Technical Appendix

A The Steady State

We denote constant, steady-state levels of variables by dropping the time subscript and assume $f_E = f_E^*$, $f_X = f_X^*$, $\tau = \tau^*$, $L = L^*$, and $Z = Z^* = 1$. Under these assumption, the steady state of the model is symmetric: $\tilde{Q} = Q = TOL = 1$ and the levels of all other endogenous variables are equal across countries.

Solving for $\tilde{z}_X$

Given the solution for the average export productivity $\tilde{z}_X$, we can obtain the cutoff level $z_X$ from $\tilde{z}_X = \nu z_X$, where $\nu \equiv \{k/|k-(\theta-1)|\}^{1/(\theta-1)}$. We can solve for $\tilde{z}_X$ as follows. The Euler equation for share holdings yields:

$$\tilde{v} = \frac{\beta (1-\delta)}{1-\beta (1-\delta)} \left( \tilde{d}_D + \frac{N_X}{N_D} \tilde{d}_X \right).$$

Combining this equation with the free entry condition $\tilde{v} = f_E w$ implies:

$$\tilde{d}_D + \frac{N_X}{N_D} \tilde{d}_X = \frac{[1-((1-\delta)\beta)]}{(1-\delta)\beta} f_E w.$$  \hspace{1cm} (A.1)

The steady-state zero profit export cutoff equation is:

$$\tilde{d}_X = w f_X \left( \frac{\theta - 1}{k - (\theta - 1)} \right).$$  \hspace{1cm} (A.2)

Also, steady-state profits from selling at home and abroad are $\tilde{d}_D = (\tilde{\rho}_D)^{1-\theta} C/\theta$ and $\tilde{d}_X = (\tilde{\rho}_X)^{1-\theta} C/\theta - w f_X$, respectively. These two equations imply:

$$\tilde{d}_D = \left( \frac{\tilde{\rho}_X}{\tilde{\rho}_D} \right)^{\theta-1} \left( \tilde{d}_X + w f_X \right).$$  \hspace{1cm} (A.3)

Optimal pricing yields $\tilde{\rho}_D = [\theta/((\theta - 1))] \tilde{z}_D^{-1} w$ and $\tilde{\rho}_X = [\theta/((\theta - 1))] \tau \tilde{z}_X^{-1} w$. Hence, $\tilde{\rho}_X/\tilde{\rho}_D = \tau \tilde{z}_D/\tilde{z}_X$, and substituting this into (A.3), we have:

$$\tilde{d}_D = \left( \frac{\tau \tilde{z}_D}{\tilde{z}_X} \right)^{\theta-1} \left( \tilde{d}_X + w f_X \right),$$
or, taking (A.2) into account,

\[ \tilde{d}_D = \left( \frac{\tau \tilde{z}_D}{\tilde{z}_X} \right)^{\theta - 1} \left( w f_X \frac{\theta - 1}{k - (\theta - 1)} + w f_X \right). \]  

(A.4)

The steady-state share of exporting firms in the total number of domestic firms is:

\[ \frac{N_X}{N_D} = \left( z_{\text{min}} \right)^k \left( \tilde{z}_X \right)^{-k} \left[ \frac{k}{k - (\theta - 1)} \right] \frac{k}{(\theta - 1) f_X}. \]  

(A.5)

Substituting equations (A.2), (A.4), and (A.5) into (A.1), using \( \tilde{z}_D = \{ k / [k - (\theta - 1)] \} \tilde{z}_{\text{min}} \), and rearranging yields:

\[ \left( \frac{\tilde{z}_X}{\tau z_{\text{min}}} \right)^{\theta - 1} \left[ \frac{k}{k - (\theta - 1)} \right]^2 + \left( \frac{\tilde{z}_X}{z_{\text{min}}} \right)^{-k} \left[ \frac{k}{k - (\theta - 1)} \right]^k \frac{k}{(\theta - 1) f_X} = \left[ 1 - \left( 1 - \delta \right) \beta \right] f_E. \]

This equation can be rewritten as:

\[ \xi_1 \left( \frac{\tilde{z}_X}{\tau z_{\text{min}}} \right)^{\theta - 1} + \xi_2 \left( \frac{\tilde{z}_X}{z_{\text{min}}} \right)^{-k} = \xi_3, \]  

(A.6)

where

\[ \xi_1 \equiv \left( \frac{\tau z_{\text{min}}}{k - (\theta - 1)} \right)^{\theta - 1} > 0, \]

\[ \xi_2 \equiv \left( \frac{k_{\text{min}}}{k - (\theta - 1)} \right)^{-k} \frac{k}{k - (\theta - 1)} > 0, \]

\[ \xi_3 \equiv \frac{1 - (1 - \delta) \beta}{(1 - \delta) \beta} f_E f_X > 0. \]

The left-hand side of equation (A.6) is a hyperbola. This guarantees existence and uniqueness of \( \tilde{z}_X > 0 \), the exact value of which we obtain numerically.

**Solving for \( \tilde{\rho}_X \)**

The law of motion for the total number of domestic firms implies:

\[ N_E = \frac{\delta}{1 - \delta} N_D. \]  

(A.7)

Steady-state aggregate accounting yields \( C = wL + N_D \tilde{d}_D + N_X \tilde{d}_X - N_E w f_E \). Using (A.1) and
(A.7), this can be rewritten as:
\[
\frac{C}{w} = L + NDfE \frac{1 - \beta}{1 - (\delta) \beta}.
\]  
(A.8)

Equation (A.2) and the expression for average export profits, \(\tilde{d}_X = (\tilde{\rho}_X)^{1-\theta} C/\theta - w_f X\), imply:
\[
\frac{C}{w} = \tilde{\rho}_X^{\theta-1} \frac{\theta k}{k - (\theta - 1)} f_X.
\]  
(A.9)

The price index equation \(ND\tilde{\rho}_D^{1-\theta} + NX\tilde{\rho}_X^{1-\theta} = 1\) yields:
\[
\frac{\tilde{\rho}_X^{\theta-1}}{ND} = \left( \frac{\tilde{\rho}_X}{\tilde{\rho}_D} \right)^{\theta-1} + \frac{NX}{ND},
\]

or, using \(\tilde{\rho}_X/\tilde{\rho}_D = \tau \tilde{z}_D/\tilde{z}_X\) and equation (A.5),
\[
\frac{\tilde{\rho}_X^{\theta-1}}{ND} = \left( \frac{\tau \tilde{z}_D}{\tilde{z}_X} \right)^{\theta-1} + \left( \frac{z_{\min}}{\tilde{z}_X} \right)^k \left[ \frac{k}{k - (\theta - 1)} \right]^k \theta - 1.
\]  
(A.10)

Together, equations (A.8), (A.9), and (A.10) yield the following equation for \(\tilde{\rho}_X\):
\[
\tilde{\rho}_X^{1-\theta} = \left[ \frac{\theta k}{k - (\theta - 1)} f_X - K^{-1} f_E \frac{1 - \beta}{(1 - \delta) \beta} \right] L^{-1}.
\]

where \(K\) is the right-hand side of equation (A.10).

**Special Case: All Firms Export**

In this case, equation (A.9) no longer holds since the zero cutoff profit condition (A.2) no longer applies. Using \(\tilde{d}_D = (\tilde{\rho}_D)^{1-\theta} C/\theta\) and \(\tilde{d}_X = (\tilde{\rho}_X)^{1-\theta} C/\theta - w_f X\), equation (A.1) can be written as:
\[
\tilde{\rho}_X^{1-\theta} \frac{C}{\theta} \left( \tau^{\theta-1} + 1 \right) - w_f X = \frac{[1 - (1 - \delta) \beta]}{(1 - \delta) \beta} f_E w,
\]

which implies:
\[
\frac{C}{w} = \tilde{\rho}_X^{\theta-1} \frac{\theta}{\tau^{\theta-1} + 1} \left\{ f_X + \frac{[1 - (1 - \delta) \beta]}{(1 - \delta) \beta} f_E \right\}.  
\]  
(A.11)

Equation (A.11) now replaces equation (A.9) when solving for \(\tilde{\rho}_X\). This yields the following expression for \(\tilde{\rho}_X\):
\[
\tilde{\rho}_X^{1-\theta} = \left[ \frac{\theta}{\tau^{\theta-1} + 1} \left\{ f_X + \frac{[1 - (1 - \delta) \beta]}{(1 - \delta) \beta} f_E \right\} - K^{-1} f_E \frac{1 - \beta}{(1 - \delta) \beta} \right] L^{-1}.
\]
Solving for the Remaining Variables

The solutions for other endogenous variables are straightforward

• \( N_D = K^{-1} \rho_X^{-1} \);
• \( \tilde{\rho}_D = \frac{\tilde{z}_X}{\tau \tilde{D}_D} \hat{\rho}_X \) using \( \hat{\rho}_X / \tilde{\rho}_D = \tau \tilde{z}_D / \tilde{z}_X \);
• \( \tilde{w} = \tilde{\rho}_X \theta - 1 \frac{1}{\theta - 1} \tau \tilde{z}_X \tilde{\rho}_X \) using (A.8);
• \( C = \tilde{w} \left[ L + N_D f_E \frac{1-\beta}{1-(\theta - 1)} \right] \) using (A.8);
• \( N_E = \frac{\delta}{1-\delta} N_D \);
• \( N_X = N_D (z_{min})^{k} (\tilde{z}_X)^{-k} \left[ \frac{k}{1-(\theta - 1)} \right]^{\frac{1}{\theta - 1}} \) using (A.5);
• \( \tilde{d}_D = \frac{1}{\beta} (\tilde{\rho}_D)^{1-\theta} C \);
• \( \tilde{d}_X = \frac{1}{\beta} (\tilde{\rho}_X)^{1-\theta} C - w f_X \);
• \( \tilde{v} = w f_E \) (using the free entry condition);
• \( 1 + r = 1/\beta \) (using the Euler equation for bond holdings).

Symmetry of the steady state ensures \( \tilde{z}_X^* = \tilde{z}_X \), \( \tilde{\rho}_X^* = \tilde{\rho}_X \), \( N_D^* = N_D \), \( N_E^* = N_E \), \( N_X^* = N_X \), \( \tilde{d}_D^* = \tilde{d}_D \), \( \tilde{d}_X^* = \tilde{d}_X \), \( \tilde{v}^* = \tilde{v} \), in addition to \( C^* = C \), \( w^* = w \), and \( r^* = r \).

B Labor Market Clearing

Recall that a firm with productivity \( z \) produces \( Z_t z \) units of output per unit of labor employed. Consider separately the labor used to produce goods for the domestic and export markets: let \( l_{D,t}(z) \) and \( l_{X,t}(z) \) represent the amount of labor hired to produce goods for each market. These only represent labor used in production; in addition, each new entrant hires \( f_{E,t}/Z_t \) units of labor to cover the entry cost, and each exporter hires \( f_{X,t}/Z_t \) units of labor to cover the fixed export cost in every period. The profits earned from domestic sales for a firm with productivity \( z \) are then given by:

\[
d_{D,t}(z) = \rho_{D,t}(z) Z_t z l_{D,t}(z) - w_t l_{D,t}(z) = \frac{1}{\theta - 1} w_t l_{D,t}(z),
\]

using \( \rho_{D,t}(z) = \frac{\theta}{\theta - 1} w_t / Z_t z \) from optimal pricing. This relationship holds for a firm with average productivity \( \tilde{z}_D \), and also for averages across all domestic firms. This implies that the average
The amount of production labor hired to cover domestic sales is \((\theta - 1)\tilde{d}_{D,t}/w_t\). The total amount of such labor hired at home is thus \(N_{D,t}(\theta - 1)\tilde{d}_{D,t}/w_t\).

The profits earned from export sales for an exporting firm with productivity \(z\) are given by:

\[
d_X(t) = Q_t\rho_X(t)\frac{Z_t z l_{X,t}(z)}{\tau_t} - w_t \left[ l_{X,t}(z) + \frac{f_{X,t}}{Z_t} \right] = \frac{1}{\theta - 1} w_t l_{X,t}(z) - w_t \frac{f_{X,t}}{Z_t},
\]

using \(\rho_X(t) = Q_t^{-1} \tau_t \frac{\theta}{\theta - 1} \frac{w_t}{Z_t}\) from optimal pricing. Note that only \(Z_t z l_{X,t}(z)/\tau_t\) export units are sold, although \(Z_t z l_{X,t}(z)\) are produced (the remaining fraction having “melted” away in an iceberg fashion while crossing the border). Again, this relationship holds for a firm with average export productivity \(\bar{z}_{X,t}\), and also for averages across all exporters. The average amount of production labor hired to cover export sales is thus \((\theta - 1)\tilde{d}_{X,t}/w_t + (\theta - 1)\bar{f}_{X,t}/Z_t\). Multiplying by \(N_{X,t}\) yields the total amount of such labor for the home economy.

The total amount of production labor hired in the home economy is then

\[
\frac{\theta - 1}{w_t} N_{D,t}\tilde{d}_{D,t}(z) + \frac{\theta - 1}{w_t} N_{X,t}\tilde{d}_{X,t} + \frac{\theta - 1}{Z_t} N_{X,t}\bar{f}_{X,t}.
\]

Adding the total amount of labor hired by new entrants, \(N_{E,t}\bar{f}_{E,t}/Z_t\), and that hired by exporters to cover the fixed costs, \(N_{X,t}\bar{f}_{X,t}/Z_t\), yields the aggregate labor demand for the home economy:

\[
L_t = \frac{\theta - 1}{w_t} N_{D,t}\tilde{d}_{D,t} + \frac{\theta - 1}{w_t} N_{X,t}\tilde{d}_{X,t} + \frac{\theta}{Z_t} N_{X,t}\bar{f}_{X,t} + \frac{1}{Z_t} N_{E,t}\bar{f}_{E,t}.
\]

Equating \(L_t\) to labor supply \((L)\) yields the equilibrium condition for home’s labor market. The derivation for foreign is analogous.

**Balanced Trade Implies Labor Market Clearing**

We now demonstrate that balanced trade under financial autarky implies labor market clearing.

Using the home price index equation \(1 = N_{D,t} (\tilde{\rho}_{D,t})^{1-\theta} + N_{X,t}^* (\tilde{\rho}_{X,t})^{1-\theta}\), the balanced trade condition \(Q_t N_{X,t} (\tilde{\rho}_{X,t})^{1-\theta} C_t^* = N_{X,t}^* (\tilde{\rho}_{X,t})^{1-\theta} C_t\) can be written:

\[
Q_t N_{X,t} (\tilde{\rho}_{X,t})^{1-\theta} C_t^* = \left[ 1 - N_{D,t} (\tilde{\rho}_{D,t})^{1-\theta} \right] C_t.
\]
This condition can be re-written as

\[ C_t = \theta N_{X,t} \left( \tilde{d}_{X,t} + w_t \frac{f_{X,t}}{Z_t} \right) + \theta N_{D,t} \tilde{d}_{D,t}, \]

since \( \tilde{d}_{D,t} = (\tilde{\rho}_{D,t})^{1-\theta} C_t / \theta \) and \( \tilde{d}_{X,t} = Q_t (\tilde{\rho}_{X,t})^{1-\theta} C_t^* / \theta - w_t f_{X,t} / Z_t \). Combining this with aggregate accounting \( (C_t = w_t L + N_{D,t} \tilde{d}_{D,t} + N_{X,t} \tilde{d}_{X,t} - N_{E,t} w_t f_{E,t} / Z_t) \) yields the labor market clearing condition for the home economy:

\[ L = \frac{\theta - 1}{w_t} N_{D,t} \tilde{d}_{D,t} + \frac{\theta - 1}{w_t} N_{X,t} \tilde{d}_{X,t} + \frac{\theta}{Z_t} N_{X,t} f_{X,t} + \frac{1}{Z_t} N_{E,t} f_{E,t}. \]

The proof for the foreign economy follows the same steps.