i=H,L} v'(l^i) l^i \frac{w_e(e,\theta^i)}{w(e,\theta^i)} \tag{3.4}$$

Eq (3.3) implicitly defines the appropriate savings wedge in this model (Eq (3.5)). It is equal to the difference between the expected marginal rate of inter-temporal substitution and the return on savings:

$$\tau_s \equiv 1 - \frac{u'(c^1)}{\beta R \left[\pi^H u'(c^{2H}) + \pi^L u'(c^{2L})\right]} \tag{3.5}$$

3.3 First best

Now, consider the optimal allocation chosen by a utilitarian social planner with complete information and subject to a resource constraint.

$$\max_{e,c^1,c^{2i},Y^i} \quad u(c^1) + \beta \sum_{i\in\{H,L\}} \pi^i \left[u(c^{2i}) - v\left(\frac{Y^i}{w(e,\theta^i)}\right)\right]$$

$$\text{s.t.}(\lambda) \qquad \sum_{i\in\{H,L\}} \pi^i \left[c^{2i} - Y^i\right] \le R(1 - e - c^1)$$

As is standard in optimal non-linear tax problems with redistributive social welfare functions, the benevolent planner would provide full-insurance by equalising the marginal utility of consumption across both productivity shocks $u'(c^{2H}) = u'(c^{2L})$. There is also a zero savings wedge in the first best allocation.

Of particular interest is the education first-order condition:[33]

$$\sum_{i\in\{H,L\}} \pi^i l^i w_e(e,\theta^i) = R \tag{3.6}$$

On the right is the opportunity cost of education – forgone labour income which earns interest. On the left is the marginal benefit of human capital investment. In the *laissez faire* and first-best allocations, marginal cost equals marginal benefit. Following Anderberg (2009), we will say there is a positive (negative) education premium when the left-hand side is larger (smaller) than the right-hand side.[34]

[33] In this section we will focus on the education FOC defined in Eq (3.6) rather than the education wedge to characterise the distortion in the second best allocation. This is done to keep the model tractable and the derivation is provided in Appendix A.2.

[34] The interpretation of a positive premium is very similar to a positive education wedge. A positive education premium implies that education should be distorted downward at the optimum, equivalent to setting $\tau_e > 0$.

3.4 Second best

Again, with incomplete information, the pareto-planner must respect incentive compatibility. However, the problem can be simplified by assuming that at the optimum, only the downward incentive compatibility constraint (high-shock mimicking the low-shock) is binding.[35]

The planner's problem can then be represented by:

$$\max_{e,c^1,c^{2i},Y^i} \quad u(c^1) + \beta \sum_{i\in\{H,L\}} \pi^i \left[u(c^{2i}) - v\left(\frac{Y^i}{w(e,\theta^i)}\right)\right]$$

$$\text{s.t.}(\lambda) \qquad \sum_{i\in\{H,L\}} \pi^i \left[c^{2i} - Y^i\right] \leq R(1 - e - c^1)$$

$$(\mu) \qquad u(c^{2H}) - v\left(\frac{Y^H}{w(e,\theta^H)}\right) \geq u(c^{2L}) - v\left(\frac{Y^L}{w(e,\theta^H)}\right)$$

3.4.1 Optimal labour wedge

The expression of the labour wedge in the second-period is the same as from Section 2, given by Eq (2.5). Once again, we derive that there is no distortion of the labour supply of the high-shock, while the labour supply of the low-shock is distorted downwards:[36]

$$\tau_l(\theta^H) = 0 \tag{3.7}$$

$$\tau_l(\theta^L) = 1 - \frac{1 - \frac{\mu}{\beta\pi^L}}{1 - \frac{\mu w(e,\theta^L)v'(l^{LH})}{\beta\pi^L w(e,\theta^H)v'(l^L)}} > 0 \tag{3.8}$$

3.4.2 Optimal savings wedge

Turning to savings, we obtain the standard result of a positive inter-temporal wedge $\tau_s > 0$ in a model with separable preferences and private information (Golosov et al., 2003). The intuition for this result is well-explained by Anderberg (2009) — any individual who under-reports their productivity shock would work less and have less consumption relative to truth-telling. Therefore, by the assumption of separable preferences, they have a higher marginal utility of consumption and greater incentive to save in the first period. For this reason, making savings less attractive ($\tau_s > 0$) relaxes the incentive constraint.

[35] See Guesnerie and Seade (1982) for a formal proof of why this assumption is valid at the constrained efficient allocation. A good explanation of the intuition behind this result is also provided in Anderberg (2009, p. 1019). Briefly, with a utilitarian social welfare function and separable preferences, any resource-feasible redistribution from individuals with higher consumption to individuals with lower consumption is welfare improving. Any such redistribution must therefore violate incentives at the optimum. Then, by the single-crossing property, each individual will strictly prefer their second period bundle (c^i, Y^i) to any feasible larger bundle $(c^{i'}, Y^{i'})$ where for each i, $c^i < c^{i'}$ and $Y^i < Y^{i'}$.

[36] Recall that $l^{LH} = \frac{Y^L}{w(e^L,\theta^H)} < l^L$. Deriviation is provided in Appendix A.2.

3.4.3 Optimal education

Finally, taking the first-order condition with respect to education gives:[37]

$$\sum_{i\in\{H,L\}} \pi^i l^i w_e(e,\theta^i) = R\left(1+\frac{\mu v'(l^{LH})l^{LH}}{\lambda R}\left[\frac{w_e(e,\theta^H)}{w(e,\theta^H)}-\frac{w_e(e,\theta^L)}{w(e,\theta^L)}\right]\right) \tag{3.9}$$

Clearly, there will be a positive (negative) education premium if the expression in brackets on the right-hand side is greater (smaller) than one. This will occur when individual productivity varies more with education at high shocks relative to low shocks.[38]

$$\frac{w_e(e,\theta^H)}{w(e,\theta^H)} > \frac{w_e(e,\theta^L)}{w(e,\theta^L)} \iff \rho_{e\theta} > 1 \tag{3.10}$$

3.5 Discussion

This gives rise to the following proposition:

Proposition 2 *When individuals are risk averse and make risky human capital investment decisions, the optimal policy is to tax (subsidise) education when education increases (decreases) wage risk. And if education is neutral with respect to wage risk, there is no distortion on education at the optimum.*

Therefore, uncertainty can also justify second best government intervention in education. Once again it is the Hicksian complementarity parameter that exclusively determines the sign of the optimal policy. But compared to the benchmark case of heterogeneity absent uncertainty summarised in Proposition 1, there are two differences.

First, with ex-ante homogeneous agents, labour income taxation has a pure insurance role and is no longer redistributive. And second, the optimal allocations are no longer history-dependent. If we extended the model in Section 2 to two periods, the agent's period one ability would be relevant for determining their period two allocation. But in this model, only the shock received in the second period is relevant.

The next section will examine whether adding a redistributive role (i.e. reintroducing heterogeneous ability) to labour income taxation changes any of these results. Intuition suggests that if education and ability are complements, adding uncertainty to the returns of human capital investment should make optimal taxes larger. This is because taxes must now play both a redistributive and insurance role. But in the case of substitutes, the insurance and redistribution motivations will conflict. It is then unclear what the sign of optimal education policy will be or what factors will determine the magnitude of the second best policy.

[37] This condition is similar to that derived in Anderberg (2009) for a continuum of productivity shocks. Importantly, it disproves the assertion in Costa and Maestri (2007) that the sign of education policy does not depend on the risk properties of human capital when there are only two productivity shocks. Therefore we affirm the conclusion in Anderberg (2009, p. 1020) that Costa and Maestri 'overlooked' the term in brackets on the RHS of Eq (3.9) in their expression for human capital at the second best optimum.

[38] $\frac{\partial}{\partial\theta}\left(\frac{w_e(e,\theta)}{w(e,\theta)}\right) = \frac{w_{e\theta}w - w_e w_\theta}{w^2}$

4 Heterogeneity and Uncertainty

This section analyses the impact of uncertainty on the design of education policy when agents which differ in ability. While the combination of heterogeneity and uncertainty has been examined before, we present a new way to model this problem which is an extension of the pure heterogeneity model from Section 2.

The advantage of our new approach is that it allows us to identify the interaction effects between heterogeneity and uncertainty on the design of education policy. This aspect is ignored in the contributions of Findeisen and Sachs (2016) and Stantcheva (2017) which both begin by assuming the presence of heterogeneity and uncertainty. We find that introducing uncertainty results in unambiguously higher education taxes and that the sign of the net wedge is no longer exclusively determined by the Hicksian complementarity.

4.1 Set up

As considered in Section 2, individuals will differ in innate ability θ^i, for $i \in \{H, L\}$. However, instead of a productivity shock as modelled in Section 3, following Del Rey and Racionero (2010), risk will be introduced through a new exogenous parameter describing the probability of successful graduation, $p \in (0, 1)$.[39]

In the first period, individuals do not work, consume c^{1i} and choose between two levels of human capital investment e^i, for $i \in \{H, L\}$ (i.e. cost of education is money as in Section 2). For now, assume this first-period expenditure is financed by private loans with interest rate R.[40] The second period wage is given by $w(e^i, \theta^i)$ for individuals who graduate successfully and is w^L for those who are unsuccessful, regardless of their type. Successful individuals of type i receive gross labour income Y^i and consume c^{2i} in the second period, while unsuccessful individuals of both ability types receive Y^U and consume c^{2U}. Following Findeisen and Sachs (2016), we will focus our attention on separating allocations where the high-ability type is incentivised to invest e^H and the low-ability type is incentivised to invest e^L. This simplification restricts attention to the design of education incentives rather than addressing the question of what the optimal level of human capital investment is for each type.

An individual of type i's lifetime utility is then given by:

[39] Allowing the probability of successful graduation to vary (increase) with ability $p(\theta)$, as is done in Del Rey and Racionero (2012), does not alter the results of the model substantially. So for simplicity, we will consider some $p \in (0, 1)$. This parameter is calibrated to historical Australian graduation data in the numerical simulation in Section 5.

[40] In Section 5 we consider alternate implementations, including income-contingent loans provided by the government.

$$u(c^{1i}) + \beta\Bigg[p(u(c^{2i}) - v\left(\frac{Y^i}{w(e^i,\theta^i)}\right)\Bigg) + (1-p)(u(c^{2U}) - v\left(\frac{Y^U}{w^L}\right)\Bigg)\Bigg] \quad (4.1)$$

4.2 Laissez faire

Absent government intervention, successful type i individuals must solve the following optimisation problem in the second period.

$$V^i(\theta^i, e^i, s^i) \equiv \max_{c^{2i}, Y^i} \quad u(c^{2i}) - v\left(\frac{Y^i}{w(e^i,\theta^i)}\right)$$
$$\text{s.t.} \quad c^{2i} \leq Y^i + Rs^i$$

On the other hand, unsuccessful individuals must solve:

$$V^U(s) \equiv \max_{c^{2U}, Y^U} \quad u(c^{2U}) - v\left(\frac{Y^U}{w^L}\right)$$
$$\text{s.t.} \quad c^{2U} \leq Y^U + Rs^i$$

In the first period, type i individuals will solve:

$$\max_{c^{1i}, e^i, s^i} \quad u(c^{1i}) + \beta\left[pV^i + (1-p)V^U\right]$$
$$\text{s.t.} \quad c^{1i} + e^i = -s^i$$

Similar to Section 2, we can use the *laissez faire* allocations to define the following wedges.

4.2.1 Wedges

Labour

$$\tau_l(\theta^i) \equiv 1 - \frac{v'(l^i)}{u'(c^{2i})w(e^i,\theta^i)} \quad (4.2)$$

Savings

$$\tau_s(\theta^i) \equiv 1 - \frac{u'(c^{1i})}{\beta R\left[pu'(c^{2i}) + (1-p)u'(c^{2U})\right]} \quad (4.3)$$

Gross Education

$$\tau_e(\theta^i) \equiv 1 - \frac{(1-\tau_l(\theta^i))\beta p l^i w_e(e^i,\theta^i)u'(c^{2i})}{u'(c^{1i})} \quad (4.4)$$

Net Education

$$t_e(\theta^i) = \frac{\tau_e(\theta^i) - \tau_l(\theta^i) + \frac{\tau_s(\theta^i)(1-\tau_l(\theta^i))}{R(1-\tau_s(\theta^i))}}{(1-\tau_e(\theta^i))(1-\tau_l(\theta^i))} \tag{4.5}$$

The net education wedge is now more complicated than in Section 2 and ensuring full-deductibility of human capital investments is no longer sufficient for a zero net wedge at the optimum. This is because with a positive savings wedge, human capital investments will be distorted upwards since they allow individuals to transfer consumption to the future without paying the savings tax. So we must use a new expression for the net wedge (Eq (4.5)) to account for the distortionary impact of the savings wedge (as well as the gross education wedge and labour wedge) on human capital investment decisions.[41]

4.3 First best

Now consider the optimisation problem faced by a utilitarian social planner with complete information and subject to a basic resource constraint.

$$\begin{aligned} &\max_{e^i, c^{1i}, c^{2U}, c^{2i}, Y^U, Y^i} && \sum_{i \in \{H,L\}} \pi^i \left[u(c^{1i}) + \beta \left[pV^i + (1-p)V^U \right] \right] \\ &\text{s.t.}(\lambda) && \sum_{i \in \{H,L\}} \pi^i \Big[-(c^{1i} + e^i) + \\ & && \frac{1}{R} \big[p(Y^i - c^{2i}) + (1-p)(Y^U - c^{2U}) \big] \Big] \geq 0 \end{aligned}$$

As before, the planner would set $\tau_l(\theta^H) = \tau_l(\theta^L) = 0$ and $\tau_e(\theta^H) = \tau_e(\theta^L) = 0$ at the first best optimum, using lump sum taxes and transfers.[42] However, this solution is not feasible in the second best because of asymmetric information. The nature of asymmetric information in this model is discussed next.

4.4 Second best

Incentive compatibility

With two-dimensions of private information[43] — exogenous ability and whether or not the agent successfully graduated — incentive compatibility is now more complex. The education period constraint (μ_1) links education decisions in the first period to labour supply decisions in the second period and ensures that high-ability types are incentivised to invest e^H in human capital rather than consume more in the first period. The downward working period incentive constraint (μ_2) is the same as in Section 2.

Technically there are analogous education and working period (upward) incentive constraints for the low-ability type. In general, whether these upward incentive constraints

[41] The expression for the net education wedge considered here is a simplification of the expression found in Stantcheva (2017), which allows for multiple periods of human capital investment.

[42] Derivation is provided in Appendix A.3.

[43] The wage and labour supply are also not observed, similar to the model in Section 2.

bind at the optimum depends on the pareto weights used in the planner's objective function. But for common social welfare functions like the utilitarian (considered here) or Rawlsian, in which redistribution takes place from the high-ability type to the low-ability type, it is usually only the downward constraints that bind. We will make this assumption for now and verify it ex-post in the numerical simulations in Section 5.[44]

Therefore, under asymmetric information, the planner must solve the following optimisation problem:

$$\max_{e^i, c^{1i}, c^{2U}, c^{2i}, Y^U, Y^i,} \quad \sum_{i \in \{H,L\}} \pi^i \Big[u(c^{1i}) + \beta \Big[pV^i + (1-p)V^U \Big] \Big]$$

$$\text{s.t.}(\lambda) \quad \sum_{i \in \{H,L\}} \pi^i \Big[-(c^{1i} + e^i) + \frac{1}{R} \Big[p(Y^i - c^{2i}) + (1-p)(Y^U - c^{2U}) \Big] \Big] \geq 0$$

$$(\mu_1) \quad u(c^{1H}) + \beta \Big[p\Big(u(c^{2H}) - v\Big(\frac{Y^H}{w(e^H, \theta^H)} \Big) \Big) + (1-p)\Big(u(c^{2U}) - v\Big(\frac{Y^U}{w^L} \Big) \Big) \Big] \geq u(c^{1L}) + \beta \Big[p\Big(u(c^{2L}) - v\Big(\frac{Y^L}{w(e^L, \theta^H)} \Big) \Big) + (1-p)\Big(u(c^{2U}) - v\Big(\frac{Y^U}{w^L} \Big) \Big) \Big]$$

$$(\mu_2) \quad u(c^{2H}) - v\Big(\frac{Y^H}{w(e^H, \theta^H)} \Big) \geq u(c^{2L}) - v\Big(\frac{Y^L}{w(e^L, \theta^H)} \Big)$$

4.4.1 Optimal labour wedge

Taking the FOCs for c^{2i} and Y^i yield the following labour wedges at the optimum.[45]

$$\tau_l(\theta^H) = 0 \tag{4.6}$$

$$\tau_l(\theta^L) = 1 - \frac{1 - \Big(\frac{\mu_1}{\pi^L} + \frac{\mu_2}{\beta p \pi^L} \Big)}{1 - \Big(\frac{\mu_1}{\pi^L} + \frac{\mu_2}{\beta p \pi^L} \Big) \frac{v'(l^{LH}) w(e^L, \theta^L)}{v'(l^L) w(e^L, \theta^H)}} > 0 \tag{4.7}$$

Once again, there is no distortion on the labour supply of the high-ability type, while the low-ability type faces a positive labour wedge.

[44]See Appendix A.4 (pg 54–55) for ex-post confirmation that the upwards constraints are satisfied at the optimum.

[45]Recall $l^{LH} = \frac{Y^L}{w(e^L, \theta^H)} < l^L$. See Appendix A.3 for the derivation.

4.4.2 Optimal gross education wedge

The education first order condition yields:[46]

$$\tau_e(\theta^H) = 0 \tag{4.8}$$

$$\tau_e(\theta^L) = 1 - \frac{1 - \frac{\mu_1}{\pi^L}}{1 - \frac{\mu_1}{\pi^L}\left[\frac{v'(l^{LH})w(e^L,\theta^L)\eta(e^L,\theta^H)}{v'(l^L)w(e^L,\theta^H)\eta(e^L,\theta^l)}\right] - A} \lesseqgtr 0 \tag{4.9}$$

$$\text{where } A = \frac{\mu_2}{\beta p \pi^L}\left[\frac{v'(l^{LH})w(e^L,\theta^L)\eta(e^L,\theta^H)}{v'(l^L)w(e^L,\theta^H)\eta(e^L,\theta^l)}\right] > 0$$

As in Section 2, the education choice of the high-ability type is not distorted, while the sign of the gross education wedge for the low-ability type is indeterminate, without making further assumptions.

4.4.3 Optimal net education wedge

$$t_e(\theta^H) = 0 \tag{4.10}$$

$$t_e(\theta^L) = B\left[C - \frac{1 - \left(\frac{\mu_1}{\pi^L} + \frac{\mu_2}{\beta p \pi^L}\right)\frac{v'(l^{LH})w(e^L,\theta^L)\eta(e^L,\theta^H)}{v'(l^L)w(e^L,\theta^H)\eta(e^L,\theta^L)}}{1 - \left(\frac{\mu_1}{\pi^L} + \frac{\mu_2}{\beta p \pi^L}\right)\frac{v'(l^{LH})w(e^L,\theta^L)}{v'(l^L)w(e^L,\theta^H)}}\right] \tag{4.11}$$

$$\text{where } B = \frac{1 - (\frac{\mu_1}{\pi^L} + \frac{\mu_2}{\beta p \pi^L})\frac{v'(l^{LH})w(e^L,\theta^L)\eta(e^L,\theta^H)}{v'(l^L)w(e^L,\theta^H)\eta(e^L,\theta^L)}}{1 - (\frac{\mu_1}{\pi^L} + \frac{\mu_2}{\beta p \pi^L})}$$

$$\text{and } C = \frac{p + (1-p)\left(1 - (\frac{\mu_1}{\pi^L} + \frac{\mu_2}{\beta p \pi^L})\right)}{R} < 1$$

4.5 Discussion

As in the preceding sections, the Hicksian complementarity continues to play a role in determining the sign of the second best education policy. However, its role is no longer deterministic. Compared to the cases of pure heterogeneity and uncertainty analysed earlier, there are two new insights.

First, compared to the case of heterogeneity absent uncertainty, the second best education taxes (subsidies) are higher (lower) for all levels of $\rho_{e\theta}$. In fact, the optimal gross education wedge under pure heterogeneity is the case where $A = 0$ in Eq (4.9). This reflects the role that uncertainty plays in generating a greater need for insurance against risky human capital investments.[47] In the numerical simulations in Section 5, we explore

[46] If instead we used $p(\theta^i)$ the optimal gross education wedge of the low-ability would be given by:
$\tau_e(\theta^L) \equiv 1 - \frac{1-\frac{\mu_1}{\pi^L}}{1-\left(\frac{\mu_1}{\pi^L}\frac{p(\theta^H)}{p(\theta^L)} + \frac{\mu_2}{\pi^L \beta p(\theta^L)}\right)\left[\frac{v'(l^{LH})w(e^L,\theta^L)\eta(e^L,\theta^H)}{v'(l^L)w(e^L,\theta^H)\eta(e^L,\theta^l)}\right]}$

[47] If we allow p to vary with ability, we find that the gross wedge is increasing in the ratio of probabilities $\frac{p(\theta^H)}{p(\theta^L)}$. One possible explanation is that when high-ability types are significantly more likely to successfully graduate relative to low-ability types, the redistribution motive for the planner is greater. Therefore, low-ability types will need more insurance to undertake the same level of human capital investment.

in more detail what model parameters affect the size of the gross education wedge at the optimum.

Second, in terms of the net wedge, we retain the result that if $\rho_{e\theta} > 1$, it follows that $t_e(\theta^l) < 0$. But the converse case may no longer hold — $t_e(\theta^L) \lesseqgtr 0$ when $\rho_{e\theta} < 1$. This result highlights the conflict between the redistributive and insurance motives of the social planner when education and ability are substitutes (or weak complements)[48]. While it is not possible to unambiguously sign the net wedge, we can, after holding $\rho_{e\theta}$ fixed at a number less than one, identify what factors will determine its magnitude at the optimum.

Distribution of ability: The higher the proportion of low-ability types, the smaller the net wedge. This is intuitive because when the proportion of low-ability types is higher, the optimal amount of redistribution needed is smaller.

Probability of success: A higher probability of successful graduation decreases the net wedge. Again this result is intuitive and reflects the fact that as p increases, there is less risk in human capital investments and hence less need for wage insurance. A similar result is derived in Stantcheva (2017). She finds that repayments are larger when income volatility or the variance of productivity shocks is larger.

Interest rate on savings: A higher return on savings increases the size of the net wedge. This result also follows from the insurance motive of the planner, as a higher savings interest rate increases the opportunity cost of human capital investment.

We can then summarise the findings of this section in the following proposition:

Proposition 3 *The case for government intervention in education by providing wage insurance through education taxes is strengthened by the combination of uncertainty and heterogeneity when education and ability are strong complements. And in the case of substitutes (or weak complements), the sign of the net education distortion is indeterminate, but its amplitude will be modulated by the distribution of ability, probability of successful graduation and the return on savings.*

While the preceding sections have only characterised the properties of the second best allocation, the question of implementation has been ignored thus far. This is what we will turn our attention to in the next section. Three different decentralised implementations will be considered: a minimum education requirement, education-dependent taxes and ICLs.

[48] Weak complements is the case where $0 < \rho_{e\theta} < 1$. Analogously, we will refer to $\rho_{e\theta} > 1$ as the case of strong complements.

5 Implementation

This section identifies three possible decentralised implementations of the second best allocation and presents numerical simulations which shed light on the design of an optimal ICL scheme.

The formulae for the net education wedges at the second-best allocation in each model emphasised the importance of the Hicksian complementarity. Therefore, determining the value of this parameter will be important for any decentralised implementation. However, empirical research on the value of this parameter has been inconclusive. Some studies find that $\rho_{e\theta}$ is positive (Arias et al., 1999; Cunha, J. J. Heckman, et al., 2006), others negative (Ashenfelter and Rouse, 1998; Girma and Kedir, 2005) and others indeterminate (Tobias, 2003). But the widespread use of multiplicatively or additively separable wage functions in empirical human capital accumulation papers (e.g. Huggett et al. (2011)) suggests that $\rho_{e\theta} = 1$ may fit the data well. For brevity and without loss of generality, we will focus on implementations of a positive education wedge (i.e. a tax) at the optimum in this section.

In the pure heterogeneity model considered in Section 2, there is no need for education loans or grants because human capital investment and labour supply occur in the same period. Therefore, the rest of this section will focus on decentralised implementations of the allocations from the two multi-period models analysed in Sections 3 and 4 — with uncertainty and the combination of uncertainty and heterogeneity respectively.

Denote the optimal level of human capital investment in the second-best allocation from Section 3 by e^*. What policy instruments can implement this optimal level of education, given savings and consumption are also optimised at s^* and c^* respectively? As in the preceding sections, we will continue to assume that individual savings and human capital are observable.[49]

5.1 Minimum education requirement

When education is observable, one simple solution is to enforce a minimum education policy which requires that $e \geq e^*$. It will then follow from the individual's optimisation problem that e^* is chosen.[50] Optimal savings could then be achieved by introducing an appropriate savings (capital) tax (Kocherlakota, 2005; Kocherlakota, 2004) based on the

[49] In many countries, education levels including tertiary qualifications are observable by governments and therefore this assumption seems reasonable. But we accept that the assumption on savings is unrealistic. Please see Costa and Maestri (2007) and Anderberg (2009) for a discussion on implementations when savings and/or education are not observable.

[50] A formal proof is provided in Costa and Maestri (2007). The intuition behind this result is that because the only binding deviations in this model involve selecting less than the optimal level of education, a minimum education requirement can implement the optimum.

second best savings wedge:

$$T_s(\theta^i, s^*) = 1 - \frac{u'(c^{*1})}{\beta R \sum_{i\in\{H,L\}} \pi^i u'(c^{*i})} \tag{5.1}$$

5.2 Taxes

One possible direct mechanism is a state-dependent tax system conditional on savings, education and agent's announcement of their productivity shock, $T(e, s, \theta^i)$.

For the tax system to implement the optimal level of education e^* irrespective of the agent's announcement, the marginal benefit of human capital investment should be independent of the agent's report of their second-period productivity shock. This will ensure that individuals self-select the correct level of education designed for them.[51]

Formally, in the model in Section 3, this requires that the following expression hold at the optimum:[52]

$$\frac{w_e(e,\theta^H)}{w(e,\theta^H)}\left[v'(l^H)l^H - v'(l^{LH})l^{LH}\right] = T_e(e,s,\theta^H)u'(c^H) - T_e(e,s,\theta^L)u'(c^L) \tag{5.2}$$

The model primitives ensure that the left-hand side is positive. This implies that the marginal tax rate on education must be higher (or the marginal subsidy smaller) for the high-ability type. Which case (higher tax or smaller subsidy) arises depends on whether education increases or decreases wage risk, as outlined in Proposition 2.

A similar result would hold in the model considered in Section 4.[53] In this setting, the education-and state-dependent income taxes could be complemented by a menu of student grants to cover first period expenditure on human capital investment: $G(\theta^i) : e^i \rightarrow \mathbb{R}$ for $i = H, L$.

The optimisation problem of a type i individual would then be given by:

$$\begin{aligned} \max_{c^{1i}, e^i, s} \quad & u(c^{1i}) + \beta\Big[pV^i + (1-p)V^U\Big] \\ \text{s.t.} \quad & G(\theta^i) \geq c^{1i} + e^i + s \end{aligned}$$

Therefore, we can conclude that education-dependent taxes — using education as a 'tag' in the optimal tax function — can implement the second best optimum allocation (in the case of non-linear, separable utility).[54] However, this implementation has the unappealing property that individuals with the same income but different levels of education pay different taxes — a violation of horizontal equity. Diamond and Saez (2011) emphasise

[51] It can be shown that $u(c^i) - v(\frac{Y^i}{w(e^i,\theta^i)}) \geq u(c^k) - v(\frac{Y^k}{w(e^i,\theta^k)})$ for $i \neq k$ in the model in Section 3.

[52] This is the same expression as in Costa and Maestri (2007). Derivation is provided in Appendix A.4.

[53] The main departure would be the treatment of savings because individual savings in the first period may differ by ability. One solution is to imagine some prohibitively high savings tax function T_s which discourages savings altogether. Alternately, Werning (2011) has shown that history-independent savings taxes can implement the desired inter-temporal consumption allocation.

[54] For a treatment of tax-based implementations with more general, non-separable utility functions please refer to Findeisen and Sachs (2016).

the importance of 'socially acceptable' policy prescriptions and thus violating horizontal equity could hinder the feasibility of this particular implementation.

5.3 Income contingent loans

An indirect mechanism that would satisfy horizontal equity is ICLs.[55] In the model analysed in Section 4, a simple re-interpretation of the grants $G(\theta^i)$ from above as loans subject to a repayment schedule demonstrates how ICLs can implement the second best optimum.

As explained in Findeisen and Sachs (2016, Corollary 4), we could take the tax system of the low-ability type $T(Y^L, e^L)$ as the common labour income tax schedule[56] and introduce an income-contingent repayment M schedule conditioned on the size of the loan and labour income $M(Y, G)$. Unlike the previous implementation, the income tax is not conditional on agents' announcements of θ^i. To ensure budget balance, the high-ability type repayment schedule would then be set equal to the difference between the optimal high-ability type and low-ability type optimal tax schedule, $M(Y^H, G(\theta^H)) = T(Y^H, e^H) - T(Y^L, e^L)$. Because the common tax schedule is optimised for the low-ability type and their choice of e^L, there is no need for a corresponding low-ability type repayment schedule.

In a similar vein, Stantcheva (2017, Proposition 4) demonstrates that in a life-cycle model where human capital investments are made in multiple periods, the second best optimum can be implemented with ICLs, history-dependent labour income taxes and history independent savings taxes.

5.3.1 Numerical simulation

Finally, we present numerical simulations to shed more light on the design of the ICL repayment schedule detailed above.[57] The simulations are based on the model combining heterogeneity and uncertainty considered in Section 4. The goal is to identify what factors determine the magnitude of the gross education wedge of the low-ability type at the second best optimum. We focus on the low-ability type wedge because the gross education wedge of the high-ability type is zero, as shown in Section 4. We can think of the gross education wedge as being roughly equivalent to the repayment rate[58] in a typical ICL.

In the simulations, we employ the following functional forms for the utility and wage

[55] This result is a technicality because horizontal equity is satisfied by combining a common labour income tax schedule with an income-contingent repayment schedule. In reality, the net effect is similar to an education-dependent tax, with higher marginal tax rates for those repaying education loans.

[56] A related implementation could involve using the tax system of the high-ability type as the common labour income tax schedule, but the implementation described in the main text seems more natural.

[57] Please refer to the attached Computational Appendix for more details on the simulation.

[58] The repayment rate specifies size of the annual repayment burden as a proportion of annual labour income.

functions:

$$u(c) = \frac{c^{1-\alpha}}{1-\alpha} \tag{5.3}$$

$$v\left(\frac{Y}{W(e,\theta)}\right) = \frac{\kappa}{\gamma}\left(\frac{Y}{W(e,\theta)}\right)^{\gamma} \tag{5.4}$$

$$w(e,\theta) = \left[\theta^{1-\rho} + e^{1-\rho}\right]^{\frac{1}{1-\rho}} \tag{5.5}$$

Eq (5.3) is the CRRA utility function, with the coefficient of risk aversion denoted by α.[59] The labour disutility function is taken from Stantcheva (2017) and is calibrated to achieve a Frisch elasticity of 0.5 Chetty (2012). We use a CES function in Eq (5.5) for the wage, with the coefficient of Hicksian complementarity denoted by ρ. Using these functional forms enables us to examine how the size of the optimal education tax varies with these parameters.

Parameter values are set as follows in the benchmark model. We will then explore the effect of changes in the values of α, ρ, and p on the gross education wedge at the optimum:

Table 5.1: Calibration*

Parameter	**Definition**	**Value**	**Source**
ρ	Hicksian complementarity	1.2	Cunha, J. Heckman, et al. (2006)
κ	Disutility of work scale	1	
γ	Disutility elasticity	3	Chetty (2012)
β	Discount factor	0.95	
R	Gross interest rate	1.05	
π^H	Proportion of high-ability	0.2	
π^L	Proportion of low-ability	0.8	
e^H	Cost of high-education	500	
e^L	Cost of low-education	300	
α	Coefficient of risk aversion	2	
p	Probability of success	0.67	Education and Training (2017)

*If the source is blank, the parameter is normalised.

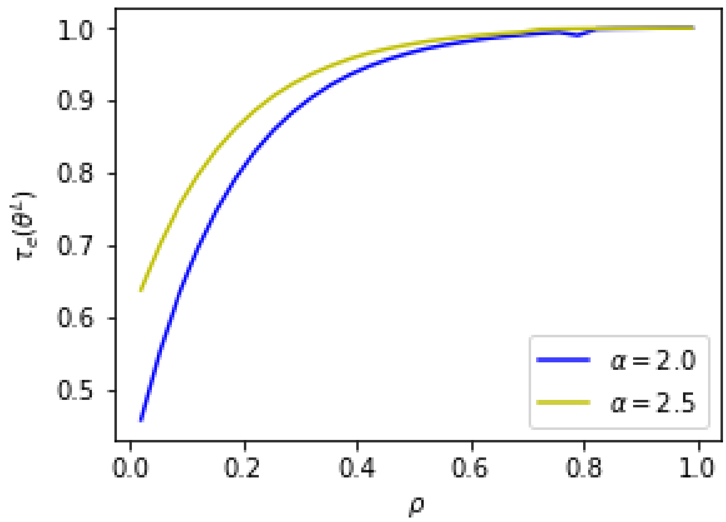
An important caveat is that because many parameters are not calibrated, readers should not place much emphasis on the size of the optimal gross education wedge. Rather, the focus should be on the sign of the wedge and how its size varies with any given parameter.

Figure 5.1 (below) illustrates how the magnitude of the gross education wedge varies with two parameters: the Hicksian complementarity and the coefficient of risk aversion. We see that the education wedge is increasing with respect to the former, while a higher level of risk aversion leads to a larger gross wedge for all values of ρ. Both of these results are intuitive. As ρ increases, education and ability are stronger complements. Therefore, higher taxes are needed for redistributive reasons. A higher level of risk aversion implies that individuals will need more insurance to undertake the same level of risky human capital investment. This explains why the education wedge is increasing in α. And this

[59] Of course, the limiting case of $\alpha = 1$ is represented by the log-utility function. To avoid this, we will focus on values of α greater than one.

effect does not diminish. Although the the two lines in Figure 5.1 appear to converge for sufficiently large values of ρ, this simply reflects the limiting case of $\tau_e(\theta^L) = 1$.

Figure 5.1: Impact of risk aversion and Hicksian complementarity

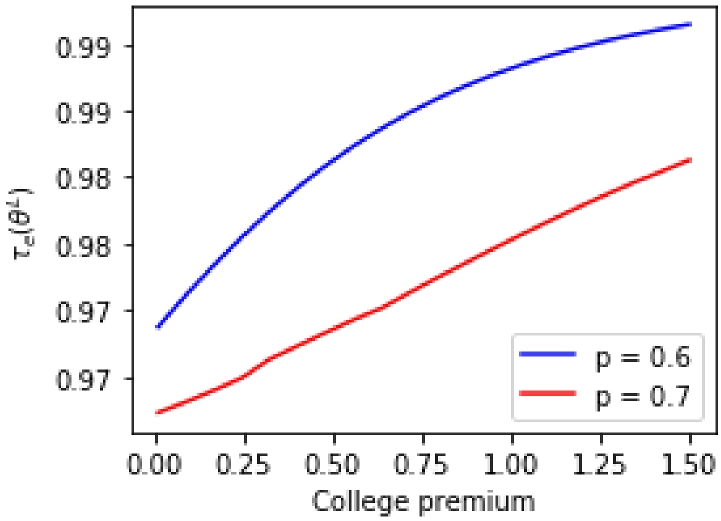


We plot values of ρ less than one to avoid the limiting case of $\rho = 1$ in the CES wage function. For the range of values of ρ considered, education and ability are complements, $w_{e\theta} > 0$, but the wage elasticity of education is decreasing in ability.

Figure 5.2 examines how the size of the gross education wedge varies with the college premium and the probability of successful graduation. Again, the results are intuitive. As the college premium rises, there is a greater need for redistribution from graduates to non-graduates and therefore education taxes are larger. One potential takeaway from this result is that budget-constrained policy makers could vary repayment rates by the college premium across different disciplines. Disciplines with higher returns to education (e.g. law and medicine) would then be subject to higher repayment rates. It is also interesting to note that the relationship between the college premium and gross education wedge is approximately linear.[60] This implies that a calibrated linear repayment schedule conditioned on the college premium (by discipline) could closely approximate the optimal solution. Finally, as human capital investments become more risky (p smaller), there is greater need for insurance and hence larger education taxes.

[60]In the Computational Appendix we directly plot the college premium against the gross wedge and find that this approximately linear relationship is robust for values of $p \geq 0.7$ — values close to those we observe in the real world (Education and Training, 2017). Below 0.7, the relationship is roughly concave.

Figure 5.2: Impact of the college premium and risky human capital investment



We have defined the college premium as the percentage gap between the wage of the successful low-ability type $w(e^l, \theta^L)$ and the wage of the unsuccessful w^L. For reference, Autor et al. (1998) estimates the college premium in the United States is 42.7%.

Therefore, that the repayment rate in an optimal ICL should increase with the Hicksian complementarity, level of risk aversion and college premium. This finding reinforces the importance of evaluating reliable estimates of these parameters to guide policy makers.

6 Conclusion

This paper has analysed the design of education policy in an optimal non-linear tax setting. It has found that both redistribution and insurance motivations can justify government intervention in education. Regardless of the motivation, the sign of optimal education policy is exclusively determined by one parameter: the Hicksian complementarity. Specifically, when education increases (decreases) exposure to risk or the wage elasticity of education is increasing (decreasing) in ability, the optimal policy is to tax (subsidise) education. This result continues to hold when both heterogeneity and uncertainty are present and education and ability are complements. In the case where education and ability are substitutes, the sign of the optimal policy is indeterminate, but its magnitude will depend on the distribution of ability, the probability of successful graduation and the return on savings.

This paper has also shown that regardless of the sign of education policy, ICLs — a particular application of education-dependent taxes — can implement the second best optimum. Therefore, both heterogeneity and uncertainty, or the combination of the two, can provide a theoretical justification for the use of ICLs to finance education.

The novel contributions of this paper to the literature on endogenous human capital accumulation with optimal non-linear taxation are two-fold. First, we proved that introducing uncertainty unambiguously increases the size of optimal taxes on education when agents differ in exogenous ability. And second, we used numerical simulations to show that the repayment rate in an optimal ICL scheme is increasing in the level of risk aversion, Hicksian complementarity and college premium. The latter conclusion suggests that empirical research into the value of these parameters and their evolution over time could shed more light on the design of optimal education policies.

Further, this paper has abstracted away from three issues which could be addressed in future work. First, we have not considered the interaction between heterogeneity, uncertainty and moral hazard in learning effort. The latter could be important in designing incentives for optimal human capital accumulation. Second, we have not modelled the 'supply' side of education. Changes in policy settings, such as introducing income-contingent loans, may have implications for the choice between public and private provision of education. And third, we have not considered differences in education quality, which are often a key factor in human capital investment decisions. Relaxing some or all of these assumptions might be a fruitful area for future work.

A Appendix

A.1 Heterogeneity

First best

$$\begin{aligned} \max_{c^i, Y^i, e^i} \quad & \sum_{i \in \{H,L\}} \pi^i \left[u(c^i) - v\left(\frac{Y^i}{w(e^i, \theta^i)}\right)\right] \\ \text{s.t.}(\lambda) \quad & \sum_{i \in \{H,L\}} \pi^i \left[Y^i - c^i - e^i\right] \geq 0 \end{aligned}$$

The FOCs of the social planner's optimisation problem are:

$$FOC(c^i): \qquad \sum_{i=H,L} \pi^i u'(c^i) - \lambda \sum_{i \in \{H,L\}} \pi^i = 0 \tag{A.1}$$

$$FOC(Y^i): \qquad -\sum_{i \in \{H,L\}} \pi^i v'(l^i) \frac{1}{w(e, \theta^i)} + \lambda \sum_{i=H,L} \pi^i = 0 \tag{A.2}$$

$$FOC(e^i): \qquad \sum_{i \in \{H,L\}} \pi^i \left[\frac{v'(l^i) l^i w_e(e, \theta^i)}{w(e, \theta^i)}\right] - \lambda \sum_{i \in \{H,L\}} \pi^i = 0 \tag{A.3}$$

Using Eq (A.2) we can re-write Eq (A.3) as:

$$u'(c^i) = v'(l^i) \frac{l^i w_e(e^i, \theta^i)}{w(e^i, \theta^i)} \tag{A.4}$$

Second best

$$\begin{aligned} \max_{c^i, Y^i, e^i} \quad & \sum_{i \in \{H,L\}} \pi^i \left[u(c^i) - v\left(\frac{Y^i}{w(e^i, \theta^i)}\right)\right] \\ \text{s.t.}(\lambda) \quad & \sum_{i \in \{H,L\}} \pi^i \left[Y^i - c^i - e^i\right] \geq 0 \\ (\mu) \quad & u(c^H) - v\left(\frac{Y^H}{w(e^H, \theta^H)}\right) \geq u(c^L) - v\left(\frac{Y^L}{w(e^L, \theta^H)}\right) \end{aligned}$$

FOCs for the high-ability type are:

$$FOC(c^H): \qquad \pi^H u'(c^H) - \lambda\pi^H + \mu u'(c^H) = 0 \tag{A.5}$$

$$FOC(Y^H): \qquad -\pi^H v'(l^H)\frac{1}{w(e^H,\theta^H)} + \lambda\pi^H - \mu v'(l^H)\frac{1}{w(e^H,\theta^H)} = 0 \tag{A.6}$$

$$FOC(e^H): \qquad \left[\pi^H + \mu\right]\left(-u'(c^H) + \pi^H v'(l^H)\frac{l^H w_e(e^H,\theta^H)}{w(e^H,\theta^H)}\right) = 0 \tag{A.7}$$

Combining Eq (A.5) and Eq (A.6) give $\tau_l(\theta^H) = 0$ as required, and then it is simple to show that $\tau_e(\theta^H) = 0$ using Eq (A.7).

FOCs for the low-ability type are:

$$FOC(c^L): \qquad \pi^L u'(c^L) - \lambda\pi^L + \mu u'(c^L) = 0 \tag{A.8}$$

$$FOC(Y^L): \qquad -\pi^H v'(l^L)\frac{1}{w(e^L,\theta^L)} + \lambda\pi^L - \mu v'(l^{HL})\frac{1}{w(e^L,\theta^H)} = 0 \tag{A.9}$$

Using Eq (A.8) and Eq (A.9) in Eq (2.5) gives the expression for the optimal labour wedge:

$$\tau_l(\theta^L) = 1 - \frac{1 - \frac{\mu}{\pi^L}}{1 - \frac{\mu v'(l^{LH})w(e^L,\theta^L)}{\pi^L v'(l^L)w(e^L,\theta^H)}} \tag{A.10}$$

Next, consider the FOC(e^L):

$$\pi^L\left[-u'(c^L) + v'(l^L)\frac{l^L w_e(e^L,\theta^L)}{w(e^L,\theta^L)}\right] + \mu\Big[u'(c^L) + v'(l^{LH})\frac{l^{LH}w_e(e^L,\theta^H)}{w(e^L,\theta^H)}\Big] = 0 \tag{A.11}$$

Simplifying Eq (A.11) using Eq (2.6) gives that:[61]

$$\pi^L(1 - \tau_e(\theta^L)) = (\pi^L - \mu) + \frac{\mu v'(L^{LH})l^L w(e^L,\theta^L)w_e(e^L,\theta^H)}{w(e^L,\theta^H)^2 u'(c^L)} \tag{A.12}$$

Rewrite the second term on the RHS of Eq (A.12) using the definition of $\eta(e^i,\theta^i)$ provided in the text:

$$\pi^L(1 - \tau_e(\theta^L)) = (\pi^L - \mu) + \frac{\mu v'(L^{LH})l^L w(e^L,\theta^L)\eta(e^L,\theta^H)}{w(e^L,\theta^H)u'(c^L)e^L} \tag{A.13}$$

Divide and multiply the same term by $\frac{w_e(e^L,\theta^L)}{w(e^L,\theta^L)}$ and again apply the definition of $\eta(e^i,\theta^i)$:

[61] We have also made use of the fact that $l^{HL}w(e^L,\theta^H) = l^L w(e^L,\theta^L)$

$$1-\tau_e(\theta^L) = \left(1-\frac{\mu}{\pi^L}\right) + \frac{\mu v'(L^{LH}) l^L w(e^L,\theta^L)\eta(e^L,\theta^H) w_e(e^L,\theta^L)}{\pi^L w(e^L,\theta^H) u'(c^L)\eta(e^L,\theta^L) w(e^L,\theta^L)} \tag{A.14}$$

Factorising Eq (A.13) using Eq (2.6) gives the expression of the gross education wedge for the low-ability type:

$$\tau_e(\theta^L) = 1 - \frac{1-\frac{\mu}{\pi^L}}{1-\frac{\mu v'(l^{LH}) w(e^L,\theta^L)\eta(e^L,\theta^H)}{\pi^L v'(l^L) w(e^L,\theta^H)\eta(e^L,\theta^L)}} \tag{A.15}$$

Finally, using Eq (A.10) and Eq (A.15) from above in Eq (2.7) gives the required expression for the net education wedge of the low-ability type.

A.2 Uncertainty

First best

$$\max_{e,c^1,c^{2i},Y^i} \quad u(c^1) + \beta \sum_{i\in\{H,L\}} \pi^i \left[u(c^{2i}) - v\left(\frac{Y^i}{w(e,\theta^i)}\right)\right]$$

$$\text{s.t.}(\lambda) \quad \sum_{i\in\{H,L\}} \pi^i \left[c^{2i} - Y^i\right] \leq R(1-e-c^1)$$

The FOCs of the social planner's optimisation problem are:

$$FOC(c^1) : u'(c^1) - \lambda R = 0 \tag{A.16}$$

$$FOC(c^{2H}) : \beta\pi^H u'(c^{2H}) - \lambda\pi^H = 0 \tag{A.17}$$

$$FOC(c^{2L}) : \beta\pi^L u'(c^{2L}) - \lambda\pi^L = 0 \tag{A.18}$$

Clearly, $u'(c^{2H}) = u'(c^{2L}) = \frac{\lambda}{\beta}$.

And after substituting Eqs (A.16-18) into Eq (3.5) it follows that $\tau_s = 0$ in the first best.

$$FOC(Y^H) : -\beta\pi^H v'(l^H)\frac{1}{w(e,\theta^H)} + \lambda\pi^H = 0 \tag{A.19}$$

$$FOC(Y^L) : -\beta\pi^L v'(l^L)\frac{1}{w(e,\theta^L)} + \lambda\pi^L = 0 \tag{A.20}$$

It is simple to show that $v'(l^H) > v'(l^L)$, and therefore individuals who receive the high productivity shock work more in the first best.[62]

[62] This follows from the assumption that the disutility of labour supply function $v(.)$ is convex.

$$FOC(e): \quad \beta\left[\frac{\pi^H v'(l^H)l^H w_e(e,\theta^H)}{w(e,\theta^H)} + \frac{\pi^L v'(l^L)l^L w_e(e,\theta^L)}{w(e,\theta^L)}\right] - \lambda R = 0 \tag{A.21}$$

Combining Eq (A.21) with Eq (A.19) and Eq (A.20) gives the required expression for optimum first best education in the text.

Second best

$$\begin{aligned}
&\max_{e,c^1,c^{2i},Y^i} && u(c^1) + \beta \sum_{i\in\{H,L\}} \pi^i \left[u(c^{2i}) - v\left(\frac{Y^i}{w(e,\theta^i)}\right)\right] \\
&\text{s.t.}(\lambda) && \sum_{i\in\{H,L\}} \pi^i \left[c^{2i} - Y^i\right] \le R(1-e-c^1) \\
&(\mu) && u(c^{2H}) - v\left(\frac{Y^H}{w(e,\theta^H)}\right) \ge u(c^{2L}) - v\left(\frac{Y^L}{w(e,\theta^H)}\right)
\end{aligned}$$

The first period FOC is:

$$FOC(c^1): \quad u'(c^1) - \lambda R = 0 \tag{A.22}$$

The FOCs for those who receive the high-shock are:

$$FOC(c^{2H}): \qquad \beta\pi^H u'(c^{2H}) - \lambda\pi^H + \mu u'(c^{2H}) = 0 \tag{A.23}$$

$$FOC(Y^H): \qquad -\beta\pi^H v'(l^H)\frac{1}{w(e,\theta^H)} + \lambda\pi^H - \mu v'(l^H)\frac{1}{w(e,\theta^H)} = 0 \tag{A.24}$$

Combining Eq (A.23) and Eq (A.24) gives that $\tau_l(\theta^H) = 0$ as required.

The FOCs for the those who receive the low-shock are:

$$FOC(c^{2L}): \qquad \beta\pi^L u'(c^{2L}) - \lambda\pi^L - \mu u'(c^{2L}) = 0 \tag{A.25}$$

$$FOC(Y^L): \qquad -\beta\pi^L v'(l^L)\frac{1}{w(e,\theta^L)} + \lambda\pi^L + \mu v'(l^{LH})\frac{1}{w(e,\theta^H)} = 0 \tag{A.26}$$

From Eq (A.26), we get that:

$$\lambda\pi^L = \beta\pi^L\frac{v'(l^L)}{w(e,\theta^L)} - \mu\frac{v'(l^{LH})}{w(e,\theta^H)} \tag{A.27}$$

After multiplying and dividing the second term on the RHS of Eq (A.27) by $\frac{v'(l^L)}{w(e,\theta^L)}$ we get that:

$$\lambda\pi^L = \frac{v'(l^L)}{w(e,\theta^L)}\left[\beta\pi^L - \frac{v'(l^{LH})w(e,\theta^L)}{v'(l^L)w(e,\theta^H)}\right] \tag{A.28}$$

Combining Eq (A.28) with Eq (A.26) and simplifying gives the expression for the low-shock optimal labour wedge in the text.

Next, substitute Eq (A.22-23) and Eq (A.25) into Eq (3.5):

$$\begin{aligned}\tau_s &= 1 - \frac{\lambda R}{\beta R\left[\pi^H \frac{\lambda}{\beta\pi^H+\mu} + \pi^L \frac{\lambda}{\beta\pi^L-\mu}\right]} \\ &= 1 - \frac{1}{\frac{\beta\pi^H}{\beta\pi^H+\mu} + \frac{\beta\pi^L}{\beta\pi^L-\mu}} \\ &> 0\end{aligned} \tag{A.29}$$

Finally, taking the FOC with respect to education:

$$\frac{\beta\pi^H v'(l^H) l^H w_e(e,\theta^H)}{w(e,\theta^H)} + \frac{\beta\pi^L v'(l^L) l^L w_e(e,\theta^L)}{w(e,\theta^L)} - \lambda R = \frac{\mu w_e(e,\theta^H)}{w(e,\theta^H)}\left[v'(l^{LH})l^{LH} - v'(l^H)l^H\right] \tag{A.30}$$

After collecting like terms from Eq (A.30):

$$\frac{v'(l^H) l^H w_e(e,\theta^H)}{w(e,\theta^H)}\left[\beta\pi^H + \mu\right] = \lambda R + \frac{\mu w_e(e,\theta^H) v'(l^{LH}) l^{LH}}{w(e,\theta^H)} - \frac{\beta\pi^L v'(l^L) l^L w_e(e,\theta^L)}{w(e,\theta^L)} \tag{A.31}$$

Substitute Eq (A.24) into the LHS of Eq (A.31):

$$\lambda\pi^H l^H w_e(e,\theta^H) = \lambda R + \frac{\mu w_e(e,\theta^H) v'(l^{LH}) l^{LH}}{w(e,\theta^H)} - \frac{\beta\pi^L v'(l^L) l^L w_e(e,\theta^L)}{w(e,\theta^L)} \tag{A.32}$$

Add and subtract $\lambda\pi^L l^L w_e(e,\theta^L)$ to both sides of Eq (A.32):

$$\lambda \sum_{i=[H,L]} \pi^i l^i w_e(e,\theta^i) = \lambda R + \frac{\mu w_e(e,\theta^H) v'(l^{LH}) l^{LH}}{w(e,\theta^H)} - \frac{\beta\pi^L v'(l^L) l^L w_e(e,\theta^L)}{w(e,\theta^L)} + \lambda\pi^L l^L w_e(e,\theta^L) \tag{A.33}$$

Substitute Eq (A.26) into the RHS of Eq (A.33):

$$\lambda \sum_{i=[H,L]} \pi^i l^i w_e(e,\theta^i) = \lambda R + \frac{\mu w_e(e,\theta^H) v'(l^{LH}) l^{LH}}{w(e,\theta^H)} - \frac{\mu w_e(e,\theta^L) v'(l^{LH}) l^{LH}}{w(e,\theta^L)} \tag{A.34}$$

Then, dividing both sides of Eq (A.34) by λ and factorising gives the optimal second best education result in the text.

A.3 Heterogeneity and uncertainty

First best

$$\max_{e^i, c^{1i}, c^{2U}, c^{2i}, Y^U, Y^i} \quad \sum_{i \in \{H,L\}} \pi^i \left[u(c^{1i}) + \beta \left[pV^i + (1-p)V^U \right] \right]$$

$$\text{s.t.}(\lambda) \quad \sum_{i \in \{H,L\}} \pi^i \Big[-(c^{1i} + e^i) + \frac{1}{R} \big[p(Y^i - c^{2i}) + (1-p)(Y^U - c^{2U}) \big] \Big] \geq 0$$

In the first best problem, the planner's FOCs for a type i individual are:

$$FOC(c^{1i}): \quad \pi^i u'(c^{1i}) - \lambda \pi^i = 0 \tag{A.35}$$

$$FOC(c^{2i}): \quad \pi^i \beta p u'(c^{2i}) - \frac{\lambda}{R} p \pi^i = 0 \tag{A.36}$$

$$FOC(Y^i): \quad -\pi^i \beta p v'(l^i) \frac{1}{w(e^i, \theta^i)} + \frac{\lambda}{R} p \pi^i = 0 \tag{A.37}$$

$$FOC(e^i): \quad \pi^i \beta p v'(l^i) \frac{l^i w_e(e^i, \theta^i)}{w(e^i, \theta^i)} - \lambda \pi^i = 0 \tag{A.38}$$

It is then simple to show that $\tau_l(\theta^i) = \tau_e(\theta^i) = 0$ for $i = H, L$ by substituting Eq(A.35-38) into Eq (4.2) and Eq (4.4).

Second best

$$\max_{e^i, c^{1i}, c^{2U}, c^{2i}, Y^U, Y^i,} \quad \sum_{i \in \{H,L\}} \pi^i \Big[u(c^{1i}) + \beta \Big[pV^i + (1-p)V^U \Big] \Big]$$

$$\text{s.t.}(\lambda) \quad \sum_{i \in \{H,L\}} \pi^i \Big[-(c^{1i} + e^i) + \frac{1}{R} \Big[p(Y^i - c^{2i}) + (1-p)(Y^U - c^{2U}) \Big] \Big] \geq 0$$

$$(\mu_1) \quad u(c^{1H}) + \beta \Big[p\Big(u(c^{2H}) - v\Big(\frac{Y^H}{w(e^H, \theta^H)} \Big) \Big) + (1-p)\Big(u(c^{2U}) - v\Big(\frac{Y^U}{w^L} \Big) \Big) \Big] \geq u(c^{1L}) + \beta \Big[p\Big(u(c^{2L}) - v\Big(\frac{Y^L}{w(e^L, \theta^H)} \Big) \Big) + (1-p)\Big(u(c^{2U}) - v\Big(\frac{Y^U}{w^L} \Big) \Big) \Big]$$

$$(\mu_2) \quad u(c^{2H}) - v\Big(\frac{Y^H}{w(e^H, \theta^H)} \Big) \geq u(c^{2L}) - v\Big(\frac{Y^L}{w(e^L, \theta^H)} \Big)$$

In the second best problem, the planner's FOCs for the high-ability type are:

$$FOC(c^{1H}): \quad \pi^H u'(c^{1H}) - \lambda \pi^H + \mu_1 u'(c^{1H}) = 0 \tag{A.39}$$

$$FOC(c^{2H}): \quad \pi^H \beta p u'(c^{2H}) - \frac{\lambda}{R} p \pi^H + \mu_1 \beta p u'(c^{2H}) + \mu_2 u'(c^{2H}) = 0 \tag{A.40}$$

$$FOC(Y^H): \quad \frac{\pi^H \beta p v'(l^H)}{w(e^H, \theta^H)} + \frac{\lambda}{R} p \pi^H - \frac{\mu_1 \beta p v'(l^H)}{w(e^H, \theta^H)} - \frac{\mu_2 v'(l^H)}{w(e^H, \theta^H)} = 0 \tag{A.41}$$

$$FOC(e^H): \frac{\pi^H \beta p v'(l^H) l^H w_e(e^H, \theta^H)}{w(e^H, \theta^H)} - \lambda \pi^H + \frac{\mu_1 \beta p v'(l^H) w_e(e^H, \theta^H)}{w(e^H, \theta^H)} + \frac{\mu_2 v'(l^H) l^H w_e(e^H, \theta^H)}{w(e^H, \theta^H)} = 0 \tag{A.42}$$

Again, it is a simple matter of combining Eq (A.39-42) to show that $\tau_l(\theta^H) = \tau_e(\theta^H) = 0$.

The planners FOCs for the low-ability type are:

$$FOC(c^{1L}): \quad \pi^L u'(c^{1L}) - \lambda \pi^L - \mu_1 u'(c^{1L}) = 0 \tag{A.43}$$

$$FOC(c^{2L}): \quad \pi^L \beta p u'(c^{2L}) - \frac{\lambda}{R} p \pi^L - \mu_1 \beta p u'(c^{2L}) - \mu_2 u'(c^{2L}) = 0 \tag{A.44}$$

$$FOC(Y^L): \quad \frac{-\pi^L \beta p v'(l^L)}{w(e^L, \theta^L)} + \frac{\lambda}{R} p \pi^L + \frac{\mu_1 \beta p v'(l^{LH})}{w(e^L, \theta^H)} + \frac{\mu_2 v'(l^{LH})}{w(e^L, \theta^H)} = 0 \tag{A.45}$$

From Eq (A.45), we get that:

$$\frac{v'(l^L)\beta p\pi^L}{w(e^L,\theta^L)} = \frac{\lambda}{R}p\pi^L + \frac{v'(l^{LH})}{w(e^L,\theta^H)}\left[\mu_1\beta p + \mu_2\right] \tag{A.46}$$

Multiply and divide the second term on the RHS of Eq (A.46) by $\frac{v'(l^L)}{w(e^L,\theta^L)}$:

$$\frac{v'(l^L)}{w(e^L,\theta^L)} = \frac{\frac{\lambda p\pi^L}{R}}{\left[\beta p\pi^L - (\mu_1\beta p + \mu_2)\frac{v'(l^{LH})w(e^L,\theta^L)}{v'(l^L)w(e^L,\theta^H)}\right]} \tag{A.47}$$

Then, combining Eq (A.47) with Eq (A.45) and simplifying yields the second best optimal labour wedge for the low-ability type expressed in the text.

Consider the education FOC of the low-ability type:

$$\frac{\pi^L\beta p v'(l^L)l^L w_e(e^L,\theta^L)}{w(e^L,\theta^L)} - \lambda\pi^L - \frac{\mu_1\beta p v'(l^{LH})l^{LH}w_e(e^L,\theta^H)}{w(e^L,\theta^H)} - \frac{\mu_2 v'(l^{LH})w_e(e^L,\theta^H)}{w(e^L,\theta^H)} = 0 \tag{A.48}$$

Using Eq (4.3) in Eq (A.48):

$$\pi^L u'(c^{1L})(1-\tau_e(\theta^L)) = \lambda\pi^L + \frac{v'(l^{LH})l^{LH}w_e(e^L,\theta^H)}{w(e^L,\theta^H)}\left[\mu_1\beta p + \mu_2\right] \tag{A.49}$$

Next, use the definition of $\eta(e^i,\theta^i)$ in the RHS of Eq (A.49):

$$\pi^L u'(c^{1L})(1-\tau_e(\theta^L)) = \lambda\pi^L + \frac{v'(l^{LH})l^L w(e^L,\theta^l)\eta(e^L,\theta^H)}{e^L w(e^L,\theta^H)}\left[\mu_1\beta p + \mu_2\right] \tag{A.50}$$

Multiply and divide the second term on the RHS of Eq (A.50) by $\frac{w_e(e^L,\theta^L)}{w(e^L,\theta^L)}$ and again apply the definition of $\eta(e^i,\theta^i)$:

$$\pi^L u'(c^{1L})(1-\tau_e(\theta^L)) = \lambda\pi^L + \frac{v'(l^{LH})l^L w(e^L,\theta^L)\eta(e^L,\theta^H)w_e(e^L,\theta^L)}{w(e^L,\theta^H)w(e^L,\theta^L)\eta(e^L,\theta^L)}\left[\mu_1\beta p + \mu_2\right] \tag{A.51}$$

Divide both sides of Eq (A.51) by $u'(c^{1l})$ and again apply Eq (4.4):

$$(1-\tau_e(\theta^L))\left[1 - \left[\frac{\mu_1}{\pi^L} + \frac{\mu_2}{\pi^L\beta p}\right]\left(\frac{v'(l^{LH})w(e^l,\theta^L)\eta(e^L,\theta^H)}{v'(l^L)w(e^L,\theta^H)\eta(e^l,\theta^L)}\right)\right] = \frac{\lambda}{u'(c^{1L})} \tag{A.52}$$

Lastly, using Eq (A.43) in Eq (A.52) and simplifying gives the optimal second best gross education wedge from the text.

Using Eq (A.43-45) and Eq (A.48) in Eq (4.3) gives the following expression for the low-ability type savings wedge:

$$\tau_s(\theta^L) = \frac{1 - \left(\frac{\mu_1}{\pi^L} + \frac{\mu_2}{\beta p \pi^L}\right)}{\left(1 - \frac{\mu_1}{\pi^L}\right)\left[p + (1-p)\left(1 - \frac{\mu_1}{\pi^L} - \frac{\mu_2}{\beta p \pi^L}\right)\right]} \tag{A.53}$$

Then, substituting $\tau_e(\theta^L)$, $\tau_l(\theta^L)$ and $\tau_s(\theta^L)$ from above into Eq (4.4) gives the optimal net education wedge for the low-ability type.

A.4 Implementation via taxes

Under asymmetric information in the uncertainty model from Section 3, incentive compatibility requires that the marginal return to human capital investments be equal under truth-telling and lying.

Under truth-telling, the marginal return to human capital investments is given by:

$$\sum_{i \in \{H,L\}} \pi^i u'(c^i)(1 - T_e(e, s, \theta^i)) - \sum_{i=H,L} \pi^i v'(l^i) l^i \frac{w_e(e,\theta^i)}{w(e,\theta^i)} = 0 \tag{A.54}$$

And if the agent misreports their productivity shock (high-shock mimicking behaviour of the low-shock), the first-order condition is:

$$\sum_{i \in \{H,L\}} \pi^i u'(c^i)(1 - T_e(e, s, \theta^i)) - \left[\pi^L v'(l^L) l^L \frac{w_e(e,\theta^L)}{w(e,\theta^L)} + \pi^H v'(l^{HL}) l^{HL} \frac{w_e(e,\theta^H)}{w(e,\theta^H)}\right] = 0 \tag{A.55}$$

Combining Eq (A.54) and Eq (A.55) gives Eq (5.2) in the text.

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