# Appendix

## A Calibration Details

### A.1 Summary of External Calibration

| | Description | Value | Target / Source |
|---|---|---|---|
| *Preferences* | | | |
| $\gamma$ | Risk Aversion | 2 | Literature |
| $\phi$ | Procrastination Decay Rate | $-\log(0.5)$ | Andersen et al. (2019) |
| *Income* | | | |
| $y_t$ | Transitory Income | $\{0.75, 0.98, 1.28\}$ | Guerrieri and Lorenzoni (2017) |
| $\mathbf{A}^y$ | Income Transition Matrix | (see text) | Guerrieri and Lorenzoni (2017) |
| *Interest Rates* | | | |
| $r_t$ | Short Rate | $\{-1\%, 0\%, 1\%, 2\%\}$ | 10-Year TIPS |
| $\mathbf{A}^r$ | Short Rate Transition Matrix | (see text) | 10-Year TIPS |
| $\omega^{cc}$ | Credit Card Wedge | 10.3% | Credit Card - 10-Yr Treasury Spread |
| $\omega^m$ | Mortgage Wedge | 1.7% | 30-Yr FRM - 10-Yr Treasury Spread |
| *Assets and Liabilities* | | | |
| $h$ | House Value | 3.1 | 2016 SCF |
| $\theta$ | Max LTV | 0.8 | Greenwald (2018) |
| $\xi$ | Mortgage Paydown | 0.035 | 20 Year Half-Life |
| $\kappa^{prepay}$ | Prepayment Fixed Cost | 0.002 | Numerical Stability |
| $\kappa^{refi}$ | Refi Fixed Cost | 0.05 | FRB Documentation |
| $\underline{b}$ | Credit Limit | $-\frac{1}{3}$ | 2016 SCF |
| *Other Structural Assumptions* | | | |
| $\lambda^F$ | Rate of Forced Refi | $\frac{1}{15}$ | 2016 CPS Avg. Moving Rate |
| $\lambda^R$ | Retirement Rate | $\frac{1}{30}$ | Average Working Life |
| - | Birth Distribution | $m_0 = \theta h, b_0 \sim U(0, y_L)$ | Lifecycle Dynamics |

Table 5: Externally Calibrated Parameters.

Notes: This table presents the model's externally calibrated parameters. See Section 5.1 for details on calibration choices.

### A.2 SCF Details

Many of our calibrated parameters rely on data from the 2016 SCF. To construct a sample of households that is consistent with our model we impose the following data filters. The head of house must be in the labor force and aged 25-66. The household must own a home (with weakly positive home equity), possess a credit card, earn no income from Social

Security nor retirement accounts, and have after-tax permanent income between the 1st and 99th percentile. On this sample, we then condition on households with a home value to permanent income ratio between the 25th and 75th percentile.

All of our variables are scaled relative to permanent income. Following Kennickell (1995), Kennickell and Lusardi (2004), and Fulford (2015) we use the SCF's question about "normal income" to measure each household's permanent income.[58] Though this is an imperfect proxy for the household's permanent income, it has the benefit of being both straightforward and respecting the household's information set. We adjust each household's normal income for 2015 federal taxes, and deduct an additional 5% for state taxes.

We use the 2016 SCF to estimate six moments that are used in our calibration: (i) permanent income; (ii) average home value to permanent income; (iii) average LTV; (iv) average credit card debt to permanent income; (v) share of households with revolving credit card debt; and (vi) average credit limit to permanent income. Moments (ii) – (v) are reported in the main text. The average after-tax permanent income for our sample of homeowners is \$95,918. The average credit limit to permanent income is 0.36.

## A.3 Estimation and Discretization of Ornstein-Uhlenbeck Processes

To calibrate our income and interest-rate processes, we assume that these processes are discretized versions of continuous-time Ornstein-Uhlenbeck (OU) processes.

Consider a generic mean-zero OU process $u(t) = \int_0^t e^{-\kappa(t-s)}\sigma dZ_s$. Process $u(t)$ has the conditional distribution $u(t+\tau)|u(t) = \mathcal{N}\left(u(t)e^{-\kappa\tau}, \frac{\sigma^2}{2\kappa}(1-e^{-2\kappa\tau})\right)$.

Assume that $u(t)$ is only observed in discrete snapshots every $\Delta$ years. Let $d_s = u(s\Delta)$ denote the $s$'th snapshot of process $u(t)$. The discrete process $d_s$ can be modeled as an AR(1) process:

$$\begin{aligned} d_{s+1} &= \rho d_s + \sigma_d \varepsilon_{s+1}, \text{ where} \\ \rho &= e^{-\kappa\Delta} \\ \sigma_d^2 &= \frac{\sigma^2}{2\kappa}(1-e^{-2\kappa\Delta}). \end{aligned}$$

Given any discrete-time AR(1) estimate, we can use the above formulas to back out the parameters of the underlying OU process: $\kappa$ and $\sigma$. We discretize the OU process using standard finite difference methods. For details, see https://benjaminmoll.com/wp-content/uploads/2020/02/HACT_Numerical_Appendix.pdf.

[58] SCF respondents are asked whether or not their 2015 income was normal. If not, they are asked to report what their total income would be if it had been normal.

# B Proofs and Additional Mathematical Details

## B.1 MPCs and MPXs over Discrete Wealth Shocks

In Section 4 the MPC and the MPX are defined over infinitesimal wealth shocks. Following Achdou et al. (2017), this section extends these definitions to discrete wealth shocks.

Let $C_\tau(x) = \mathbb{E}\left[\int_0^\tau c(x_t)dt \mid x_0 = x\right]$ denote total expected consumption from time 0 to time $\tau$. Recall that $x = (b, m, y, r^m, r, e)$. Let $x + \chi$ be shorthand for the vector $(b + \chi, m, y, r^m, r, e)$, i.e. $x + \chi$ is point $x$ plus a liquid wealth shock of size $\chi$.

For a discrete liquidity shock of size $\chi$ the MPC is defined as:

$$MPC_\tau^\chi(x) = \frac{C_\tau(x+\chi) - C_\tau(x)}{\chi}.$$

The MPX is defined as:

$$MPX_\tau^\chi(x) = MPC_\tau^\chi(x) + \frac{s}{\nu + r_0}\left(\frac{\mathbb{E}\left[c(x_\tau) \mid x_0 = x+\chi\right] - \mathbb{E}\left[c(x_\tau) \mid x_0 = x\right]}{\chi}\right).$$

Total consumption $C_\tau(x)$, which is used in the MPC calculation, can be calculated numerically using a Feynman-Kac formula (see Lemma 2 of Achdou et al. (2017) for details). To calculate the MPX we also need to solve for the expected consumption rate at time $\tau$, $\mathbb{E}\left[c(x_\tau)|x_0 = x\right]$. Again, a Feynman-Kac formula can be used to solve for this directly.[59]

## B.2 Naive Present Bias: Passing to Continuous Time

Here we present a heuristic derivation of naive IG preferences as the continuous-time limit of a model where decisions are made discretely. Assume that the current self (self 0) lives for a discrete length of time, denoted $\Delta$. For ease of exposition, we make the non-standard assumption that time becomes continuous starting with the next self at time $\Delta$. Since the naive present-biased household incorrectly perceives that all future selves will discount exponentially, continuation-value function $v(x)$ characterizes the equilibrium starting at time $\Delta$. The current self discounts all future selves by $\beta$, so the current-value function for the

[59] The Feynman-Kac formula for $C_\tau(x)$ is provided in Achdou et al. (2017). The Feynman-Kac formula for $\mathbb{E}\left[c(x_\tau)|x_0 = x\right]$ is specified slightly differently. Here, $\mathbb{E}\left[c(x_\tau)|x_0 = x\right]$ is given by $\Gamma(x, 0)$, where $\Gamma(x, 0)$ satisfies the PDE $0 = (\mathcal{A}\Gamma)(x, t)$ subject to the terminal condition $\Gamma(x, \tau) = c(x)$.

naive present-biased household is given by:

$$
\begin{aligned}
w(x_0) &= \max\left\{ \max_c \; u(c)\Delta + \beta e^{-\rho\Delta}\mathbb{E}_0[v(x_\Delta)],\; w^*(x_0) - e_0 \right\} \quad \text{with} \\
w^*(x) &= \max\left\{ w^{prepay}(x), w^{refi}(x) \right\} \\
w^{prepay}(x) &= \max_{b',m'} \; w(b', m', y, r^m, r, e) \quad \text{s.t. prepayment constraint (4) holds} \\
w^{refi}(x) &= \max_{b',m'} \; w(b', m', y, r + \omega^m, r, e) \quad \text{s.t. refinancing constraint (5) holds}
\end{aligned}
\tag{14}
$$

Equation (14) captures the consumption/adjustment decisions made by the household at time 0. In the left branch of the first line the household does not adjust, and chooses consumption rate $c$ over the next $\Delta$ units of time to maximize the current-value function. In the right branch of the first line the household pays effort cost $e_0$ and fixed monetary cost $\kappa^i$ to discretely adjust their mortgage. Importantly, this discrete-time value function is written such that there is no delay to refinancing (i.e., the current self benefits from refinancing).[60] Though this is unrealistic – there are time delays in conducting a cash-out refinance – we write the Bellman in this way to emphasize that our results do not rely on assumptions about temporal delays in refinancing.

Suppressed inside equation (14) is the discretized evolution of state variable $x$ from time 0 to time $\Delta$. For this heuristic analysis, the only relevant law of motion is for liquid wealth $b$. The evolution of $b$ can be discretized as follows for small $\Delta$:

$$
\begin{aligned}
b_\Delta &= R_0^b \left[ b_0 + y_0\Delta - (r_0^m + \xi)m_0\Delta - c_0\Delta \right], \text{ where} \\
R_0^b &= \begin{cases} 1 + r_0\Delta & \text{if } b_0 + y_0\Delta - (r_0^m + \xi)m_0\Delta - c_0\Delta \geq 0 \\ 1 + (r_0 + \omega^{cc})\Delta & \text{if } b_0 + y_0\Delta - (r_0^m + \xi)m_0\Delta - c_0\Delta < 0 \end{cases}.
\end{aligned}
$$

Discrete-time Bellman equation (14) can be used to derive the current-value function in continuous time. Taking the time-step $\Delta$ to its continuous-time limit, we see that the term $u(c)\Delta$ drops out of the current-value function, leaving:

$$
w(x) = \max\left\{ \beta v(x),\; w^*(x) - e \right\}.
$$

This recovers equation (9) in the main text.

For the consumption decision, equation (14) implies that consumption is given by the

[60] To see how the value function is written in this way, note that refinancing gives the current self the current-value function of $w^*$. As the first line of equation (14) shows, this value function consists of an undiscounted utility flow earned for the current self, $u(c)\Delta$.

following first-order condition:[61]

$$u'(c(x)) = \beta e^{-\rho\Delta} \frac{\partial}{\partial b} \mathbb{E}_0[v(x_\Delta)].$$

Taking $\Delta \to 0$ recovers equation (10):

$$u'(c(x)) = \beta \frac{\partial v(x)}{\partial b}.$$

Moving on to the mortgage adjustment policy functions, we first consider the decision of choosing $(b', m')$ conditional on adjustment. If adjusting, the household chooses $b'$ and $m'$ to maximize $w(b', m', y, r^{m'}, r)$, subject to either the prepayment constraint (4) or the refinancing constraint (5). Since $w^*(x) = \beta v^*(x)$ in the continuous-time limit, maximizing $w^*$ is equivalent to maximizing $v^*$. Hence, $\beta$ does not affect the choice of $b'$ and $m'$. This proves clause 1 of Proposition 2.

Next we discuss the effort cost and the decision of whether or not to adjust ($\mathfrak{R}$). First, assume that effort cost $e_0 = 0$. In the continuous-time limit, $w(x_0) = \beta v(x_0)$ when $e_0 = 0$. Thus, the $\beta < 1$ household will choose to adjust at the same points in the state space as the exponential household. This proves clause 2b of Proposition 2.

Next, assume $e_0 = \varepsilon$. If the household adjusts their mortgage at time 0 they earn value function $w^*(x_0) - \varepsilon$. If the household delays until time $\Delta$, they earn value function:

$$\begin{aligned} w(x_0) &= \max_c \; u(c)\Delta + \beta e^{-\rho\Delta} \mathbb{E}_0[v(x_\Delta)] \\ &\geq \max_c \; u(c)\Delta + \beta e^{-\rho\Delta} \mathbb{E}_0[v^*(x_\Delta) - \varepsilon]. \end{aligned}$$

In going from the first to the second line we impose that the household refinances at time $\Delta$, and therefore the $\geq$ sign is introduced. Taking $\Delta \to 0$ yields:

$$w(x_0) \geq \beta(v^*(x_0) - \varepsilon) = w^*(x_0) - \beta\varepsilon. \tag{15}$$

Adjusting in the current period yields a value of $w^*(x_0) - \varepsilon$ while waiting yields at least $w^*(x_0) - \beta\varepsilon$. Thus, for all $\beta < 1$ the naive agent perceives waiting to be optimal whenever $e_t = \varepsilon$. As equation (15) shows, waiting allows the effort cost to be discounted by $\beta$. This proves clause 2c of Proposition 2.

[61] We ignore difficulties such as the kink in the budget constraint when taking this first-order condition.

## B.3 Additional Proofs

**Proof of Corollary 1.** Recall that, with naiveté, the perceived continuation-value function of a $\beta < 1$ household equals the value function of an exponential $\beta = 1$ household and solves (8′). Assume that the household does not refinance at time $t$ so that the perceived continuation-value function $v(x_t)$ is characterized by a standard HJB equation. This HJB equation is given by the left branch of (8′), which we write here as

$$\rho v(x) = \max_c \; u(c) + \frac{\partial v(x)}{\partial b}(y + rb + \omega^{cc} b^- - (r^m + \xi)m - c) + (\mathcal{B}v)(x) \tag{16}$$

where the operator $(\mathcal{B}v)(x)$ is short-hand notation for lines two to seven of (8′). Recall that we use hat-notation to denote the policy functions that naive households perceive for future selves, and denote by $\widehat{c}(x)$ and $\widehat{s}(x) = (y + rb + \omega^{cc} b^- - (r^m + \xi)m - \widehat{c}(x))$ the corresponding perceived consumption and liquid saving policy functions. In contrast, denote by $c(x)$ (from Proposition 1) and $s(x) = (y + rb + \omega^{cc} b^- - (r^m + \xi)m - c(x))$ the *actual* policy functions.

The following observation is important in the proof below: the HJB equation for the perceived continuation-value function (16) features the *perceived* policy functions $\widehat{c}(x), \widehat{s}(x)$ rather than the *actual* policy functions. But what determines the evolution of liquid wealth $b$ are the *actual* policy functions.

Differentiate (16) with respect to $b$ and use the envelope theorem:

$$(\rho - r(b))\frac{\partial v(x)}{\partial b} = \frac{\partial^2 v(x)}{\partial b^2}\widehat{s}(x) + \frac{\partial}{\partial b}(\mathcal{B}v)(x). \tag{17}$$

Define the marginal continuation-value of wealth $\eta(x) \equiv \frac{\partial v(x)}{\partial b}$. From (17) it satisfies

$$(\rho - r(b))\,\eta(x) = \frac{\partial \eta(x)}{\partial b}\widehat{s}(x) + (\mathcal{B}\eta)(x). \tag{18}$$

If $\beta = 1$, from Itô's formula, the right-hand side of (18) also governs the expected change in the marginal value of wealth: $\mathbb{E}_t[d\eta(x_t)] = \left[\frac{\partial \eta(x_t)}{\partial b}\widehat{s}(x_t) + (\mathcal{B}\eta)(x_t)\right] dt$. But with $\beta < 1$ this is no longer true: the evolution of $b$ is governed by the *actual* drift $s(x)$ rather than the *perceived* drift $\widehat{s}(x)$ and so

$$\mathbb{E}_t[d\eta(x_t)] = \left[\frac{\partial \eta(x_t)}{\partial b}s(x_t) + (\mathcal{B}\eta)(x_t)\right] dt. \tag{19}$$

Therefore, evaluating (18) along a particular trajectory $x_t$, we have

$$(\rho - r(b_t))\,\eta(x_t) = \frac{1}{dt}\mathbb{E}_t[d\eta(x_t)] - \frac{\partial \eta(x_t)}{\partial b}(s(x_t) - \widehat{s}(x_t)).$$

Rearranging

$$\begin{aligned}\frac{1}{dt}\mathbb{E}_t[d\eta(x_t)] &= (\rho - r(b_t))\,\eta(x_t) + \frac{\partial \eta(x_t)}{\partial b}(s(x_t) - \widehat{s}(x_t)) \\ &= (\rho - r(b_t))\,\eta(x_t) + \frac{\partial \eta(x_t)}{\partial b}(\widehat{c}(x_t) - c(x_t)) \\ &= (\rho - r(b_t))\,\eta(x_t) + \frac{\partial \eta(x_t)}{\partial b}\left(\beta^{\frac{1}{\gamma}} - 1\right) c(x_t)\end{aligned}$$

Finally, recalling that $\eta(x) \equiv \frac{\partial v(x)}{\partial b}$, the first-order condition is $u'(c(x)) = \beta\eta(x)$ and therefore

$$\begin{aligned}\frac{1}{dt}\mathbb{E}_t[du'(c(x_t))] &= (\rho - r(b_t))\,u'(c(x_t)) + \frac{\partial u'(c(x_t))}{\partial b}\left(\beta^{\frac{1}{\gamma}} - 1\right) c(x_t) \\ &= (\rho - r(b_t))\,u'(c(x_t)) - u''(c(x_t))c(x_t)\left(1 - \beta^{\frac{1}{\gamma}}\right)\frac{\partial c(x_t)}{\partial b} \\ &= \left[\rho + \gamma\left(1 - \beta^{\frac{1}{\gamma}}\right)\frac{\partial c(x_t)}{\partial b} - r(b_t)\right] u'(c(x_t)),\end{aligned}$$

where going from the second line to the third line uses that, with CRRA utility, the coefficient of relative risk aversion is $\gamma = \frac{u''(c(x_t))c(x_t)}{u'(c(x_t))}$. Dividing by $u'(c(x_t))$, we have (11).□

# C Durables Extension: MPCs and MPXs

To bridge the gap between consumption and expenditure, this Appendix proposes an extension with durables that is isomorphic to the baseline model of Section 2. This extension allows us to study the marginal propensity for expenditure (MPX) in addition to the marginal propensity to consume (MPC). The simple technology that we develop here can be applied to a wide range of economic models.

**Setup with Durables.** The household can now consume two different goods: durables and non-durables. Let $n_t$ denote non-durable consumption, which the household purchases as a flow. The price of non-durable consumption is normalized to 1. To consume durables, the household must purchase a stock of durables $D_t \geq 0$. Durable stock $D_t$ provides durable consumption as a flow, and depreciates at rate $\nu$ with $\nu + \min\{r_t\} > 0$. In keeping with our partial equilibrium analysis, the price of durables is exogenous and we normalize it to one.[62]

Our extension with durables is isomorphic to our baseline model under three (strong) assumptions.

**Assumption 1** *The household always consumes a constant share of durables and non-durables. Let $s$ denote the durable share of total consumption.*

This assumption can be micro-founded, for example by assuming that utility is Leontief.

**Assumption 2** *The durables market is perfectly liquid, i.e. households can buy and sell durables freely at price $p_t \equiv 1$ (there are no transaction costs nor time delays). Further, households can borrow (short-term) against durables at the market rate $r_t$ rather than the more expensive credit card rate $r_t + \omega^{cc}$.*

The rationale for this assumption should be clear: if durables were illiquid we would not be able to map the extension with durables into our baseline model. Given that the durables price is constant and durables are liquid, the user cost of durables equals $r_t + \nu$.

**Assumption 3** *The durable stock $D_t$ provides a durable consumption flow of $f_t D_t$. The consumption flow per unit of durable equals the user cost of durable consumption $f_t = r_t + \nu$.*

A non-standard implication of this assumption is that the consumption flow per unit of durable $f_t$ varies over time with the interest rate $r_t$. This dependence is required to maintain

[62] This obviously rules out any dependence of the durable price on the interest rate $r_t$. If the price of durables varied with $r_t$, this would provide households with an additional asset that can be used to hedge against interest rate shocks. Since our goal is to provide an extension with durables that is isomorphic to our baseline model – which does not feature such an asset – we also shut down its existence here.

isomorphism with our original model. It is quantitatively minor because we calibrate $\nu \gg r_t$ so that changes in $r_t$ only result in very small changes in the durable flow $f_t = \nu + r_t$.

These three assumptions mean that we do not need to keep track of the household's durable stock separately from their total liquid wealth.

We continue to use $c_t$ to denote the household's total consumption. From Assumption 1, durable flow $f_t D_t$ composes share $s$ of total consumption, and non-durable consumption flow $n_t$ composes share $1 - s$ of total consumption:

$$f_t D_t = s \times c_t \qquad \text{and} \qquad n_t = (1 - s) \times c_t. \tag{20}$$

Additionally, using Assumption 3 the durable stock required to generate total consumption flow $c_t$ is

$$D_t = \frac{s}{r_t + \nu} c_t. \tag{21}$$

In this extended model with durables, the household holds two types of liquid assets: liquid bank holdings and durables. We continue to use $b_t$ to denote the household's bank holdings. To keep track of household spending on durables, we introduce process $\psi_t$ to record the household's cumulative spending on durables from time 0 to time $t$. Process $\psi_t$ is a right-continuous "jump-drift" process. The household's budget constraint is:

$$db_t = \left[y_t + r_t b_t + \omega^{cc}(b_t + D_t)^- - (r_t^m + \xi) m_t - n_t\right] dt - d\psi_t, \tag{22}$$

$$dm_t = -\xi m_t dt, \tag{23}$$

$$dD_t = -\nu D_t dt + d\psi_t. \tag{24}$$

The household's purchase of durables at time $t$ is denoted $d\psi_t$. Durables purchases can be lumpy so that $\psi_t$ and $D_t$ jump discontinuously at time $t$ (this is why we work with cumulative spending $\psi_t$). Because the household can borrow against durables at interest rate $r_t$ (Assumption 2), the household only pays the credit card borrowing wedge $\omega^{cc}$ on liquid debt not backed by durables $(b_t + D_t)^-$. Finally, the borrowing constraint is now applied to the household's total liquid wealth holdings:

$$b_t + D_t \geq \underline{b}.$$

The household will never hold more durables than are required for consumption. Durables are costly to hold because they depreciate and because interest payments are sacrificed (i.e., the cost of holding wealth in durables relative to bank holdings is $\nu + r_t$). Since we assumed that $\nu + \min\{r_t\} > 0$ it is always beneficial to hold the minimum amount of durables necessary for consumption.

**Isomorphism to Baseline Model.**

**Proposition 4** *Under Assumptions 1 to 3, the extension with durables is isomorphic to our baseline model. In particular, total liquid wealth $\ell_t = b_t + D_t$ evolves as*

$$\dot{\ell}_t = y_t + r_t \ell_t + \omega^{cc} \ell_t^- - (r_t^m + \xi) m_t - c_t, \qquad \ell_t \geq \underline{b} \tag{25}$$

*which is identical to the law of motion for $b_t$ in equation (1) and the borrowing constraint $b_t \geq \underline{b}$.*

**Proof.** Summing (22) and (24), we have

$$db_t + dD_t = \left[ y_t + r_t b_t + \omega^{cc} (b_t + D_t)^- - (r_t^m + \xi) m_t - n_t - \nu D_t \right] dt,$$

or in terms of $\ell_t = b_t + D_t$

$$d\ell_t = \left[ y_t + r_t \ell_t + \omega^{cc} \ell^- - (r_t^m + \xi) m_t - n_t - (\nu + r_t) D_t \right] dt.$$

Using (20):

$$d\ell_t = \left[ y_t + r_t \ell_t + \omega^{cc} \ell^- - (r_t^m + \xi) m_t - c_t + (f_t - \nu - r_t) D_t \right] dt.$$

Finally using that $f_t = r_t + \nu$ from Assumption 3 we have (25). ■

We briefly discuss the role of Assumption 3 ($f_t = \nu + r_t$) in the proof. This assumption ensures that the cost of total consumption is held constant in the model with and without durables. In the model of the main text, consumption of $c_t$ costs $c_t$. In this model with durables, total consumption $c_t$ is composed of durable flow $f_t D_t$ and non-durable consumption $n_t$. For the isomorphism to hold, we need the total cost of durable plus non-durable consumption to still equal $c_t$. The household consumes non-durables of $n_t = (1-s)c_t$, which costs $(1-s)c_t$. In contrast, the cost of non-durable consumption is the user cost $(r_t + \nu)D_t$. Thus we need the durable consumption flow $f_t D_t$ to equal its cost $(\nu + r_t)D_t$, and hence $f_t = r_t + \nu$.

**Proof of Proposition 3: Calculating the Marginal Propensity for Expenditure (MPX).** The household begins at point $x$ in the state space. Assume that the household is hit with a discrete wealth shock of size $\chi$ at $t = 0$. Starting from point $x$ and for a particular realization of the various shocks faced by the household, we let $\{c_t, n_t, d\psi_t, D_t\}$ denote the household's path of total consumption, non-durable consumption, durable spending, and the durable stock, respectively. We let $\{c_t', n_t', d\psi_t', D_t'\}$ denote the alternate path that these variables would have taken if the wealth shock had never occurred.

In the proof that follows, we derive expressions that hold for a discrete wealth shock of size $\chi$ and for particular shock realizations. To obtain the statement in Proposition 3, we will later consider an infinitesimal wealth shock $\chi \to 0$ and take an expectation over different shock realizations.

The total marginal propensity for expenditure (MPX) is the sum of the MPX on non-durable consumption (NDMPX) and the MPX on durable consumption (DMPX):

$$MPX_\tau^\chi(x) = NDMPX_\tau^\chi(x) + DMPX_\tau^\chi(x). \tag{26}$$

The MPX on non-durable consumption, denoted $NDMPX_\tau^\chi$, is:

$$NDMPX_\tau^\chi(x) = \frac{\int_0^\tau n_t dt - \int_0^\tau n_t' dt}{\chi}. \tag{27}$$

The MPX on durable consumption, denoted $DMPX_\tau^\chi$, is:

$$DMPX_\tau^\chi(x) = \frac{\left(\int_0^\tau d\psi_t + \int_0^\tau r_t D_t dt\right) - \left(\int_0^\tau d\psi_t' + \int_0^\tau r_t D_t' dt\right)}{\chi}. \tag{28}$$

The household's expenditure on durables is defined to include both the household's spending on new durables ($\int_0^\tau d\psi_t$) and also the interest payments that are sacrificed by holding durables ($\int_0^\tau r_t D_t dt$).[63]

Our benchmark model does not differentiate between the consumption of durables and non-durables. To map aggregate consumption into the MPX in (26), the next step is to rewrite the $NDMPX$ and the $DMPX$ in terms of total consumption $c_t$ using the assumption of constant shares (20). For non-durables, substituting $n_t = (1-s) \times c_t$ into (27)

$$NDMPX_\tau^\chi(x) = (1-s)\frac{\int_0^\tau c_t dt - \int_0^\tau c_t' dt}{\chi}. \tag{29}$$

For durables, we use both the budget constraint (24) or $d\psi_t = dD_t + \nu D_t dt$ and (28) to get

$$DMPX_\tau^\chi(x) = \frac{\int_0^\tau (r_t + \nu)(D_t - D_t')dt + D_\tau - D_\tau'}{\chi} = \frac{s\int_0^\tau (c_t - c_t')dt + D_\tau - D_\tau'}{\chi}, \tag{30}$$

[63] Though the inclusion of interest payments in the expenditure definition is not necessary, it is consistent with the empirical literature that imputes expenditure from household balance sheet data (e.g., Fagereng et al., 2019). The MPX formula can also be recomputed without the inclusion of interest rate payments. Since $r_t \approx 0$, the difference between these two approaches is quantitatively minor.

where the second equality uses (21). Combining equations (29) and (30) yields:

$$MPX_\tau^\chi(x) = \frac{\int_0^\tau (c_t - c_t')dt + D_\tau - D_\tau'}{\chi}, \tag{31}$$

and finally using (21) we have

$$MPX_\tau^\chi(x) = \frac{\int_0^\tau (c_t - c_t')dt + \frac{s}{\nu + r_t}(c_\tau - c_\tau')}{\chi}. \tag{32}$$

The first term in equation (32) is the MPC. The second term is the adjustment factor to go from the MPC to the MPX, which captures the additional stock of durables $D_\tau - D_\tau'$ that the household needs at time $\tau$ in order to increase consumption from $c_\tau'$ to $c_\tau$.

Since equation (32) holds state by state for an arbitrary shock of size $\chi$, it will also hold in expectation for an infinitesimal shock. This gives Proposition 3. ■

**Calibration of $s$ and $\nu$.** We need to calibrate two parameters: durable depreciation rate $\nu$ and durable share $s$. We calibrate the durable depreciation rate from the 2016 BEA Fixed Assets Accounts Tables. Table 1.1 reports a consumer durables stock of \$5,155.3 billion. Table 1.3 reports depreciation of \$1,033.3 billion. This implies a depreciation rate of $\nu = -\log\left(1 - \frac{1033.3}{5155.3}\right) = 0.22$. This calibration means that durables have a half-life of 3.15 years.

To calibrate $s$, we again use the consumption data recorded in the 2016 NIPA report. The 2016 NIPA Report (Table 2.4.5) documents that total household consumption expenditures (in billions) are \$12,748.5. This is broken into durable goods of \$1,352.6, non-durable goods of \$2,643.3, and services of \$8,752.6. From services we subtract spending on housing and utilities of \$2,355.3, because our model abstracts from home spending.

Assuming that households are in the steady state, all durable expenditures are made to offset depreciation.[64] Thus, $\nu D = 1352.6$. Assuming $r_0 = 0$ for simplicity, the restriction that $f \equiv \nu$ implies that a household's total durable expenditures of $\nu D_t = f D_t = s c_t$. We also know $n_t = (1 - s)c_t$. Letting both non-durable goods and services compose "non-durables", we have $n_t = 2643.3 + 8752.6 - 2355.3 = 9040.6$. Total consumption is given by $c_t = \nu D + n_t = 1352.6 + 9040.6 = 10393.2$. Now, the durable share can be imputed from $\frac{\nu D}{c_t}$, yielding an estimate of:

$$s = \frac{1352.6}{10393.2} = 0.13.$$

As a robustness check on our method, we could also have calibrated the durable share

[64] This assumption is not too far off. Total durable spending is 1,352.6, while depreciation is 1,033.3.

$s$ using the estimates of Parker et al. (2013). Parker et al. (2013) use the Consumer Expenditure Survey to track spending out of the 2008 Economic Stimulus Payment. They report a quarterly MPX of 52%. We can map these estimates into equation (13). Over one quarter, non-durable spending increases by 12%. Assuming that consumption increases to a constant rate following the shock, a quarterly MPC of 12% implies that the consumption rate increases by 48% of the stimulus payment (i.e., $12\% \times 4$). Setting $r = 1\%$, we arrive at:

$$MPX^{\chi}_{\frac{1}{4}} = 0.52 = 0.12 + \left(\frac{s}{0.23}\right) 0.48.$$

Plugging in for $\nu$ and solving yields a calibration of:

$$s = 0.19.$$

This is comparable to our benchmark calibration using the NIPA tables, though calculated using a different methodology. In particular, the NIPA table calculation corresponds to an average durable share, while the calculation here corresponds to a marginal durable share.

**Discussion of Model Shortcomings.** Though our model with durables is highly tractable, it is also stylized. The model with durables fails to provide an accurate description of reality in two important scenarios: negative wealth shocks and interest rate shocks. Given these considerations, we will only use our model with durables to study the MPX out of positive wealth shocks, holding interest rates constant.

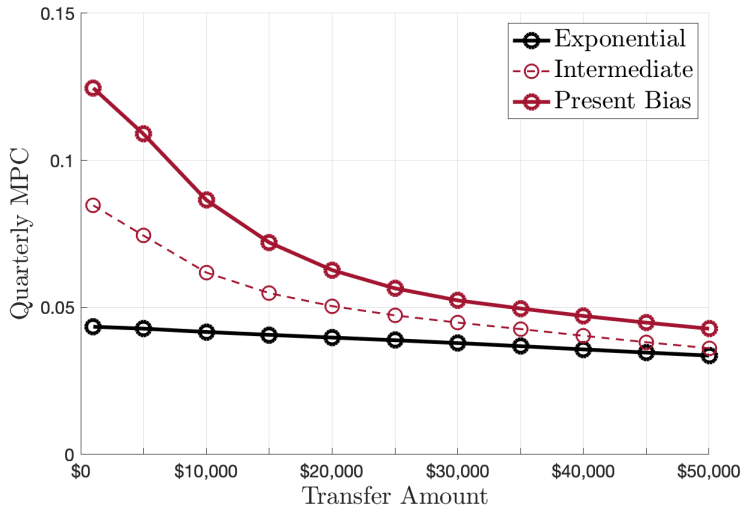
On negative wealth shocks, Berger and Vavra (2015) find that the durable response to wealth shocks is asymmetric. Households buy durables in response to positive wealth shocks, but wait for durables to depreciate in response to negative wealth shocks. In our model with liquid durables, the response to positive and negative wealth shocks is symmetric.

On interest rate shocks, the assumption that durables provide a consumption flow of $f_t = \nu + r_t$ means that the durable stock needed to attain a given consumption flow varies with $r_t$. This assumption is reverse engineered, and is not intended to capture reality. McKay and Wieland (2019) provide a richer model detailing the channels through which durables interact with interest rate changes. For this reason, we only study the MPX holding $r_t$ constant.

# D Additional Results

## D.1 Model Solution Details: MPCs

(a) Quarterly MPCs Across Transfer Amounts




(b) Present-Bias Benchmark: MPCs over Liquid Wealth

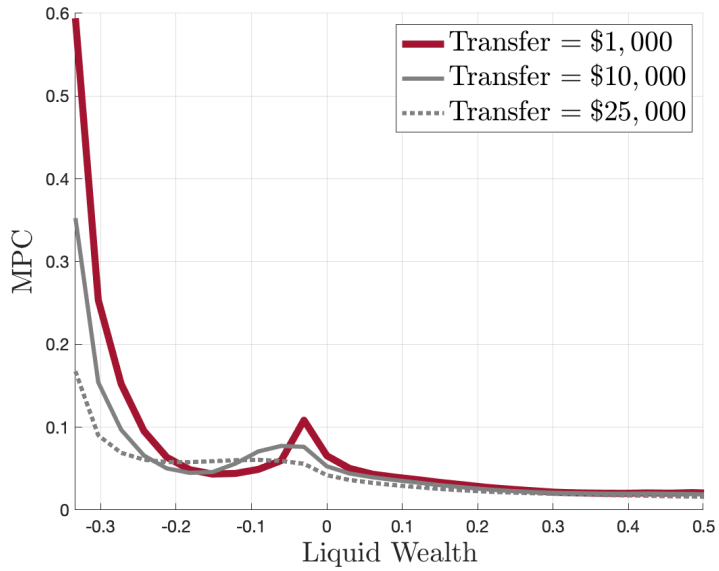



Figure 10: MPCs Across Transfer Amounts.

Notes: For the three calibration cases, the top panel plots quarterly MPCs out of transfers ranging from $1,000 to $50,000. The bottom panel replicates the MPC analysis in Figure 3 for the Present-Bias Benchmark calibration across transfer amounts of $1,000 (benchmark), $10,000, and $25,000.

## D.2 Model Solution Details: Steady State Distributions

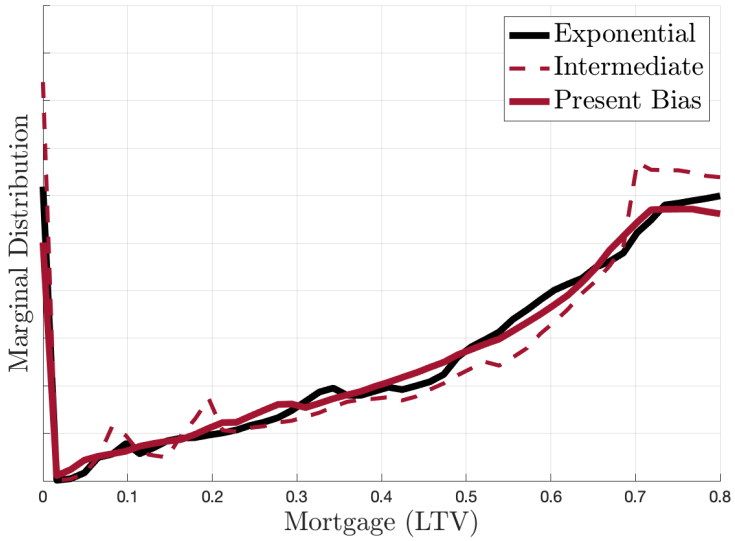



Figure 11: LTV Distribution.

Notes: This figure shows the steady state distribution of households over the LTV ratio.

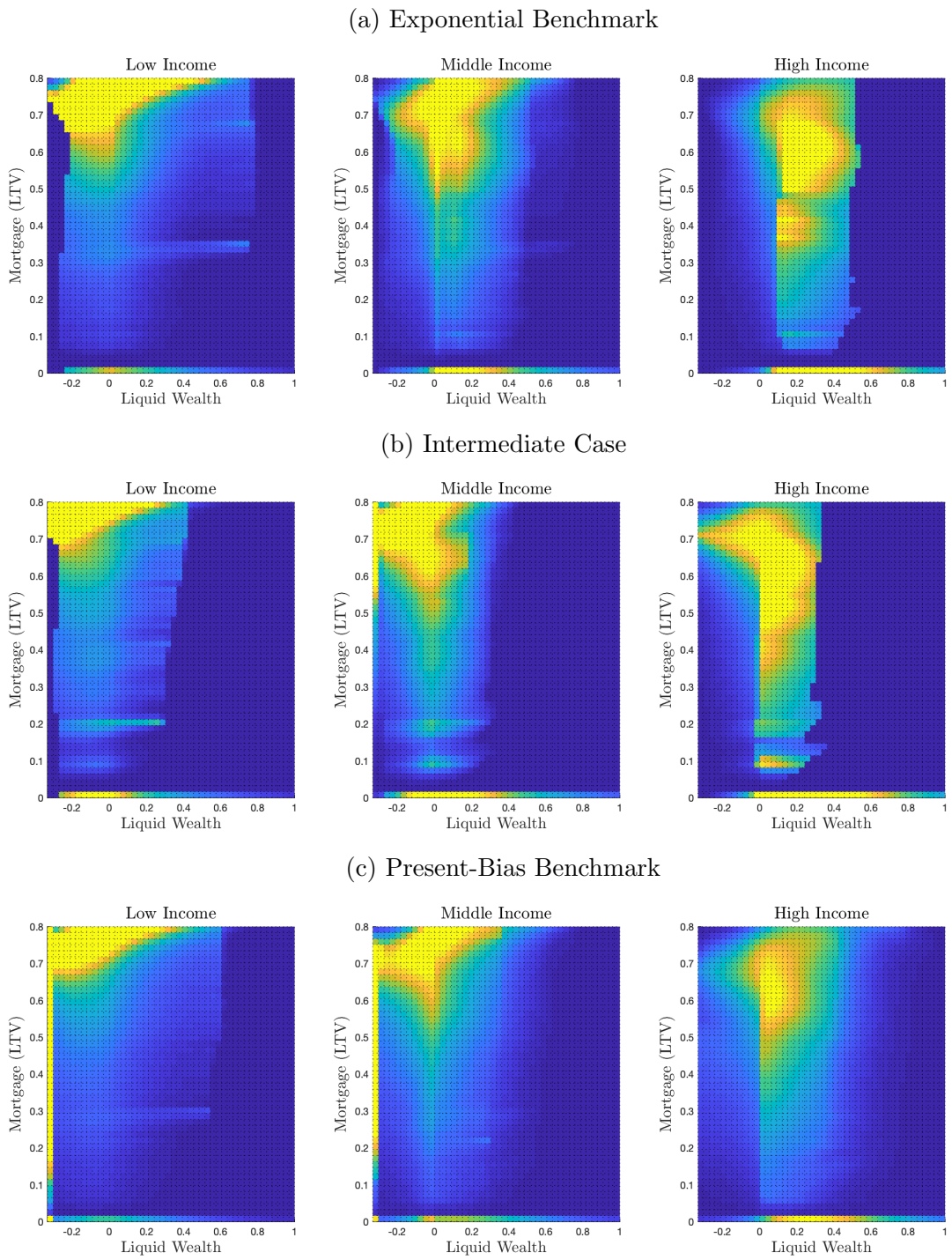



Figure 12: Steady State Distribution.

Notes: For the three calibration cases, this figure presents the full steady state distribution over income, liquid wealth, and mortgage debt. Dark blue regions are rarely encountered, while light yellow regions feature large masses of households.

## D.3 Monetary Policy: Refinancing Dynamics

Figure 13 plots the adjustment regions following an interest rate cut from 1% to 0%. This figure replicates the phase diagrams seen earlier, but now for the case of $r_t = 0\%$ and $r_t^m = 1\% + \omega^m$. Thus, Figure 13 plots the adjustment regions for households with a mortgage rate that is above the rate they can refinance into.

As above, the red region marks where households take a cash-out refinance and the blue region marks where households prepay their mortgage. The gray region indicates where households conduct a rate refinance, defined as the household increasing their mortgage balance by less than 5% during the refinance.

Relative to the steady state adjustment regions, the interest rate cut causes the red cash-out region to expand drastically. In particular, households with larger LTVs are more likely to refinance. Intuitively, households with larger mortgages have more to gain by reducing their mortgage interest payments.

Table 6 presents details of the refinancing decision. The first row lists the share of households who find themselves in a refinancing region at the time of the interest rate cut. Conditional on refinancing, the second row lists the share of households who extract equity when refinancing. The next four rows list the share of households who have actually refinanced within 1 quarter, 1 year, 2 years, and 3 years following the interest rate cut. While refinancing is instant in the Exponential Benchmark and the Intermediate Case, procrastination means that refinancing occurs slowly in the Present-Bias Benchmark.

| | Exponential | Intermediate | Present Bias |
|---|---|---|---|
| Share Refi Region (On Impact) | 73.1% | 68.5% | 74.9% |
| (Share Cash Out) | 81.0% | 66.8% | 77.3% |
| $\frac{1}{4}$ Year Realized Refi | 75.2% | 71.0% | 13.6% |
| 1 Year Realized Refi | 80.0% | 76.5% | 42.0% |
| 2 Year Realized Refi | 84.5% | 81.2% | 62.7% |
| 3 Year Realized Refi | 87.8% | 84.6% | 74.3% |
| Average Refi Amount | 0.31 | 0.17 | 0.29 |

Table 6: Refinancing Details.

Notes: For the three calibration cases, this table summarizes details of the household refinancing decision following an interest rate cut from 1% to 0%.

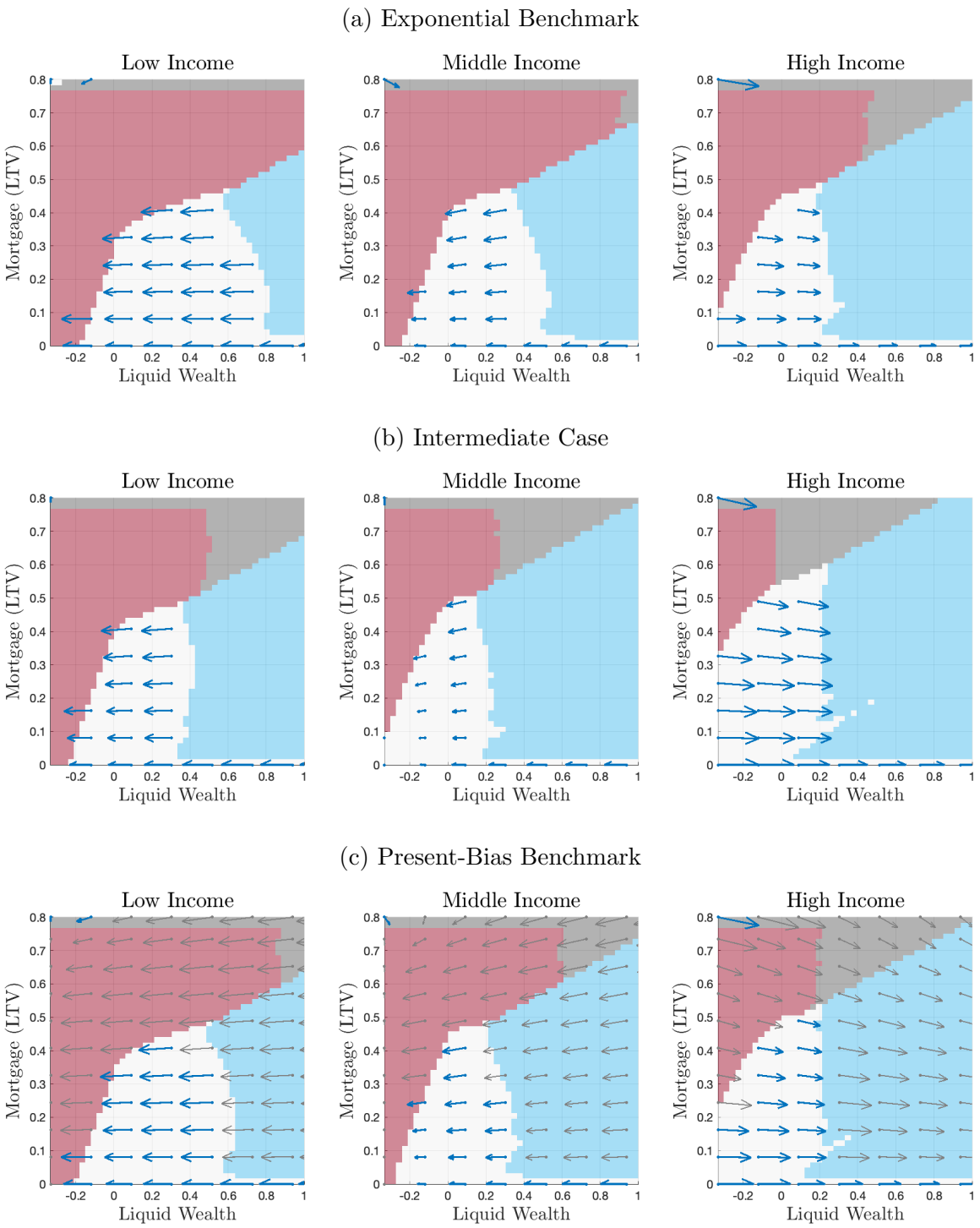



Figure 13: Rate-Cut Phase Diagrams

Notes: For the three calibration cases, this figure presents the phase diagrams for households who can refinance into a lower mortgage interest rate (see Figure 1 for phase diagram details).

## D.4 Monetary Policy and Refinancing Procrastination

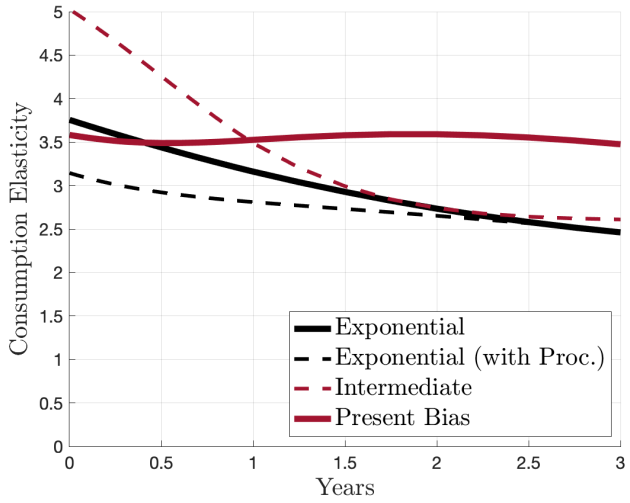



Figure 14: Consumption Response to Monetary Policy: Procrastination Sensitivity.
Notes: This figure adds a fourth case to the benchmark monetary policy analysis (Figure 6). This fourth case augments the Exponential Benchmark case with refinancing procrastination. The dashed black presents the consumption response to monetary policy in this exponential calibration with refinancing procrastination.

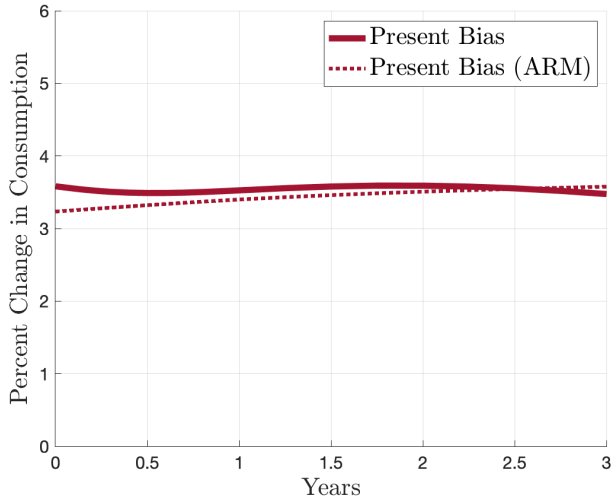



Figure 15: Monetary Policy under FRMs versus ARMs
Notes: For the Present-Bias Benchmark calibration, this figure compares the consumption response to monetary policy under FRMs (solid line) versus ARMs (dotted line). The interest rate is cut by 2% in the ARM experiment, compared to 1% in the FRM experiment, since monetary policy produces larger movements in ARM rates than long-duration FRM rates.

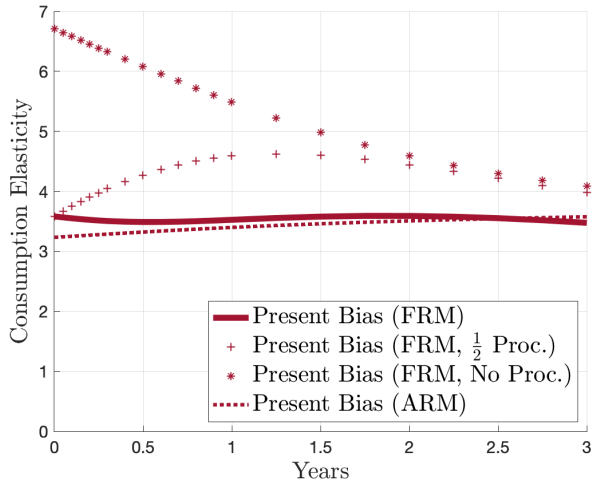



Figure 16: Monetary Policy with Procrastination Reduction.

Notes: This figure presents the consumption response to monetary policy in the Present-Bias Benchmark across varying levels of refinancing procrastination. The +'s assume that policymakers are able to halve the expected duration of procrastination at the time of the rate cut. The $*$'s make refinancing immediate at the time of the rate cut. The baseline consumption response under FRMs (solid red line) and ARMs (dotted red line) are presented for comparison.

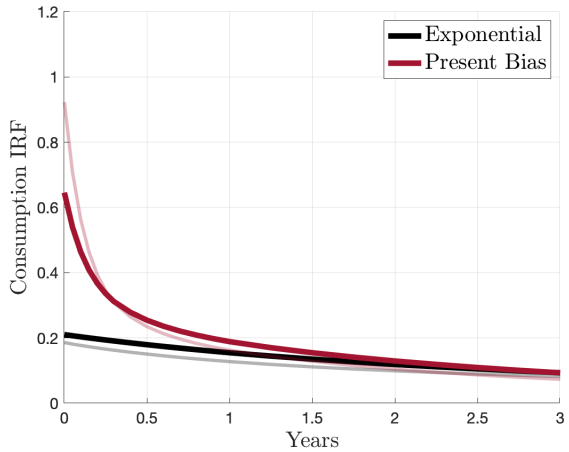
## D.5 Details on Aggregate House Price and Income Shocks

This section provides additional results for the analysis in Section 7.

**House Price Shocks.** Section 7.1 of the main text discusses the sensitivity of monetary policy to house price shocks. Here we provide further details on the fiscal policy experiment. Figure 17 plots the consumption response to fiscal stimulus in the negative (left) and positive (right) shock case. The corresponding MPCs are reported below in Table 7.

As the left panel of Figure 17 illustrates, the negative house price shock weakens the consumption response in the Present-Bias Benchmark over the first quarter, but strengthens the consumption response thereafter. The right panel of Figure 17 shows that the opposite is true of fiscal policy following a positive house price shock. In both cases, present bias strongly amplifies the consumption response to fiscal policy.

(a) -25% House Price Shock




(b) +25% House Price Shock

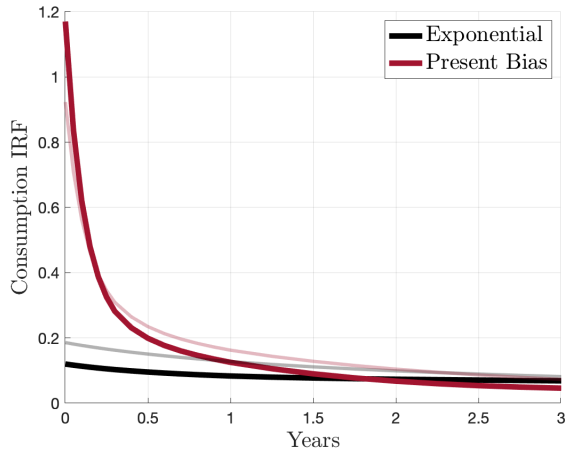



Figure 17: Fiscal Policy and House Price Shocks.

Notes: This figure plots the IRF of aggregate consumption to a $1,000 fiscal transfer that immediately follows a house price shock of -25% (left) or +25% (right). The transparent lines plot to the baseline case in Figure 4, and are included for reference.

| | Exponential | Present Bias |
|---|---|---|
| Baseline (No Shocks) | | |
| 1 Year MPC | 15% | 28% |
| 2 Year MPC | 26% | 41% |
| 3 Year MPC | 35% | 49% |
| -25% House Price Shock | | |
| 1 Year MPC | 18% | 28% |
| 2 Year MPC | 31% | 43% |
| 3 Year MPC | 41% | 53% |
| +25% House Price Shock | | |
| 1 Year MPC | 10% | 26% |
| 2 Year MPC | 17% | 35% |
| 3 Year MPC | 24% | 40% |

Table 7: Fiscal Policy MPCs and House Price Shocks

Notes: This table presents the aggregate MPC from a $1,000 fiscal transfer that is given immediately after a ± 25% house price shock.

**Income Shocks.** Section 7.2 of the main text outlines the effect of aggregate income shocks on monetary and fiscal policy. For this aggregate income shock experiment, the left panel of Figure 18 plots the consumption response to monetary policy and the right panel plots the consumption response to fiscal policy. As described in the main text, the consumption response is almost identical to the baseline case in Section 6.

(a) Monetary Policy (b) Fiscal Policy

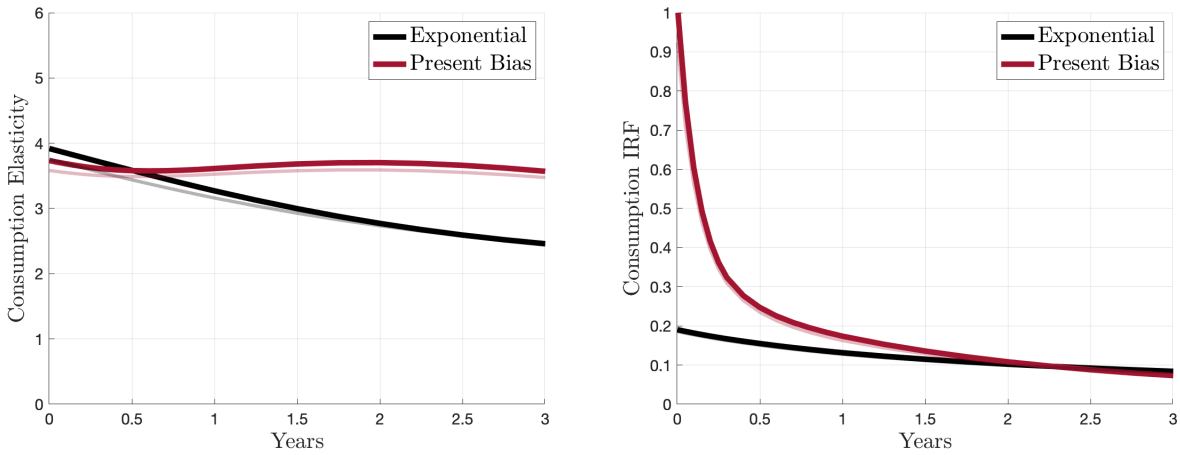



Figure 18: Fiscal and Monetary Policy Following a Negative Income Shock
Notes: This figure plots the consumption response to monetary (left) and fiscal (right) policy that is implemented immediately following a transitory 5% decline in aggregate income.