Announcements

▶ Midterm #2 in class on November 15
▶ Have posted old exams and problem sets
▶ Review Session currently scheduled for 11/13, 4-5:15pm, Rubenstein 304
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Roadmap

▶ Finishing up confidence intervals, reviewing interpretation
▶ Extending testing framework to data that is structured in pairs
▶ Common in medical/public health studies
▶ This finishes up the standard suite of hypothesis tests
▶ Move on next time to Analysis of Variance and Chi-Square Tests
Roadmap

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Reviewing Confidence Intervals for Means

Let's review construction of CIs for confidence mean.

For CI's: Leverage fact that:

\[ P(-1.96 \leq \bar{X} - \mu \leq 1.96) = 0.95 \]

\[ P(\bar{X} + 1.96 \times \hat{SE} \geq \mu \geq \bar{X} - 1.96 \times \hat{SE}) = 0.95 \]

This gives us the 95% confidence interval for \( \bar{X} \):

\[ \bar{X} \pm 1.96 \times \hat{SE} \]

Can rewrite as general formula for a \( (1 - \alpha) \)% confidence interval:

\[ \bar{X} \pm z_{\alpha/2} \times \hat{SE} \]

Where we use \( z_{\alpha/2} \) from standard normal (or \( t_{\alpha/2} \)).
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Calculating Confidence Intervals for Other Quantities of Interest

- CLT means we can calculate confidence intervals for any estimator that approximates the Normal distribution as $n$ goes up.
- Means
- Difference in means
- Proportions
- Difference in proportions
- All follow general form of $\text{Point Estimate} \pm z_{\alpha/2} \hat{\text{SE}}$
- $z_{\alpha/2} \hat{\text{SE}}$ refers to the margin of error.
- CI's most generally are: $\text{Point Estimate} \pm \text{Margin of Error}$
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How to Interpret Confidence Intervals?

Confidence intervals (CIs) are one of the most misinterpreted estimators. Calculation of confidence intervals depends on the sampling distribution (from CLT). Different samples will yield different confidence intervals. With some samples, the calculated CI would “capture” the true population parameter. With some samples, the calculated CI would not “capture” the population parameter. For all of the CIs we could calculate with repeat sampling, 95% of them would cover the true population parameter.
How to Interpret Confidence Intervals?

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How to Interpret CIs?

▶ Show this with simulation
▶ For sake of simulation, assume observations come from Normal distribution w/ mean 1 and variance of 10
▶ I sample 500 observations, 100 times
▶ For each sample, calculate 95% CI

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How to Interpret CIs?

500 observations drawn from N(1,10)

95% Confidence Interval
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How to Interpret CIs?

For all of the CIs we could calculate with repeat sampling, 95% of them would cover the true population parameter. Confidence intervals are one of the most frequently misinterpreted estimators. "There is a 95% probability that this interval I've calculated contains the true population parameter." → Not correct: Once you have calculated the CI, it either contains the true value or not. Also, the population parameter is a constant (it has a numerical value, but you just don't know what it is!). "95% of the confidence intervals I calculate using this formula using repeated sampling will contain the true population parameter." → Correct!
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CIs and Sample Size

Notes on sample size:

▶ Small samples → Follow same rules as hypothesis tests for when to switch to Student’s $t$ distribution

▶ Larger sample size → Will shrink standard errors → Will lead to smaller CIs

▶ Ex) Doubling sample size will reduce the width of the confidence interval for a sample mean by a half

$\bar{X} \pm \frac{\sigma}{\sqrt{n}}$

$\bar{X} \pm \frac{\sigma}{\sqrt{4n}} \rightarrow \bar{X} \pm \frac{1}{2} \frac{\sigma}{\sqrt{n}}$

▶ Problems may ask you to calculate minimum sample size, given $\alpha$ and standard deviation
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CIs and Sample Size

What happens to CIs when \( n \) goes up?

- 500 observations drawn from \( \mathcal{N}(1,10) \)
- 95% Confidence Interval
CIs and Sample Size

Show this graphically → What happens to CIs when $n$ goes up?
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CIs and Sample Size

Show this graphically → What happens to CIs when \( n \) goes up?

1500 observations drawn from N(1,10)
CIs versus Hypothesis Tests

- Close relationship between CIs and HTs
- If value $A$ not in 95% CI → would be rejected by a two-sided hypothesis test at the 5% level
- If value $A$ in 95% CI → would not be rejected by a two-sided hypothesis test at the 5% level
- A 95% confidence interval is all of null hypotheses that would not be rejected at the 0.05 level
- A $(1 - \alpha)$% confidence interval is all of null hypotheses that would not be rejected at the $\alpha$ level
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The diagram illustrates the relationship between confidence intervals (CIs) and hypothesis tests. The area under the curve from $-\alpha/2$ to $\alpha/2$ represents $(1 - \alpha)$ CI, where $\alpha$ is the significance level. This indicates the probability of the CI containing the true parameter value.
CIs versus Hypothesis Tests

Both CIs and HTs are useful tools in your inference toolkit.

- **HTs** are useful when comparing groups or trying to test a theory.

- **CIs** are useful for thinking about the range of possible values, providing additional information about a single sample.

Many people prefer CIs:

- They can give you information about all possible null hypotheses that would be rejected (as opposed to one), conditional on the $\alpha$ value.

- Many find the margin of error intuitive.

- They are extremely widely used (although many incorrectly interpret...
Both CIs and HTs useful tools in your inference toolkit
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Paired Data

We have covered differences in means and differences in proportions. Have assumed that two groups we were comparing were independent. Two independent samples were simple random samples from two distinct populations. Or, independent samples drawn from same population (with pooled standard error for sample mean).

What happens when we can’t assume independence any more?
Paired Data

- We have covered differences in means and differences in proportions.
Paired Data

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Paired Data

- We have covered differences in means and differences in proportions.
- Have assumed that two groups we were comparing were independent.
  - Two independent samples were simple random samples from two distinct populations.
  - Or, independent samples drawn from same population (with pooled standard error for sample mean).
- What happens when we can’t assume independence any more?
Public Opinion Paired Data Example

A recent General Social Survey asked 2 questions of 1,492 Americans under hypothetical scenario that government suspected a terrorist act about to happen:

Question #1: Do you believe the authorities should have the right to tap people's phone conversations?

Question #2: Do you believe the authorities should have the right to stop and search people on the street at random?

Two questions asked after the other

Same respondents → two questions
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Impairment Paired Data Example

- Study done to test effects of cell phone on driving impairment
- Undergraduates study
  - Each UG asked to conduct two driving simulations
- Reaction times measured when (a) talking on cell phone versus (b) not talking on cellphone
- Same student for two conditions
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Sunscreen Paired Data Example

Sun exposure/skin cancer of increasing concern in Australia

Australian scientists interested in testing effectiveness of sun screen lotion

Asks volunteers to (a) use sunscreen on one arm, but (b) no sunscreen on the other, then go about daily lives for 6 months

Degree of skin damage measured 6 months later

Each person \(\rightarrow\) one arm in group 1 and one arm in group 2
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Paired Data

Paired data are dependent → observation in group 1 "matches" an observation in group 2. Either the same person/subject or somehow have pair/match in other group (spouse, twins, different arms). Look for clear link between subject in one group and subject in other (are they same or pair?). Commonly used in longitudinal studies in which a person's response is observed over time (before/after) and cross-over studies in which subject receives both control and treatment. More frequently used in medical/public health studies.
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Why Use Paired Data?

1. Addresses potential sources of difference
   - Keeps fixed factors that could potentially affect analysis
   - Ex) Sunscreen example → design takes into account fact that people have different levels of sun exposure
   - Particularly useful for making causal inferences (more in API 202)

2. Because of this standard errors of the difference in the groups might be smaller than if using independent samples
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Mean example looking at taxi times

An HKS student often goes to Logan Airport on Thursday evenings, wants to find the fastest route. Conducts a study to determine the fastest of two possible routes using a random sample of 14 cab drivers. Each hired driver drives from HKS to the airport on two different Thursday evenings using one of the two routes. Question: Difference in travel times between Routes A and Route B?
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Same steps as regular hypothesis test.

However: Define new random variable:

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where \( X_d \) has \( n \) observations (just like \( X_1 \) and \( X_2 \)).

We use \( X_d \) and \( n \) in our analysis.
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How does this differ from independent (non-paired) difference in means?

From our discussion of random variables:

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\text{Var}(X_1 - X_2) = \text{Var}(X_1) + \text{Var}(X_2) - 2\text{Cov}(X_1, X_2)
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So variance of \(X_1 - X_2\) for independent samples is

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Note: This is what makes standard error for dependent pairs usually smaller than for non-dependent pairs
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\[ \bar{X}_d = -0.59 \]
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\[ X_d = -0.59 \]

\[ s_d = 7.67 \]
Mean example looking at taxi times

Step 1: Null and Alternative Hypotheses

Null: No difference between two routes

$$\mu_d = 0$$ (the same as $$\mu_1 - \mu_2 = 0$$)

Alternative: Some difference between two routes

$$\mu_d \neq 0$$ (the same as $$\mu_1 - \mu_2 \neq 0$$)
Mean example looking at taxi times

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Mean example looking at taxi times

- Step 2: Collect sample data

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\( \bar{X}_d = -0.59 \)

\( s_d = 7.67 \)
Mean example looking at taxi times

Step 3: Calculate appropriate test statistic

Because we have redefined quantity of interest, analysis straightforward

Note: Because of small \( n \), use Student's \( t \) distribution \( w/ n - 1 \) degrees of freedom (same rules as before)

For \( \bar{X}_d \) and \( H_0: \mu_d = 0 \):

\[
t \frac{\bar{X}_d - \mu_d}{s_d / \sqrt{n}} = \frac{-0.59767}{0.288} = -0.288
\]
Mean example looking at taxi times

- Step 3: Calculate appropriate test statistic

\[ t_{df} = \frac{\bar{X}_d - \mu_d}{s_d / \sqrt{n}} \]

\[ t_{df} = -0.59 \]
Mean example looking at taxi times

- Step 3: Calculate appropriate test statistic
- Because we have redefined quantity of interest, analysis straightforward

\[ t = \frac{\bar{X}_d - \mu_d}{s_d / \sqrt{n}} \]

\[ t_{13} = -0.59 \]

\[ t_{14} = -0.288 \]
Mean example looking at taxi times

- Step 3: Calculate appropriate test statistic
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t = \frac{-0.59}{7.67 / \sqrt{14}} = -0.288
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$$t_{13} = \frac{-0.59}{7.67/\sqrt{14}} = -0.288$$
Mean example looking at taxi times
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- Step 4: Calculate p-value

\[ p\text{-value} = 2 \times P(t_{13} < -0.288) \]

\[ p\text{-value} = 0.7771 \]
Mean example looking at taxi times

- Step 4: Calculate p-value
- Here, using a two-sided test
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Mean example looking at taxi times

Step 5: Decide whether or not to reject the null hypothesis and interpret results

$p$-value = 0.7771 → Reject? Do not reject?
Mean example looking at taxi times

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Mean example looking at taxi times

Can also calculate the confidence interval:

\[ \hat{X} \pm \frac{z}{2} SE \]

Because we're using the \( t \) distribution, the 95% CI would be equal to:

\[ \hat{X} \pm t_{13} \frac{z}{2} SE \]

\[ = -0.59 \pm 2.160 \times 7.67 \sqrt{14} \]

\[ = -0.59 \pm 4.428 \]

\[ = (-5.018, 3.838) \]
Mean example looking at taxi times

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\[
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Mean example looking at taxi times

CI is $[-5.018, 3.838]$.

How to interpret?

If we repeat this study many times, the true mean difference will lie in 95% of our confidence intervals.

What does it mean for 0 to be in the 95% confidence interval?

→ Inconclusive whether one route "faster" on average than other.
Mean example looking at taxi times

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Mean example looking at taxi times

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Paired Tests for Proportions
Paired Tests for Proportions

- Tests works similarly for paired test of proportions

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Usually only have a contingency table

So we estimate the standard error slightly differently
Paired Tests for Proportions

- Tests works similarly for paired test of proportions
- However, for proportions we sometimes don’t have entire table of individual observations
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Proportion example from public opinion

Each person asked 2 questions: (1) ok for gov't to tap phone, or (2) ok for gov't to conduct random stops

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<tr>
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<th>Random Stop on St</th>
<th>Tap Phone Yes</th>
<th>Tap Phone No</th>
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<tbody>
<tr>
<td>Yes</td>
<td>494</td>
<td>335</td>
<td></td>
</tr>
<tr>
<td>No</td>
<td>126</td>
<td>537</td>
<td></td>
</tr>
</tbody>
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Question: Does the true proportion answering yes to the first question differ significantly from the second question?
Proportion example from public opinion

- Work through this using public opinion example (difference in proportion) from GSS data on government oversight given suspected terrorist activity

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<th>Yes</th>
<th>No</th>
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<td>Q1: Tap Phone</td>
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- Work through this using public opinion example (difference in proportion) from GSS data on government oversight given suspected terrorist activity
- Each person asked 2 questions: (1) ok for gov’t to tap phone, or (2) ok for gov’t to conduct random stops
- Results as follows:

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<th>Q1: Tap Phone</th>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q2: Random Stop on St</td>
<td>Yes</td>
<td>494</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>126</td>
</tr>
</tbody>
</table>
Proportion example from public opinion

- Work through this using public opinion example (difference in proportion) from GSS data on government oversight given suspected terrorist activity
- Each person asked 2 questions: (1) ok for gov’t to tap phone, or (2) ok for gov’t to conduct random stops
- Results as follows:

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</tr>
<tr>
<td>No</td>
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</tr>
<tr>
<td></td>
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<td>335</td>
</tr>
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<td>126</td>
<td>537</td>
</tr>
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- Question: Does the true proportion answering yes to the first question differ significantly from the second question?
Proportion example from public opinion

Although we have paired design, steps of hypothesis test the same

▶ Step 1: Null and Alternative Hypotheses
▶ Same as before
→ null usually some variant of “no difference”
▶ Null: Proportions of respondents answering “yes” to both questions equal
▶ \( H_0: \pi_1 - \pi_2 = 0 \)
▶ Alternative: Proportions of respondents answering “yes” to both questions unequal
▶ \( H_a: \pi_1 - \pi_2 \neq 0 \)
▶ What is more appropriate here, one-tailed or two-tailed?
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Step 2: Collect data
Proportion example from public opinion

- Step 2: Collect data
- Assume done for us
Proportion example from public opinion

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- Sample of $n = 1492$ U.S. adults
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Sample of \( n = 1492 \) U.S. adults

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</thead>
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<tr>
<td>Yes</td>
<td>494 ((n_{11}))</td>
<td>335 ((n_{12}))</td>
<td></td>
</tr>
<tr>
<td>No</td>
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<td>537 ((n_{22}))</td>
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Proportion example from public opinion

Question #1: Proportion of people in sample who believe authorities should be able to tap phones:
\[ \hat{\pi}_1 = \frac{494 + 335}{1492} = 0.556 \]

Question #2: Proportion of people in sample who believe authorities should be able to randomly stop and search people on street:
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Proportion example from public opinion

- Calculate some basic point estimates of “yes” answers:

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- In proportions case → Oftentimes don’t have that data
Proportion example from public opinion

▶ Interested in distribution of $\hat{\pi}_1 - \hat{\pi}_2$
▶ By CLT, this should be normally distributed
▶ From earlier lecture, if independent, then $\hat{\pi}_1 - \hat{\pi}_2 \sim N(\pi_1 - \pi_2, \pi_1(1 - \pi_1)n + \pi_2(1 - \pi_2)n)$
▶ However, here dependent, so $\hat{\pi}_1 - \hat{\pi}_2 \sim N(\pi_1 - \pi_2, \pi_1^2 + \pi_2^2 - (\pi_1 - \pi_2)^2)n$
▶ where these refer to cell proportions (not conditional proportions)
▶ (Proof in appendix)
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(Proof in appendix)
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Proportion example from public opinion

This gives us a test statistic:

$z = \hat{\pi}_1 - \hat{\pi}_2 - (\pi_1 - \pi_2) \sqrt{\hat{\pi}_{12} + \hat{\pi}_{21} - (\hat{\pi}_{12} - \hat{\pi}_{21})^2 / n}$

When the null is true, $\pi_1 - \pi_2 = 0$, we have some options:

1) Use this expression for the standard error
2) Simplify test using McNemar's Test for comparing dependent proportions (medicine/public health)

$z = n_{12} - n_{21} \sqrt{n_{12} + n_{21}}$

where this approximately comes from standard Normal.

Intuition borrows from Binomial distribution – explanation of McNemar's test statistic in Appendix.
Proportion example from public opinion

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Calculating McNemar's test statistic:

\[ z = \sqrt{\frac{n_{12} - n_{21}}{n_{12} + n_{21}}} = \sqrt{\frac{335 - 126}{335 + 126}} = 9.7341 \]

where this approx comes from a standard normal distribution
Calculating McNemar’s test statistic:
Proportion example from public opinion

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$p$-value = $2 \times P(Z \leq -9.7341)$

$p$-value < 0.01

Step 5: Decide whether or not to reject the null hypothesis and interpret results

Question: What is your conclusion?

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Proportion example from public opinion

To calculate confidence interval, follow same formula

\[ \hat{\pi}_1 - \hat{\pi}_2 \pm z_{\alpha/2} \times SE[\hat{\pi}_1 - \hat{\pi}_2] \]

Using full form of standard error

\[ \hat{\pi}_1 - \hat{\pi}_2 \pm z_{\alpha/2} \times \sqrt{\hat{\pi}_1(1-\hat{\pi}_1)/n - \hat{\pi}_2(1-\hat{\pi}_2)/n} \]
Proportion example from public opinion

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Proportion example from public opinion

Using our example

For 95% CI:

\[0.556 - 0.416 \pm 1.96 \sqrt{\frac{335}{1492}} - \left(\frac{335}{1492} - \frac{126}{1492}\right)^2] 0.14 \pm 0.018

So 95% is (0.122, 0.158)

Does it include 0?

What does this mean substantively?
Proportion example from public opinion

- Using our example

\[
\text{For 95\% CI: } 0.556 - 0.416 \pm 1.96 \sqrt{\frac{1492 - 126}{1492}} - \left( \frac{126}{1492} \right) \frac{2}{1492} = 0.14 \pm 0.018
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Next Time

- Starting Analysis of Variance tests
Appendix: Standard Error for Paired Diff in Proportion

Note: Notation here slightly different than used in main part of slides

\[
\text{Var}[\hat{\pi}_2 - \hat{\pi}_1] = \text{Var}[\hat{\pi}_{01} - \hat{\pi}_{10}]
\]

\[
= \text{Var}[\hat{\pi}_{01}] + \text{Var}[\hat{\pi}_{10}] - 2\text{Cov}[\hat{\pi}_{01}, \hat{\pi}_{10}]
\]

\[
= \frac{1}{n\pi_{01}(1 - \pi_{01})} + \frac{1}{n\pi_{10}(1 - \pi_{10})} + \frac{2}{n\pi_{01}\pi_{10}}
\]

\[
= \frac{1}{n}(\pi_{01} + \pi_{10} - (\pi_{01} - \pi_{10})^2)
\]
Appendix: McNemar’s Test

Under $H_0$, $n_{12}$ (or equivalently $n_{21}$) has a binomial distribution with parameters $n_\ast = n_{12} + n_{21}$ and "success" probability 0.5.

Under $H_0$, the mean of $n_{12}$ is $0.5n_\ast$.

Under $H_0$, the variance of $n_{12}$ is $0.5(1 - 0.5)n_\ast$.

Using CLT, we can approximate the binomial with Normal: $n_{12} \sim N(0.5n_\ast, 0.5(1 - 0.5)n_\ast)$.

Standardizing, we get $z = \frac{n_{12} - 0.5n_\ast}{\sqrt{0.5(1 - 0.5)n_\ast}}$.

Using $n_\ast = n_{12} + n_{21}$ can further simplify to $z = \frac{n_{12} - n_{21}}{\sqrt{n_{12} + n_{21}}}$, which approximates standard Normal.
Appendix: McNemar’s Test

- Under $H_0$: $n_{12}$ (or equivalently $n_{21}$) has a binomial distribution with parameters $n^* = n_{12} + n_{21}$ and “success” probability 0.5.
Appendix: McNemar’s Test

- Under $H_0$: $n_{12}$ (or equivalently $n_{21}$) has a binomial distribution with parameters $n^* = n_{12} + n_{21}$ and “success” probability 0.5
  - Under $H_0$: the mean of $n_{12}$ is $0.5n^*$
Appendix: McNemar’s Test

- Under $H_0$: $n_{12}$ (or equivalently $n_{21}$) has a binomial distribution with parameters $n^* = n_{12} + n_{21}$ and “success” probability 0.5
  - Under $H_0$: the mean of $n_{12}$ is $0.5n^*$
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$$n_{12} \sim N(0.5n^*, 0.5(1 - 0.5)n^*)$$
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- where this approximates standard Normal
Next Time

- Starting Analysis of Variance tests