Inflation, Black Market Exchange Rates, and Economic Growth

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ABSTRACT

This paper considers a model of a small open economy with a black market in foreign currency. The presence of the black market makes foreign assets an attractive alternative to domestic capital and agents hold a portfolio comprised of both assets. The return on domestic capital is linked to the monetary growth and inflation through a reserve requirement imposed on banks which forces them to hold a fraction of deposits on reserve in the form of domestic cash. In this way, we link economic growth to such macroeconomic factors as inflation, capital flight, reserve requirements, and the black market premium.

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INFLATION, BLACK MARKET EXCHANGE RATES, AND ECONOMIC GROWTH

1. INTRODUCTION

Among the more interesting variables included in empirical studies on economic growth is the black market exchange rate premium (Barro [5]; Barro and Sala-I-Martin [7]; Fischer [15]; Fry [17]). In these studies, the black market premium is intended to serve as a measure of market distortions more broadly defined, and there appears to be a small but significant negative relationship between economic growth and the black market premium. Figure 1, which shows a scatter diagram of average growth residuals and black market premia for 69 non-OECD countries for the years 1960-89, summarizes some of this evidence.[1]

-Insert Figure 1-

Despite a large body of empirical evidence focusing on international trade factors as determinants of growth rates, there is surprisingly little theoretical work exploring the role of open macroeconomic factors - such as the exchange rate - in economic growth. And while there are theoretical studies that show level effects of the black premium, there has been no study, to our knowledge, that links growth and the black market premium. This omission in the literature is important, for without a theoretical basis to relate the aforementioned empirical findings, the premium is a merely a proxy, pretty much left to measure whatever observers wish to claim it measures. In turn, these findings provide little support for policy initiatives aimed at addressing distortions in developing financial markets, and they do not shed much light on how growth and the black premium are related to other macroeconomic factors, such as inflation.[2]

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1. The growth residuals in Fig.1 were constructed in the following manner. We used the data from the Barro and Lee[6] data set which has a panel structure - that is, growth rates over 5-year intervals (1960,65,70,75,80,85-89) and items such as investment-GDP ratios in the initial years (60,65,...), proxy for initial human capital, black market exchange rates, and the like. We ran a panel regression of growth rates on initial per capita GDP, initial investment to GDP, intial human capital, and intial black market premium. These regressions were done using various country samples and time periods. We experimented with excluding OECD and excluding those with small premia. The coefficient on the premium showed some variation across different samples but qualitatively, they were the same, a small negative coefficient with a t-statistic of around 5. The growth residuals in Fig.1 represent the residuals from a panel regression of the growth rate on all the above items except the premium. In Fig.1, one can see a weak, negative correlation between growth and the black premium and the dominance of countries with low premia.

2. The *ad hoc* nature of these ‘Barro regressions’ is well-established in the literature. See, for example, Temple [27].

The purpose of this paper is to develop a dynamic general equilibrium model which can be used to study the relationship between economic growth and black market exchange rates. Along the way, we are able to link economic growth to a host of other factors relevant to the macroeconomic performance of small developing countries, including inflation, capital flight, and reserve requirements. In this regard, the paper spans the literature of a number of different areas, including growth and inflation, inflationary finance in open economies, parallel currency markets, and the relation between black and official exchange rates. The model can be also viewed within the context of the literature on the fiscal theory of the price level, in that money, growth, and prices are determined in accordance with an exogenous fiscal policy.

A currency black market arises in our model due to government imposed restrictions which prevent agents from legally holding foreign assets. Despite such restrictions, some agents in the economy diversify their asset portfolio, choosing to hold both a domestic intermediated asset and an illegal off-shore account. The presence of the black market makes foreign assets an attractive alternative to domestic capital. Agents use the black market to acquire the necessary hard currency needed to establish a foreign bank account, as well as to remit their foreign asset earnings, converting these back into local currency or into goods in the domestic market.[3] The real return of the black, foreign account is different from the return of the same account held legally *outside the country*. Here we establish an alternative to the usual explanations of capital flight based on risk. Agents diversify and hold foreign assets which, if held outside the country, may command a return lower than the real return on domestic assets. They are willing to do this since the *internal yield* - that is, the yield inclusive of the change in the black market premium, makes these assets competitive with higher yielding home assets.

We associate the black market premium in our model with a specific distortion in the economy, one due to government restrictions on which assets agents can and cannot legally hold. Because of these restrictions, certain trades cannot be conducted at the official exchange rate and are, instead, conducted through the parallel

3. For simplicity, we assume the cost of smuggling goods inside or out of the country is prohibitively high. See Barnett [3] or Mourmouras and Barnett [25] for recent treatments of currency black markets and goods smuggling.

currency market. It is precisely trades in this market - the diversification of savings - that has a negative impact on domestic capital accumulation and economic growth. On the other hand, economic growth itself generally has a positive effect on the demand for these assets, if only to increase overall saving in the economy. This suggests the premium and growth are positively related. Indeed, both these theoretical possibilities are borne in artificial time-series generated by the model.

The return on domestic capital is linked to the growth rate in the money supply and inflation through a requirement imposed on banks which forces them to hold a fraction of deposits on reserve in the form of domestic cash. Reserve requirements and their role in inflation and economic growth have been the focus of a number of recent theoretical growth studies of closed economies, including Bhattacharya et.al. [8]; Chari, Jones and Manuelli [10]; Haslag [20]; and Haslag and Young [22]. Other studies of reserve requirements and inflation include Brock [9]; Freeman [16]; Haslag [21]; and Wallace ([28],[29]).

Like several of the studies mentioned above, production and growth are introduced in the model through a simple AK production function. The underlying source of uncertainty in the model is due to shocks in the production coefficient A, which allows us to consider trend growth and deviations from trend. Unlike the other studies, however, the growth rate is not determined solely by the technical coefficient A, factor shares, and the nation's savings rate or rate of time preference. Here, as indicated, growth depends on the extent to which the economy channels its savings to the black market. The margin of participation in this market is determined along two lines - firstly, the agents' portfolio choice, and secondly, along an extensive margin, the number of people willing to break the law and saving in the form of an illegal account.

So far, we have said little about the inflationary side of the model. We assume, as does Barro [4], that government purchases are a fixed percentage of GNP. The revenue needed to finance these purchases is raised entirely through seigniorage. The seigniorage base is determined endogenously through the portfolio choice of agents. This creates a natural link between growth and inflation - higher inflation lowers the return on domestic intermediated assets. Agents in turn allocate more of their saving to the black market and less to domestic capital, thereby lowering the growth rate. This negative correlation between inflation and growth is readily observable in numerical simulations of the model.

Finally, we use the model simulations to address several questions broadly related to currency black markets. First, what is the relationship between the official and black market exchange rates? The empirical evidence in Lane [24] suggests that changes in official and black market exchange rates track each other quite well; our numerical simulations lead us to a similar conclusion. Second, it is natural to ask, what harm do these markets do? While the negative aspects of these markets, higher inflation and lower physical capital accumulation, seem apparent, they do provide a useful role by allowing agents to circumvent artificial restrictions on portfolio choice imposed by the government. We explore these welfare considerations in a numerical example of the model.

The rest of this paper proceeds as follows. Section 2 lays out the general structure of the model. Existence and the construction of an equilibrium are established in Section 3. Here we impose sufficient restrictions on government spending and on the reserve requirements to ensure portfolio diversification and that the reserve requirement is binding at each date and state. We also discuss some restrictions on the technology, preferences, and government policy to ensure economic growth. In Section 4, we study some numerical examples of the model and contrast the results with those generated by a closed economy (no black market) version of the model. Section 5 contains some concluding remarks.

2. THE MODEL

2.1. Preliminaries.

Consider an overlapping generations model of a small, open economy. Time is discrete and indexed by $t = 1, 2, \ldots$. Each generation $t \geq 1$ consists of a continuum of agents, of measure one. Agents within a generation are identical; each is endowed with 1 unit of time in the first period of life which is supplied inelastically to local 'firms' - competitive, infinitely lived institutions that produce the single consumption good each period using labor and capital. Young agents use a portion of their wages to finance consumption in the current period, the rest is saved in order to finance consumption in the second period of life.

All agents save (legally) in the form of a local intermediated bank deposit. These intermediaries (banks) are competitive and are restricted to making loans to local firms only. Banks are required to hold a minimum fraction of their deposits in the form of a cash reserve requirement. This cash requirement is in the form of the home currency, and as it turns out, it will be binding on each intermediary. Young agents receive a state-dependent return on deposits equal to a weighted return on money and the loan rate faced by firms.[4]

Alternatively, a portion of the population, of measure η, save clandestinely, using an illegal, off-shore account. Since the home currency is assumed to be inconvertible on bona fide foreign exchange markets, deposits to this account must be in the form of foreign currency. Agents acquire these funds in the home black (or more appropriately, parallel) market for foreign currency. They also remit foreign currency earnings from these off-shore accounts through the parallel market. The cost of laundering funds to off-shore accounts is a fixed proportion θ of the amount deposited overseas. These costs are meant to reflect the bribes, fees, and transactions costs that are necessarily incurred in setting up an off-shore account. For simplicity, it is assumed that these fees are paid to foreigners, or equivalently, leave the country completely. Although illegal, it is assumed the parallel market is tolerated by law enforcement officials; agents face no risk that transactions in the market will be confiscated, nor do they face the possibility of penalties or fines for dealing in the market. It may seem odd to think of this market as illegal - after all, we have modeled it as one in which there is no specific risk in participation, either in fines or confiscation. The parameter θ is meant to capture some aspect of the illicit nature of the market, but to be sure, it is incomplete. Mourmouras and Barnett [25] and Soller and Waller [26] consider environments in which agents face the risk of confiscation for holding foreign currency. We forgo this type of individual uncertainty and focus instead on aggregate uncertainty in the model. It is a fact, however, that in many developing countries with currency controls, enforcement of existing currency laws is 'mild', if not nonexistent, and currency dealers generally operate openly (Grosse [19]). In light of this observation, it seems wholly appropriate that the overriding source of uncertainty confronting agents in the model stems from aggregate, not individual risk.

We do not offer a deep explanation regarding the determinants of the fraction of the population $1 - \eta$ that

4. In many ways, intermediated bank deposits in this model resemble mutual funds which have, in their charters, a restriction on holdings of foreign assets and a minimum portfolio allocation of home cash.

choose not to participate in the black market. This could be modeled explicitly by assuming different costs of transacting in illicit markets across agents or by assuming some agents receive sufficiently negative utility by breaking the law. In either case, these frictions would need to be infinitely large, since there is economic growth. We will, however, vary the size of η to gain insight into how this market affects the welfare of agents. Preferences are represented by a log-linear utility function. This assumption together with the assumption that agents have no second period endowment implies that the level of (gross) savings is independent of the rate of return. Additional assumptions on the production function and government spending (discussed below) ensure that the country's growth rate and inflation rate depend on rates of return only to the extent that they affect portfolio shares.

Firms in this model hire workers and borrow funds for capital purchases from an intermediary. Factor markets are competitive and each factor is paid its marginal product. Growth is introduced into the model by assuming a capital externality exists so that the country's aggregate output can be expressed as a simple AK production function. The production parameter, A, is assumed to follow a first-order autoregressive process in deviations from trend:

$$A_t - A = \rho(A_{t-1} - A) + \epsilon_t, \quad t \geq 2, \tag{1}$$

with $0 \leq \rho < 1$ and the initial $A_1 = A$. The random variable ϵ_t can take on one of two values, $\epsilon_t \in \{-\epsilon, \epsilon\}$, which we label as state 1 and state 2, respectively. It has a probability transition matrix $\Pi = \begin{bmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix}$, where, for example, π_{12} is the probability that next period's state is state 2, given the current state is 1. This formulation, along with additional assumptions on the values of A in each state allows trend growth as well as deviation from trend. Below, we impose additional assumptions on $A, \rho,$ and ϵ which ensure positive economic growth.

Despite the assumption of a simple stationary forcing variable ϵ_t, the equilibria studied here are nonstationary and stochastic. The parameter $\sigma_t \equiv \{\epsilon_{t-s}\}$, for $s \leq t$, summarizes a history of realizations of ϵ, and the parameterization $(i;\sigma_t)$ below refers to the conditional history for date $t+1$, with $i = 1, 2$ indexing the date $t+1$ realized state of ϵ_{t+1}. Without loss of generality, we define $\sigma_1 \equiv \epsilon$. To simplify much of our discussion, a formal notation, indexing all variables by either σ_t or $(i;\sigma_t)$, is used only when necessary to highlight the fact that next period's variables are stochastic and take on one of two possible values. For example, $r^b(1;\sigma_t)$ denotes the real return to the black market asset in period $t+1$, state 1, conditional on a history of past state realizations, $\sigma_t \equiv \{\epsilon_{t-s}\}_{s=0}^{s=t-1}$.

2.2. Agents

Given history σ_t and wage income, w_t, a young agent among the cohort of η agents that participate in the black currency market at date t chooses the level of consumption c_t, saving, s_t, a portfolio l_t (the share of savings allocated to home assets), and consumption plans for each possible state at date $t+1$, x_t to:

Maximize $\quad u(c_t) + \beta E_t\, u(x_t) \qquad (2)$

subject to: $\quad c_t + s_t \le w_t$

$$x(i;\sigma_t) \le r^d(i;\sigma_t)\, l_t\, s_t + r^b(i;\sigma_t)\,(1-l_t)\, s_t/(1+\theta);$$

$$c_t \ge 0,\; x(i;\sigma_t) \ge 0,\; s_t \ge 0,\; 0 \le l_t \le 1,$$

where $r^d(i;\sigma_t)$, $r^b(i;\sigma_t)$ denote the real return on the intermediated deposits and illegal black market accounts, respectively. The term for saving, s_t, includes fees θ incurred at date t in setting up a black, off-shore account.[5]

Assuming the utility function for the agent is $u(c) = \ln c$, the agent's saving function is given by

$$s_t = \frac{\beta\, w_t}{1+\beta}. \qquad (3)$$

Let j denote the state at date t, the portfolio choice l_t is given by

$$l_t = \begin{cases} 1 & \textit{when } 1+\theta > \Gamma_d \\[2ex] \dfrac{(1+\theta)[\pi_{j;1} r^b(2;\sigma_t) r^d(1;\sigma_t) + \pi_{j;2} r^b(1;\sigma_t) r^d(2;\sigma_t)] - r^b(1;\sigma_t) r^b(2;\sigma_t)}{[(1+\theta) r^d(1;\sigma_t) - r^b(1;\sigma_t)][r^b(2;\sigma_t) - (1+\theta) r^d(2;\sigma_t)]} & \textit{otherwise} \\[2ex] 0 & \textit{when } (1+\theta)\Gamma_b < 1 \end{cases} \qquad (4)$$

where $\Gamma_d \equiv \dfrac{\pi_{j,1}\, r^b(1;\sigma_t)}{r^d(1;\sigma_t)} + \dfrac{\pi_{j,2}\, r^b(2;\sigma_t)}{r^d(2;\sigma_t)}$, and $\Gamma_b \equiv \dfrac{\pi_{j,1}\, r^d(1;\sigma_t)}{r^b(1;\sigma_t)} + \dfrac{\pi_{j,2}\, r^d(2;\sigma_t)}{r^b(2;\sigma_t)}$. The derivation of l_t is

5. Specifically, for the portfolio decision l_t, the fees incurred are $\dfrac{\theta(1-l_t)\, s_t}{1+\theta}$.

available from the authors upon request.

The remaining $1 - \eta$ agents do not diversify their saving internationally and simply save $\frac{\beta}{1 + \beta}$ of their wage income, as suggested by eq.(3), in the form of the domestic intermediated asset.

2.3. Firms

Firms in this economy are assumed to be infinitely lived institutions that hire labor and borrow funds from agents through an intermediary to produce the single consumption good in the economy. Each firm's production function is Cobb-Douglas, with increasing returns that are external to the firm. Capital depreciates fully each period.

We adopt a standard timing convention, assuming firms observe next period's productivity shock *before* making their capital and labor decisions (see, for example, Altuğ and Labadie [2]). Firms face state-contingent loan and wage rates, r_{t-1} and w_t, respectively. A representative firm chooses capital k_t and labor n_t to maximize

$$A_t\, B(K_t)\, n_t^{\alpha} k_t^{1-\alpha} - w_t\, n_t - r_{t-1}\, k_t, \tag{5}$$

where K_t is the aggregate capital stock at date t and $B(K_t) \equiv K_t^{\alpha}$. Setting $k_t = K_t$, and $n_t = 1$ (as required by labor market clearing), factor payments satisfy:

$$w_t = \alpha\, A_t\, K_t \tag{6}$$

$$r_{t-1} = (1 - \alpha)\, A_t. \tag{7}$$

2.4. Intermediaries

Intermediaries act merely as competitive go betweens for agents and firms in the model, earning zero profits. They accept deposits from young agents and lend to local firms. Intermediaries face a reserve constraint which forces them to hold a portion λ of their deposits in the form of the home currency. When binding, deposits at date t pay a date $t+1$ state-contingent, weighted average return on home loans and currency, $r^d(i;\sigma_t)$,

$$r^d(i;\sigma_t) = \lambda\, r^m(i;\sigma_t) + (1 - \lambda)\, r(i;\sigma_t), \tag{8}$$

where $r^m(i;\sigma_t) \equiv \frac{P(\sigma_t)}{P(i;\sigma_t)}$ is the real return on home currency, $P(\sigma_t)$ ($P(i;\sigma_t)$) is the nominal price level at date t (date $t+1$), and λ is the reserve requirement.

Note that the (unintermediated) return on home capital at date $t+1$ is $r(i;\sigma_t) = (1-\alpha)A(i;\sigma_t)$. Given the law of motion for $A(i;\sigma_t)$ as described in eq. (1), this return can be greater than or less than A. To ensure bindingness, we assume $r^m(i;\sigma_t) < r(i;\sigma_t)$ for each state. Additional restrictions (listed below) on the reserve requirement and on the size of the government deficit to be financed through seigniorage ensure this inequality obtains in equilibrium.

2.5. Black Off-Shore Accounts

Although illegal, agents can access foreign accounts by acquiring the necessary hard currency on a parallel foreign exchange market. Off-shore accounts pay a return of R_t^*, expressed in terms of foreign currency. Let $e^b(\sigma_t)$ $(e^b(i;\sigma_t))$ denote the value of the foreign currency in terms of the home currency on the black market at date t $(t+1)$. The realized real return of off-shore accounts, ex-processing fee θ, is

$$r^b(i;\sigma_t) = \frac{e^b(i;\sigma_t)\,R_t^*/P(i;\sigma_t)}{e^b(\sigma_t)/P(\sigma_t)} = \frac{e^b(i;\sigma_t)\,R_t^*\,P(\sigma_t)}{e^b(\sigma_t)\,P(i;\sigma_t)}\,. \tag{9}$$

The right-hand side of the eq.(9) turns out to be the change in the black premium times the world real interest rate (see eq. (9a) below).

2.6. The Government

The government plays three roles in this economy. First, it imposes currency restrictions on agents, prohibiting individuals by law from holding foreign currency and making the home currency inconvertible on world foreign exchange markets. The force of this assumption is that all legal international trade must be conducted through the monetary authority. Agents wishing to import goods legally must obtain foreign currency at the official exchange rate of e_t units of domestic currency per unit of foreign currency. Similarly, anyone legally exporting goods must surrender their foreign export earnings to the central bank in exchange for domestic currency at the official rate of e_t.[6] Without loss of generality, the monetary authority is assumed to conduct an official exchange rate policy such that purchasing parity is maintained and individuals engaged in legal goods trade earn

6. In the language of international trade economists, the currency arrangement here is essentially one in which the home currency is *convertible on trade account transactions only*.

zero profits. That is, $e_t = P_t / P_t^*$, where P_t, (P_t^*) is the price of the good in the home (world) market at date t. Here the force of the type of distortion in the currency markets we are considering becomes apparent. The black market arises not because of tariffs, quotas or other restrictions in real trade - which are generally acknowledged to have level but not necessarily growth effects - but rather, it is due to asset portfolio restrictions facing agents which do have growth effects. This notion is in keeping with the observation by Agénor [1] who notes: "the role of asset composition in the determination of the parallel-market rate" seems to be important in explaining black market exchange rates and premia. This is not to say that distortions in the goods market are unimportant in the study of currency black markets, but simply that they are not needed for our analysis. The assumption that PPP holds stresses the point that a black market premium reflects a distortion in *intertemporal international trade*, and that is the focus of attention in this model. On the whole, the country does not run legal trade surpluses or deficits - legal imports are paid for by legal exports, along the lines of a clearing arrangement. For simplicity, it is assumed the cost of smuggling goods is prohibitively high.

Second, the central bank imposes a reserve requirement λ on member banks. This reserve requirement ensures a minimum inflationary tax base for the government.

Third, the government purchases goods each period. Following Barro [4], government spending at each date is assumed to be a constant fraction g, $0 < g < 1$, of home output,

$$G_t = g Y_t. \tag{10}$$

The government finances its purchases through seigniorage. The government's budget constraint may be written as:

$$G_t = \frac{M_t - M_{t-1}}{P_t}, \qquad t \geq 1. \tag{11}$$

2.7. Market-Clearing

At each date, three markets must clear - the two markets for home assets, money and capital, and the black foreign currency market. The remaining goods market clears by Walras' Law. Let s_t denote each agent's real savings, and M_t the nominal money stock at date t. The conditions for the home assets are

$$\lambda\,[\,\eta\, l_t + (1-\eta)\,]\, s_t = \frac{M_t}{P_t} \tag{12}$$

$$(1-\lambda)\,[\,\eta\, l_t + (1-\eta)\,]\, s_t = K_{t+1}. \tag{13}$$

Let F_t denote the stock of black currency at date t, and $v_t \equiv e_t^b / e_t$ denote the (gross) black market premium. The market-clearing condition for the black currency market is

$$\eta\,(1 - l_t)\, s_t = \frac{e_t^b F_t}{P_t}.$$

Using the PPP condition, $P_t = e_t P_t^*$, this can be rewritten as

$$\eta\,(1 - l_t)\, s_t = \frac{v_t F_t}{P_t^*}. \tag{14}$$

The law-of-motion for the stock of black currency can be derived using eq. (14). At date t, agents collectively incur $\frac{\theta\,\eta\,(1-l_t)\,s_t}{1+\theta}$ amount of fees or bribes in setting up off-shore accounts. From eq.(14), this amounts to $\frac{\theta}{1+\theta}\frac{v_t F_t}{P_t^*}$, or, expressed in terms of black currency, $\frac{P_t}{e_t^b}\frac{\theta}{1+\theta}\frac{v_t F_t}{P_t^*} = \frac{\theta}{1+\theta} F_t$, using the PPP condition and the definition of the black premium. The remaining stock is deposited abroad, earning $\frac{R_t^* F_t}{1+\theta}$. The stock of the black foreign currency follows the path

$$F_{t+1} = \frac{R_t^* F_t}{1+\theta}, \tag{15}$$

with F_1 given.

Given the home money-market condition (12), the government budget constraint can be rewritten as:

$$\begin{aligned} G_1 &= \lambda\,[\,\eta\, l_1 + (1-\eta)\,]\, s_1 - \frac{M_0}{P_1}, \qquad t = 1. \\ G_t &= \lambda\,[\,\eta\, l_t + (1-\eta)\,]\, s_t - \frac{P_{t-1}}{P_t}\,\lambda\,[\,\eta\, l_{t-1} + (1-\eta)\,]\, s_{t-1}, \qquad t \geq 2, \end{aligned} \tag{16}$$

where M_0 is the initial home money stock.

At this point, we can describe the equilibrium price sequences. From eq.(16), home prices satisfy

$$P_1 = \frac{M_0}{\lambda\,[\,\eta\, l_1 + (1-\eta)\,]\,s_1 - G_1} \tag{17}$$

$$r^m(i;\sigma_t) = \frac{P(\sigma_t)}{P(i;\sigma_t)} = \frac{\lambda\,[\eta\, l(i;\sigma_t) + (1-\eta)]\, s(i;\sigma_t) - G(i;\sigma_t)}{\lambda\,[\eta\, l(\sigma_t) + (1-\eta)] s(\sigma_t)}, \qquad t \geq 1 \tag{18}$$

Using eq. (18), along with (9), the real return on home intermediated deposits satisfies

$$r^d(i;\sigma_t) = \lambda\, r^m(i;\sigma_t) + (1-\lambda)\, r(i;\sigma_t), \tag{19}$$

where $r(i;\sigma_t) \equiv (1-\alpha)A(i;\sigma_t)$.

The real return on off-shore accounts, independent of fees, is given by eq. (9). Using the PPP condition and the definition of the premium v_t, this can be rewritten as

$$r^b(i;\sigma_t) = v(i;\sigma_t)\, r_t^* / v(\sigma_t), \tag{9a}$$

where $r_t^* \equiv P_t^*\, R_t^* / P_{t+1}^*$ is the real return on the foreign deposit *outside the home country*. The fact that home agents must use the black market in order to get and to remit hard currency means that the real return on foreign assets inside and outside the country may differ at any given date.[7] Note too that the return $r^b(i;\sigma_t)$ is state and history dependent, even though r_t^* is not.

Using the black market clearing condition (14) and law-of-motion (15), the real return on illegal off-shore accounts can be written as

$$r^b(i;\sigma_t) = \frac{(1+\theta)\,(1 - l(i;\sigma_t))\, s(i;\sigma_t)}{(1 - l(\sigma_t))\, s(\sigma_t)}. \tag{20}$$

An equilibrium consists of stochastic processes for prices, interest rates, allocations of goods, capital, and monies, and monetary growth rates such that:

- the allocations are optimal for agents and firms when faced with those interest rates and prices;
- the allocations satisfy the market-clearing conditions;
- the government budget constraint is satisfied at each date t.

A *binding equilibrium* is an equilibrium in which the return on home cash, $r^m(\sigma_t)$**,** is less than the return on home unintermediated capital, $r(\sigma_t)$, for all histories σ_t, $t \geq 2$.

7. Our framework prohibits foreigners from participating in the black market. Since the home currency is inconvertible on world currency markets, any profits foreigners can make by selling hard currency on the black market would necessarily need to exit the home country through the legal export of goods. However, legal exports require evidence of payment in hard currency, which foreign traders no longer have once they have sold hard currency on the black market.

3. EQUILIBRIA

We begin our discussion on the existence of an equilibrium by placing some limits on government spending and on the reserve requirements:

Assumption 1. Seigniorage and Reserve Requirements.

(*i*) $\frac{(1-\eta)\lambda\beta\alpha}{1+\beta} < g < \frac{\lambda\beta\alpha}{1+\beta}$.

(*ii*) $0 < \lambda < 1$.

Assumptions 1i) ensures that the government spending is not so large that it exceeds all domestic savings, held as cash reserves, $G_t < \lambda s_t$, and yet is not so small that it can be financed entirely by the savings of $(1-\eta)$ population alone, $(1-\eta)\lambda s_t < G_t$. Assumption 1ii) rules out Friedman-type deposits backed with 100% cash. In effect, Assumption 1ii) ensures some fraction of home savings will be allocated to home capital.

Given Assumption 1, it is convenient to use the following characterization for government spending G_t:

$$G_t = \lambda[\eta \underline{l} + (1-\eta)] s_t, \tag{21}$$

where $\underline{l} \equiv \frac{g(1+\beta)}{\lambda\alpha\beta\eta} - \frac{1-\eta}{\eta}$. This is derived from eq.(10), together with the government budget (16) and the condition $P_{t-1}/P_t = 0$ (an infinite inflation at date t). Note that Assumption 1i) ensures $0 < \underline{l} < 1$.

Two important concepts stand behind $\underline{l}$. First, as seen by (16), it represents the smallest possible portfolio value of l_t, given that the government relies on seigniorage to finance its government deficit. From eq.(11) and the market-clearing condition (12), the government budget constraint can be written as:

$$G_t = (1 - \frac{1}{z_t}) \lambda [\eta l_t + (1-\eta)] s_t, \quad t \geq 1, \tag{22}$$

where $z_t = M_t / M_{t-1}$ is the gross growth rate in the money supply at date t. When $l_t = \underline{l}$, the country must impose an inflation tax equal to 1, or equivalently, set $z_t = \infty$, in order to finance purchases G_t. Secondly, the value $\underline{l}$ also represents the greatest degree of currency substitution (or *dollarization*) possible in equilibrium. Any equilibrium portfolio sequence $\{l(\sigma_t)\}$ satisfies $\underline{l} < l(\sigma_t) \leq 1$, for all histories σ_t.

Next, denote the *(gross) growth rate in the capital stock*, from date t to t + 1, as $\gamma_K(\sigma_t)$, where

$$\gamma_K(\sigma_t) \equiv \frac{K_{t+1}}{K_t}$$

$$= \frac{\alpha\beta A(\sigma_t)(1-\lambda)[\eta\, l(\sigma_t) + (1-\eta)]}{1+\beta}. \tag{23}$$

Eq.(23) is derived from the market-clearing conditions for the capital market, eq.(13), noting that saving for each agent, $s_t = \dfrac{\alpha\beta A(\sigma_t) K_t}{1+\beta}$. Capital at date $t+1$, as well as the capital growth rate, does not depend on the realized state at date $t+1$, since households select their portfolio allocations before observing the state at $t+1$ (see eq.13).

Output, on the other hand, does depend on the realized state at date t +1. Since aggregate production at date t is $Y_t = A(\sigma_t)K_t$, and at date t + 1, $Y(i;\sigma_t) = A(i;\sigma_t)K_{t+1}$, the growth in output is

$$\gamma_Y(i;\sigma_t) = \frac{A(i;\sigma_t)}{A(\sigma_t)}\gamma_K(\sigma_t)$$

$$= \frac{\alpha\beta A(i;\sigma_t)(1-\lambda)[\eta\, l(\sigma_t) + (1-\eta)]}{1+\beta}. \tag{24}$$

It turns out to be convenient to write the returns on money and black off-shore assets in terms of the growth rate $\gamma_Y(i;\sigma_t)$. Since $s(i;\sigma_t)/s(\sigma_t) = \gamma_Y(i;\sigma_t)$, the return on home money, $r^m(i;\sigma_t)$, can be written as

$$r^m(i;\sigma_t) \equiv \frac{l(i;\sigma_t) - \underline{l}}{l(\sigma_t) + (1-\eta)/\eta}\,\gamma_Y(i;\sigma_t)$$

$$= \frac{\alpha\eta\beta A(i;\sigma_t)(1-\lambda)(l(i;\sigma_t) - \underline{l})}{1+\beta}. \tag{25}$$

From (20), the return on the off-shore asset is

$$r^b(i;\sigma_t) = \frac{(1+\theta)(1-l(i;\sigma_t))}{(1-l(\sigma_t))}\,\gamma_Y(i;\sigma_t)$$

$$= \frac{\alpha(1+\theta)\beta A(i;\sigma_t)(1-\lambda)(1-l(i;\sigma_t))[\eta\, l(\sigma_t) + (1-\eta)]}{(1-l(\sigma_t))(1+\beta)}. \tag{26}$$

As can be observed from eqs. (25) and (26), the returns on home currency and off-shore asset are increasing and decreasing functions, respectively, of the date $t+1$ portfolio $l(i;\sigma_t)$. The return on the off-shore account is an increasing function of the date t portfolio, $l(\sigma_t)$ as well. Eq. (26), together with its counterpart, eq. (9a), suggests that growth and the black market premium may be positively, not negatively, correlated.

A binding equilibrium requires $r^m(i;\sigma_t) < r(i;\sigma_t)$, for each history $\sigma_t,\ t \geq 1$. We would also like agents to diversify their savings and for this economy to exhibit economic growth. We approach each in turn below.

The reserve requirement is binding if

$$\frac{\alpha\eta\beta A(i;\sigma_t)(1-\lambda)(l(i;\sigma_t)-\underline{l})}{1+\beta} < (1-\alpha)A(i;\sigma_t),$$

for all $\sigma_t, t \geq 1$. Since the left-hand side of the inequality is increasing in the portfolio $l(i;\sigma_t)$, a condition sufficient for this inequality to hold is

$$\frac{\alpha\eta\beta(1-\lambda)(1-\underline{l})}{(1-\alpha)(1+\beta)} < 1.$$

Using the definition for $\underline{l}$, this condition places a lower bound on g:

$$\frac{\lambda[(1-\lambda)\beta\alpha - (1-\alpha)(1+\beta)]}{(1-\lambda)(1+\beta)} < g. \tag{27}$$

Diversification, at the very minimum, requires that the home asset is dominated in rate of return by the off-shore account in one state i, and dominates the off-shore account in the other state.[8] This boils down to showing that for *any current portfolio* $l(\sigma_t)$, there exists a portfolio $l(i;\sigma_t)$ such that $(1+\theta)r^d(i;\sigma_t) < r^b(i;\sigma_t)$, as well as a portfolio $l(i';\sigma_t)$ with $(1+\theta)r^d(i';\sigma_t) > r^b(i';\sigma_t)$. The latter case is easy to show; with an upper bound of 1 for $l(i';\sigma_t)$, there always exists a value for $l(i';\sigma_t) \in (\underline{l};1)$ such that $(1+\theta)r^d(i';\sigma_t) > r^b(i';\sigma_t)$. To establish the former, it is enough to show

$$(1+\theta)(1-\alpha)(1-\lambda)A(i;\sigma_t) < \frac{\alpha(1+\theta)\beta A(i;\sigma_t)(1-\lambda)(1-\underline{l})[\eta l(\sigma_t)+(1-\eta)]}{(1-l(\sigma_t))(1+\beta)},$$

since $r^b(i;\sigma_t)$ is decreasing, and $r^d(i;\sigma_t)$ increasing, in $l(i;\sigma_t)$ (See Figure 2 below). The left hand side of the above inequality is $r^d(i;\sigma_t)$, the right hand side, $r^b(i;\sigma_t)$, each evaluated at $l(i;\sigma_t) = \underline{l}$. To ensure this inequality holds for all possible $l(\sigma_t)$, we assume

8. The equilibrium conditions impose additional constraints on the magnitudes of the two returns. These are discussed later in the main text below.

$$(1+\theta)(1-\alpha)(1-\lambda)A(i;\sigma_t) < \frac{\alpha(1+\theta)\beta A(i;\sigma_t)(1-\lambda)[\eta \underline{l} + (1-\eta)]}{(1+\beta)},$$

which is the right-hand side of the inequality evaluated at $l(\sigma_t) = \underline{l}$, or equivalently,

$$\lambda(1-\alpha) < g, \tag{28}$$

using the definition $\underline{l} \equiv \frac{g(1+\beta)}{\lambda\alpha\beta\eta} - \frac{1-\eta}{\eta}$.

Heuristically, conditions eq.(27) and eq.(28) state the following. First, in order for the reserve requirement to be binding, there must be sufficient inflation, or monetary growth, in this economy. In this model, monetary growth is tied to seigniorage, so a high enough level of government spending, or g, will ensure just that. Second, since economic growth is positively linked to the proportion of savings $l(\sigma_t)$ allocated to home capital formation, and since the return on off-shore accounts is positively related to economic growth (since the demand for black currency is an increasing function of income), the return $r^b(i;\sigma_t)$, at its 'min-max', will depend critically and positively on $\underline{l}$. In turn, the minimum portfolio $\underline{l}$ is determined in part by the size of government spending g.

Formally, these two conditions are stated in Assumption 2.

Assumption 2. Binding Equilibria and Portfolio Diversification.

$$Max\{\lambda(1-\alpha); \frac{\lambda[\alpha\beta(1-\lambda) - (1+\beta)(1-\alpha)]}{(1+\beta)(1-\lambda)}\} < g.$$

The assumptions regarding economic growth are a little more arbitrary. The process (1) is bounded; $A_t \in (A - \frac{\epsilon}{1-\rho}; A + \frac{\epsilon}{1-\rho})$ for all t. We require $A_t > 0$, so assume $A - \frac{\epsilon}{1-\rho} > 0$. Next, we would like the model to be rich enough to display positive trend growth, deviations from trend, and possibly prolonged periods of negative economic growth. All these seem characteristic of the output/growth process of developing economies. To accomplish this, we assume the transition probabilities are symmetric, with $\pi_{11} = \pi_{22} = \pi$ and $\pi_{12} = \pi_{21} = (1-\pi)$, and $\pi \geq 1/2$. This ensures the conditional expectation of the shock parameter ϵ_{t+1} is non-zero at any point in time (provided $\pi > 1/2$), but the average, or unconditional expectation of the production coefficient is A.

Establishing conditions that ensure the average growth rate is positive is a bit more problematic. Since the growth rate at any date depends on the portfolio l_t (see eq.(24)), this necessarily involves making some assumption regarding the 'average' portfolio value. In turn, however, the 'average' portfolio will depend on exactly the assumptions we make regarding trend economic growth. We bypass this problem here, assuming instead that positive growth is possible for *some* portfolio value. The same will be true of negative economic growth. These assumptions are presented formally below. In Section 4, we consider numerical examples which display positive average growth rates.

Assumption 3. Economic Growth.

(i) $A - \frac{\epsilon}{1-\rho} > 0.$

(ii) $\pi_{11} = \pi_{22} = \pi,\ \pi_{12} = \pi_{21} = (1-\pi)$, and $\pi \geq 1/2.$

(iii) $\frac{\alpha\beta A(1-\lambda)}{1+\beta} > 1.$

(iv) $\frac{\alpha\beta((1-\lambda)(\eta \underline{l} + (1-\eta)))}{(1+\beta)}[A - \frac{\epsilon}{1-\rho}] < 1.$

Assumption 3 (iii) ensures the economy would experience positive economic growth on average if the currency black market did not exist. Assumption 3 (iv), on the other hand, states that if there is a sufficiently bad shock and the economy is sufficiently dollarized, the economy experiences negative economic growth.

Existence

We are now ready to formally establish the existence of a binding equilibrium and discuss some of its properties.

Proposition. *Given Assumptions 1 - 3, a binding equilibrium exists for any* $\hat{l}(\sigma_1) \in (\underline{l}, 1)$.

The proof of the proposition involves constructing a portfolio sequence $\{\hat{l}(\sigma_t)\}$ such that

i) the allocations satisfy the market clearing conditions;

ii) when faced with the returns $\{\hat{r}^d(i;\sigma_t);\ \hat{r}^b(i;\sigma_t)\}$, determined by eqs. (19), (25), and (26), evaluated at $\{\hat{l}(\sigma_t)\}$, the agents' optimal portfolio choice as described by (4) is consistent with each corresponding portfolio value $\hat{l}(\sigma_t)$ of the sequence $\{\hat{l}(\sigma_t)\}$.

A formal proof of the proposition is presented in the appendix The proof follows the along the lines of the algorithm below.

Step 1: Pick an initial (date 1) portfolio in the permissible range, $(\underline{l}, 1)$.

Step 2: Suggest a possible distribution for future (date 2) portfolios.

Step 3: This distribution generates a distribution for returns.

Step 4: Establish that the suggested distribution in step 2 and the portfolio agents select when facing the return distribution in step 3 is consistent with the initial (numerical) value of the selected portfolio in 1.

Step 5: Repeat steps 2 - 4 for each date and state, taking the portfolio from the previous period as given, in place of step 1.

In constructing an equilibrium, we use the following which describe the returns at each date as functions of the current and state-contingent future portfolios (Step 3),

$$(1+\theta)\, r^{d}(l(\sigma_t); l(i;\sigma_t)) \equiv \frac{(1+\theta)\,\alpha\,\eta\,\beta\,\lambda A(i;\sigma_t)(1-\lambda)\,(l(i;\sigma_t)-\underline{l})}{1+\beta} + (1+\theta)\,(1-\lambda)\,(1-\alpha)A(i;\sigma_t) \tag{29}$$

$$r^{b}(l(\sigma_t); l(i;\sigma_t)) \equiv \frac{\alpha\,(1+\theta)\,\beta A(i;\sigma_t)(1-\lambda)\,(1-l(i;\sigma_t))[\eta\, l(\sigma_t) + (1-\eta)]}{(1-l(\sigma_t))(1+\beta)}. \tag{30}$$

The first expression is derived from eq.(25) and the second is eq.(26). Given a portfolio for the previous date, these returns are linear functions of the future portfolio, $l(i;\sigma_t)$. The return on intermediated deposits is bounded below due to the AK production technology. It attains the lower bound at $l(i;\sigma_t) = \underline{l}$, and equals $(1-\lambda)(1-\alpha)A(i;\sigma_t)$ at that point (represented as $(1-\lambda)r$ in Figure 2 below). The return on the offshore account, on the other hand, attains its highest value at $l(i;\sigma_t) = \underline{l}$ and equals 0 at the upper bound for the portfolio, $l(i;\sigma_t) = 1$. This return depends positively on the value of the portfolio for the previous period, $l(\sigma_t)$, so the return $r^{b}(l(\sigma_t); l(i;\sigma_t))$ is bounded below by $r^{b}(\underline{l}; l(i;\sigma_t))$ for each $l(i;\sigma_t)$ in the interval $(\underline{l}; 1)$. (This is the dashed line in Figure 2). As mentioned, Assumption 2 ensures that $r^{b}(\underline{l}; l(i;\sigma_t))$ evaluated at $l(i;\sigma_t) = \underline{l}$

is greater than the lower bound on intermediated deposits, $(1-\lambda)(1-\alpha)A(i;\sigma_t)$, ensuring there is a value $l^*(i;\sigma_t)$ in the interval $(\underline{l};1)$ such that $r^b(l(\sigma_t);l^*(i;\sigma_t)) = r^d(l(\sigma_t);l^*(i;\sigma_t))$. (The condition used in Assumption 2 for this argument, namely, the inequality (28), is nothing more than a condition that ensures that the slope of the dashed line in Figure 2 is greater $\dfrac{(1-\lambda)(1-\alpha)A(i;\sigma_t)}{1-\underline{l}}$ in absolute value).

The critical value $l^*(i;\sigma_t)$ is instrumental in constructing the portfolio distribution in Step 2. Values lower (higher) than $l^*(i;\sigma_t)$ ensure the return on the offshore asset is greater (lower) than the intermediated deposit. The equilibrium portfolio distribution will have one portfolio less than $l^*(i;\sigma_t)$ in one state, greater than $l^*(i;\sigma_t)$ in the other.

Note that the equilibrium is not unique. Each equilibrium may be indexed by the initial portfolio l_1 which is indeterminant, the only requirement placed on it is that it be in the interval $(\underline{l};1)$.

- Insert Figure 2 -

4. NUMERICAL EXAMPLES AND COMPARISONS TO CLOSED ECONOMIES

In this section we study numerical simulations of the model to gain insight into its underlying qualitative properties. We categorize this study into 4 basic subsections, Official and Black Exchange Rates, Growth and Black Market Premium, Growth and Inflation, and Welfare and Black Market Participation. These examples are presented in three separate formats, discussed in detail below. Before turning to each of these discussions, however, a few technical comments regarding the simulation procedure are in order.

The artificial series are generated by simulating the model period by period for $t = 1, \ldots, T$. At each date t we generate a draw for the value of ϵ_t from the two-element set $\{-\epsilon, \epsilon\}$ according to the probability matrix Π. We then solve the market-clearing conditions for date t. The model does not have a closed form solution and has to be solved numerically. It is, however, simple enough to be "triangulated" into one equation and one unknown - the portfolio, eq.(4), and the distribution weight, $\delta(\sigma_t)$ (see eqs.(37) and (38) in the appendix). That is, given a guess for $\delta(\sigma_t)$, all the other variables in the model can be derived using all equations except eq.(4). The system is iterated over $\delta(\sigma_t)$ until this last equation is satisfied.

Unless otherwise indicated, we assume the values $A = 5.6; \alpha = 0.75; \beta = 0.95; \theta = 0.1; \rho = 0.8;$ $K_0 = 10; g = 0.08; \eta = .8; \lambda = .31; r* = 1.05; \epsilon = 0.2,$ and $\Pi = \begin{bmatrix} .6 & .4 \\ .4 & .6 \end{bmatrix}$. Without the black market, this economy has a maximum potential growth rate of approximately 66% per period and a minimum growth rate near 16%. The unconditional average growth rate is 41%. With the black market, the minimum growth rate is reduced to -18%.

Official and Black Market Exchange Rates

Our first study examines the relationship between official and black markets in the model. In a recent study on the determination of nominal exchange rates, Lane [24] provides evidence in the form of a scatter plot of the average depreciations of the two exchange rates over the period 1974-92 for 102 countries that suggests official and black market rates are very tightly linked. His plot (Fig. 4 in Lane [24]) shows essentially a near 45^0 line out of the origin.

Fig.3 below shows a similar plot with data generated by our model. We generate 250 sets of realizations, each of length $T = 30$. For each set, the average rates of official and black depreciation of home currency (increase in e_t and e_t^b) over the T periods were calculated. While our plot shows a little more variation in the sample, it looks remarkably similar to the plot shown in Lane.

- Insert Figure 3 -

Growth and Black Market Premium

We next turn to the issue of the relationship between growth and the black market premium. In a cross-section of actual economies, we can think of growth and the premium as being driven by a vector of external factors, which differs across countries. These factors include different monetary and fiscal polices, as well as differences in the percentage of the population participating in the black market and in the cost of bribes and laundering fees. Here we focus on one of these factors, differences in the policy parameter g. In this experiment, we generate series of length $T = 50$ each for a representative sequence of the shock term $\{\epsilon_t\}$, allowing the policy parameter g to vary across series. We then treat the collection of observations we obtain from each series for each particular date as a single cross sectional observation. Each cross-section was plotted (for a total of 50 scatter plots) to draw some inference about the relation between growth and the premium. A sampling of the results of these simulations is given in Figures 4a and 4b below.

We could draw no inference of a systematic relationship between these variables in our first results (not shown), which used observations on the growth rate and the *level* of the black market premium. However, keeping with the notion that the black market premium is a distortion of intertemporal trade (see eq. (9a)), we looked at the relationship between growth and the *change* in the black premium.[9] Fig. 4a shows scatter plots of growth and change in premium across different values of g for a sampling of 10 dates.

- Insert Figure 4 -

A casual, armchair-sort of analysis of the type of growth-premium relationships that might occur in actual economies readily suggests two possibilities: i) allocations to the black market raises the black market premium and siphons savings away from domestic capital markets, leading to lower economic growth, and ii) economic growth increases the demand for all assets, including black, offshore accounts, thereby increasing the black market premium. Just how these possibilities play out in terms of the growth rate and changes in the premium is difficult to say, it depends on which impact outweighs the other. (Note too that in the first case, it is entirely possible that increased allocations to the black market *lower* the premium, not raise it, due to the growth effect on the premium). While at any point in time, one of the effects may dominate the other, a cross-sectional time series plot should capture both possibilities. The plot that emerges from Fig. 4a appears to do just that. In the first and third quadrants, the growth effect (both positive and negative) outweigh the asset allocation effect; we have observations of positive growth and rising premiums (quadrant 1) and of negative growth and falling premiums (quadrant 3). The second and fourth quadrants illustrate observations of positive growth and a falling premium and negative growth and rising premium, respectively, suggesting cases where the asset allocation effect dominates the growth effect.

The model is rich enough to study a number of different numerical comparison exercises. One question that comes to mind is how do the equilibria change with a mean-preserving increase in the variance of the production coefficient A_t. This can be accomplished in several ways - changing the transition probability matrix

9. The parallel between this and a growth-inflation relationship seems quite natural.

Π, changing the value of ϵ, or by changing the autocorrelation coefficient ρ in the law of motion for A_t, $A_t - A = \rho(A_{t-1} - A) + \epsilon_t$. The latter has the effect of also changing the persistence of the shock term ϵ_t.

Fig. 4b depicts scatter plots for two period, $t = 40$, $t = 50$ for two different values of ρ. The plot lines on the top refer to the case with no persistence in the difference, $\rho = 0$, while the lower plot lines refect an unconditional mean-perserving increase in the spread of A_t, with $\rho = .8$. As evident from Fig. 4b, the same sort of growth-black premium relationship appears in the two plots, the impact of a higher variance in A_t seems manifested in terms of larger changes in the financial variable, the black premium, and less so in terms of growth.

Variations in the reserve requirement λ across economies produce a similar sort of V-shaped pattern in the plots.

- Insert Figure 4 -

Growth and Inflation

There is substantial interest in the growth effects of inflation; Barro [5], DeGregario [11], Fisher [15], Gomme [18], examine some of the empirical evidence, Chari, Jones, and Manuelli [10], Espinosa and Yip ([12], [13]), Haslag [20], Haslag and Young [22], and Ireland [23] represent recent theoretical work in this area, just to name a few. To our knowledge, there are few theoretical studies examining the relationship between growth and inflation in an international context - Fisher [14] is a notable exception.

To begin, it is useful to examine what our model has to say about growth and inflation if there were no black market. The gross inflation rate is simply the inverse of the gross return on money, $r^m(i;\sigma_t)$; in the case with no black market, the inflation rate is $P(i;\sigma_t)/P(\sigma_t) = \lambda s(\sigma_t)/(\lambda s(i;\sigma_t) - G(i;\sigma_t))$, and since $G(i;\sigma_t) = g\,Y(i;\sigma_t)$, $s(\sigma_t) = \alpha\beta Y(\sigma_t)/(1+\beta)$, and $s(i;\sigma_t) = \alpha\beta Y(i;\sigma_t)/(1+\beta)$, this may be written as

$$\frac{P(i;\sigma_t)}{P(\sigma_t)} = \frac{\lambda\beta\alpha}{\lambda\beta\alpha - g(1+\beta)}\,\frac{1}{\gamma_Y(i;\sigma_t)}. \qquad (31)$$

Without the black market, the growth rate in output is

$$1 + \gamma_Y(i;\sigma_t) = \left[\frac{A(i;\sigma_t)}{A(\sigma_t)}\right]\left[\frac{\alpha\beta A(\sigma_t)(1-\lambda)}{1+\beta}\right] = \frac{\alpha\beta A(i;\sigma_t)(1-\lambda)}{1+\beta}, \qquad (32)$$

where the second term in the square brackets is the growth rate in the capital stock (see eq.(23), setting the participation variable $\eta = 0$). It follows that

$$\frac{P(i;\sigma_t)}{P(\sigma_t)} = \left[\frac{\lambda}{\lambda\beta\alpha - g(1+\beta)}\right]\left[\frac{1+\beta}{(1-\lambda)A(i;\sigma_t)}\right]. \tag{33}$$

From eqs.(32) and (33), we see that $\partial(P(i;\sigma_t)/P(\sigma_t))/\partial g > 0$ and $\partial(1+\gamma_Y(i;\sigma))/\partial g = 0$.

Taking the derivative of the inflation rate, eq. (33), with respect to λ, we have

$$\partial(P(i;\sigma_t)/P(\sigma_t))/\partial\lambda = \frac{-(1+\beta)[g(1+\beta) - \alpha\beta\lambda^2]}{A(i;\sigma_t)(1-\lambda)^2[g(1+\beta) - \alpha\beta\lambda]^2} \gtrless 0 \tag{34}$$

and $\partial(1+\gamma_Y(i;\sigma))/\partial\lambda < 0$.

The intuition behind the derivative (34) is clear. The impact of a change in the reserve requirement on the inflation rate is the sum of two effects, the first we will refer to as a tax base effect, the second as a growth effect. The tax base effect refers to the impact of the change in λ on the first square bracket in eq. (33). An increase in λ, all else the same, increases the tax base and thereby lowers the inflation tax necessary to finance the government expenditures; from eq. (33), we see the first square bracket is decreasing in λ at an increasing rate. The growth rate effect, on the other hand, refers to the fact that a higher reserve requirement reduces the amount of savings allocated to the capital asset, thereby lowering the growth rate. A lower growth rate, in turn, raises the inflation rate; from eq. (33), the second square bracket is increasing in λ at an increasing rate. It follows that for 'low' reserve requirements, the tax base effect dominates the growth rate effect and the inflation rate is decreasing in λ. On the other hand, if λ is 'high' enough, the growth rate effect dominates the tax base effect, and inflation is increasing in λ.[10]

Turning to our open economy setting, we have

$$P(i;\sigma_t)/P(\sigma_t) = \lambda(\eta\, l(\sigma_t) + (1-\eta))s(\sigma_t)/(\lambda(\eta\, l(i;\sigma_t) + (1-\eta))s(i;\sigma_t) - G(i;\sigma_t)) \tag{35}$$

which, along with the growth rate,

$$1 + \gamma_Y(i;\sigma_t) = \frac{\alpha\beta A(i;\sigma_t)(1-\lambda)[\eta\, l(\sigma_t) + (1-\eta)]}{1+\beta}$$

the inflation rate can be written as

10. The inflation rate is increasing for values of λ sufficiently close to 1, since $g(1+\beta) < \alpha\lambda\beta$.

$$\frac{P(i;\sigma_t)}{P(\sigma_t)} = \left[\frac{\lambda(\eta l(\sigma_t) + (1-\eta))}{(\lambda(\alpha\beta\eta l(i;\sigma_t) + (1-\eta)) - (1+\beta)g}\right]\left[\frac{1+\beta}{(1-\lambda)[\eta l(\sigma_t) + (1-\eta)]A(i;\sigma_t)}\right] \tag{36}$$

Comparing growth and inflation with and without the black market, we see

- Growth is not affected by the value of government spending in the closed economy, no black market case, but it can be affected by the size of g through the impact of g on the portfolio.

- Growth and its direct effect on inflation is always lower in the open economy setting (the second term in square brackets in eq. (36) is always greater than its counterpart in eq. (33), so all else the same, inflation should be higher in the open economy setting).

- The tax base portion of the inflation rate in the open economy (the first term in square bracket in eq. (36)) may be higher or lower than its counterpart in the closed economy (the counterpart in eq.(33)).

In light of these remarks, consider Figures 5a below, which shows a single cross-sectional plot for economies of our model that differ only according to the size of government expenditure parameter g. A distinct, nonlinear negative relationship between growth and inflation appears in this figure, which is absent in the cross-sectional plot of the closed economy counterpart of the model (the horizontal plot in the northwest section of Fig. 5a). Fig. 5a also illustrates how large an impact asset substitution can have on the inflation and growth rates; compare, for example, the variance in the inflation rates across the closed economy sample with that of the open economy sample.

Similarly, variations in the size of the reserve requirement across countries also suggests a negative growth-inflation relationship, as seen in Fig. 5b.[11] This is in contrast to the cross-section plot for the closed-economy counterpart to the model, which suggests that growth and inflation are *positively* related. As in Fig. 5a, the key difference between the two reflects the effects of the portfolio allocation in the open economy on growth and the tax base.

- Insert Figure 5 -

11. Haslag [21] provides some empirical evidence of a negative growth-reserve requirement relationship.

Plots of cross sections for different dates for each of these experiments give different slopes from those shown in Figs. 5a and 5b, but the negative growth and inflation relationship holds up across periods.

Fig 5a illustrates a surprising result regarding the policy variable g. In this model, government spending has no direct effect on agents' welfare, nor does it have any direct impact on the production process. As evident from eq.(32), government spending has no impact on the growth rate in the closed economy case. Since the plot in Fig. 5a shows a negative growth-inflation relationship in the open economy setting, it is tempting to associate a higher g with a lower growth rate, since a higher level of government spending requires more seigniorage. In fact, this conjecture does not necessarily hold up; in Fig. 5a, a movement along the points on the curve (from top to bottom) represents a *decrease* in g across economies. The reason behind this result stems from the simple version of the fiscal theory of the price level we have described for this open economy setting.[12] The fiscal requirement sets limits on the extent of asset substitution permissible in equilibrium. Higher values of g are associated with higher minimum portfolio requirements $\underline{l}$. This in turn can lead to higher growth, on average, and a lower inflation rate. (In Fig. 5a, the closed economy points are consistent with the standard argument; higher values of g are associated with higher inflation rates). It turns out that higher levels of government spending can have a welfare-improving effect on agents, at least for some generations, despite the fact that government consumption has no welfare-enhancing properties, per se. (See Fig. 6).

- Insert Figure 6 -

Welfare and Black Market Participation

Our last numerical study revolves around the central question, what harm do currency black markets do? While the deleterious effects of the black market on economic growth and on inflation may be apparent to some, there are subtle aspects of the equilibrium that challenge this conventional wisdom. In addition, the presence of the market does offer some welfare-enhancing advantages. In the present model, the currency black market allows agents to circumvent the government imposed portfolio restriction, allowing them to diversify their savings. In

12. This aspect of the model is discussed in more detail in the section on welfare below.

the presence of domestic technology shocks, the diversification of savings should have a beneficial impact on the welfare of agents.

To gain a better understanding of how the level of participation in the black market affects agents' welfare, we consider a numerical analysis of the impact of different values of the participation variable η on the composite *ex ante* utility of agents for each generation $t = 1, 2, 3, \ldots T$, for a given realization of the random variable ϵ_t. (To make comparisons across different values of η, we keep the set of ϵ_t drawn the same). Fig.7 summarizes the results of this exercise, for values of $\eta = .3$, $.7$, and 1.0, and $T = 50$. Table 1 provides some summary statistics. Results similar to those presented here were obtained for different values of the primitives, and for longer time horizons T.

- Insert Figure 7 -

This set of results is interesting. A conventional reading of this experiment suggests that lower black market participation (lower η) raises the primary inflationary base, the amount of aggregate savings that is not subject to asset substitution, and, given a value of the portfolio, reduces the inflationary tax. This in turn raises the return on the domestic intermediated asset as well as increases the domestic growth rate. However, an alternative possibility can present itself. In this model, a larger primary inflationary base is an increase in the inflation tax base supplied inelastically to the government, since savings do not depend on the rate of return. The government in turn can levy a high inflation tax, since it does not need to rely as much on the portion of aggregate savings that is not inelastically given, i.e, the portion subject to asset substitution and summarized by $l(\sigma_t)$. A high inflation tax still generates agreement in equilibrium - the higher the inflation, the lower the return on intermediated domestic assets, the lower the portfolio allocation to domestic assets (a lower $l(\sigma_t)$) and the lower the rate of economic growth. The latter, of course, also raises the inflation rate. In actuality, an entire Laffer Curve sort of tax rates-tax base configurations is possible.

Table 1 contains summary statistics regarding average growth rates, average premiums, average money growth rates, and average inflation rates for the three different values of η for a representative sequence of realizations of ϵ_t (i.e., the realizations of the technology parameter A_t is the same across economies). The example illustrates the sort of tradeoff with market participation, inflation, and growth described above. The economy attains higher average growth but also higher inflation with full participation verses 70% black market participation. Decreasing the degree of black market participation even further, to 30%, reduces average growth and raises the inflation rate.

η	Ave. Growth in Y (in Percent)	Ave. Log Premium	Ave. Growth in M (in Percent)	Ave. Inflation Rate (in Percent)
1.0	5.71	- .16	404.82	398.42
0.7	5.41	.61	379.03	373.05
0.3	1.92	2.49	425.65	423.70

Table 1. Summary Statistics for Different Levels of Black Market Participation

As it turns out in this example, the economy with a lower extensive participation rate in the black market does experience poorer overall economic performance (higher inflation, lower growth) on average than its counterpart economies with greater degrees of access to the black market. In our monetary/fiscal arrangement, the monetary authority, in a sense, passively sets monetary policy in accordance with the fiscal authority (g) *and* the decision of private agents ($l(\sigma_t)$). Greater access to the black market, in this example, 'forces' the monetary authority to become more competitive on average; by this we mean that the inflation rate is lower and the return to the intermediated deposit on average is higher with greater black market participation. It turns out, in fact, that in the case of full access ($\eta = 1$), the discipline imposed on the central bank can virtually *shut down* participation in the black market - at several dates in this example, the portion of savings allocated to the black market is near zero.[13]

As shown in Fig.7, the composite, *ex ante* utility of agents is initially higher with the lower participation rate ($\eta = .3$). At these dates, capital and output do grow faster in the $\eta = .3$ economy as oppose to the other two cases. Near date $t = 40$, however, there is a series of negative shocks to the production technology. All three

13. As a directive for policy, our example is less robust. There are, of course, realizations in which the economy with lower extensive participation in the black market outperforms on average, the other economies, along these two measures of economic performance.

economies experience a reduction in growth rates, the most dramatic case being the $\eta = .3$ economy. That economy then gets mired on the 'wrong side' of the Laffer Curve - agents pull out of domestic intermediated asset in a big way due to the persistence of the shock, growth becomes negative, and the central bank is forced to levy a large inflation tax to generate agreement between the agents' portfolio choice and the government's revenue needs, as dictated by the government's budget constraint, eq.(22). The composite welfare of agents born during these periods is lower than that in the other two economies.

- Insert Figure 7 -

5. CONCLUSION

The empirical literature of macroeconomic factors in growth includes currency black market variables but has done so in a reduced form, or unstructured, manner. The black market premium is taken, explicitly or implicitly, to be an indicator of the degree of distortion in the real trade and foreign exchange markets. How these distortions actually affect growth is not described in an explicit model. Implicitly perhaps, one might think that there are some readily available explanations in the large theoretical literature on smuggling and black markets in foreign exchange. This literature, however, has mostly focused on the interaction with official rates, and on welfare effects and policy implications of black markets in foreign exchange in settings that do not permit a discussion of growth.

Our model tries to capture some aspects of the interaction between black markets in foreign exchange and growth in a dynamic general equilibrium setting. While it ignores some real world features like smuggling or foreign borrowing, it does include important elements of monetary policy as practiced in some developing countries with currency black markets, including reserve requirements and government deficits financed by seigniorage, and it captures some of the links between growth, portfolio choice, returns on domestic assets, and inflation.

Through numerical simulations of the model, we study four aspects of the model: how are official and black market exchange rates related, what is the relationship between growth and the black market premium, what is the relationship between growth and inflation, and how is welfare affected by the degree of extensive participation in the black market. As regard to the first and third, our model yields predictions consistent with some recent empirical evidence on official and black market rates and on growth and inflation. Official and black market exchange rates are really tightly linked in our simulations and display a scatter plot very similar to actual data. Our model also adds to the growing theoretical arguments that economic growth is negatively correlated with inflation. The simulations also suggest that this relationship is nonlinear. Unlike most of the theoretical research in this area, however, our model makes this claim within an open economy setting.

Our numerical examples appear less consistent with patterns observed in actual data when it comes to growth and black market premium correlations.[14] Our simulations suggest little systematic relationship between growth and the level of the premium. We provide evidence here to suggest that changes in output more appropriately may be more correlated with changes in the black market premium. We offer some plausible theoretical arguments why growth and changes in the premium may be negatively or positively correlated.

Our study of the welfare implications of the currency black market also turns up some interesting results. Here we illustrate, by way of counterexample, that it is entirely possible that policies designed to eliminate the black market, through greater capital controls and tighter access to currency black markets, may lead to more savings being allocated to the black market, provided of course that these controls are, in some sense, incomplete. Instead of leading to lower inflation and higher growth, the economy achieves just the opposite, thereby reducing overall welfare. Greater extensive participation in the black market forces the central bank to levy a lower inflation tax, thereby increasing the amount of savings allocated to domestic intermediated capital, through a lower degree of black market participation along the intensive margin. This in turn increases economic growth.

Finally, our model adds to the understanding of how economic growth is related to open macroeconomic factors such as exchange rates and currency substitution. Our examples show that growth and inflation in an open economy can be quite different from that in a closed economy. In light of the international focus of the empirical growth literature, it is surprising there is not more in the theoretical literature on this subject. In turn, these sort of applications of applied theory offer promising avenues for future empirical work.

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14. The parameters of the model were chosen to give a wide range of growth rates. By exploring high growth rate scenarios we observe a positive growth-premium relationship that may not be observed often in actual data.

APPENDIX

Proof of the proposition.

Select any $\hat{l}(\sigma_1) \in (\underline{l}, 1)$, and define $\tilde{l}(\sigma_1)$ as the value that equates (29) and (30),

$$(1+\theta) r^d(\hat{l}(\sigma_1); \tilde{l}(\sigma_1)) = r^b(\hat{l}(\sigma_1); \tilde{l}(\sigma_1)).$$

Assumption 2 guarantees $\tilde{l}(\sigma_1)$ exists for any $\hat{l}(\sigma_1) \in (\underline{l}, 1)$. Note too that the value of $\tilde{l}(\sigma_1)$ is independent of the realized state at date 2.

For date t = 2, let $\hat{l}(i;\sigma_t)$, for i =1, 2, be defined as weighted averages

$$\hat{l}(1;\sigma_t) = \delta(\sigma_t)\tilde{l}(\sigma_t) + (1-\delta(\sigma_t)) \tag{37}$$

$$\hat{l}(2;\sigma_t) = \delta(\sigma_t)\underline{l} + (1-\delta(\sigma_t))\tilde{l}(\sigma_t), \tag{38}$$

where $\delta(\sigma_t) \in (0,1)$ is a value to be determined. Using eqs. (29), (30), the interior solution for $l(\sigma_t)$ in eq.(4) may be expressed as a function $l(l(\sigma_t);\delta(\sigma_t))$. We wish to establish there exists a $\hat{\delta}(\sigma_1) \in (0,1)$ with $\hat{l}(\sigma_1) = l(\hat{l}(\sigma_1);\hat{\delta}(\sigma_1))$.

We have

$$\lim_{\delta(\sigma_1)\to 0} (1+\theta) r^d(\hat{l}(\sigma_1);\hat{l}(1;\sigma_1)) > \lim_{\delta(\sigma_1)\to 0} r^b(\hat{l}(\sigma_1);\hat{l}(1;\sigma_1)) = 0 \tag{39a}$$

$$\lim_{\delta(\sigma_1)\to 0} (1+\theta) r^d(\hat{l}(\sigma_1);\hat{l}(2;\sigma_1)) = \lim_{\delta(\sigma_1)\to 0} r^b(\hat{l}(\sigma_1);\hat{l}(2;\sigma_1)) \tag{39b}$$

$$\lim_{\delta(\sigma_1)\to 1} (1+\theta) r^d(\hat{l}(\sigma_1);\hat{l}(1;\sigma_1)) = \lim_{\delta(\sigma_1)\to 1} r^b(\hat{l}(\sigma_1);\hat{l}(1;\sigma_1)) \tag{39c}$$

$$\lim_{\delta(\sigma_1)\to 1} (1+\theta) r^d(\hat{l}(\sigma_1);\hat{l}(2;\sigma_1)) < \lim_{\delta(\sigma_1)\to 1} r^b(\hat{l}(\sigma_1);\hat{l}(2;\sigma_1)) \tag{39d}$$

These inequalities yield

$$\lim_{\delta(\sigma_1)\to 0} l(\hat{l}(\sigma_1);\delta(\sigma_1)) = \infty$$

$$\lim_{\delta(\sigma_1)\to 1} l(\hat{l}(\sigma_1);\delta(\sigma_1)) = -\infty.$$

The continuity of the interior solution (4) together with these limits ensure a supporting weight $\hat{\delta}(\sigma_1) \in (0,1)$ exists.

The procedure is then repeated for each date and state, $t = 2,3,\ldots,$ taking $\hat{l}(\sigma_t)$ as given. This produces a sequence of weights for each date and state, $\{\hat{\delta}(\sigma_t)\}$ which yields a stochastic equilibrium for this economy.

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