Lecture 7:
Random Variables and Probability Distributions
API-201Z

Maya Sen

Harvard Kennedy School
http://scholar.harvard.edu/msen
Announcements

Problem Set #4 posted

We'll be posting several practice exams and practice problems in advance of midterm.
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Roadmap

By now, be comfortable w/ summary statistics in Stata/R, basic probability, conditional probability, independence, LOTP, Bayes' Rule

Now:

▶ Introduce probability distributions, which are foundational for statistical inference
▶ Random variables
▶ Give examples of discrete and continuous random variables
▶ Walk through probability distributions for discrete random variables (continuous next time)
▶ Introduce Bernoulli processes
Roadmap

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What are random variables?

Random phenomenon: A situation where we know what outcomes could happen, but we don't know which particular outcome did or will happen.

Ex) Flipping a coin 3 times
Ex) Randomly calling 5 U.S. Senators
Ex) Lady Bristol choosing 4 cups of tea out of 8

Random variables:
1. Numerical outcomes of random phenomenon

Ex) Number of Heads in 3 coin flips
Ex) Number of female Democrats you call
Ex) Number of 4 cups Lady identifies correctly
Can take on a set of possible different numerical values, each with an associated probability.

RVs denoted by capital letters – e.g., $X$ or $Y$

Note: Random variable = number (e.g, number of heads), random event = event (e.g., tossing heads)

As before, “random” does not mean “haphazard.”
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Example: You toss a coin 3 times

Random phenomenon → act of tossing the coin 3 times

Random variable → Sequence of outcomes of the flips, e.g. HTH, is not a random variable (not a number)

But we could make it into a random variable, $X$, by making it # of Heads in 3 flips

Note: Sample space can have many different random variables defined on it (e.g., number of tails flips, $Y$)

Random variable → one of several possible mappings from sample space (HTH) into a number (in this case, 2)
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Discrete Random Variables

A discrete random variable can take on only integer (countable) number of values (usually within an interval).

Examples:

- Outcomes when you roll a roulette wheel
- Number of times a particular word is used in a document
- Number of Democrats in U.S. Senate

In theory, you could count up these values, given finite amount of time.
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Continuous Random Variables (next time)

A continuous random variable that can take on any value (usually within an interval) on the real number line.

Examples:
- Annual rainfall in Bangladesh
- Time it takes to run a 100m dash
- Time until next earthquake in Japan

→ Cannot count these quantities, given finite amount of time

Much of intuition we cover today applies to both
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How do we describe random variables?

- We describe distribution of random variables using probability distributions.
- Probability distributions represent information about how likely various outcomes of $X$ are.
- Two common ways of representing probability distributions for discrete random variables are via:
  1. Probability mass functions ($f(x)$)
  2. Cumulative mass functions ($F(x)$)
- Similar for continuous distributions: probability density functions, cumulative density functions.
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Probability mass functions (PMFs)

A function that defines the probability of each possible outcome:

\[ f(x) = P(X = x) \]

where \( X \) is the random variable, \( x \) possible value it could take.

Provides you with all possible outcomes and probability of each outcome.

Three ways of describing PMFs:
- Table
- Relative frequency histogram (bar plot)
- Formula itself
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**Discrete Example**

Let $X$ be the number of heads in 3 coin flips.

Sample space of possible outcomes would look like:

<table>
<thead>
<tr>
<th>Event</th>
<th>Value of $X$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) TTT</td>
<td>0</td>
</tr>
<tr>
<td>2) TTH</td>
<td>1</td>
</tr>
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<td>3) THT</td>
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<tr>
<td>4) HTT</td>
<td>1</td>
</tr>
<tr>
<td>5) THH</td>
<td>2</td>
</tr>
<tr>
<td>6) HTH</td>
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Let $X$ be the number of heads in 3 coin flips.

The probability mass function distribution would look like:

- $f(x) = 1/8$ for $x = 0$
- $f(x) = 3/8$ for $x = 1$
- $f(x) = 3/8$ for $x = 2$
- $f(x) = 1/8$ for $x = 3$

All probabilities, between 0 and 1, inclusive.

Probabilities add up to 1.

Note: You'd only know this distribution if you repeated the random phenomenon over and over again!
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- All probabilities, between 0 and 1, inclusive
- Probabilities add up to 1
- Note: You'd only know this distribution if you repeated the random phenomenon over and over again!
Discrete Example

- Let $X$ be the number of heads in 3 coin flips
- Probability mass function distribution would look like

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Discrete Example

- Let $X$ be the number of heads in 3 coin flips
- Probability mass function distribution would look like

\[
\begin{array}{cccc}
0 & 1 & 2 & 3 \\
\hline
f(x) & \frac{1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{1}{8} \\
\end{array}
\]

- All probabilities, between 0 and 1, inclusive
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- All probabilities, between 0 and 1, inclusive
- Probabilities add up to 1
- Note: You’d only know this distribution if you repeated the random phenomenon over and over again!
Discrete Example

Could also represent this graphically:

\[
\begin{array}{cccc}
X = 0 & X = 1 & X = 2 & X = 3 \\
0.0 & 0.1 & 0.2 & 0.3 & 0.4 & 0.5 \\
\end{array}
\]

Again note: Heights of bars add up to 1
Discrete Example

- Could also represent this graphically:
Discrete Example

- Could also represent this graphically:

![Probability Mass Distribution]

- Again note: Heights of bars add up to 1.
Discrete Example

- Could also represent this graphically:

![Probability Mass Distribution]

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Cumulative Mass Function

Probability mass function describes the probability of each possible outcome, \( f(x) = P(X = x) \).

A cumulative mass function (CMF) gives us the probability that a random variable \( X \) takes on a value less than or equal to some particular value \( x \):

\[
F(x) = P(X \leq x) = \sum_{X \leq x} P(X = x)
\]

Can also represent CMFs using:
- Table
- Relative frequency histogram
- Formula itself
Probability mass function describes the probability of each possible outcome, $f(x) = P(X = x)$
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Can also represent CMFs using:
- Table
- Relative frequency histogram
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Discrete Example

Let $X$ be the number of heads in 3 coin flips.

Universe of possible outcomes would look like:

<table>
<thead>
<tr>
<th>Event Value of $X$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) TTT 0</td>
</tr>
<tr>
<td>2) TTH 1</td>
</tr>
<tr>
<td>3) THT 1</td>
</tr>
<tr>
<td>4) HTT 1</td>
</tr>
<tr>
<td>5) THH 2</td>
</tr>
<tr>
<td>6) HTH 2</td>
</tr>
<tr>
<td>7) HHT 2</td>
</tr>
<tr>
<td>8) HHH 3</td>
</tr>
</tbody>
</table>
Discrete Example

- Let $X$ be the number of heads in 3 coin flips

  - Universe of possible outcomes would look like:
    - 1) TTT 0
    - 2) TTH 1
    - 3) THT 1
    - 4) HTT 1
    - 5) THH 2
    - 6) HTH 2
    - 7) HHT 2
    - 8) HHH 3
Discrete Example

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Discrete Example

Let $X$ be the number of heads in 3 coin flips.

The probability mass function distribution would look like:

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<tr>
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Let $X$ be the number of heads in 3 coin flips
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Discrete Example
And then we can also represent the CMF graphically:
Discrete Example

- And then we can also represent the CMF graphically:

![Probability Mass Distribution](image1)

![Cumulative Mass Distribution](image2)
There are 5 candidates for 2 job openings

3 of the candidates are women and 2 are men: W1, W2, W3, M1, M2

Let $X$ be a random variable that is the # of women hired

$X \in \{0, 1, 2\}$

What is the PMF of $X$?

What is the CMF of $X$?
Sex Discrimination Example

- There are 5 candidates for 2 job openings
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Sex Discrimination Example

10 possibilities in the sample space:

- (W1, W2)
- (W1, W3)
- (W1, M1)
- (W1, M2)
- (W2, W3)
- (W2, M1)
- (W2, M2)
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Sex Discrimination Example

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- (W1, W2) (W1, W3) (W1, M1) (W1, M2)
Sex Discrimination Example

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- (W1, W2)  
- (W1, W3)  
- (W1, M1)  
- (W1, M2)  
- (W2, W3)  
- (W2, M1)  
- (W2, M2)  
- (W3, M1)  
- (W3, M2)  
- (M1, M2)
Sex Discrimination Example

10 possibilities in the sample space:

- (W1, W2) (W1, W3) (W1, M1) (W1, M2)
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Sex Discrimination Example

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Sex Discrimination Example

(W1, W2) (W1, W3) (W1, M1) (W1, M2)
(W2, W3) (W2, M1) (W2, M2)
(W3, M1) (W3, M2)
(M1, M2)

Probability mass function distribution:

\[ X = 0 \quad X = 1 \quad X = 2 \]

Cumulative mass function distribution:

\[ X \leq 0 \quad X \leq 1 \quad X \leq 2 \]
Sex Discrimination Example

- (W1, W2) (W1, W3) (W1, M1) (W1, M2)
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Sex Discrimination Example

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  \[
  \begin{array}{ccc}
  X = 0 & X = 1 & X = 2 \\
  f(x) & ? & ? & ? \\
  \end{array}
  \]

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  F(x) & ? & ? & ? \\
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Sex Discrimination Example

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Sex Discrimination Example

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- Probability mass function distribution:

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Sex Discrimination Example

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Sex Discrimination Example

Can use this information to address substantive questions

Probability mass function distribution:

\[ X = 0 \]
\[ X = 1 \]
\[ X = 2 \]

\[ f(x) = \begin{array}{c}
\frac{1}{10^7} \quad \frac{1}{10^1} \\
\end{array} \]

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Would you be concerned if 1 or fewer women were hired?

Would you be concerned if exactly 1 woman was hired?

Would you be concerned if no women were hired?
Sex Discrimination Example

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X &= 0 & f(x) &= \frac{1}{10} \\
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\end{align*}
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Key features of probability distributions

Similar to a sample of data, we can calculate useful info about probability distribution:

- Expected value
- Variance
- Standard deviation

Use Greek letters to denote these population parameters. Can't actually measure these empirically, but can calculate them based on what we think would happen if we repeated the random process over and over again.

Conceptually distinct from (but completely analogous to) sample parameters from earlier.
Key features of probability distributions

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Expected value

Expected value: a measure of the center of a probability distribution.

Denoted as $\mu_E[X] = \mu_X = \sum_{\text{all } x} x p(x)$.

Sum of all possible values, each weighted by its relative probability.

Substantively: If we repeated random process repeatedly and averaged values together, what would we expect to find?
Expected value

- **Expected value**: a measure of the center of a probability distribution

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E[X] = \mu_X = \sum_{x \text{ all } x} x p(x)
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$$E[X] = \mu_X$$

$$= \sum_{all \ x} xp(x)$$
Expected value

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Variance and Standard Deviation

Variance: Measure of the spread of a probability distribution:

\[
\sigma^2 = \sum_{\text{all } x} (x - \mu)^2 p(x)
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Standard deviation: Square root of the variance

\[
\sigma = \sqrt{\sigma^2}
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Sex Discrimination Example
Sex Discrimination Example

Probability mass function distribution from earlier:

\[ X = 0 \]
\[ X = 1 \]
\[ X = 2 \]

\[
f(x) = \frac{1}{10^6} /rac{1}{10^3} /rac{1}{10^2}
\]

\[
\mu = (10 \times 0) + (6 \times 1) + (3 \times 2) = 12
\]

\[
\sigma^2 = (0 - 12)^2 + (1 - 12)^2 + (2 - 12)^2 = 360
\]

\[
\sigma = 3.5
\]
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\[
\mu = \left( \frac{1}{10} \times 0 \right) + \left( \frac{6}{10} \times 1 \right) + \left( \frac{3}{10} \times 2 \right) = \frac{12}{10} = \frac{6}{5}
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\sigma^2 = \left( 0 - \frac{12}{10} \right)^2 \frac{1}{10} + \left( 1 - \frac{12}{10} \right)^2 \frac{6}{10} + \left( 2 - \frac{12}{10} \right)^2 \frac{3}{10}
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\[
= \frac{144}{1000} + \frac{24}{1000} + \frac{192}{1000} = \frac{360}{1000} = \frac{9}{25}
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Joint Probability Distributions

- Have focused on distribution of one random variable, $X$.
- A joint probability mass function describes behavior of 2 discrete random variables, $X$ and $Y$, at the same time.
- Lists probabilities for all pairs of possible values of $X$ and $Y$.
- Can represent as relative frequency table, joint frequency histogram, or formula.
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Joint Probability Distributions

- Written as $P(X = x, Y = y)$
- Can also have conditional probabilities: $(X = x | Y = y)$
- And can use joint probability $P(X = x, Y = y)$ to back out marginal probability distribution, $P(X = x)$ or $P(Y = y)$

Ex) Suppose in Boston there is relationship between floods & snowfall in a given year
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Association between floods and snowfalls

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<th>2</th>
<th>3</th>
<th>4</th>
<th>Total</th>
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<tr>
<td>0</td>
<td></td>
<td>0.15</td>
<td>0.05</td>
<td>0</td>
<td>0</td>
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What is probability of 2 floods and 3 snowstorms in a randomly chosen year?
Association between floods and snowfalls

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What is probability of 2 floods and 3 snowstorms in a randomly chosen year?
## Association between floods and snowfalls

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What is the marginal probability mass function of $Y$?
Association between floods and snowfalls

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Association between floods and snowfalls

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- What is the **conditional probability mass function** of $Y$ conditional on $X = 2$?
Association between floods and snowfalls

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| Total               | 0.3 | 0.25| 0.2 | 0.15| 0.1 | 1.00|

What is the **conditional probability mass function** of \( Y \) conditional on \( X = 2 \)?
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What is the conditional probability mass function of $Y$ conditional on $X = 2$?
Probability Distribution Families

We can write down some PMFs and CMFs for easy examples. Real world much more complicated. Fortunately, don't have to calculate our own PMFs/CMFs for every random variable we are interested in. Generations of statisticians have derived generalizable PMFs, CMFs for many frequently occurring processes. This includes processes frequently seen in social sciences/policy. Also includes processes frequently seen in natural sciences (e.g., Normal distribution). → Wikipedia a great resource.
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Bernoulli Distribution
A single coin flip well-known example of one of most basic probability distributions, a Bernoulli distribution/trial.
Bernoulli Distribution

- A single coin flip well-known example of one of most basic probability distributions, a Bernoulli distribution/trial
- Need:
Bernoulli Distribution

- A single coin flip well-known example of one of most basic probability distributions, a Bernoulli distribution/trial
- Need:
- A trial (e.g., coin flip) that has a probability of success \( (p) \) and a probability of failure \( (1 - p) \)
Bernoulli Distribution

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- Need:
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- For Bernoulli processes, the PMF:

\[
f(x) = \begin{cases} 
  p & \text{for } x = 1 \\ 
  1 - p & \text{for } x = 0 
\end{cases}
\]
Bernoulli Distribution

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- Need:
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- For Bernoulli processes, the PMF:

  \[ f(x) = \begin{cases} 
  p & \text{for } x = 1 \\
  1 - p & \text{for } x = 0 
  \end{cases} \]

- which can be rewritten as

  \[ f(x) = p^x(1 - p)^{1-x} \]
Binomial Distribution

A series of Bernoulli trials results in a binomial distribution.

Ex) Series of coin flips

Need:

- Only two possible outcomes at each trial (success and failure)
- Constant probability of success, $p$, for each trial
- A fixed number of trials $n$ that are independent
- $X$ represents the number of successes in $n$ trials
Binomial Distribution

- A series of Bernoulli trial results in a binomial distribution
A series of Bernoulli trials results in a binomial distribution

Ex) Series of coin flips
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Binomial Distribution

For a binomial process, denote $X \sim \text{Binomial}(n, p)$

Has a PMF of:

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

where $\binom{n}{x} = \frac{n!}{x! (n-x)!}$

and $n! = n \times (n-1) \times \cdots \times 1$

and a CDF of:

$$P(X \leq x) = \sum_{k=0}^{x} \binom{n}{k} p^k (1-p)^{n-k}$$
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$$\binom{n}{x} = \frac{n!}{x!(n-x)!}$$

and $n! = n \times (n - 1) \times (n - 2) \times \ldots \times 1$

and a CMF of:

$$P(X \leq x) = \sum \binom{n}{x} p^x (1 - p)^{n-x}$$
### Binomial Distribution

**Binomial probabilities (continued)**

Entry is $P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$

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- Poisson Distribution:
  - $X$ is # of events per unit of time
  - Probability that 25 aviation incidents occur in a given year
  - Probability that 5 MBTA buses stop at JFK bus stop in an hr

- Geometric Distribution:
  - $X$ is the # of Bernoulli trials needed to get one “success”
  - Probability that HKS needs to ask 5 former ambassadors to join faculty before one says “yes”
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Next Time

- Continuous probability distributions