Penultimate Draft: "Leibniz's Formal Theory of Contingency," *Logical Analysis and History of Philosophy*: From Leibniz to Kant, eds. Katherine Dunlop and Samuel Levey, forthcoming. Please cite published version.

Leibniz's Formal Theory of Contingency

Abstract

This essay argues that, with his much-maligned "infinite analysis" theory of contingency, Leibniz is onto something deep and important—a tangle of issues that wouldn't be sorted out properly for centuries to come, and then only by some of the greatest minds of the twentieth century. The first section places Leibniz's theory in its proper historical context and draws a distinction between Leibniz's logical and meta-logical discoveries. The second section argues that Leibniz's logical insights initially make his "infinite analysis" theory of contingency more rather than less perplexing. The last two sections argue that Leibniz's meta-logical insights, however, point the way towards a better appreciation of (what we should regard as) his formal theory of contingency, and its correlative, his formal theory of necessity.

Introduction

Leibniz's views on logic and truth might seem to commit him to the view that all true propositions are necessarily true. Leibniz assumes that every proposition can be cast in subject-predicate form. A sentence such as "Peter is a denier of Christ" wears its logical form on its sleeve, while the logical form of, say, "Adam and Eve love each other" might be more perspicuously expressed by "Adam loves Eve" and "Eve loves Adam." Leibniz further maintains that "in all true affirmative propositions, necessary or contingent, universal or singular, the notion of the predicate is always in some way included in that of the subject—*praedicato inest subjecto*" (GP 2:56/FW 111-112; see also A VI.iv.223). Thus, the proposition *Peter is a denier of Christ* seems to be true if and only if the predicate expressed by "is a denier of Christ" is contained in the subject expressed by "Peter." Finally, Leibniz also maintains that for every genuine subject there is a complete concept containing all and only those predicates that will be true of that subject. Given these three commitments, it is hard to see how any proposition might be contingent. If the sentence "Peter is a denier of Christ" expresses a true proposition, it seems that there must be a complete concept corresponding to "Peter," that

that complete concept must contain the predicate *is a denier of Christ*, and that that is sufficient for the proposition *Peter is a denier of Christ* to be true. How then could the sentence "Peter is a denier of Christ" be contingent?

It is a measure of Leibniz's brilliance—or madness—that he offers not one, but (at least) two theories of contingency. The first theory—his *hypothetical necessity theory*—is relatively plain sailing.[1] It effectively weakens his theory of truth by suggesting that predicate containment is a necessary but not sufficient condition for a proposition's being true. For the proposition *Peter is a denier of Christ* to be true, not only must the complete concept corresponding to Peter contain the predicate *is a denier of Christ*, but Peter must also be created. *Peter is a denier of Christ* is thus hypothetically necessary in the sense that it must be the case that if Peter exists, then he denies Christ. But *Peter is a denier of Christ* is nonetheless contingent because Peter's existence is itself contingent. There are, of course, well-known objections to Leibniz's "first" theory of contingency. One might worry, for example, that given Leibniz's system, Peter's existence might itself be necessary, and so the proposition *Peter is a denier of Christ* might turn out to be not just hypothetically, but absolutely necessary after all. Likewise, one might object that Leibniz's hypothetical necessity theory of contingency won't meet the demands of his theodicy. If, for example, the only way for Peter not to deny Christ is for Peter not to exist then it might seem that Peter cannot be responsible for denying Christ. Whatever one thinks of the ultimate merits of Leibniz's first theory of contingency, however, it at least has the following virtue: it's easy to see what Leibniz is getting at, to see how he could think that the contingency of contingent propositions might be rooted in the contingent existence of their subjects.

Leibniz's second theory of contingency, his *infinite analysis* theory, or, as we will call it, his *formal theory* of contingency, may well seem to lack even the minimal virtue of intelligibility. In developing this "second" theory, Leibniz suggests that the distinction between necessary and contingent propositions may be drawn in purely formal terms, gesturing to a contrast between propositions that admit of a finite formal analysis to identities and propositions for which the process of formal analysis "proceeds to infinity" (A VI.iv.1656/L 265). There is little consensus, however, over how these various hints might be pulled together into a theory of contingency.[2] Indeed,

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[1] See, for example, A VI.iii.128; GP 7:235. For discussion and further texts see Adams (1994, 10-22) and Sleigh (1990, 80-83).

[2] For recent discussions of Leibniz's formal theory of contingency, see especially Hawthorne and Cover (2000), Merlo (2012), Rodriguez-Pereyra and Lodge (2011) and Steward (2014). For classic discussions see also Adams (1994); Blumenfeld

in a characteristically blunt assessment, Jonathan Bennett has suggested that it is high time we simply threw in the towel:

> Nobody has made respectable sense of what Leibniz says about finite and infinite analysis of subject-concepts. Furthermore, even if he did succeed in that, nobody thinks the result would have anything to do with contingency as we understand it, or, therefore, that it could satisfy Leibniz's need to defend contingency (in our sense) so that God has real choices to make. We should drop the matter. It is too late in the day to expect the mystery to be cleared up, and I guess that if Leibniz or scholarship did remove the veil, we would conclude that the search had not been worth our trouble. I mean: worth our trouble as philosophers. It is different for antiquarians. (Bennett 2001, 329)

While not unsympathetic with Bennett's frustration, in what follows we intend to flout his advice. Section 1 places Leibniz's formal theory of contingency in its proper—if often neglected—context and draws a distinction between his logical and meta-logical insights. Section 2 argues that closer attention to Leibniz's logical insights should make his formal theory of contingency—at least as it has been standardly interpreted—seem more, rather than less, perplexing. Section 3 argues that Leibniz's meta-logical insights, however, point the way towards a better understanding of his formal theory of contingency. Section 4 argues the same for Leibniz's correlative formal theory of necessity. The essay as whole thus aims to show that, with his formal theories of contingency and necessity, Leibniz was, after all, onto something genuinely complex, puzzling and profound—a tissue of ideas that wouldn't be sorted out for centuries to come, and then only by some of the greatest minds of the twentieth century. Removing the veil, we think, will prove well worth the trouble.

1. Language, Logic, and Meta-Logic

Leibniz's formal theory of contingency develops against the backdrop of his often-neglected interest in ideal languages. The dream of creating, or discovering, an ideal language is perhaps as old as the myth of the tower of Babylon.[3] It was given a distinctive shape, however, near the close of the

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(1985); Curley (1972); Carriero (1993), (1995); Hacking (1973), (1974), (1978); Ishiguro (1972, 171-203); Maher (1980); and Mates (1986, 105-117).

[3] For a helpful overview of Leibniz's thinking about language see Rutherford (1994). For an engaging overview of the ideal language tradition, see Eco (1997).

thirteenth century by the Franciscan philosopher Raymond Lull. Lacking a traditional university education, Lull conceived a radically non-Aristotelian "art" of reasoning based on primitive principles, symbolic representations, and algorithmic rules. Although initially viewed with suspicion, Lull's work soon inspired related efforts by Renaissance figures such as Nicholas of Cusa (1401-64) and Guillaume Postel (1510-81). It was carried further by famous early modern figures such as Bacon, Descartes and Spinoza through their work on philosophical method, as well as by less-well-known figures such as Johann Heinrich Alsted, Johann Heinrich Bisterfeld, Jan Amos Comenius and Athanasius Kircher through their work on combinatorial arts (for background, see Antognazza (2009, 62) and Arthur (2006, 29)). While still a young man, Leibniz himself became enthralled with the search for a perfect language (W 17), writing his Habilitation thesis, *Dissertatio de Arte Combinatoria*, on the combinatorial possibilities of fundamental concepts. A steady stream of works on logic, method, and combinatorics followed over the course of his long career. Some fifty years after his initial inspiration, Leibniz was still working out what he saw as the fantastic promise of an ideal language (Antognazza 2009, 64).

Leibniz thought that the construction of an ideal language would have to involve at least three essential steps. First, primitive concepts would have to be identified. Leibniz maintains that the primitive concepts represented in a truly ideal language would be absolutely fundamental and unanalyzable into more basic, simpler concepts (A VI.iv.590/AG 26; A VI.iv.1569/AG 57). Not insignificantly, however, he also allows for the possibility of a relatively ideal language, a language whose primitive concepts would be only relatively, or provisionally, basic. In any case, primitive concepts are to be contrasted with derivative concepts. Derivative concepts are built up from more basic concepts, and can conversely be resolved or analyzed into those more basic concepts (A VI.iv.540/L 230). In an ingenious analogy, Leibniz suggests that primitive concepts might be likened to prime numbers and derivative concepts to composite numbers. Just as a prime number cannot be divided by any other prime number, so primitive concepts cannot be decomposed into other concepts. And just as composite numbers can be reached by multiplying prime numbers and, conversely, be decomposed into prime numbers by division, so too derivative concepts can be constructed from primitive concepts and can be decomposed, ultimately, back into primitive concepts (A VI.iv.289/P 37).

Second, the construction of an ideal language would require the formulation of a suitable system of symbols. Leibniz's success with his infinitesimal calculus had impressed upon him the importance of helpful notation (see, for example, A VI.iv.910). He recognized that well-chosen symbols may make conceptual relationships almost self-evident, while poorly-chosen symbols can obscure even the simplest of conceptual connections. Leibniz hoped that someday a wholly new language might be devised that would make the conceptual relations of our everyday discourse utterly transparent. In the nearer term, however, he commended "another less elegant road already open to us" that would not "have to be built completely new" (A VI.iv.965/W 52). Picking up on the analogy mentioned above, Leibniz suggests that the names of prime numbers might be used to denote primitive concepts and the names of composite numbers might be used to denote derivative concepts. Anyone capable of performing multiplication or division could then easily discern the relationships between primitive and derivative concepts. So, for example, Leibniz suggests that we might let "6" stand for the derivative concept *man*, "3" stand for the primitive concept *rational*, and "2" stand for the primitive concept *animal*. Anyone familiar with the relevant symbolism, and capable of performing division, could then recognize immediately that the concept *man* contains the concepts *rational* and *animal* (A VI.iv.201/L 238).[4]

Third, the construction of an ideal language would require identifying a set of rules that would allow users of the language to formally, mechanically, or "blindly" manipulate symbols in order to make new discoveries and draw conclusions without fear of error (A VI.iv.587/AG 25). A helpful example of the sort of rules Leibniz has in mind is provided by his *Addenda to the Specimen of the Universal Calculus* (1679-86) (GP 7:221-227/P 40-46). Allowing italicized lower case letters to stand for concepts, Leibniz suggests that "the transposition of letters in the same term changes nothing: e.g. *ab* coincides with *ba*, or 'rational animal' and 'animal rational.'" Likewise, he suggests that "[r]epetition of the same letter in the same term is superfluous, such as *b* is *aa*, or *bb* is *a*: man is an animal animal, or man man is an animal. For it is enough to say, *a* is *b*, or man is an animal." Given such

[4] Leibniz's suggestion here anticipates Gödel numbering, the technique used by Kurt Gödel to treat a formal system as a mere system of signs with a specified mapping from signs to numbers in order to mathematically study the syntax of Peano Arithmetic. Although it is known that Gödel checked out the Gerhardt volume containing Leibniz's *Dissertatio de arte combinatoria* in 1929 (van Atten and Kennedy (2015, 124, fn 67), see also van Atten (2015)), and that Gödel studied Leibniz's work extensively in the early 1930s, around the time of his incompleteness results, there is no direct evidence that Gödel was inspired in this particular respect by Leibniz (Menger 1994, 210). Special thanks to Mark van Atten for helpful discussion of this point.

rules, users of Leibniz's ideal language would be licensed in asserting *ab* when given *ba* and *a* when given *aa*. Indeed, they could be confident of such substitutions even without knowing what concepts the terms *a* and *b* stand for. A user of Leibniz's ideal language could thus carry out inferences algorithmically just as a mathematician, following the methods of Leibniz's infinitesimal calculus, may "blindly" or mechanically find derivatives of (say) even complex polynomial expressions.

Leibniz's aspirations for an ideal language may at first glance seem fanciful and utopian (see, for example, A VI.iv.6-7/W 16-17). A closer look, however, reveals a more intriguing, if still optimistic, picture. For, on the one hand, closer scrutiny shows that Leibniz was less starry-eyed about the ideal language tradition and its short-term prospects than is commonly supposed. He takes Lull to task, for example, for his seemingly arbitrary choice of primitive concepts (A VI.i.193/W 53). He expresses amazement at the logical lacunae in the purported demonstrations of his contemporaries (A VI.iv.705/W 37). He acknowledges that, in practice, the construction of an ideal language may depend essentially on experiments and observations (A III.i.331-332/L 166; G 1:193-99/L 187). Thus, although characteristically optimistic, Leibniz is far from naïve either about the uneven work of his predecessors in the ideal language tradition or about the difficulties that would need to be surmounted in order to realize the dream of an ideal language. Furthermore, and on the other hand, a closer scrutiny of Leibniz's work on ideal languages suggests that he has far better grounds for his optimism—or at least his enthusiasm—than has often been recognized. Mostly importantly for our purposes, Leibniz's pursuit of an ideal language put him solidly on the path of three startling advances that would, some 200 years later, come to revolutionize the study of mathematics, logic, and, what we now call, computer science.

The first of those advances concerns the nature of demonstration itself. With his syllogistic logic, Aristotle had introduced the notion of a *formal* demonstration, that is, of a demonstration that is truth-preserving in virtue of its form rather than the meanings or denotations of the terms involved (see, for example, A III.ii.449-452/L 192-194). So profound was Aristotle's influence that his syllogistic logic was still being taught to students in Leibniz's day. Many of them evidently hated it (see, for example, Locke (1975, IV.xvii.4)). Sensing that not all formal reasoning could be, or had to be, fit into the straightjacket of syllogistic form, efforts were made to articulate new methods of demonstration that are intuitive rather than formal. Descartes's *Rules for the Direction of the Mind* (*Regulae ad directionem ingenii*) and Antoine Arnauld and Pierre Nicole's *Logic, or Art of Thinking* (*La logique, ou l'art de* *penser*) are perhaps the most famous examples of such efforts. Leibniz too recognized that not all valid reasoning could be, or had to be, fit to the procrustean bed of syllogistic form. Unlike so many of his contemporaries, however, Leibniz held Aristotle's logic in high regard, praising it as "one of the most important [discoveries], to have been made by the human mind" (NE 478). Rather than abandoning formal demonstration in favor of intuitive reasoning, Leibniz proposed to expand formal reasoning to include any form that "has been demonstrated in advance so that one is sure of not going wrong with it" (NE 479 see also G 1:194/L 187). His efforts led him to devise a general system of logic inspired by the rules of algebra (see, for example, A VI.iv.739-788/P 47-87 and A VI.iv.845-855/P 122-130). Some 150 years later, George Boole would follow the same inspiration in constructing what is now known as Boolean Logic. When, half a generation later, Gottlob Frege sought to expand the rules of formal reasoning even further, he presented himself as continuing Leibniz's efforts, explaining that what he "wanted to create was … a *lingua characteristica* in Leibniz's sense" (van Heijenoort (1967, 2); see also Davis (2000, 48-52) and Kluge (1977)).

The second advance that can be traced back to Leibniz's pursuit of an ideal language concerns the notion of decidability. For Leibniz, the pursuit of an ideal language was a matter of not only theoretical but also practical interest. Eager to bridge the religious, political and social rifts of his time, Leibniz hoped that a perfect language might allow disputes to be settled in a foolproof, automatic, algorithmic manner like disputes over simple calculated sums. Indeed, he hoped that, armed with an ideal language—"the greatest instrument of reason"—that "when there are disputes among persons, we can simply say: Let us calculate, without further ado, and see who is right" (A VI.iv.964/W 51; see also A VI.iv.913). In suggesting that arguments couched in an ideal language might be guaranteed to be resolved, Leibniz anticipated an idea notably not taken up even by Frege. For all its brilliance, Frege's *Begriffsschrift* offers no way of knowing—apart from success—whether a conclusion can be derived from a given set of premises (see Goldfarb (2001)). Questions concerning decidability came into their own during the late-nineteenth and early-twentieth centuries with the explicit development of meta-logic and meta-mathematics. Gödel's incompleteness results, together with the precise mathematical explication of the notion of decidability, led for instance to a proof of the remarkable result that mathematics (as encapsulated in Peano Arithmetic) is undecidable if consistent—that is, informally, that there is no computational procedure for telling, given any formula of Peano Arithmetic, whether or not that formula is derivable in Peano Arithmetic. Fueled by such results, contemporary consensus now holds that no language could possibly guarantee that all disputes—or even just all mathematical disputes—could be solved algorithmically. Remarkably, however, Gödel, an ardent student of Leibniz's philosophy (see Goldfarb (2011) and Parsons (2010)), seems to have shared Leibniz's optimistic outlook, insisting that "Leibniz did not in his writings about the *Characteristica universalis* speak of a utopian project," and maintaining that "he [Leibniz] had developed his calculus of reasoning to a large extent, but was waiting with its publication till the seed could fall on fertile ground" (Schilpp 1999, 153).[5]

Finally, the third advance that can be traced back to Leibniz's pursuit of an ideal language concerns the notion of computability. Leibniz recognized that if reasoning could be carried out in a "blind," algorithmic manner, then even a machine should, in principle, be able to carry out formal inferences. Around 1671, apparently inspired by a mechanistic pedometer, Leibniz resolved to construct just such a machine (Leibniz 1685). Within a couple of years, he had invented the first calculator—now known as the Step Reckoner—capable of performing all four arithmetical operations: addition, subtraction, multiplication and division. In doing so, Leibniz's machine improved upon Blaise Pascal's calculating machine, the Pascaline, principally in its ability to solve—to mechanistically calculate—problems of multiplication and division. In subsequent studies, Leibniz developed further plans for machines that would be capable of solving even more complex problems, including algebraic equations (Couturat 1901, 115). Leibniz's work on calculating machines adds a concrete dimension to his intuitive thinking about decidability. In thinking through the design and construction of various calculating machines, Leibniz couldn't but be confronted with questions we

[5] In a revision, quoted by van Atten and Kennedy (2003, p. 433), Gödel remarks that the universal characteristic "if interpreted as a formal system" does not exist. Although difficult to interpret, Gödel may have thought that mathematics is decidable provided that formal systems are supplemented by a kind of mathematical intuition or experience. If that is correct, his views may be remarkably in tune with Leibniz's views as we interpret him. In brief, our Leibniz draws a distinction between propositions that decidable without ordinary experience and propositions decidable only with the aid of ordinary experience. Gödel's incompleteness results raise difficulties in connection with the class of propositions that Leibniz thought decidable without the aid of experience. Gödel's optimism is grounded in the thought that that class of propositions, although not decidable without appeal to experience of any kind, might nonetheless be decidable by appeal to some kind of extra-ordinary, "rational," experience and "extrinsic" justifications. For related discussion see Parsons (2010, 185).

naturally think of as questions of computability: given a certain input, will a particular machine (computer) be guaranteed to yield, or "halt" with, the correct output? Given, for example, a problem of multiplication or division, in the case of the Pascaline, the answer is "no." In the case of Leibniz's Step Reckoner, the answer is "yes." Interestingly, our contemporary understanding of decidability, although slightly different and more precise than Leibniz's, was similarly refined in part by thinking in terms of idealized machines. Today, we think of the decidability of, say, a formal system, in terms of there being a computational procedure for determining whether any given formula is derivable in that formal system. The relevant notion of "computability," used at first in an intuitive sense, was made more precise in the 1930's in terms of an abstract model of computing machines generally known today as "Turing Machines."[6] In seeing an intimate connection between formal logic on the one hand and machines capable of carrying out algorithmic procedures on the other, Leibniz had an early glimpse of the powerful combination that, with time, would come to fuel the computer revolution that surrounds us today.[7]

At some risk of whiggishness, we have emphasized the sophistication of Leibniz's work on ideal languages for two reasons. First, although we think that Leibniz's formal theories of contingency and necessity are best understood against the backdrop of his interest in ideal languages, that interest itself might be thought a cause for embarrassment, a further reason to be dismissive of Leibniz's second theory of contingency. But Leibniz has nothing to be embarrassed about here. His work on ideal languages is, to be sure, imperfect, incomplete and not fully settled. But it is also deep, insightful and far ahead of its time. A better sense of the difficulty of the issues Leibniz was grappling with, and the considerable advances he made in thinking them through, should clear the way for a more sympathetic assessment of the essential background to his formal theories of contingency and necessity. Second, a sense of what Leibniz was on to with his work on ideal languages is

[6] Turing machines were first described by Alan Turing (1937). Though their proposals turned out to be equivalent, Gödel (1986) and Alonzo Church (1936) also independently offered precise mathematical explications of computability.

[7] For an engaging discussion of the path from Leibniz's thinking about language and logic to Turing's discoveries, see Davis (2000). Although not noted by Davis, Leibniz and Turing also shared a common interest in mechanical cryptography, Leibniz evidently having quickly realized that his *Machina arithmetica* could easily be modified to serve as a *Machina deciphratoria* (A I.ii.125, A IV.iv.27, A IV.iv.68, A III.ii.449-450/L 192). For discussion of Leibniz's interest in cryptography, see Rescher (2012) and also Beeley (2014, 111-122).

crucial to recognizing a distinction that will be central to our discussion to follow. Some aspects of Leibniz's thinking about ideal languages concern what we may think of as issues belonging to logic per se. His thinking about primitive concepts, proper symbolism, and above all formal demonstration, for example, belongs to this branch of his work. Other aspects of Leibniz's thinking about ideal languages concern what we may think of as issues belonging to meta-logic. Leibniz's thinking about whether or not problems, questions and arguments are resolvable by means of an ideal language, or by the operations of algorithmic machines, belongs to this branch of his thinking. With that rough distinction in mind—the distinction between Leibniz's logical and meta-logical insights—we will argue, in the next section, that Leibniz's insights in logic make his formal theory of contingency, at least as it has been standardly interpreted, more rather than less puzzling. In subsequent sections, we'll argue that Leibniz's insights in meta-logic, however, finally point the way towards a better understanding of his formal theories of contingency and necessity.

2. A Logical Theory of Contingency?

Leibniz's understanding of logic in general, and of formal demonstration in particular, are natural places to start in trying to make sense of his formal theories of necessity and contingency. And, indeed, there are a number of well-known passages in which Leibniz seems to encourage just such an approach. So, for example, in a well-known piece that has been entitled "On Contingency" and dated to the mid-1680's, Leibniz writes:

> And with this secret the distinction between necessary and contingent truths is revealed … namely that in necessary propositions, when the analysis is continued indefinitely, it arrives at an equation that is an identity; that is what it is to demonstrate a truth with geometrical rigor. But in contingent propositions one continues the analysis to infinity through reasons for reasons, so that one never has a complete demonstration, though there is always, underneath, a reason for the truth, but the reason is understood completely only by God, who alone traverses the infinite series in one act of mind. (A VI.iv.1650/Adams (1994, 26), see also A VI.iv.1515-1516/PM 96-98)

Passages such as this suggest a natural interpretation of Leibniz's formal theories of necessity and contingency. The interpretation's core thought is that a proposition is *necessary* if and only if its demonstration requires (only) a finite number of steps. A proposition is *contingent* if and only if its demonstration would require an infinite number of steps. For ease of exposition, let's call attempts to understand Leibniz's infinite analysis theories of necessity and contingency along these lines *logical* interpretations.

It is relatively easy to see how, drawing on Leibniz's logical studies, one might begin to flesh-out a logical interpretation for at least some necessary propositions. We might suppose, for example, that the proposition *5 = 2 + 3* could be demonstrated in a finite number of steps by appealing to definitions and self-evident rules of valid substitution and inference.[8] Beginning with the statement "5 = 2 + 3" we could appeal to the definition of "2" and rules of substitution to arrive at the statement "5 = 1 + 1 + 3." Appealing to the definition of "3" and rules of substitution we could arrive next at the statement "5 = 1 + 1 + 1 + 1 + 1." Finally, appealing to the definition of "5" and rules of substitution we could arrive at the identity statement "1 + 1 + 1 + 1 + 1 = 1 + 1 + 1 + 1 + 1." In three formally valid steps, we would thus have moved from a statement expressing the proposition *5 = 2 + 3* to "an identical equation." And in this case at least, a logical interpretation seems to give us the right result. We think that "5 = 2 + 3" expresses a necessary proposition, and, as we've just seen, it is indeed possible to demonstrate that *5 = 2 + 3* in a finite number of formally valid steps.

It is harder, but not impossible, to see how a logical interpretation might be similarly developed for at least some putatively contingent propositions. Consider, for example, the proposition *Peter is a denier of Christ*. Drawing roughly on Leibniz's own examples, we might represent Peter's complete concept with the letters "mdy," letting "m" represent the predicate "is a man," "d" the predicate "is a denier of Christ," and "y" the predicate expressing the conjunction of all the other predicates contained in Peter's complete concept.[9] Substituting definitions, we could rewrite the statement

[8] See, for example, Leibniz's *Specimen of a Universal Calculus* 1679 (A VI.iv.280-288/P 33-39) and *Addenda to the Specimen of the Universal Calculus* 1679-86? (A VI.iv.289-296/P 39-46). For helpful discussion, see Rescher (1954) and Levey (2014). Note, on the following reconstruction, Leibniz takes for granted the associativity of addition, a point noted and discussed by Frege in his *Foundations of Arithmetic* (Frege 1980, 7-8).

[9] That a predicate such as the one expressed by "y" should be permissible should be clear both from Leibniz's general understanding of the relationship between signs and derivative concepts as well as from his explicit statements, e.g.: "one letter can be put which is equal to this conjunction of several (just as for the term 'rational animal' we put, for sake of brevity, the one term 'man'); and for the composite term ab or abc, found in the predicate, there can be substituted the simple term a" (A VI.iv.283-284/P 35).

expressing the proposition to be demonstrated as "mdy is d." [10] Leibniz accepts a transposition rule of the form *ab is ba*, writing, for example, that "The transposition of letters in the same term changes nothing: e.g. ab coincides with ba, or, 'rational animal' and 'animal rational'" (A VI.iv.293/P 43). Applying Leibniz's transposition rule thus yields "dmy is d." We may then introduce "z = my" as a definition, and appeal to substitution to get "dz is d." It is clear that Leibniz would see this as being as good as a reduction to an explicit "primitive truth" or "axiom." He tells us, for example, that "'ab is a' is always true," (A VI.iv.754 /P 58) and describes "ab is a," e.g. "A rational animal is an animal" as belonging to "propositions true in themselves" (A VI.iv.292/P 42).[11] We might—indeed, probably should—stop our derivation here. If, however, we wish to push on, Leibniz's logic does provide resources for showing that "ab is a" is logically equivalent to an explicit identity statement. We may introduce "z is z" as an identity, combine to get "dzz is dz," [12] and eliminate repetition to get "dz is dz." [13] Since the inclusion expressed by "dz is dz" is symmetric, we may finally conclude with an explicit identity statement, that is, "dz = dz."[14] Given Leibniz's logic, it therefore

[10] "Is" here denotes predication rather than identity (denoted above by "="). On Leibniz's intensional approach (see Levey (2014, 112)), it would seem that a proof of an identity statement should make the mutual inclusion of both sides of the equation explicit, while a proof of predicative statement should only need to make the inclusion of the predicate in the subject explicit. Intuitively, a proof of an identity statement such as "a = b" should show that a is included in b and that b is included in a, while a proof of a predicative statement such as "a is b" should only need to show that b is included in a (even if a is not included in b).

[11] Similarly, in his *Addenda to the Specimen of the Universal Calculus 1679-86?*, Leibniz writes: "[W]hen I say that the proposition 'ab is a' is always true, I understand to be true not only the example 'A rational animal is an animal (taking 'animal' to be signified by a, and 'rational' by b), but also the example 'A rational animal is rational (taking 'rational to be signified by a, and 'animal' by b)" (A VI.iv.289/P 40). See also *General Inquiries about the Analysis of Concepts and Truths 1686*, (A VI.iv.755/P 58).

[12] See Leibniz's *Addenda to the Specimen of the Universal Calculus*: "(4) From any number of propositions it is possible to make one proposition, by adding together all the subjects into one subject and all the predicates into one predicate. From *a* is *b*, *c* is *d* and *e* is *f* we get *ace* is *bdf*" (A VI.iv.293/Park 43).

[13] See Leibniz's *Addenda to the Specimen of the Universal Calculus*: "(3) Repetition of the same letter in the same term is superfluous, such as *b* is *aa*, or *bb* is *a*: man is an animal animal, or man man [*homo homo*] is an animal. For it is enough to say, *a* is *b,* or, man is an animal" (A VI.iv.293/Park 43).

[14] See Leibniz's *Addenda to the Specimen of the Universal Calculus*: "If *a* is *b* and *b* is *a*, then *a* and *b* are said to be 'the same.' For example, every pious man is happy, and every

seems that the proposition expressed by "Peter is a denier of Christ" may similarly be reduced to an identity statement in a manner closely analogous to the way in which "5 = 2 + 3" may be reduced to an identity statement.[15]

In contrast to our earlier example, however, in this case our derivation seems to yield the wrong result. We generally think that "Peter is a denier of Christ" expresses a contingent proposition. But, as we've just seen, it now seems that it too can be reduced to an identity statement in a finite number of formally valid steps. Although we've come at it from a slightly different angle, this is essentially the same difficulty famously identified by Robert Adams as the problem of the Lucky Proof:

> Even if infinitely many properties and events are contained in the complete concept of Peter, at least one of them will be proved in the first step of any analysis. Why couldn't it be Peter's denial? Why couldn't we begin to analyze Peter's concept by saying, "Peter is a denier of Christ and…? (1994, 34)

The problem of the Lucky Proof brings out a deep and central difficulty for logical interpretations of Leibniz's formal theory of contingency. For what the problem shows most centrally is that Leibniz appears to be committed to the existence of finite demonstrations of contingent propositions. And if that is right, then, of course, it cannot be the case that a proposition is contingent if an only if its demonstration must involve infinitely many steps.

In response to the problem of the Lucky Proof, many commentators have argued that Leibniz is not, in spite of appearances, committed to there being finite proofs of contingent propositions. In making those arguments, commentators have, for the most part, followed one of two broad strategies. The first and most dominant strategy looks to non-formal considerations in order to, as Robert Adams puts it, place "some sort of restriction on what counts as a step in an analysis of an individual concept" (1994, 34). The

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happy man is pious, therefore 'happy man' and 'pious man' are the same" (A VI.iv.294/Park 43).

[15] Notice that the rules of combination and elimination used above could be used to turn even a false statement of the form "a is b" into an identity statement "ab = ab" (we can add "a is a" and "b is b" to "a is b" to get "aab is abb" (see footnote 13), then eliminate repetitions to get "ab is ab" (see footnote 14) and thereby conclude "ab = ab" (see footnote 15). So, it would seem that in order to properly "reduce" a sentence to an identity in Leibniz's sense, the sentence we "reduce" to an identity should be "logically equivalent" to the identity, and not merely logically imply it. In our example above, our two sentences ("mdy is d" and "dz = dz") are indeed logically equivalent (for the trivial reason that they are both axioms on Leibniz's logic). But "a is b" and "ab = ab" are not logically equivalent for any arbitrary a and b.

driving idea here is to appeal to non-formal considerations in order to rule out otherwise valid inferences and thereby to block otherwise possible finite proofs of contingent propositions. So, for an example, it has been suggested that the analysis of the subject Caesar in the proposition *Caesar crosses the Rubicon* should have to follow the causal order of the appetites that lead Caesar to cross the Rubicon. Assuming that there are infinitely many such appetites, a story can then be told according to which Caesar crosses the Rubicon cannot be demonstrated in a finite number of step.[16] By drawing on non-formal considerations, such as the causal structure of a subject's appetites, one might thus hope to solve the problem of the Lucky Proof by showing that there are, after all, no permissible finite demonstrations of contingent propositions.

Although the non-formal strategy may be initially tempting, as a development of the logical approach to Leibniz's formal theory of contingency it faces, what seems to us, a devastating dilemma. For the non-formal considerations appealed to in order to block otherwise permissible finite demonstrations must be understood as placing constraints either (i) on what counts as a valid inference itself or (ii) on what counts as a legitimate analysis. But if, taking the first horn, the non-formal constraints are thought of as constraining what counts as a valid inference, then those constraints would undermine Leibniz's advanced understanding of logical demonstration itself. For if, for example, the inference from "a is ab" to "a is ba" may be beholden to, say, the order of Caesar's appetites, or, more generally, what "a" and "b" mean or signify, then we have utterly abandoned the notion of a formal inference, and with it Leibniz's advanced understanding of the very notion of formal demonstration. If, however, taking the second horn, the non-formal considerations are understood as additional constraints on what counts as an analysis, then the distinction between contingent and necessary truths is not really being drawn in terms of Leibniz's logic after all. For, on this horn, contingent propositions will still admit of finite demonstrations even if they don't admit of finite analyses in some non-logical sense of analysis. To take the second horn is not to defend a logical interpretation of Leibniz's second theory of contingency but rather to deny that he has a logical theory of contingency after all.

The second, and recently resurgent, strategy for responding to the problem of the Lucky Proof appeals to additional formal considerations

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[16] This version of the non-formal strategy is developed in Cover and Hawthorne (2000). For specific discussion and criticism, see Bennett (2001, 327-329), Rodriguez-Pereyra and Lodge (2011) and Stewart (2014, 37-38).

rather than non-formal considerations. Drawing inspiration from Leibniz's critique of the ontological argument, its core idea is that a complete proof requires, in effect, two phases: first a reduction of the relevant terms to an identity statement, and second a demonstration that the concepts involved are themselves consistent.[17] On this *consistency strategy*, it can be allowed that even contingent propositions, for Leibniz, may admit of finite reductions to identity statements, that, for example, the statement "Peter is a denier of Christ" might be reduced to an identity statement via a finite number of substitutions and inferences. But, proponents of this strategy will insist, such finite reductions do not show that contingent propositions admit of finite proofs or demonstrations. Rather they will maintain that for "Peter is a denier of Christ" to be demonstrated it is necessary to show that the concept of Peter is itself consistent, and given that the concept of Peter is infinitely complex, one might suppose that such a consistency check will itself require infinitely many steps. The consistency strategy thus suggests that Leibniz can allow that in the case of contingent propositions there may be, as it were, lucky finite reductions (first phase) while still denying that there are lucky finite demonstrations (first and second phase).

Although skillfully developed by its proponents, the consistency strategy faces long-standing, well-discussed difficulties.[18] Most importantly, it implies that any proposition involving an infinitely complex concept will be contingent. In that case, however, too many propositions would seem to count as contingent. In particular, even identity statements involving complete concepts, such as "Peter = Peter," would appear to be contingent. And that raises both philosophical and textual difficulties. It presents a philosophical difficulty because —as Gonzalo Rodriguez-Pereyra and Paul Lodge put it— "It is natural to think of identities as necessary" (2011, 231, fn. 42). If the morning star is identical to the evening star, then it is presumably the case that the morning star is necessarily identical to the evening star. It presents a textual difficulty because it conflicts with clear assertions made by Leibniz that identity statements are necessary. He tells us, for example, that "An animal is an animal" is true in itself (A VI.iv.292/P 42) and that "identical propositions are necessary without any understanding or resolution of the terms, for I know that A is A regardless of what is understood by A" (G 1:194/L 187). Even beyond such philosophical and textual difficulties,

[17] This avenue of response is developed by Patrick Maher (1980, 238-239), Hawthorne and Cover (2000, 153-156) and especially Rodriguez-Pereyra and Lodge (2011).

[18] For critical discussion, see Hawthorne and Cover (2000, 155-156), Maher (1980, 239), Merlo (2012) and Steward (2014, 32-37).

however, it is furthermore unclear why Leibniz himself would have adopted the two-stage, or two-requirement, understanding of demonstration presupposed by the consistency strategy. Or rather it is hard to see why he would have adopted it other than to specifically block the problem of the Lucky Proof, a problem that he shows no sign of having recognized. In addition to giving rise to philosophical and textual difficulties, the consistency strategy thus seems to us both ad hoc and under-motivated by Leibniz's own concerns.

Although a natural place to start in trying to make sense of his formal theory of contingency, Leibniz's advanced understanding of logic turns out, on closer inspection, to make his second theory of contingency seem more rather than less puzzling. For Leibniz's understanding of logic makes it difficult to see how he could possibly think that contingent propositions do not admit of finite demonstrations. One might, of course, suppose that the trouble here lies with Leibniz. One might suppose that Leibniz simply failed to see a rather obvious and fatal objection to a theory of contingency that he enthusiastically, but somewhat naively, entertained for decades. But that seems to us implausible. More likely, we have not yet fully understood what exactly Leibniz is trying to get at with his formal theory of contingency. In the next two sections, we will suggest that looking to Leibniz's meta-logical insights provides a more plausible, intuitive picture of what he is up to with his formal theories of contingency and necessity, a picture, incidentally, on which the Problem of the Lucky Proof, as a problem, simply never arises.

3. A Meta-Logical Theory of Contingency

As noted above, Leibniz's interest in ideal languages led him to reflect not only on the construction of formal systems of logic but also on what might be established by means of such formal systems. Leibniz's interest in meta-logical considerations suggests an alternative way of understanding what he was trying to get at with his formal theory of contingency. Framed in terms of his intuitive understanding of the notion of decidability, the core thought would be that a proposition is contingent if there is no algorithmic, formal procedure guaranteed to discover (and output) its proof. Even if we were equipped with a perfect language, we would have no algorithmic means, no method analogous to multiplication or division, that we could apply in order to arrive with certainty at a proof of the relevant proposition. Put alternatively in terms of his intuitive understanding of computability, the core thought would be that no machine could be built by us that, working from definitions alone, and following an algorithmic procedure, would be guaranteed to halt at a proof of a contingent proposition. Even if we were equipped with a fantastically expanded and idealized version of Leibniz's Step Reckoner—a modern computer, if you like—our "rational calculator" might run forever without arriving at a proof of a contingent proposition. For ease of expression, let us call such an interpretation of Leibniz's formal theory of contingency, a *meta-logical* interpretation.[19]

On a meta-logical interpretation, Leibniz's proposed demarcation of contingent propositions would be formal, not epistemic: whether or not something is decidable or computable is a formal matter, not an epistemic one. But, of course, the suggestion that contingent propositions are not, in the relevant sense, decidable or computable does have epistemic implications, and those implications might be seen as furnishing the intuitive idea behind Leibniz's technical demarcation. For, of course, it is quite plausible that the truth of contingent propositions cannot be established by appeal to non-empirical axioms, definitions and formal procedures alone. If we want to know whether or not Peter is a denier of Christ, my dog is a Labradoodle, or it rained last Tuesday, it seems our reasoning must at some point touch base with experience. Someone must witness Peter's betrayal, examine my dog, or get caught in a downpour. Leibniz, of course, couldn't think that that intuitive idea alone might provide a fully satisfying demarcation of contingent truths. For he allows that God can know that Peter is a denier of Christ, that my dog is a Labradoodle, and that it rained last Tuesday *a priori*; that is to say, Leibniz allows that even contingent truths are, strictly speaking, knowable even in the absence of ordinary experience. With his formal theory of contingency, however, Leibniz could still offer an objective, non-epistemic demarcation of contingent propositions that nonetheless explains, and thus can draw support from, the intuitive idea that contingent propositions are knowable by us—by finite creatures—only with the aid of experience. Again for ease of exposition, let us call that intuitive idea the *driving idea* behind Leibniz's formal theory of contingency.

A meta-logical interpretation and its driving idea fit nicely—although not necessarily uniquely—with many of Leibniz's texts. Consider, for

[19] We leave open the possibility that, for Leibniz, a contingent proposition might be decidable by an infinitely complex procedure and that an infinitely complex machine might be guaranteed to halt at a proof of a contingent proposition. And, in fact, perhaps a monad's law of the series is just such a procedure, and the monad, or the natural machine that is its body, is just such a machine. On this point, see also Hacking (1978, 191). Special thanks to Tomas Feeney for drawing our attention to this point.

example, Leibniz's letter to Henry Oldenburg dated December 28, 1675 (A III.i.326-334/L 165-166). In it, Leibniz makes mention of his "combinatorial characteristic," which he describes as "a higher science" than algebra but similar to it in that with it "we cannot err even if we wish and that truth can be grasped as if pictured on paper with the aid of a machine." He boasts to Oldenburg that "nothing more effective" than his new method "can well be conceived for perfecting the human mind and that if this basis for philosophizing is accepted, there will come a time, and it will be soon, when we shall have as certain knowledge of God and the mind as we now have of figures and numbers and when the invention of machines will be no more difficult than the construction of geometric problems." Having extolled the virtues of his characteristic, however, and most remarkably for our purposes, Leibniz goes on to imply that we should not expect his characteristic, on its own, to settle contingent truths about nature. He writes "when these [combinatorial] studies have been completed … men will return to the investigation of nature alone, which will never be entirely completed," adding later "Once men carry our [combinatorial] method through to the end, therefore, they will always philosophize in the manner of Boyle," that is, empirically (A III.i.332/L 166). In perfect keeping with the spirit of a meta-logical interpretation, Leibniz, in his letter to Oldenburg, thus suggests that, even once perfected, his combinatorial characteristic won't provide an algorithmic procedure guaranteed to settle contingent propositions. According to Leibniz, an ideal language will be of great use in establishing the truth of contingent propositions—it will be a great tool to mathematicians and natural philosophers alike. But it will be of practical use in settling contingent propositions only when supplemented by experience.

Another significant text is provided by Leibniz's letter to Herman Conring of 19 March 1678 (G 1:193-199/L 186-191). In it, Leibniz attempts to respond to Conring's "criticisms regarding analysis and demonstration." Focusing on demonstration, Leibniz first explains that "only identities are indemonstrable;" he maintains that even axioms are strictly speaking demonstrable even though "they are mostly so clear and easy that they do not need demonstration." Leibniz then provides an account of his understanding of demonstration in which he makes it clear that he sees the demonstration of contingent propositions as presupposing empirical observation:

> [I]t is clear that demonstration is a chain of definitions. For in the demonstration of any proposition, nothing is used but definitions, axioms (with which I here include postulates), theorems which have been demonstrated previously, and observations. Since the theorems again must themselves be demonstrated, and axioms, except for

> identities, can also all be demonstrated, it follows that all truths can be resolved into definitions, identical propositions, and observations—though purely intelligible truths do not need observations. After the analysis has been completed, it will become manifest that the chain of demonstrations begins with identical propositions or observations and ends in a conclusion but that the beginning is connected with the conclusion through intervening definitions. In this sense I said that a demonstration is a chain of definitions. (G 1:194/L 187)

Leibniz's account of demonstration here again fits remarkably well with a meta-logical interpretation and its driving idea. Setting aside formal identities, and taking a practical context for granted, Leibniz thinks that all propositions can be demonstrated. Necessary propositions (to which we'll return in the next section) can be demonstrated ultimately from definitions alone (since postulates and axioms can themselves be demonstrated). Contingent propositions, however, can be demonstrated ultimately from definitions only with the aid of empirical observations. Leibniz's ideal language is not guaranteed to settle the truth of all propositions on it is own. And, in particular, in the case of contingent propositions it serves only as a tool aiding experience, not as an algorithmic method for establishing contingent propositions a priori.

A final text worth considering here has been dated to 1680 and entitled "Precepts for Advancing the Sciences and Arts" (A VI.iv.692-713/W 29-46). In this work, Leibniz makes more explicit how he sees his work on ideal language as dovetailing with his views on scientific method more generally. As is so often the case, Leibniz presents himself as staking out a moderate, intermediate position. Against what he sees as immoderate empiricism, which he clearly associates with the Royal Society, Leibniz emphasizes the importance of formal tools, including logic and mathematics. He thus reports, for example, "they confessed to me in England that the great number of experiments they have amassed gives them no less difficulty than the lack of experiments gave the ancients" (A II.i.554/W. xxiii). Leibniz's emphasis on formal tools, however, should not be taken to imply that he thinks that all contingent propositions can be established by pure reasoning alone. On the contrary, he emphasizes, often with a barb directed at Descartes and his followers, that progress is generally to be made in the sciences by joining formal tools to careful observation and choice experiments. As emerges clearly in *Precepts for Advancing the Sciences and Arts*, the theory of perspective and of musical harmonies and dissonances serve, for Leibniz, as scientific paradigms. They are powerful theories rooted in a few observations or experiments with implications drawn out by careful reasoning. Leibniz thus rejects immoderate rationalism no less than immoderate empiricism, an attitude that lies behind his famous comment "I prefer a Leeuwenhoek who tells me what he sees to a Cartesian who tells me what he thinks. It is … necessary to add reasoning to observations" (Leibniz 1690, 641). Leibniz's understanding of science, as set out in his *Precepts for Advancing the Sciences and Arts* thus once again fits nicely with both a meta-logical interpretation of his formal theory of contingency and its driving idea. Even armed with an ideal language, and fully exploiting formal tools, it will be possible for us to establish contingent propositions of science only with the aid of experience.

Beyond textual considerations, a meta-logical interpretation also provides an elegant resolution to the problem of the Lucky Proof. As we've seen, the problem of the Lucky Proof originally arose in the context of logical interpretations. In that context, it raises a rather obvious worry that directly challenges the guiding thought of such interpretations, namely, that a proposition is contingent if and only if it does not admit of a finite demonstration. In the context of a meta-logical interpretation, however, things look significantly different. On a meta-logical interpretation, Leibniz's formal theory of contingency isn't particularly invested in whether or not there are finite demonstrations of contingent propositions. It is concerned rather with the question of whether or not there is an algorithmic procedure guaranteed to find finite proofs. Given a meta-logical interpretation, it is therefore not so surprising that Leibniz never seems to have worried about the possibility of a Lucky Proof. Moreover, a meta-logical interpretation suggests two, closely related, ways in which Leibniz could have responded to the problem of the Lucky Proof if he had considered it. On the first way, he could allow that lucky proofs are possible but deny that they represent a threat to his formal theory of contingency. For if a lucky proof is precisely a proof that one luckily "hits" upon without following an algorithmic procedure, then such proofs do not present even a prima facie a challenge to Leibniz's formal theory of contingency on a meta-logical interpretation. On the second way, Leibniz could maintain that the very notion of a lucky proof is incoherent. In order to take this route, he would have to stipulate that a proof essentially involves a sequence of definitions arrived at by application of an algorithmic process. Given such an understanding of what a proof is, a lucky proof would be as incoherent for Leibniz as the notion of an infinite proof is for many today. The first way of responding to the problem of the Lucky Proof would cleave closer to our understanding of what a proof is; the second way would allow Leibniz to deny that contingent propositions admit of finite proofs. Both ways of responding are sufficient to undercut the problem of the Lucky proof, both draw on Leibniz's attention to meta-logical considerations, and both would rest on purely formal considerations. On a meta-logical interpretation, the problem of the Lucky Proof is no problem at all.

A meta-logical interpretation of Leibniz's formal theory of contingency also offers insight into a worry that has recently been revived by Rodriguez-Pereyra and Lodge. They suggest that behind the problem of the Lucky Proof "there is a deeper and more substantive problem about Leibniz's infinite analysis conception of contingency:"

> For even if we are unlucky and it takes a long time to uncover a particular predicate in the definition of a subject, it will always be uncovered in some finite number of steps. The point can be seen more clearly if we associate each one of the infinitely many concepts constituting Peter's concept with a natural number and we imagine that our analysis uncovers those constituent concepts according to the order of natural numbers. Then no matter what number the concept 'denier of Christ' is associated with, it will take only a finite—but probably very large—number of steps to reach this concept from the beginning of our analysis. In this case, although the full decomposition of the infinitely complex 'Peter' will not be completeable in a finite number of steps, every concept composing 'Peter' can be found in 'Peter' after a finite number of steps. (Rodrigues-Pereyra and Lodge (2011, 223), see also Mayer (1980, 239))

Rodriguez-Pereyra and Lodge call this allegedly deeper problem *the problem of the Guaranteed Proof*. It represents a prima facie challenge to proof-based interpretations because it suggests that every contingent proposition should admit of a finite proof. That needn't worry us, of course. We have already argued that, on a logical interpretation, Leibniz's formal theory of contingency is doomed to failure. But the problem of the Guaranteed Proof also represents a prima facie challenge to a meta-logical interpretation because it suggests that there might be an algorithmic procedure—for example "unpacking" in the order of the natural numbers—that would be guaranteed to find a finite proof for any contingent proposition. If that were the case, then Leibniz's formal theory of contingency would be doomed to failure even on a meta-logical interpretation.

It has been claimed that the problem of the Guaranteed Proof is, in fact, a non-starter. On the way to offering his own (extra-formal) solution to the problem of the Lucky Proof, Giovanni Merlo suggests that the problem of the Guaranteed Proof rests on a rather elementary confusion:

> Think again of a bag containing infinitely many marbles, each numbered with a different natural number. Sure enough, we can imagine a sequence of draws in which marble '2' is hit upon after finitely many attempts (here is one sequence: 1, 3, 7, 11, **2**, 34, …). But of course there are many 'unlucky' sequences as well: think of any sequence going from marble '150' onward. Rodriguez-Pereyra and Lodge invite us to "imagine that our analysis uncovers [the] constituent concepts according to the order of natural numbers." Now, sure enough, if our analysis unfolds according to the order of natural numbers, concept number '56' will be hit upon after 56 steps. But the point is precisely that whether or not our analysis unfolds according to the order of natural numbers is a matter of *luck*: there are vastly many 'unlucky' analyses that evolve randomly and take infinitely long detours … So even if there is a problem of lucky proof, I do not think this problem generalizes into a problem of guaranteed proof. (Merlo 2012, 14)

Supposing that the analysis of complete concepts is analogous to drawing marbles from a bag, Merlo denies that there is a Problem of the Guaranteed Proof. For he maintains that there are infinitely many procedures that would not result in a proof of a given contingent proposition. Supposing that the predicate needed to complete a proof is numbered by 2, we might "draw" predicates from the bag forever without finding 2 if, for example, we were to begin our search with 3 and follow the order of the natural numbers.

Although instructive, we think that neither side in the debate over the Problem of the Guaranteed Proof has quite put its finger on the deep lesson the challenge offers. Both sides assume that the concepts contained in Peter's complete concept can be "numbered" with the natural numbers (cf. Merlo (2012, 32-33, fn 20)). If this means that the concepts contained in Peter's complete concept can be exhaustively listed in the order of the natural numbers in an effective manner—if they are recursively enumerable, as it were[20]—then Rodriguez-Pereyra and Lodge will be right in suggesting that any contained, sought-after predicate can be found in a finite number of steps.

[20] More technically, a set of integers is said to be "recursively enumerable" if and only if it is the domain of some partial recursive function. Sets that are recursively enumerable have a *search procedure*: an effective procedure that, applied to an integer n, eventually terminates if n is in the set but does not terminate if n is not in the set. Metaphorically, we only need to look through the integers, consecutively, and terminate the process once we find the integer that represents the concept p, and keep looking, infinitely, if p isn't part of the complete concept.

The fact that there would still be infinitely many procedures that will fail to find the sought-after predicate would be entirely beside the point. If, however, the assumption that the concepts contained in Peter's complete concept are "numbered" isn't meant to imply that they can be exhaustively listed in the order of the natural numbers (or in a similarly well-behaved sequence)—that is, if they are not recursively enumerable—then Merlo will be right in suggesting that there is no guarantee that a contained, sought-after predicate will be found in a finite number of steps. The deep lesson of the problem of the Guaranteed Proof is that there will be a guaranteed finite proof if the domain of predicates specified by a genuine subject's complete concept, or (perhaps equivalently) the domain of proofs of contingent propositions, can be recursively enumerated. And that deep lesson is exactly what we should expect given a meta-logical interpretation of Leibniz's formal theory of contingency. For if, for example, the domain of predicates contained in Peter's complete concept can be recursively enumerated, then it should be possible to algorithmically "search" for precisely the predicate needed in order to construct a finite proof of any true proposition concerning Peter. But if, as the meta-logical interpretation suggests, Peter's complete concept is not recursively enumerable, then there should be no such procedure; an attempt to reduce "Peter is a denier of Christ" to an explicit identity statement might go on forever. On a meta-logical interpretation not only does the problem of the Lucky Proof go away but the problem of the Guaranteed Proof vanishes as well.[21]

All interpretations of Leibniz's formal theory of contingency will have to be, to some extent, both constructive and speculative. On this topic especially, Leibniz's texts suggest a work in progress, an incomplete project with details still being worked out. Nonetheless, there is much to recommend a meta-logical interpretation of Leibniz's formal theory of contingency. As we argued in the first section, Leibniz's work on ideal languages provides him with the resources for thinking, even if without contemporary precision,

[21] Incidentally, we can also now see more precisely what, by Leibniz's lights, is illegitimate about our derivation above of Peter's denial of Christ. In a sense, the proof itself is fine; it is formally valid by the rules of Leibniz's logic. But in assuming that Peter's complete concept can be represented with the letters "mdy," where "d" represents the predicate "is a denier of Christ," we effectively assume that Peter's complete concept is recursively enumerable, that is, put more intuitively, that there is a procedure that has allowed us to find within Peter's complete concept precisely the predicate we are looking for and write down "p = mdy". Absent that assumption, we might apply Leibniz's rules of substitution and inference forever without arriving at an explicit identity statement.

about contingency in either logical or meta-logical terms. As we argued in the second section, however, attempts to understand Leibniz's formal theory of contingency in logical terms alone seem doomed to failure. In contrast, a meta-logical approach to Leibniz's formal theory of contingency can be seen as being motivated by an intuitive idea, namely, that the truth of contingent propositions cannot be established with certainty by appeal to non-empirical axioms, definitions and formal procedures alone—in order to settle contingent truths, we must, in practice, appeal to experience. That intuitive, driving idea, in turn fits well with key Leibnizian texts, and, furthermore, solves and provides insight into the most important, long-standing challenges associated with Leibniz's formal theory of contingency. Without wishing to deny that there are competing threads and lacuna in his treatment, we suggest that, all in all, a meta-logical interpretation provides the most promising account of Leibniz's formal theory of contingency. In the next section, we'll argue that a meta-logical approach also provides an intuitive, if surprisingly flawed, account of Leibniz's formal theory of necessity.

4. A Meta-logical Theory of Necessity

Propositions, for Leibniz, are either contingent or necessary: if a proposition isn't contingent, it's necessary. Leibniz's formal theory of contingency thus implies a formal theory of necessity, and our interpretation of Leibniz's formal theory of contingency implies an interpretation of his formal theory of necessity. Framed in terms of his intuitive understanding of the notion of decidability, on our reading, a proposition, for Leibniz is necessary if a perfect language would provide us with an algorithmic means, a method analogous to multiplication or division, that we could apply in order to arrive with certainty at a proof of the relevant proposition. Framed in terms of his intuitive understanding of computability, on our reading, a proposition, for Leibniz is necessary if a machine could, in principle, be built by us that, working from non-empirical definitions and axioms alone, and following an algorithmic procedure, would be guaranteed to halt at a proof of that proposition. A suitably sophisticated descendent of Leibniz's Step Reckoner, for example, would be guaranteed to find a proof of any necessary proposition provided that we turned the crank—or left the power on—long enough. The meta-logical interpretation of Leibniz's formal theory of contingency thus has a natural correlative, namely, a meta-logical interpretation of his formal theory of necessity.

On a meta-logical interpretation, Leibniz's proposed demarcation of the class of necessary propositions would again be formal rather than epistemic. Nonetheless, it too would have epistemic implications that might be seen as providing the intuitive idea or drive behind his formal distinction. For on a meta-logical interpretation, Leibniz's formal theory of necessity would imply that we can know necessary truths by appeal to non-empirical axioms, definitions and formal procedures alone, that is to say, his formal theory of necessity implies that we can know necessary truths without appeal to ordinary experience. And that, of course, should seem quite plausible. For we do seem to be able to know necessary truths such as *2 + 2 = 4* and the Pythagorean theorem without, say, counting our fingers or measuring bits of paper. As before, Leibniz cannot think that the epistemic property of being knowable independently of ordinary experience might itself provide a satisfying criterion of necessary truth. For he thinks that God at least can know both necessary and contingent propositions without the aid of ordinary experience. Nonetheless, Leibniz's formal demarcation of necessary propositions entails, and can thus draw support from, the intuitive idea that necessary propositions seem to be knowable by non-empirical means. Echoing our terminology from the previous section, we might call this intuitive epistemological thought the *driving idea* behind Leibniz's formal theory of necessity.

That driving idea might be further fleshed out by distinguishing two senses in which necessary propositions, for Leibniz, may be said to be analytic. In one rough, but common sense, a proposition may be said to be *semantically* analytic if it is true simply in virtue of the meanings of the terms used to express the proposition. "Tricycles have three wheels," for example, might be thought to express a proposition that is true in virtue of the meanings of the terms used to express it. Analyticity in this sense suggests one possible ground for holding that analytic propositions can be known without the aid of experience: if analytic propositions are true simply in virtue of the meanings of the terms used to express them, then we might expect that anyone who understood those meanings would be in a position to thereby recognize the truth of the proposition they express. As competent speakers of English, for example, we are in a position to know that tricycles have three wheels in virtue of our understanding the terms "tricycle" and "wheel"; we don't need to conduct experiments or carry out observations in order to know how many wheels tricycles have. Although Leibniz doesn't use the expression "analytic" as we do,[22] he could nonetheless agree that all true

[22] See, for example, NE 524. Interestingly, Gödel, possibly inspired by Leibniz, draws a distinction similar to the one we're drawing here. Gödel shares Leibniz's intuition that mathematics should be analytic. In light of the undecidability results, however,

necessary propositions are analytic in this first sense, that is, that all true necessary propositions may be said to be true in virtue of the meanings of the terms used to express them. In this sense, however, all true contingent propositions will also be analytic for Leibniz. The proposition "Peter is a denier of Christ" is, for him, no less true in virtue of the meanings of the terms used to express it than is the proposition expressed by "Tricycles have three wheels." This first sense of analyticity therefore does nothing to distinguish between necessary and contingent truths as Leibniz understands them.

In another broad, but intuitive sense, a proposition might be said to be *formally* analytic if it is derivable from axioms and definitions. Although less familiar today, formal analyticity was once championed by Rudolf Carnap and gestured at by other logical positivists.[23] This second sense of analyticity suggests a different ground for thinking that analytic propositions can be known to be true without the aid of ordinary experience: if analytic propositions are derivable from axioms and definitions alone, then we might expect that anyone armed with those axioms and definitions will be in a position to, at least in principle, establish an analytic proposition without consulting the empirical world, indeed even without grasping the meanings of the terms used to express that proposition.[24] As competent logicians, we may know that $2 + 2 = 4$ by appealing to axioms, definitions, and derivation rules, and we may do so even if we do not grasp the meanings of "2", "+", "=" and "4;" indeed we may establish that $2 + 2 = 4$ even if we are non-conscious computers performing mechanistic operations. If we say that a proposition is derivable only if it can be established by a finite derivation following a general algorithmic procedure or rule, then formal analyticity will serve as a

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he understandably maintains that mathematics should be semantically rather than formally analytic (Gödel 1944). For related discussion, see Parsons (2014, 132-133).

[23] More precisely, Carnap proposed that analyticity might be understood as a property of sentences of formal languages with an analytic sentence (an "L-true" sentence) being one that is true in virtue of the syntactic (or, in his later period, semantic) rules of the relevant formal system. See, for example, Carnap (1934), (1939, 13); cf. Ayer (1934, 70, fn. 1). Carnap's later definition of analyticity is equivalent, in more modern terms, to Tarski's inductive definition of truth.

[24] We set aside here complications generated by so-called "conceptual role semantics." If the meanings of terms are constituted by their inferential roles, then the distinction between what we are calling semantic and formal theories of analyticity would (likely) collapse. In that case, but only in that case, we could say that contingent propositions are not semantically analytic for Leibniz. For further discussion of these issues, see for instance Boghossian, (1996).

distinguishing feature of analytic truths as Leibniz understands them. For Leibniz, necessary truths are formally analytic, contingent truths are not.

The view that at least mathematical propositions are formally analytic was plausible enough that it was widely embraced in the early part of the twentieth century, most prominently by David Hilbert and his followers.[25] Nonetheless, Leibniz himself may have had some reason for concern. His thinking about the modal status of propositions in terms of finite and infinite analyses seems to have been sparked by his early work on infinite numerical series.[26] And, indeed, it is easy to see something analogous between the suggestion that, say, 2 = 1/1 + 1/3 + 1/6 + 1/10 + 1/15 + 1/21 etc. and the suggestion that (say) Peter = an early leader of the Christian Church, a denier of Christ, a martyr under Emperor Nero, etc. But the analogy also raises a rather obvious puzzle, a puzzle that has been surprisingly overlooked by Leibniz's commentators (although, see Sleigh (1990, 87)). To the extent that we think that statements such as "2 = 1/1 + 1/3 + 1/6 + 1/10 + 1/15 + 1/21 etc." express truths, we are inclined to think that they express necessary truths. If the sum of the reciprocal triangular numbers equals 2, then presumably it equals 2 necessarily. But, in spite of the analogy, or, indeed, paradoxically *because* of the analogy, it seems we are—on Leibniz's formal theory of necessity—supposed to draw exactly the opposite conclusion in the case of statements involving complete concepts and their predicates. That is to say, the analogy is supposed to support the disanalogous conclusion that statements such as "Peter = an early leader of the Christian Church, a denier of Christ, a martyr under Emperor Nero, etc." express not necessary but rather contingent truths. For the sake of shorthand, let us call this apparent difficulty, the *Surprising Tension*.

There is textual evidence that Leibniz recognized the Surprising Tension himself. In a difficult passage from the main text of his most developed treatment of logic, his *General Inquires about the Analysis of Concepts and Truths* (1686), Leibniz writes:

> [I]f, when the analysis of the predicate and of the subject has been continued, a coincidence can never be proved, but it does at least appear from the continued analysis (and the progression and its rule which arise from it) that a contradiction will never arise, then the proposition is possible. But, if in analyzing it, it appears from the rule

[25] For an engaging discussion of Hilbert and his program see Davis (2000, 83-106).

[26] Leibniz's studies of infinite series can be found in A VII.iii. For helpful, but advanced, introductions to those studies see the Akademie editors' introduction (*Einleitung*) to that volume as well as Arthur (2006). For a more accessible introduction see Arthur (2014, 86-89).

> of progression that the reduction has reached a point at which the difference between what should coincide is less than any given difference, then it will have been proved that the proposition is true. If, on the other hand, it appears from the progression that nothing of this sort will ever arise, then it has been proved to be false—that is to say, in the case of necessary propositions. (C 374/P 63-64; cf. A VI.iv.760-761)[27]

On a natural reading, Leibniz means to suggest here that where an infinite series converges on a limit, a proposition stating the equality of that series with that limit expresses a necessary, true proposition; and, conversely, where

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[27] We have followed C 374 and P 63-64 in rendering the last sentence of this passage. Couturat uses corner brackets "<…>" to "enclose words or phrases added by Leibniz," and renders the last sentence: "<*sin contra apparet ex progressione tale quid nunquam oriendum, demonstratum esse falsam* <*scilicet in necessariis.*>>" It is clear from checking the manuscript (LH 4 7C Bl.24v) that that is what Leibniz wrote. Couturat's conjecture that the fragment "*sin … necessariis*" was added after the next line of text was written is also plausible from the manuscript as the fragment appears to be squeezed in between two lines of text. His conjecture that the fragment "*scilicet in necessariis*" represents a further addition is, in our opinion, not implausible but also not strongly supported by the manuscript. Those words do appear more cramped in the manuscript, and could be a still later addition, but the writing might also be cramped simply because of the size of the words appearing below them, and perhaps that is the more likely explanation. In the Akademie edition, however, the end of the sentence is rendered as "*scilicet in* <*contingentibus*>" which would raise difficulties for our interpretation of the passage above. The Akademie edition rendering, however, represents both a small typographical error and an editorial decision. It represents a typographical error insofar as, given the conventions of the Akademie edition, the word "*contingentibus*," as an editorial addition, should be enclosed in square brackets rather than corner brackets, which are used in the Akademie edition to indicate a conjecture concerning words that cannot be made out with certainty. It represents an editorial decision insofar as it replaces in the main text what Leibniz wrote ("*necessariis*") with a conjecture about what he meant to write ("*contingentibus*"). There is no doubt about what Leibniz actually wrote ("*necessariis*"). We respectfully disagree with the editorial decision. The replacement ("*contingentibus*") would indeed make better sense of the main text itself, but the original reading ("*necessariis*") makes far better sense overall once Leibniz's marginal note is taken into consideration. Although it is impossible to know with certainty even from the manuscript, our own considered view is that Leibniz wrote the fragment "*sin … necessariis*" after the sentence that follows it, later noticed the philosophical tension we discuss above, and subsequently added the marginal note beginning with "*Dubium* …". We are grateful to Dr. Stephan Meier-Oeser of the Leibniz-Forschungsstelle Münster for invaluable discussion of this passage and for providing us with a copy of the manuscript.

an infinite series either converges on a different limit, or no limit at all, then the same statement expresses a necessary, false proposition. As a view concerning the truth conditions and modal status of a certain class of propositions, that all seems reasonable enough. Leibniz, however, sees that it stands in tension with his formal theory of necessity. In the margin of the passage just quoted, he writes:

> A doubtful point: is everything true which cannot be proved false, or everything false which cannot be proved true? What, then, of the cases of which neither of these holds? It must be said that both truth and falsity can always be proved, at any rate by an analysis which is carried to infinity. But then it is contingent, i.e. it is possible that it is true, or that it is false. The same is the case with concepts: namely, that in an analysis which is carried to infinity they are manifestly true or false, that is, to be admitted to existence, or not. (C 374/P 64, fn 1; cf. A VI.iv.761, fn 30)

Leibniz's marginal remark is prompted, we conjecture, from his seeing a tension between his first thought that propositions stating the equality of infinite series with limit values are either necessarily true or necessarily false, and the implication of his formal theories of contingency and necessity that such propositions should be reckoned to be contingently true or contingently false. If that's right, it appears that Leibniz himself may have recognized that the very sorts of examples that seem to have sparked his formal theory of necessity stand in tension with one of our most basic intuitions concerning the modal status of propositions, namely, that mathematical propositions are necessarily true or necessarily false.

A fictionalist thread in Leibniz's thinking about infinite series might be thought to provide one way out of the Surprising Tension.[28] Leibniz often speaks as if infinite series literally sum to the limits they approach, as, for example, when he tells us that "1 + 1/3 + 1/6 + 1/10 + 1/15 + 1/21 etc. *égal à* 2" (A VII.iii.368). But his considered view is more nuanced. He maintains that to suggest that the sum of the reciprocal triangular numbers sums to exactly 2 would be to imply that there is some last term, some infinieth term that, as it were, makes up the difference between the sum of some finite series and 2 itself. But, of course, he recognizes that there is no such infinieth term. He therefore concludes that equations such as "2 = 1/1 + 1/3 + 1/6 + 1/10 + 1/15 + 1/21 etc." are "not rigorously true" (A VI.iii.502). He insists instead that "Whenever it is said that a certain infinite

[28] For discussions of Leibniz's fictionalism, see Arthur (2014, 79-85), Jesseph (2015), and Levey (1998, 79-82) and (2008).

series of numbers has a sum … all that is being said is that any finite series with the same rule has a sum, and that the error always diminishes as the series increases, so that it becomes as small as we would like (A VI.iii.503). Given this fictionalist thread in his thinking about infinite sums, Leibniz could maintain that propositions expressed by statements such as "2 = 1/1 + 1/3 + 1/6 + 1/10 + 1/15 + 1/21 etc." are not necessarily true because they are not, strictly speaking, true at all. Rather they are to be understood as licensing the expression of a series of finitely derivable statements, such as "2 = 1/1 + 1/3 + 1/6" all of which might, in keeping with his formal theory of necessity, be counted as being either necessarily true or necessarily false.

Leibniz's fictionalism, however, can't provide a satisfying resolution to the Surprising Tension for at least two reasons. First, all propositions that count as contingent by the lights of Leibniz's formal theory, should, by the lights of his fictionalism, count as, strictly speaking, false.[29] For, as we've seen, Leibniz's formal theory suggests, for example, that the predicate *is a denier* can never be reached by means of a finite algorithmic analysis. It is thus analogous to an infinieth term in a numerical series. But in that case, Leibniz's fictionalism implies that any proposition expressed by "Peter is a denier of Christ" must, taken strictly, be false, just as any proposition expressed by "2 = 1/1 + 1/3 + 1/6 + 1/10 + 1/15 + 1/21 etc." must, taken strictly, be false. Second, Leibniz's fictionalism, in general, sits uncomfortably with his formal theories of necessity and contingency. For the deep lesson of Leibniz's fictionalism would seem to be that all statements that would appear to involve infinite series should be interpreted as expressing finitely complex propositions. But the distinction Leibniz draws with his formal theories of necessity and contingency depends essentially on at least some statements being interpreted as expressing infinitely complex propositions. Far from lending support to his formal theories of necessity and contingency, an appeal to Leibniz's fictionalism would thus seem to only raise additional puzzles and concerns.[30]

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[29] Leibniz could also maintain that they are not well-formed and, thus, strictly speaking neither true nor false. Since consideration of this alternative would clutter, but not otherwise affect our line of argument, we will ignore it in what follows. Likewise, for the thought that we can imagine similar statements that would be, as it were, be true by default, e.g. "2 > 1/1 + 1/3 + 1/6 + 1/10 + 1/15 + 1/21 etc."

[30] The nature of Leibniz's fictionalism is, alas, open to debate. It has been suggested to us in particular that the point of Leibniz's fictionalism is that "2 = 1/1 + 1/3 + 1/6 + 1/10 + 1/15 + 1/21 etc. is true even though we cannot get to 2 by summing step by step." If that were correct (but see A VI.iii.502) then Leibniz could think that statements such as "Peter is a denier" are also true even though we cannot get to an

Perhaps anticipating such difficulties, late in the *General Inquires*, Leibniz appears to embrace a different response to the Surprising Tension. Seeming more confident and settled, he writes:

> (133) A true necessary proposition can be proved by reduction to identical propositions, or by reduction of its opposite to contradictory propositions; hence its opposite is called 'impossible.'
>
> (134) A true contingent proposition cannot be reduced to identical propositions, but is proved by showing that if the analysis is continued further and further, it constantly approaches identical propositions, but never reaches them. …
>
> (135) So the distinction between necessary and contingent truths is the same as that between lines which meet and asymptotes, or between commensurables and incommensurable numbers. (A VI.iv.776/P 77)

In this passage, Leibniz seems ready to bite the bullet implied jointly by his formal theory of necessity and his understanding of propositions involving infinite series. Pressed by the Surprising Tension, one could, after all, simply give up the conviction that all mathematical propositions are necessarily true or necessarily false. Leibniz thinks that some mathematical propositions can be finitely, algorithmically demonstrated, and those are to be said to be necessarily true or necessarily false. Leibniz thinks other mathematical propositions cannot be so demonstrated. In the passage just above he appears ready to say that those mathematical propositions are to be said to be contingently true or contingently false. Although high-handed, this response to the Surprising Tension is, in a way, an improvement over the previous response. It at least would provide us with one right result: the analogy between propositions involving infinite series and propositions involving complete concepts would be upheld and both kinds of propositions could be counted as true. Nonetheless, this route would clearly come at a high cost. For the thought that mathematical propositions, if true or false, must be necessarily true or necessarily false, might reasonably be regarded as a touchstone upon which the success or failure of a theory of the modal status

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explicit identity statement step by step. That would do better for statements such as "Peter is a denier," since it could be both contingent and true, but it would nonetheless stoke difficulties elsewhere. For, on such an account, mathematical statements such as "2 = 1/1 + 1/3 + 1/6 + 1/10 + 1/15 + 1/21 etc." would themselves turn out to be contingent. Leibniz's fictionalism, on this proposed alternative, would thus be too permissive, that is, it would make even paradigmatically necessary propositions contingent. We would like to thank [edited for blind review].

of propositions can be measured. Biting this bullet would seem to be more reckless than brave.

Nowadays, of course, we don't think that demonstrations of summations involving infinite series must require infinitely many steps. Nonetheless, insofar as the Surprising Tension arises from the undecidability of some mathematical propositions, we can recognize that Leibniz was right to be worried about the implications of his formal theory of necessity. As we've noted, intuitive notions of derivability, decidability and computability were given rigorous formulation with the explosion of meta-logic in the late nineteenth and early twentieth centuries. Using those rigorous formulations mathematicians were able to prove, roughly, that given a (sufficiently strong) formal system, there must be some mathematical propositions that are true, and presumably necessarily true, but for which there cannot, contrary to the implication of Leibniz's formal theory of necessity, be a computational procedure that will output their proof from the formal system (for a more precise characterization of Gödel's results, see for instance Raatikainen (2015)). Leibniz may have been wrong in seeing a worry on the horizon arising specifically from infinite series, but he was surely right to worry that all mathematical truths might not admit of finite demonstrations as his formal theory of necessity requires.

If the present account of Leibniz's formal theories of contingency and necessity is on track, its greatest irony is that of the two halves of Leibniz's proposal, it is the half concerning necessary truths that should be reckoned the more suspect. Commentators have generally seen Leibniz's logical commitments as pushing him into the arms of necessitarianism. His formal theory of contingency has been scrutinized and is often viewed as a last ditch, unsuccessful effort to resist the conclusion that all propositions are necessary. Leibniz's formal theory of necessity, in contrast, has generally been ignored, its success, as it were, seemingly guaranteed by the failure of his formal theory of contingency. As we read him, however, this familiar story gets things nearly backwards. Leibniz has a plausible and, in spirit at least, even successful formal theory of contingency. Although we may trifle over details, Leibniz is essentially correct in suggesting that contingent propositions cannot be algorithmically demonstrated. Furthermore, as we read him, Leibniz has an interesting, substantive, and even plausible theory of necessity. Again, while we may trifle over details, it is easy to sympathize with his view that all necessary propositions should be demonstrable from non-empirical definitions and axioms, that is, that they should be formally analytic. Developments since Leibniz's time have done nothing to undermine his formal theory of contingency; we should still agree with Leibniz that in order to determine the truth of contingent propositions we must ultimately consult with experience. Developments since Leibniz's time have, however, undermined his formal theory of necessity. If standard interpretations of Gödel's incompleteness results are correct, we should no longer agree with Leibniz that all necessary propositions must be demonstrable from definitions and axioms alone. Leibniz's formal theory of necessity thus seems to be less well founded than his formal theory of contingency, although it should be added immediately that even its most serious difficulties are evident now only in light of some of the most astounding results to have ever occurred in the history of mathematics and logic.

5. Conclusion

The aim of the present essay has been to offer a novel interpretation of Leibniz's notorious "infinite analysis" theory of contingency. In the broadest strokes, we've argued that Leibniz's theory must be understood against the backdrop of his lifelong interest in ideal languages, his formal understanding of logical demonstration, and his intuitive grasp of the meta-logical notions of decidability and computability. According to Leibniz's formal theory of necessity, necessary propositions are guaranteed to be demonstrable by means of an algorithmic, formal procedure and therefore are guaranteed to be knowable, in principle, a priori even by finite creatures. According to Leibniz's formal theory of contingency, contingent propositions are not guaranteed to be demonstrable by means of an algorithmic, formal procedure and therefore are not guaranteed to be knowable, even in principle, a priori by finite creatures. Placed in their proper context, and with an appreciation of the sophistication of Leibniz's logical efforts, we can see Leibniz's formal theories of necessity and contingency for what they are, namely, genuinely profound attempts to draw a distinction between necessary and contingent propositions in terms of the formal properties of the statements that would be used to express them in an ideal language. Although it is, of course, still possible to object to Leibniz's theory, most obviously by challenging its background assumptions and by drawing on the full resources of contemporary mathematical logic, standard objections of the sort that we might have expected Leibniz to appreciate, simply fall away. While not unassailable, Leibniz's formal theories of necessity and contingency turn out to be, when properly understood, both surprisingly plausible and eerily prescient.

Contra Bennett (2001, 329), it is, we think, hard to see how Leibniz's efforts to draw a formal distinction between necessary and contingent propositions should not be of interest to philosophically engaged historians of philosophy (as opposed, presumably, to mere "antiquarians"). For Leibniz's efforts in this regard lie at a crucial, if still not well-understood, intersection of philosophical concerns that date from Leibniz's earliest philosophical insights. His formal theories of necessity and contingency are fed, and in turn feed into, his thinking about truth, language, logic, thought, and the foundations of mathematics. And that is just with respect to Leibniz's own work. Taking a broader view, Leibniz's formal theories of necessity and contingency, and the foundations upon which they rest, are important themes in the long conceptual history of logic, they are characters that emerge in a time of crisis, are unduly neglected in Leibniz's own time, and reemerge in new guise in the twentieth century renaissance of mathematical logic. Anyone unable to find something of philosophical interest here is unlikely to find it in any historical setting.

Finally, it is even easy, we think, to see how Leibniz's efforts to draw a formal distinction between necessary and contingent propositions might be of interest even to non-historically-minded contemporary philosophers. As is the case with many of his most important mathematical studies, Leibniz enjoyed both the daunting challenge but also immense freedom of working on enduring foundational issues before a dominant consensus had formed. This absence of constraints can make understanding his efforts difficult. Setting out on his own, he often chases down dead ends, explores contradictory paths, and abruptly changes his mind. But the absence of constraints also means that he often considers connections, leads and possibilities that we, conditioned by consensus, are apt to overlook. Many of those options have, of course, been closed down for good reason, and in such cases we gain the most from Leibniz's forays by emerging with a better appreciation of why consensus has formed in the way that it has. But it is always possible that in some of those options Leibniz saw something to which we are now blind, a possibility or way forward that we might better glimpse by standing on his shoulders. The likes of Frege, Gödel, and Carnap all thought that in connection with the tissue of concerns discussed above Leibniz had had profound insights that are merely waiting to be rediscovered. They might still be right.

Abbreviated References to Leibniz's Frequently Cited Works

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AG = *Philosophical Essays*, R. Ariew and D. Garber, eds. and trans. (Indianapolis: Hackett, 1989).

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G = *Die Philosophische Schriften von Gottfried Wilhelm Leibniz*, G C. I. Gerhardt, ed., (Berlin: Weidmann, 1875-90; repr. Hildesheim: Olms, 1960). Reference is to volume and page.

GM = *Mathematische Schriften von Gottfried Wilhelm Leibniz*, C. I. Gerhardt, ed., (Berlin: A. Asher; Halle: H.W. Schmidt, 1849-63). Reference is to volume and page.

L = L. E. Loemker, ed. and trans., *Philosophical Papers and Letters 2nd edition* (Dordrecht: Reidel, 1969)

LDB = *The Leibniz-Des Bosses Correspondence*, Brandon C. Look and Donald Rutherford, ed. and trans. (New Haven: Yale University Press, 2007). Reference is to original language page.

LDV = *The Leibniz-De Volder Correspondence*, Paul Lodge, ed. and trans. (New Haven: Yale University Press, 2013). Reference is to original language page.

LH = Reference to Leibniz's manuscripts as catalogued in Bodemann (1895). First digit after "LH" refers to Bodemann's chapter divisions, with subsequent digits refer to successive divisions of the manuscripts as made by Bodemann.

NE = *Nouveaux essais sur l'entendement* in A VI.vi. English translation in P. Remnant and J. Bennett, ed and trans., *New Essays on Human Understanding* (Cambridge: Cambridge University Press 1996). Reference is to page number, which is the same in both editions.

P= *Leibniz: Logical Papers*, G.H.R. Parkinson, ed. and trans. (Oxford: Clarendon Press, 1966).

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