The Macroeconomic Impact of Fertility Changes in an Aging Population*

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Abstract

We assess the impact of continued low fertility in China, versus a rebound in fertility due to the relaxation of the one child policy, on demographic and macroeconomic outcomes in a dynamic general equilibrium framework. We build a rich model of household dynamics with young and elderly dependents, human capital investment, public health insurance, pensions, private savings, and intra-family transfers. Both the model steady state and transitions are calibrated using aggregate and micro level data for China. We find that in short run (sixty years in our benchmark experiment), income per capita is lower with a fertility rebound due primarily to a higher youth dependency rate. In the long run, higher fertility leads to a reduction in the old-age dependency ratio and lowers the tax rate required to pay for old-age pensions and health care. On the other hand, higher fertility also results in a reduction in female labor supply, savings, and educational attainment, affecting output adversely. Our results suggest that higher fertility in China is unlikely to offset the negative macroeconomic effects of population aging in the short-run and potentially could have negative long-run effects as well.

JEL classifications: E62, H55, I13, J11, J13
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1 Introduction

Population aging and an associated slowdown in economic growth is a major concern in many countries. Rising old age dependency ratios may increase the private burden of caring for elderly parents and threaten the fiscal sustainability of pay-as-you-go pension and public healthcare systems. This is particularly true in China, which has recently expanded its partially funded pension and health insurance systems into rural areas. While such social insurance programs may overcome market failures and improve welfare (Bairoliya et al., 2017), they may not be as sustainable as a fully funded personal account system (Feldstein and Liebman, 2008). As population aging is driven more by below replacement fertility than longer life spans (Bloom et al., 2010), it seems natural to propose higher fertility rates as one of the potential remedies (Banister et al., 2010; Turner, 2009).

China’s fertility decline has been hastened by its one child policy and fertility is now well below replacement at a fairly low level of income, raising the prospect that China will get old, slowing economic growth, before it gets rich. It may be, therefore, that relaxing fertility restrictions in China improves individual welfare, by allowing families to have the number of children they want, while also improving macroeconomic performance. In 2015, China moved to a universal two child policy which has been forecast to raise the total fertility rate (TFR) from the current level of near 1.5 children per woman to 1.8 by 2030 (Zeng and Hesketh, 2016). We analyze the effects of this policy change on demographic and macroeconomic outcomes relative to a counterfactual of a continuation of fertility at the current level of 1.5 children per woman.

While fertility policy in China has been much stricter than elsewhere, many countries maintain policies aimed at either increasing or reducing fertility to improve economic growth and social welfare. Figure 1 shows that in 2009, 60% of all the countries for which data was available had policies to influence fertility, where 38% of the countries had policies to reduce fertility. Reductions in fertility from high levels can lead to a demographic dividend and economic growth through a reduction in the youth dependency ratio, increased investment in children’s education and health, increased female labor force participation, and higher saving rates (Bloom et al., 2003; Canning et al., 2015). On the other hand, 22% of all countries in the world (for which data was available) had policies in place to increase fertility (UN, 2011).

Bloom et al. (2010) show in a theoretical framework that a reduction in fertility below replacement levels can result in a sharp decline in the working-age share of the population and potential slow down of economic growth. Aging could also substantially increase the tax burden of health care and pension programs due to declining support ratios and increased health expenditures per capita (Christiansen et al., 2006; Bloom et al., 2011; Seshamani and Gray, 2004). However, declining fertility can induce higher investments in health and human capital which can offset some of the negative effects of aging by raising average effective labor supply (Fougère and Méréte, 1999; Lee and Mason, 2010b,a; Prettner et al., 2013). It can also induce higher physical capital accumulation by encouraging workers to save for retirement rather than rely
on their children for old-age support (İmrohoroglu and Zhao, 2016). In the light of these potential countervailing mechanisms, the macroeconomic effects of the recent relaxation of fertility controls by the Chinese government are unclear. Moreover, the macroeconomic outcomes may differ in the short run versus the long run.

In order to quantitatively assess the recent policy change in China in a general equilibrium framework, we use an overlapping generations (OLG) model featuring inter-generational altruism to mimic the important role of family in China in providing social insurance. The unit of analysis in the model is a household composed of several generations living together and engaging in various economic activities. While we treat fertility as exogenous, we allow for human capital accumulation to capture the quality-quantity trade-off as it is an important mechanism to determine the effect of fertility changes on macroeconomic outcomes. We allow for public subsidies on education, health insurance, social security and private savings and model uncertainty in survival, labor productivity and medical expenditures. The government operates public pension and health insurance programs and subsidizes primary, secondary and college education in the model. While pension payments are financed through labor income taxes, public spending on health insurance and education is jointly financed through consumption taxes.

Our quantitative exercise yields four main insights. First, we find that the impact of fertility on output in the long run crucially depends on how fertility affects saving, education, and female labor supply. If savings and physical capital accumulation is the
only behavioral mechanism (i.e. education and female labor supply are held constant), an increase in fertility can increase income per capita in the long run by reducing the old age dependency rate and the taxes needed to finance public pensions and health care. However, if we allow for even modest effects on female labor supply and investment in children’s education, income per capita is lower in the long run with higher fertility. An important point is that increasing the working-age share of the population is not the same as maximizing income per capita—income is affected not just by labor supply but by human and physical capital investments that may move in opposite directions to labor supply.

Second, the short run effects of higher fertility along the transition path to the long run involve a lower level of income per capita than with no fertility increase. In the short run, the higher fertility rate increases the youth dependency ratio and these children require consumption, child care, and education, while not producing any output until they reach working-age. The surprising point here however is how long the short run lasts. In our benchmark experiment, it takes approximately sixty years for the working-age share to increase with higher fertility. While the first cohort of children from the fertility shock enters the workforce at around age 15, higher fertility means more children coming after them and it takes a considerable period of time for the age structure to reach a new steady state.

Third, through alternate fertility experiments, we find that the effect of fertility changes on long run income per capita is not monotonic. While moving from a total fertility rate of 1.5 to 1.8 lowers income per capita in the long run, reducing fertility to 1.2 also lowers income per capita. At this very low fertility level the increased female labor supply and education effects from lower fertility are insufficient to counterbalance the negative effect of population aging. This is consistent with Lee et al. (2014) who find that the total fertility rate that maximizes consumption for China is slightly lower than the replacement fertility. Whereas both very low or very high fertility rates have adverse economic effects.

Finally, we find that there are significant externalities (both positive and negative) associated with higher fertility through taxes. Higher fertility on one hand reduces the fiscal burden of financing old-age pensions and medical expenditures by increasing the fiscal support ratio. On the other hand, it also lowers the education subsidies per child under the assumption of a fixed government budget for education. Our results indicate that failure to account for these general equilibrium effects can result in biased predictions about how fertility affects macroeconomics.

Our paper contributes to a growing body of related literature on demography and economic growth in China. First, demographics have been shown to have important implications for savings in China. There is empirical evidence that fertility has a negative effect on savings at the household level (e.g. Banerjee et al. 2010; Ge et al. 2012; Choukhmane et al. 2013; Banerjee et al. 2014). At the aggregate level, Modigliani and Cao (2004) use time series data from China to argue that fertility influenced savings
over the past several decades through changes in demographic structure. Structural
OLG models have since been used to analyze and quantify the link between demo-
graphics and the observed increases in aggregate savings in China (e.g. Curtis et al.
2015; Banerjee et al. 2014; He et al. 2015; Choukhmane et al. 2013). With a two-way
altruism model most closely related to ours, İmrohoroglu and Zhao (2016) find the
interaction of demographics, productivity growth, and uncertain long-term care of el-
derly parents to be an important driver of Chinese savings rates. This is consistent
with Chamon and Prasad (2010) who find evidence in support of rising average sav-
ings rates due to rising private burden of both health care spending and education.
However, İmrohoroglu and Zhao (2016) abstract from human capital considerations
and the role of children more broadly. Using a general equilibrium model of endoge-
 nous fertility decisions, Liao (2013) looks at the welfare effects of relaxing fertility
constraints in China but abstracts away from some key modeling details. For instance,
this paper is not able to match the evolution of age-structure over time due to a simple
demographic structure. Matching the precise evolution of age-distribution is crucial in
pinning down the demographic dividend, hence the short-run and the long-run effects
of fertility changes. It also abstracts away from the government programs on education,
pensions and healthcare which have assumed a significant role in China in the recent
times. Our general equilibrium effects indicate that the tax externality associated with
these public transfer programs is significant.

We include an endogenous schooling decision in our model as fertility has been the-
theoretically and empirically linked to human capital investments in China. Using Chinese
twin births for identification, Li et al. (2008) find that higher fertility significantly re-
duces educational attainment and enrollment while Rosenzweig and Zhang (2009) also
find reductions in schooling progress, expected college enrollment, and school grades.
Compared to savings, the impact of demographics on human capital accumulation in
China has received far less attention in the structural macro literature. An exception
is Choukhmane et al. (2013) whose partial equilibrium model predicts that changing
demographics lead children of the one-child policy generation to have at least 20% higher human capital compared to their parents.\(^2\) Meng (2003) and Chamon and
Prasad (2010) also highlight the potential role of underdeveloped financial markets in
amplifying savings motives under demographic change, particularly in terms of educa-
tion spending. Importantly, we restrict the borrowing capacity of families and allow
for an interaction between demographics and public spending on education through a
government budget constraint. Finally, as previous studies have established important
connections between demographics and the macroeconomic fluctuations since the end
of China’s centralized economy, we turn our eye to the future. In the wake of renewed
interest in relaxing the restrictive fertility policies in China, we examine the impli-
cations of changes in demographic structure moving forward under alternate fertility
paths.

\(^2\)Banerjee et al. (2014) shows that general equilibrium effects can be quantitatively important in
a model of aggregate fertility.
Our paper is also more broadly tied to the literature using general equilibrium models for analyzing an array of interesting problems like optimal taxation (Chamley, 1985), industrial pollution (Tietenberg, 1973), sovereign default (Mendoza and Yue, 2012), explaining cross country differences in fertility (Manuelli and Seshadri, 2009) and so on. An important point in our paper is that we assess the economic effects of the relaxation of the one child policy, not the welfare effects. Families may enjoy having additional children, and these children may improve their parent’s utility level, even if it lowers income per capita and their economic circumstances. In addition, a welfare analysis would have to take into account the utility of the children born due to the policy change which raises difficult ethical questions of measuring welfare with different population sizes (Blackorby et al., 2005).

The remainder of this paper is organized as follows. Section 2 provides a discussion on demographics, savings and human capital in China. Section 3 builds the dynamic general equilibrium model. Section 4 discusses our calibration strategy. Section 5 compares the fit of the model with the data. Section 6 discusses our fertility experiments in detail. Sections 7 provides a discussion of the main results. Section 8 contains some sensitivity analysis and section 9 provides concluding remarks.

2 Background

In this section, we provide some background information on demographics, savings rates and human capital investments in China. We begin with a brief discussion on the mechanics linking fertility to an economy’s demographic structure and provide some time-series data and future projections for China under section 2.1. Our theory posits two important behavioral responses to such changing demographics. First, a change in household savings rate due to the altered age structure in economies where intra-household transfers are a key source of insurance. Second, changes in human capital investments primarily in response to changes in private and public budgetary constraints. These behavioral mechanisms are briefly detailed along with supporting data in section 2.2.

2.1 Demographics

One of our primary focuses in this paper is how fertility changes affect the age dependency structure across an economy over time. Fertility changes affect the working-age share of the population by altering both the old-age and child dependency ratios. For instance, an increase in fertility lowers the working-age share by increasing the number of child dependents per worker. On the other hand, higher fertility in the long run also reduces the number of retirees or older dependents per worker, thereby increasing the working-age share of the population. On net, the long run effect of fertility on working-age share depends on the relative strengths of these two opposing forces.

\(^3\)Working-age share is defined as fraction of population ages 15 to 64.
Figure 2 provides an illustrative example of the theoretical relationship between total fertility rate and working-age share of the population in the long run steady state. When there is a marginal decrease in fertility from a very high rate, the reduction in the number of child dependents outweighs the increased share of retirees in the long run, resulting in an overall increase in working-age share. This is the case typical in many developing countries (including China prior to the 1990s). However, indefinite declines in fertility ultimately result in a lower working-age share in the long run as the relative number of retirees increases. Moreover, when fertility rates are very low, there is a potential for substantial increases in working share from relatively small increases in fertility in the long run.

Figure 3 shows the U.N. estimates (1960-2015) and medium variant projections (2015-2100) for total fertility rate and working-age share in China. Between 1970 and 2000 there was a sharp decline in fertility largely attributed to China’s one-child policy. Correspondingly, over this time-frame there was a steep rise in the working-age share driven by the drop in child dependents. However, now that the one-child generation is moving into the workforce, there is a projected decline in working-age share in the coming decades due to a sharp rise in the number of old-age dependents per worker.

Figure 2: Stable Long run Relationship Between Fertility Rate and working-age Share

2.2 Savings and Human Capital

Our framework allows fertility changes to operate through multiple channels to impact individual and aggregate savings and human capital investments. For historical context, Figure 4a shows the net national savings rate in China from 1978 to 2013. Savings rates have generally increased since the early 1980s. There are a number of theoretical channels that have been proposed linking the change in demographics to

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4For this example, we hold age-specific survival rates constant at current levels in China. Survival probabilities are taken from UN life table estimates for 2010-15 (UN, 2015). To obtain an analytic solution, we assume mothers give birth to all children at age 29.
the sustained increase in savings rate over this time-frame.\textsuperscript{5} First, there is a composition effect which operates mechanically through the changing age structure in the economy. For example, as savings is primarily coming from unconsumed labor income, a rise in working-age share can induce an increase in the aggregate savings rate. Second is the reduction in old-age support provided by children due to reductions in fertility. Fewer children might result in increased reliance on private savings for financing old age consumption and long-term care (İmrohoroglu and Zhao, 2016). Third is the child expenditure channel. Having fewer child dependents in the household frees up additional resources which may, in part, be saved. Moreover, raising children has a significant labor force participation cost, particularly for women (Bloom et al., 2009). However, if families are altruistic towards their children and their descendants, a sustained decline in fertility may lead individuals to save less as there is a decline in the transfer requirements for future generations of children. This latter channel has received considerably less attention in the literature, but is incorporated in our two-way altruism model.

In addition to influencing savings in physical capital, we hypothesize that fertility also plays a role in human capital investment decisions. To illustrate this point, Figure 4b shows the estimated share of 25-29 year-olds in China with completed secondary or college education from 1960-2010. Secondary completion rose substantially over the entire time-frame but experienced the sharpest growth during the 1980s and 90s. College completion also started to increase in the 90s and is largely believed to continue to rise substantially into the foreseeable future. The primary channel of influence we focus on is through the child expenditure channel. For example, some households may not be wealthy enough to send a large number of children to school. A decline in fertility would thus promote investments in human capital by relaxing the household budget

\textsuperscript{5}We highlight the primary mechanisms here but refer readers to excellent theoretical analyses in Banerjee et al. (2014) and Choukhmane et al. (2013).
constraint. The fertility rate may also influence individuals indirectly by altering the budgets of government programs. For example, if government outlays on education are fixed, the decreased share of school-aged individuals accompanying lower fertility results in a decrease in the per student private cost of education. Lastly, there are general equilibrium effects that will influence the strength of the above mechanisms. For example, a decline in after-tax wage relative to interest rate reduces the returns to schooling and has a dampening effect on average educational attainment.

![Figure 4: Savings and Human Capital in China Over Time](image)

**Data Source:** China (2014); Barro and Lee (2013)

In the next section, we build our dynamic general equilibrium model which captures all these important mechanisms linking fertility with demographics, savings and human capital accumulation decisions.

### 3 Model

Consider an economy populated by a large number of households that each consist of overlapping generations of a family. Household members are altruistic towards each other and make decisions as a single economic unit. Over time, children in the family grow up, have children of their own, and eventually replace their parents in the household. In this way each household in the economy is an infinitely lived dynasty. As children grow up, they accumulate human capital by attending school. As individuals age, they eventually face medical expenditure and mortality risk. Time is discrete and in each period a new generation of individuals is born.
3.1 Technology

Aggregate output in the economy ($Y$) is assumed to be produced by a representative firm using the technology:

$$Y_t = A_t K_t^\alpha N_t^{1-\alpha} \quad \alpha \in (0, 1),$$

where $K_t$ and $N_t$ are the aggregate capital stock and labor inputs (measured in efficiency units) in period $t$, $A_t$ is total factor productivity, and $\alpha$ is the capital share. Output can be consumed ($C_t$), invested in physical capital ($I_t$), expended on education ($E_t$), or expended on medical care ($M_t$):

$$Y_t = C_t + I_t + E_t + M_t.$$

Finally, letting $\delta$ equal per-period depreciation, the law of motion of capital is given by:

$$K_{t+1} = (1 - \delta) K_t + I_t.$$

3.2 Households

Demographic Structure

The economy is populated by overlapping generations of individuals residing in family households. Each individual lives through four stages of life—child, young adult, old adult, and elderly. As a child, an individual simply consumes and spends a fraction of childhood in primary school. Young adulthood begins by either continuing in school (secondary school and eventually college) or entering the labor market. Regardless of educational choice, after schooling is complete, the remainder of the young adult stage is spent working. As an old adult, an individual continues to work in the labor market and consume. Old adults eventually begin to receive public pension benefits as well. Finally, the elderly continue to receive pension payments but are assumed retired from the labor force. It is upon becoming elderly that individuals begin to face medical expenditure and mortality risk.

The economic decision making unit is a household. Each household in the economy is indexed by “household age” $j = 1, ..., J$. This index completely defines the age structure of the entire household. At each age $j$, all members of the household pool their resources to maximize a joint objective function. Following Laitner (1992), individuals derive utility from their own lifetime consumption and from the utility of other household members and descendants.

Regardless of age $j$, each household always consists of one old adult and their $n$ young adult children which is the implied model fertility rate.\footnote{Note that this is a single-sex model so $n$ is always half of the true fertility rate observed in the data. However, to avoid confusion, through the rest of this analysis we will make no distinction between the two and show calibration results with the true fertility rate.} At household age...
$j = J_c < J$, each young adult has $n$ offsprings of their own, who live with the household as children until age $J$. After age $J$, the young adult siblings become old adults and split into $n$ separate households with their own young adult children. The siblings are assumed to evenly split household assets and share the continuing financial burden of their now elderly parent. This implies that $\frac{1}{n}$ elderly parents are included in each household, conditional on survival. Moreover, any pension income received by the elderly parent is distributed evenly to old adult children to help finance consumption and medical care for the elderly.

Note that due to the birth of children and mortality risk faced by the elderly members of the family, a household can have four different compositions. One where all four generations are present; where only young and old adults are present (children are not born as yet and elderly have deceased); where children, young, and old adults are present and elderly have deceased; and finally where young adults, old adults, and elderly are present. Figure 5 summarizes the timeline for both households and individuals in the model. The figure highlights the rich demographic structure available in this framework—an individual’s life may overlap with that of his children, parents, grandchildren, grandparents, great-grandchildren, and great-grandparents.

**Figure 5: Evolution of Generations and Timeline**

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**Labor Earnings**

In each model period, every young and old adult is endowed with one unit of productive time. Young adults either spend this time in school, taking care of children, or supply it inelastically to the labor market. More specifically, a young adult of age $j < J_c$ earns the following pre-tax labor income:

$$w\left(1 - \kappa_{e_j}\right)e_y e_j \eta_y,$$
where \( w \) is the competitive wage rate, \( \epsilon_j \) is age-specific life-cycle productivity, and \( \eta_y \) is a permanent idiosyncratic shock realized by an individual upon becoming a young adult \((j = 1)\). Labor productivity is also conditional on the young adult’s level of education \( e_y \). Moreover, \( \kappa_{e_y} \) indicates whether a young adult of education type \( e_y \) is enrolled in school at age \( j \). Specifically, \( \kappa_{e_y} = 1 \) if the young adult is enrolled in school and \( \kappa_{e_y} = 0 \) otherwise.

Schooling is assumed complete by the time young adults have their own children at age \( J_c \). As such, a young adult of age \( j \geq J_c \) earns the following pre-tax labor income:

\[
we_y \epsilon_j \eta_y (1 - n \theta_f),
\]

where \( n \) is the number of children they have and \( \theta_f \) is the time-cost of raising their children. Even though our model is gender-neutral, \( \theta_f \) captures in a simple way, the effect of fertility on female labor supply.

Old adults supply their unit of time inelastically to the labor market. As such, an old adult of age \( j \) earns the following pre-tax labor income:

\[
we_o \epsilon_{j+J} \eta_o,
\]

where \( \eta_o \) is the permanent idiosyncratic shock that they received when they were a young adult. In this way, the productivity shock \( \eta \) remains constant throughout an individual’s life. Moreover, we assume the productivity shock of a young adult \((\eta')\) is correlated with their parent’s shock \((\eta)\) through a finite-state Markov chain with stationary transitions over time:

\[
\Gamma_t^n (\eta', E) = \text{Prob}(\eta' \in E \mid \eta) = \Gamma^n (\eta', E), \quad \forall t.
\]

**Education**

We model three discrete choices for educational attainment—primary school, secondary school, and college. All children exogenously enter primary school at age \( J_p \) and are in school for the remainder of childhood (i.e. through household age \( J \)). However, at age \( J \), households decide if children will drop out of school and enter the labor market the following period as young adults, will continue their education through secondary school, or will continue through college. Primary school requires an annual tuition cost of \( \theta_p \) that is entirely subsidized by the government. Continuing education beyond primary school incurs an age-specific tuition cost of \( \theta_j \) which may be fully or partially subsidized. It is important to note that education level is chosen prior to realization of an individual’s productivity shock \( \eta \). This implies that idiosyncratic returns to education are uncertain at the time when schooling decisions are made.

**Medical Expenditures and Mortality**

Elderly individuals of age \( j \) survive to age \( j + 1 \) with positive probability \( \psi_j \). At the end of period \( J \), they die with probability one. Conditional on being alive, the elderly
are characterized by a medical expenditure state $x \in \mathcal{X}$. Conditional on expenditure state, households are required to finance medical expenditure $m_x$ for the care of their elderly parent. The elderly are assumed to start in the lowest medical expenditure state $\bar{x}$. The medical expenditure state then evolves stochastically over the remaining life-cycle. The stochastic process follows a finite-state Markov chain with stationary transitions over time. The Markov process is assumed to be identical and independent across individuals:

$$
\Gamma_t^x (x', \mathcal{X}) = \text{Prob} (x' \in \mathcal{X} \mid x) = \Gamma_x^x (x', \mathcal{X}), \quad \forall t,
$$

where $x$ is the current medical expenditure state and $x'$ is that of the following period.

### 3.3 Government

The government operates three programs in the model. First, a pay-as-you-go social security system which is defined by pension benefits $SS$ for each old adult above age $J_{ss}$ and for all surviving elderly. Pension benefits are determined by a replacement rate $b_s$ of national average earnings. Second, the government subsidizes the health care of the elderly by covering a fraction $b_h$ of their medical expenditure bill. Finally, the government provides a subsidy for education. The cost of primary school is fully covered by the government. For secondary school and college, the government covers a fraction $\lambda_j$ of the total tuition cost $\theta_j$. As a majority of public revenues in China are collected through direct or indirect consumption taxes, we assume public spending on education and health care is financed with a proportional tax on individual consumption $\tau$. However, as the Chinese pension system is primarily financed with labor income taxes, we assume the social security budget is balanced through a proportional tax on labor income $\tau_{ss}$.

### 3.4 Decision Problem

At any given time, a household can be characterized by a vector of state variables $\zeta = (a, x, d, e_y, e_o, \eta_y, \eta_o, j)$, where $a$ denotes current holdings of one-period, risk-free assets, $x$ is elderly member’s medical expenditure state, $d$ is an indicator for whether the elderly is deceased, $e_y$ and $e_o$ are education levels of the young and old adults respectively, $\eta_y$ and $\eta_o$ are productivity levels of young and old adult respectively, and $j$ is the age of the household. Given this state vector, a household chooses total consumption $c$, and next period assets $a'$, to maximize the present utility of the household plus the expected discounted utility of all future periods of the family dynasty. In period $J$, the education level of the next generation of adults $e'_y$ is also chosen. The decision problem facing a household of age $j < J$ may be written:

$$
\max_{c,a'} \left\{ \tilde{n}u \left( \frac{c}{\bar{n}} \right) + \beta E_{x'd'} [\nu (a', x', d', e_y, e_o, \eta_y, \eta_o, j + 1)] \right\}
$$
subject to:

\[ c(1 + \tau) + a' = y(1 - \tau_{ss}) + a(1 + r) + \frac{(1 - d)}{n}(SS - (1 - b_h)m_x) + SS(j \geq J_{ss}) - n\kappa_{j\epsilon_g}(1 - \lambda_j)\theta_j \]

\[ a' \geq 0, c > 0 \]

where \( y \) refers to total household labor income given by:

\[ y = \begin{cases} \text{we}_o\epsilon_{j+1}\eta_o + n(1 - \kappa_{\epsilon_gj})\text{we}_y\epsilon_j\eta_y & \text{if } j < J_c \\ \text{we}_o\epsilon_{j+1}\eta_o + n[\text{we}_y\epsilon_j\eta_y(1 - n\theta_j)] & \text{if } j \geq J_c, \end{cases} \]

and \( \tilde{n} \) is the number of adult equivalents in the household:

\[ \tilde{n} = n + 1 + \frac{(1 - d)}{n} + \gamma nn(j \geq J_c) \]

where \( \gamma \) is the consumption requirement of a child relative to an adult.

The current period utility of an individual is given by \( u(\cdot) \) and value function \( V(\cdot) \) is the total expected discounted utility of arriving in a period of time with a given state vector. Note that expectations are taken with respect to the stochastic process for the medical expenditure state and the survival risk of the elderly. The first constraint is the household budget constraint. Note that the total private cost of education for each of the \( n \) young adults at age \( j \) is given by \( (1 - \lambda_j)\theta_j \). Also note the role of the elderly in the decision problem. Conditional on being alive \((d = 0)\), households have access to \( \frac{1}{\tilde{n}} \) of the elder’s pension income \( SS \) but are also responsible for the same fraction of the elder’s unsubsidized medical care \( (1 - b_h)m_x \). Finally, note that households also face a no borrowing constraint \((a' \geq 0)\).

In period \( J \), the decision problem facing a household may be written:

\[
\nu(a, x, d, e_y, e_o, \eta_y, \eta_o, j) = \max_{c, a', e_y'} \tilde{n}u\left(\frac{\tilde{n}}{n}\right) + n\beta E_{q_y}[\nu(\frac{a'}{n}, x', d', e_y', e_o', \eta_y', \eta_o', 1)]
\]

subject to:

\[ c(1 + \tau) + a' = y(1 - \tau_{ss}) + a(1 + r) + \frac{(1 - d)}{n}(SS - (1 - b_h)m_x) + SS \]

\[ a' \geq 0, c > 0, \]

and \( y \) and \( \tilde{n} \) are defined as above. Expectations over next period’s value function are now taken with respect to the productivity shock of the children \( \eta_i \) who will become young adults. Moreover, in the following period young adults become old adults so we have \( \eta_y = \eta'_o \) and \( e_y = e'_o \).
3.5 Definition of Stationary Competitive Equilibrium

Let \( a \in \mathbb{R}_+ \), \( x \in \mathcal{X} = \{x_1, x_2, \ldots, x_n\} \), \( d \in \mathcal{D} = \{0, 1\} \), \( e_y, e_o \in \mathcal{E}_d = \{e_1, e_2, \ldots, e_n\} \), \( \eta_y, \eta_o \in \mathcal{E} = \{\eta_1, \eta_2, \ldots, \eta_n\} \), \( j \in \mathcal{J} = \{1, 2, \ldots, J\} \) and \( \mathcal{R} = \mathbb{R}_+ \times \mathcal{X} \times \mathcal{D} \times \mathcal{E}_d \times \mathcal{E} \times \mathcal{X} \times \mathcal{J} \). Let \( B(\mathbb{R}_+) \) be the Borel \( \sigma \)-algebra of \( \mathbb{R}_+ \) and \( P(\mathcal{X}), P(\mathcal{D}), P(\mathcal{E}_d), P(\mathcal{E}), P(\mathcal{J}) \) the power sets of \( \mathcal{X}, \mathcal{D}, \mathcal{E}_d, \mathcal{E}, \mathcal{J} \) respectively. Let \( \Sigma_\mathcal{R} \equiv B(\mathbb{R}_+) \times P(\mathcal{X}) \times P(\mathcal{D}) \times P(\mathcal{E}_d) \times P(\mathcal{E}) \times P(\mathcal{J}) \). Let \( \mathcal{M} \) be the set of all finite measures over the measurable space \( (\mathcal{R}, \Sigma_\mathcal{R}) \).

**Definition 1.** Given fiscal policies of the government \( \{\lambda, b_s, b_h, \tau, \tau_{ss}\} \) and a fertility rate \( n \), a stationary competitive equilibrium is a set of value functions \( \Phi(\zeta) \), households’ decision rules \( \{c(\zeta), a'(\zeta), e_y(\zeta)\} \), prices \( \{r, w\} \), tax rates \( \{\tau, \tau_{ss}\} \), pension benefits \( \{SS\} \), and time-invariant measure of households \( \Phi(\zeta) \in \mathcal{M} \) such that:

1. Given fiscal policies and prices, household’s decision rules solve household’s decision problem.

2. Prices \( w \) and \( r \) satisfy:
   \[
   r = A \alpha \left( \frac{N}{K} \right)^{1-\alpha} - \delta \\
   w = A (1 - \alpha) \left( \frac{K}{N} \right)^{\alpha}.
   \]

3. Individual and aggregate behavior are consistent:
   \[
   K = \int a'(\zeta) \Phi(d\zeta) \\
   N = \int (e_\eta e_j + n \left( 1 - \kappa_\eta e_j \right) e_y e_j \eta_y) (1 - n \theta_j (j \geq J_c)) \Phi(d\zeta).
   \]

4. Goods market clears\(^7\):
   \[
   \int \left( c(\zeta) + \frac{1-\alpha}{\alpha} m_x + n \kappa_{e_j} e_j \theta_j \right) \Phi(d\zeta) + n^2 \theta_p \Phi(d\zeta(j \geq J_y)) = AK^\alpha N^{1-\alpha} - \delta K.
   \]

5. Measure of households satisfy:
   \[
   \Phi(a', \bar{x}, 0, e'_y, e'_o, \eta'_y, \eta'_o, 1) \\
   = n \sum_{\zeta, a' = a(\zeta)/n, e'_y = e_y(\zeta), e'_o = e_o, \eta'_y = \eta_y} \Gamma_{\mathcal{E}_d}^n \left( \eta'_y, \mathcal{E} \right) \Phi(\zeta) \text{ for } j = J.
   \]
   \[
   \Phi(a', x', d', e_y, e_o, \eta_y, \eta_o, j + 1) \\
   = \frac{1}{n^{j+1}} \sum_{(a,x,d,a' = a(\zeta))} \Gamma^x(x', \mathcal{X}) \Psi(d', d_j) \Phi(a, x, d, e_y, e_o, \eta_y, \eta_o, j) \text{ for } j < J.
   \]

---

\(^7\)Let \( \Phi(d\zeta(j \geq J_y)) \) denote the total measure of primary school aged children.
6. Government budget for education and medical expenses balances:
\[\int \left( b_h \frac{(1 - d)}{n} m_x + \lambda_j n \kappa r x_j \theta_j \right) \Phi(d\zeta) + \int n^2 \theta_p \Phi(d\zeta(j \geq J_p)) = \tau \int c(\zeta) \Phi(d\zeta).\]

7. Social security budget balances:
\[\tau_{ss} w N = \int SS \Phi(d\zeta(j \geq J_{ss})) + \int dSS \Phi(d\zeta),\]
where: \[SS = \frac{b_{wN}}{\int (1+n(1-\kappa e_y)) \Phi(d\zeta)}.\]

4 Calibration

We use a calibrated version of the model to understand the effect of demographic transition on macroeconomic variables in China in both the short run and the long run. Since the Chinese economy has undergone massive changes in the last five decades, we calibrate our model economy in two stages. First, we calibrate our initial steady state to match some key features of the Chinese economy and demographics circa the 1960s. Next we calibrate the transition economy to match some key changes in the Chinese economy between 1960 and 2010.

In our calibration exercise, we take some parameter values directly from the literature or estimate them using micro level survey data. For instance, we use data from the China Health and Retirement Longitudinal Study (CHARLS) to estimate the stochastic process for elderly medical expenditures. Other parameters we estimate jointly using our general equilibrium model by minimizing the distance between certain data and model moments. The following subsections lay out the details of our calibration exercise.

4.1 Demographics

Each model period is assumed to represent one calendar year. In order to capture the demographics accurately, we model the entire life cycle of an individual from ages 0 to 98. The final “household age” index is set to \( J = 29 \). We set \( J_c = 17 \) implying young adults have children at real age 29. Table 1 gives the number and age-structure of different generations living in a household. Finally, we set \( J_{ss} = 19 \) implying that old adults begin receiving pension payments at real age 60.

---

\( \Phi (\zeta (j \geq J_{ss})) \) denotes the total measure of old adults who have reached pension claiming age.

Both fertility and mortality rates were relatively stable during this time. Fertility only started declining dramatically in the 1970’s and there were major improvements in life expectancy at birth in the late 1960’s (UN, 2015). Hence steady state is a reasonable approximation for this period.

\( \Phi (\zeta (j \geq J_{ss})) \) denotes the total measure of old adults who have reached pension claiming age.

\( \Phi (\zeta (j \geq J_{ss})) \) denotes the total measure of old adults who have reached pension claiming age.
Table 1: Household Composition

<table>
<thead>
<tr>
<th>Stage</th>
<th>Individual’s age (yrs.)</th>
<th>Number</th>
<th>Household ages lived</th>
</tr>
</thead>
<tbody>
<tr>
<td>Children</td>
<td>0-11</td>
<td>$nn$</td>
<td>17 \ldots J</td>
</tr>
<tr>
<td>Young Adults</td>
<td>12-40</td>
<td>$n$</td>
<td>1 \ldots J</td>
</tr>
<tr>
<td>Old Adults</td>
<td>41-69</td>
<td>1</td>
<td>1 \ldots J</td>
</tr>
<tr>
<td>Elderly</td>
<td>70-98</td>
<td>$\frac{1}{n}$</td>
<td>1 \ldots Death</td>
</tr>
</tbody>
</table>

In our initial steady state we set $n = 2.7$ so that the implied fertility rate matches the UN estimate for China in 1965-70 of 5.4.\textsuperscript{11} Fertility along the transition from 1960-2010 is shown in Figure 6. We maintain the initial fertility rate for 10 years then gradually reduce the rate to 1.5 between 1970 and 2000 to approximate the declines estimated by the UN.\textsuperscript{12}

Figure 6: Fertility Rates: 1960-2010

\[ u(c) = \frac{c^{1-\sigma}}{1-\sigma}. \]

\textsuperscript{11}We adjust the UN total fertility rate for under-five mortality to obtain the estimate of 5.4. According to UN (2015), the total fertility rate and under-five mortality rates in China between 1965 to 1970 were 6.3 (per woman) and 143 (per 1000 live births) respectively.

\textsuperscript{12}For computational convenience, we reduce fertility to achieve the constant cohort growth rate implied by a fertility rate of 1.5 (given by 0.75\textsuperscript{2}). This by construction takes $J = 29$ periods in the model to reach a fertility rate of 1.5. Alternately, we could feed the fertility paths directly from the data. However, we would still need to adjust the cohort growth rates to achieve a stable demographic structure in the long run.
The parameter $\sigma$ controls risk aversion and is set to a value of 2, implying an inter-temporal elasticity of substitution of 0.5. As children consume fewer resources than adults, we set the child consumption weight $\gamma = 0.3$ following the OECD-modified consumption equivalent scale (Hagenaars et al., 1994).

### 4.3 Medical Expenditures and Mortality

We assume there are two possible realizations of the elderly medical expenditure state $x$—high and low. We estimate transition probabilities between states and associated medical expenditures $m_x$ using data from the 2011 and 2013 waves of the CHARLS, a nationally representative survey of Chinese residents ages 45 and older. We first divide surveyed individuals over the age of 70 into percentiles based on reported annual total medical expenditures.$^{13}$ As has been documented in other countries, the expenditure distribution is highly skewed with a thin right tail driven by a limited number of catastrophic events. As such, we categorize those in the bottom 90 percentiles of the expenditure distribution as our low expenditure state. Analogously those in the top 10 percentiles are categorized into the high expenditure state. Annualized transition probabilities between states across the two waves of the CHARLS are shown in the first columns of Table 2.$^{14}$ We next compute the mean expenditures among those categorized into the high/low expenditure state using the 2013 wave. We set low/high medical expenditures $m_x$ to be a constant share of output per capita in every model period. The last column of Table 2 shows the estimated average expenditures as a share of output per capita from the CHARLS.

<table>
<thead>
<tr>
<th>$x$</th>
<th>Transition probability</th>
<th>Mean expenditures (% GDP per capita)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low (Low)</td>
<td>High (High)</td>
</tr>
<tr>
<td>Low</td>
<td>0.94</td>
<td>0.06</td>
</tr>
<tr>
<td>High</td>
<td>0.53</td>
<td>0.47</td>
</tr>
</tbody>
</table>

Survival probabilities are taken from UN life table estimates (UN, 2015). As the initial steady state is calibrated to match key features of the Chinese economy circa the 1960s, we use age-specific survival probability estimates for 1965-70 as a starting point in the model. However, age-specific mortality rates have significantly improved in China over the past decades and are projected to continue to improve into future. As a simple means of capturing this improvement in the model, we linearly decrease the mortality risk along our transition path from 1970 to 2100 to reach the levels projected

$^{13}$Medical expenditures in the two waves are deflated to 2010 value and include both inpatient and outpatient costs.

$^{14}$As waves in the CHARLS are multiple years apart, reported values have been adjusted to an annual transition.
by UN for 2095-2100. Figure 7 shows the corresponding set of UN survival probability estimates used in our initial and final steady states.

Figure 7: Survival Probabilities

4.4 Labor Productivity

We use data from the China Family Panel Studies (CFPS) to estimate age-specific labor productivity $\epsilon_j$ over potential working years of the life-cycle (ages 12 to 69).\footnote{The CFPS is a longitudinal survey of Chinese families and communities. While the CHARLS provides excellent data on the medical spending of the elderly in China, it does not include young enough individuals to estimate life-cycle productivity profiles.} We use the 2010 and 2012 waves and regress log of hourly income on age, age-squared and an individual fixed effect to obtain our life-cycle productivity estimates. Due to lack of observations, we assume productivity is constant prior to age eighteen. Figure 8 plots the estimated life-cycle profile of labor productivity. Productivity increases steadily until age fifty, at which point it begins to decline throughout the remainder of an individual’s working life.

Figure 8: Labor Productivity Profile
We estimate the Markov chain for the stochastic component of productivity $\eta$ by assuming an underlying AR(1) process in logs:

$$\ln(\eta') = \rho \ln(\eta) + \epsilon_\eta, \quad \epsilon_\eta \sim N\left(0, \sigma^2_\eta\right).$$

We then use the Tauchen method to approximate this process with a Markov chain over eight discrete states. Parameters governing the stochastic process $\rho$ and $\sigma_\eta$ are jointly estimated using the predictions of the model (see section 4.8 for details).

Finally, recall that children in the model (ages 0-11) cost their parents a fraction of their labor time endowment $\theta_f$. Following Bloom et al. (2009) we set $\theta_f = 0.16$.16

### 4.5 Education

We assume primary education lasts for six years (age 6-11), secondary for six years (age 12-17), and college for four years (age 18-21).17 Empirical estimates suggest very low returns to education in China in the 1970s—in the range of 0-3% (Yang, 2005; Fleisher and Wang, 2005). As such, after normalizing education-specific productivity $e$ for primary education to one, we use a 3% annual return to secondary school and college for our initial (1960) steady state. The compression of wages is largely attributed to equalization policies carried out under the centrally planned economy, with particular downward pressure on more educated workers. Following economic reforms of the early 1980s, there was a steep rise in the returns to schooling.18 Most recent estimates have found overall annual returns in the 10-20% range (e.g. Li (2003); Li and Luo (2004); Zhang et al. (2005); Fang et al. (2012)). Heckman and Li (2004) estimated annual returns to college close to 10%. Moreover, the average return globally across countries is estimated at around 10% (Psacharopoulos and Patrinos, 2004). As such, to capture the decompression of wages in China over time, we assume annual returns over primary school increase to 15% by 2010 for secondary schooling and 10% for college. The initial and final education-specific productivity estimates are given in Table 3.

<table>
<thead>
<tr>
<th>Level</th>
<th>Ages</th>
<th>Productivity ($e$)</th>
<th>Annual total tuition ($\theta$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1960</td>
<td>2010+</td>
</tr>
<tr>
<td>Primary</td>
<td>6-11</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Secondary</td>
<td>12-17</td>
<td>1.19</td>
<td>2.31</td>
</tr>
<tr>
<td>College</td>
<td>18-21</td>
<td>1.34</td>
<td>3.39</td>
</tr>
</tbody>
</table>

16Bloom et al. (2009) estimate that each birth reduces a woman’s total labor supply by 1.9 years over her reproductive life. We convert this to an annual time cost for parents in the model.
17This implies $\kappa_j = I\{j \leq 6\}$ for those choosing secondary school and $\kappa_j = I\{j \leq 10\}$ for college, where $I\{}$ is the indicator function.
18See Fang et al. (2012) for a good review of the literature.
In the initial (1960) steady state we set the total annual tuition of each education category \((\theta_p, \theta_s, \theta_c)\) to match the respective total costs as a share of output per capita reported in China for the year 1965 (China Education Statistical Yearbook, 1988)\(^{19}\). For example, 1960 college tuition was set to match a reported cost per college student of 379% of GDP per capita. We assume college tuition remains constant over time which endogenously gives a reasonable tuition cost of 180% of GDP per capita in 2010 and 88% in the long run steady state. However, the cost of primary and secondary school relative to college has risen considerably over time (China Statistical Yearbook, 2012)\(^{20}\). As such, we adjust the cost of primary and secondary schooling to match their cost relative to college in 2010. The last two columns of Table 3 show the total tuition costs used in the model. We assume that the productivity returns and total tuition cost of primary, secondary and college education are constant from 1960 to 1980, then rise linearly from 1980 to 2010. We also conduct sensitivity analysis on our modeling assumptions on the changes of educational costs and returns over time.

### 4.6 Technology

We set \(\alpha\) to match the long run average capital share of income (1970-2010) for China while the depreciation rate \(\delta\) is set to 10%. Total factor productivity \(A\) is normalized to one, even though we allow for labor augmenting TFP change by increasing returns to education over time as suggested in the literature.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital share (\alpha)</td>
<td>0.48</td>
<td>Feenstra et al. (2015)</td>
</tr>
<tr>
<td>Period depreciation (\delta)</td>
<td>0.10</td>
<td>Chow and Li (2002)</td>
</tr>
<tr>
<td>Factor productivity (A)</td>
<td>1</td>
<td>Normalization</td>
</tr>
</tbody>
</table>

### 4.7 Government Policies

The government operates the pension and health insurance programs and subsidizes primary, secondary and college education in the model economy. The medical expenditure reimbursement rate is set at \(b_h = 0.7\). This is the estimated rate for urban workers in China and the target rate for rural workers as well (Yip et al., 2012). The pension replacement rate is set as 35% of national average earnings \((b_s = 0.35)\), the target rate for the pay-as-you component of the current urban system (OECD, 2010).

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\(^{19}\)Reports total cost of primary, secondary and tertiary education per student. We transform them into shares of GDP per capita of 8%, 37%, and 379%. 1965 is the only available year prior to 1978.

\(^{20}\)Reports total enrollment and total spending on primary, secondary and college education, which we use to calculate per student costs. Primary and secondary were 19% and 28% the per student cost of college in 2010 compared to 2% and 10% in 1965.
Finally, government subsidizes primary, secondary and college education at the rates of $\lambda_p$, $\lambda_s$ and $\lambda_c$ respectively. As schools operated under a centralized economy and wages were highly compressed in the 1960s, we assume all three levels of education to be fully subsidized by the government in the initial steady state ($\lambda_i = 1$, $\forall i$). Note that even with no private tuition cost, there remains a time cost of attending secondary school and college.

As primary school is still primarily funded through various levels of government we maintain $\lambda_p = 1$ throughout all analyses. However, to reflect changing demographics, total costs, returns to schooling, and public policies we allow the private tuition cost of secondary school and college to change over time. After 2010, we fix government expenditures on education at 4.3% of GDP. This matches the latest empirical estimates and is near the government’s stated long-term goal of 4% (China, 2014; Tsang, 1996). After subtracting the entire cost of primary school from the government’s budget, the remainder is split in a 60/40 ratio between secondary school and college in order to determine the respective subsidy rates.\textsuperscript{21} This approach implies that in every model period post-2010, the private tuition cost $((1 - \lambda) \theta)$ of secondary and college education is endogenously determined by aggregate output and the number of enrolled students. In contrast, we exogenously assume the private tuition cost of secondary and college increases linearly from 1960 (free) to 2010. We allow taxes to adjust to ensure the government budget clears each period along the transition.\textsuperscript{22}

4.8 Estimation of Other Parameters

We use the model to jointly estimate three remaining parameters—$(\beta, \rho, \sigma)$—by targeting relevant empirical data moments in the initial steady state. We estimate the discount factor $\beta$ by targeting the average capital-output ratio (1960-70) of 3.23 in the Penn World Tables 8.1 (Feenstra et al., 2015). For estimating the persistence and standard deviation of the labor productivity shock, we target the inter-generational income mobility and the income Gini coefficient, respectively. Gong et al. (2012) provide estimates of inter-generational income mobility in urban China for father-son, father-daughter, mother-son and mother-daughter. We use a simple average of these for our targeted moment. The income Gini coefficient for China in 1981 is taken from the World Bank Development Indicators.\textsuperscript{23} Table 5 provides a summary of all parameter estimates along with data and model moments. The model does an excellent job of matching the data moments.

\textsuperscript{21}Available data shows public spending has stayed at a relatively stable 60/40 ratio between 1996-2011 (China, 2014).

\textsuperscript{22}We could alternately fix the government budget starting in 1960 and let the private cost be determined endogenously throughout the transition. However, due to a very small share of college educated in 1960, we face convergence issues in transitions if we allow private cost of education to be determined endogenously between 1960 and 2010.

\textsuperscript{23}No estimates are available prior to 1981.
Table 5: Estimation Results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Moment</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor $\beta$</td>
<td>0.944</td>
<td>K/Y</td>
<td>3.23</td>
<td>3.16</td>
</tr>
<tr>
<td>Persistence of prod. shock $\rho$</td>
<td>0.62</td>
<td>Inter-generational income mobility</td>
<td>0.60</td>
<td>0.60</td>
</tr>
<tr>
<td>S.D. of prod. shock $\sigma$</td>
<td>0.43</td>
<td>Income Gini</td>
<td>0.29</td>
<td>0.29</td>
</tr>
</tbody>
</table>

5 Model-Fit 1960-2010

While our benchmark model is calibrated to match the data in terms of mortality, income mobility, etc., it is useful to compare predictions along some other dimensions not targeted during calibration. First, the model is able to match the changing demographic structure rather well. This is evident from Figure 9 showing the old-age and child dependency ratios both in the model and the data (UN, 2015).\textsuperscript{24} These dependency ratios were not targeted directly in our calibration exercise. As fertility and mortality rates were not completely stable prior to 1960, the initial steady state somewhat over-predicts these ratios. However, as our demographic structure evolves over time, it become quite similar to the data. This is perhaps unsurprising as we feed in fertility rates and age-specific mortality rates that approximate the data along the transition. Nonetheless, it is reassuring that estimates seem reasonable.

Figure 9: Model Fit: Demographics

(a) Old-Age Dependency Ratio

(b) Child Dependency Ratio

Figure 10 shows the primary, secondary and college share from 1970-2010 both in the model and the data.\textsuperscript{25} The model slightly over-predicts the initial secondary share.

\textsuperscript{24}The UN old-age dependency ratio is defined as the ratio of people older than 64 to those aged 15-64. The child dependency ratio is defined as the ratio of people aged 0-14 to those aged 15 to 64. It should be noted that the young dependents in our model correspond to ages 0-11 but we adopt the same definition as UN in these graphs for consistency.

\textsuperscript{25}Data is average schooling attainment for population aged 25+ from Barro and Lee (2013).
and under-predicts the college and primary shares. However, the trends in schooling shares are quite similar over time.

Figure 10: Model Fit: Education Shares

(a) Primary Share

(b) Secondary Share

(c) College Share

The model also does well in matching the average age-specific consumption, labor earnings and transfer profiles from the National Transfer Accounts (NTA) data (Lee and Mason, 2011).26 Figure 11 shows these average age-specific profiles for the year 2002 in both the data and the model.27 In this figure, consumption is expressed as a percentage of age 45 consumption, and earnings and transfers as a percentage of age 45 earnings.

Similar to the data, we have an increase in consumption through early years of life as education expenditures and consumption requirements rise. This is followed by a

26 To be consistent with the NTA data, consumption here also includes total public and private medical expenditures and total public and private education expenditures. Labor earnings are before-tax and do not include pensions, transfers are defined as labor earnings less consumption for each age.

27 Model predictions are shown assuming an indefinite fertility of 1.5 after 2010.
sustained level of consumption throughout much of the individual’s working life before there is an increase due to health expenditures later in life. In contrast to the data, our model maintains the elevated consumption throughout old-age as health expenditures remain high. Moreover, our model over-predicts the gap between mid and late-life consumption as we do not incorporate health spending prior to becoming elderly. In terms of the labor earnings profile, the model matches the data well, though the rise in income is somewhat less steep and the peak a few years later.

The life-cycle deficit is defined as consumption less labor income and gives a sense of how the model does in predicting intra-household transfers. Life-cycle deficits become positive only a few years later in the model compared to the data, while they return negative a few years later as well. Compared to the data, the deficit is somewhat lager in the model during the school-aged years and for the elderly. This is primarily due to the over-prediction of education and health spending during the respective age ranges.

Figure 11: Model Fit: Consumption, Savings, Earnings and Transfers

The level and trend direction are approximately correct for the savings rate, though the model under-predicts the large increase observed in the 2000s (Figure 12d). This
is broadly consistent with the estimates of Choukhmane et al. (2013) and Curtis et al. (2015), who find demographics to explain 30-60% of the rise in savings over this time frame.\textsuperscript{28}

6 Fertility Experiments

In order to understand the short and long run effects of fertility changes on macroeconomic outcomes in China, we analyze two different future fertility scenarios. First, we assume that the 2010 fertility of 1.5 is maintained indefinitely into the future. Second, beginning in 2010, we assume an unexpected gradual increase to a long-term fertility rate of 1.8, which is somewhat higher than the current average in China.\textsuperscript{29} Both fertility paths are identical along all other exogenous dimensions.\textsuperscript{30} The fertility rates fed along both transition paths are given in Figure 12 along with a comparison to the UN projections (medium variant). These two paths — 1.5 and 1.8 — are henceforth referred to as the “low” and “high” fertility scenarios. Under section 7.3, we also look at some alternate fertility scenarios — 1.2 and 2.

**Figure 12: Fertility Rates Along Transition**

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\textsuperscript{28}Numerous additional mechanisms have been proposed to explain the additional rise in savings including rising sex imbalances (Wei and Zhang, 2011), long-term care requirements and productivity growth (İmrohoroglu and Zhao, 2016), housing cost (Chamon and Prasad, 2010), and rising income uncertainty (Chamon et al., 2013).

\textsuperscript{29}Similar to implementing the decline in fertility from 1960-2000, we change the cohort growth rate in 2010 to that implied by a long run fertility rate of 1.8 (given by 0.97) and back out the implied transition fertility rate. This is done in order to achieve a stable demographic structure in the long run.

\textsuperscript{30}However, as detailed in the calibration section, they both differ from the initial steady state in terms of mortality, tuition cost, returns to schooling and public outlays on education. Sensitivity of our results to these changes are conducted in section 8.
7 Results

As our theory outlined above has several countervailing mechanisms, we begin our quantitative analysis by discussing the long run and short run macroeconomic outcomes in our endogenous human capital model (benchmark) as outlined above. We then compare the differences in macroeconomic outcomes across high and low fertility scenarios for two other models — 1) a simplified model featuring exogenous human capital and no time cost of raising children, 2) a model with exogenous human capital but a time cost of raising children as in our benchmark. Calibration details of the alternate model specifications are available in Appendix A.2. It should be noted that, for a given fertility rate, the old-age and child-dependency ratios and the working-age share are the same across all three models as the demographic structure is identical in all three frameworks. The following sub-sections discuss our benchmark results and then compare across the three models how the long run and short run macroeconomic outcomes respond to the changes in the fertility rate.\(^{31}\) This is followed by analyses of alternate fertility scenarios and general equilibrium effects.

7.1 Benchmark Model

Figure 13 shows the transition paths for select macroeconomic variables for both fertility scenarios for the benchmark model. All per capita variables have been normalized to output in the low fertility final steady state. First, notice there are macroeconomic increases along both fertility paths going forward from 2010. However, comparing across fertility paths, we find overall macroeconomic declines when moving from a long-term fertility rate of 1.5 to 1.8. Table 6 gives percentage change in select variables across fertility scenarios in the long run steady state. Despite a higher working-age share in the long run, there is a decline of 2.5% in per capita effective labor supply and 1.5% in capital, resulting in lower long term per capita output (2.0%) and consumption (4.2%).

In our model, demographic changes and exogenous improvements in returns to education since the 1980’s result in an increase in both college and secondary education shares and a drop in primary share over the coming decades under both fertility scenarios (figure 14). After 2070, when the primary education share goes to zero, further improvements in educational attainment are brought about by a decline in secondary share and a further increase in college share. However, the gains in college share are smaller under the higher fertility scenario.

The behavioral response to higher fertility is influenced by several additional factors when human capital is endogenous in addition to savings. First, some poor households may not be willing to send additional children to school without the ability to borrow funds for interim consumption, leading them to reduce their choice of educational attainment due to the household borrowing constraint. Moreover, recall that educational

\(^{31}\)See tables A.2 and A.3 in Appendix A.3 for initial (1960) and final steady state results for all three models.
Figure 13: Transition Dynamics: Benchmark

(a) Working-age share

(b) Effective labor supply per capita

(c) Capital per capita

(d) Output per capita

Table 6: Final Steady State Results

<table>
<thead>
<tr>
<th>Variables</th>
<th>% Change low to high fertility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output per capita</td>
<td>-2.03</td>
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<tr>
<td>Consumption per capita</td>
<td>-4.28</td>
</tr>
<tr>
<td>Capital per capita</td>
<td>-1.52</td>
</tr>
<tr>
<td>Effective labor per capita</td>
<td>-2.50</td>
</tr>
<tr>
<td>Savings rate</td>
<td>43.71</td>
</tr>
<tr>
<td>Investment labor per capita</td>
<td>3.64</td>
</tr>
<tr>
<td>Public medical expenditures</td>
<td>-18.54</td>
</tr>
<tr>
<td>SS payments</td>
<td>-17.64</td>
</tr>
<tr>
<td>After-tax wage</td>
<td>6.96</td>
</tr>
<tr>
<td>Investment per capita</td>
<td>3.64</td>
</tr>
</tbody>
</table>
returns are uncertain at the time schooling decisions are made, making human capital a riskier investment than physical capital. As our functional form for preferences exhibits decreasing absolute risk aversion, this implies that human capital investment will weakly increase with household wealth, even if the household borrowing constraint does not bind. Thus, long run declines in wealth reduce education both by placing additional budgetary pressure on families and making them less willing to invest in risky human capital. Fertility changes also influence human capital investment decisions by altering the education budget of the government. In our model, government outlays on education are fixed as a share of output so an increase in the share of school-aged children leads to an increase in the per student private cost of secondary school and college (refer to figure A.1e in appendix A.3). If families are unable or unwilling to invest in lumpy school tuition despite relatively high returns, they may shift resources to physical capital as an alternate means of investment.

With higher fertility, there is a higher transfer requirement for future generations of children. There is also a shift from human capital to physical capital investments
due to the aforementioned reason. Together these lead to a 43.7% increase in the long run savings rate which corresponds to 3.6% increase in average annual investments in physical capital. However, despite the increase in savings rate, there is a small decline in average capital stock due to lost income from the long run decline in human capital.

Finally, there are declines in fiscal outlays for elderly social insurance programs with higher fertility. Specifically, there is an 18.5% reduction in public outlays on medical expenditures as share of output, which places downward pressure on the consumption tax. There is also a 17.6% decline in social security pension payments as a share of output, and correspondingly a drop in social security labor income tax. Combining this reduction in distortionary income tax with a small increase in the ratio of physical to human capital results in a net increase in the after-tax wage of more than 7%. Nonetheless, this increase in the private returns to schooling is not able to counteract the negative aggregate effects of higher fertility on education previously detailed. All together, we find that in these experiments the favorable macroeconomic effects of higher fertility—larger working-age share, additional savings for future children, and reduced public spending on pensions and health care—are outweighed by reductions in average educational attainment and capital stock in the long run steady state.

7.2 Model Comparisons

We now discuss key differences in our benchmark model results with two alternate models of exogenous human capital. In the first model, we shut down both the human capital investment channel available to the households as well as the effect of fertility on female labor supply through a time cost of raising children (labeled “Exg. HC”). In the second model, we only shut down the aforementioned investment channel while still allowing for time cost of raising children (labeled “Exg. HC + TC”). Since we are primarily interested in comparing the differences between high and low fertility scenarios, figure 15 shows the percentage difference in key macroeconomic variables between high and low fertility paths \(x_t = \frac{x_{1.8} - x_{1.5}}{x_{1.5}} \times 100\) for the three models.\(^{32}\)

Due to identical demographic structure, the effect of increased fertility on working-age share is the same across the three models — an immediate decline due to increased child dependents followed by an increase in the long run. However, there is divergence in the impact of fertility on other macroeconomic outcomes. In the model with exogenous human capital and no time cost of children, the effect of the fertility change on effective labor supply, capital, and output is driven by the effects on working-age share over the transition. About 70 years from the start of transition, the effective labor supply in the 1.8 fertility path exceeds that of the 1.5 fertility path, with capital and output following suit a few years later. Adding a time cost of children exogenously lowers labor supply — and hence capital and output — throughout the transition, but the pattern of results remain unchanged. The addition of endogenous human capital further lowers effective

\(^{32}\)The transition paths for the two models with exogenous human capital are provided in figures A.2 and A.3 in appendix A.3.
labor supply by reducing average educational levels.

In the long run, higher fertility maintains roughly 2% higher effective labor, capital, and output in the exogenous human capital model without time cost. Adding a time cost lowers this to around 1% but maintains the positive long run impact of higher fertility. In contrast, in our benchmark model, the long run increase in working-age share in the higher fertility path is not able to compensate for the loss in education shares. As a result, effective labor, capital, and output are around 2% lower in the long run.

7.3 Alternate Fertility Scenarios

Our benchmark experiment found lower short and long run macroeconomic outcomes under a high (1.8) compared to a low (1.5) fertility path. In order to test the generalizability of these results, we feed two alternate long-term fertility paths (1.2 and 2.0)
into our benchmark model. In Figure 16 shows results for all four fertility paths for key macroeconomic variables. In the short run (2010-2080), there is a negative monotonic relationship between fertility and outcomes — lower fertility results in higher levels of average effective labor, capital stock, and output. This is primarily driven by the short run increase in working-age share accompanying lower fertility, though eventually there are marginal increases in education as well. In the long run, higher fertility affects macroeconomic variables through two countervailing mechanisms — an increase in working-age share and a decline in human capital. When moving from 1.5 to 1.8 to 2.0 fertility, the latter mechanism dominates the increase in working-age share, resulting in an overall negative effect of higher fertility in the long run. However, with a very low fertility of 1.2, the increase in college share is unable to make up for the large decline in working-age share. As a result the long run output per capita is lower than in the 1.5 case. These results indicate that while a fertility rate that maximizes the long run working-age share (2.0) may not maximize long run output, it is also true that a fertility rate that maximizes average human capital (1.2) may not maximize output either. Rather, it is the delicate balance between the working-age share and human capital levels that determine long run output.

7.4 General Equilibrium Effects
In our model households respond to changing fertility not only due to changes in household composition, but also due to economy-wide spillover effects. For example, the major positive externality of higher fertility in the long run is a reduction in the old-age dependency ratio and consequently the fiscal burden of financing old-age pensions and medical expenditures (i.e. lower payroll and consumption taxes). However, higher fertility also results in an increase in the per child private cost of schooling under the assumption of a fixed government budget for education. In order to understand the importance of these general equilibrium spillover effects, we run our fertility experiments under a partial equilibrium setting. More precisely, we simulate the behavior of households facing a long run fertility rate of 1.8 but keep prices (interest rate, wages, payroll tax, consumption tax, pension benefit and private tuition) fixed at 1.5 levels. Table 7 shows the steady state results of this partial equilibrium analysis for each of our three models. The columns marked “G.E” are our original general equilibrium results while those marked “P.E” are the same experiments conducted under the partial equilibrium setting.

When keeping human capital exogenously fixed, we find that higher fertility results in lower long run macroeconomic outcomes in partial equilibrium. This implies there are sizable positive externalities (primarily through taxes) associated with higher fertility. However, our endogenous human capital model additionally includes the negative

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33 Alternate fertility paths are fed in the same way as the 1.8 fertility path. Refer to section 6 for details.
34 Per capita variables have been normalized by the final steady state output per capita in the 1.5 fertility case.
externality on private tuition cost. In this case, the partial equilibrium results are somewhat better than general equilibrium in terms of educational attainment, capital and output (e.g. a 2.21% increase in output as opposed to a 2.04% decline). Nonetheless,
Table 7: Externality Effects

<table>
<thead>
<tr>
<th></th>
<th>Exg. HC</th>
<th>Exg. HC + TC</th>
<th>Benchmark</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>G.E.</td>
<td>P.E.</td>
<td>G.E.</td>
</tr>
<tr>
<td>Output per capita</td>
<td>1.95</td>
<td>-2.91</td>
<td>1.15</td>
</tr>
<tr>
<td>Consumption per capita</td>
<td>0.59</td>
<td>-6.39</td>
<td>0.23</td>
</tr>
<tr>
<td>Consumption per adult equivalent</td>
<td>2.72</td>
<td>-4.41</td>
<td>1.89</td>
</tr>
<tr>
<td>Capital per capita</td>
<td>1.84</td>
<td>-8.02</td>
<td>1.01</td>
</tr>
<tr>
<td>Effective labor per capita</td>
<td>2.05</td>
<td>2.05</td>
<td>1.29</td>
</tr>
<tr>
<td>College share (%)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Notes: The table reports % change in variables between high and low fertility under both the general and partial equilibrium fertility experiments in the long run steady state.

consumption measures remain worse under partial equilibrium due to higher payroll and consumption taxes. Overall, these experiments reveal that general equilibrium effects are quite significant and the failure to account for them can result in biased estimates of the effects of fertility on macroeconomic outcomes.

8 Sensitivity Analysis

In this section, we test the sensitivity of our results to various modeling assumptions pertaining to education, health insurance, social security and mortality changes.

8.1 Education

Since a main focus of our analysis is to understand how fertility changes interact with the human capital margin in both the short and long run, we have given significant consideration to modeling the education system in China. Towards this goal, we have captured changes in the returns to schooling, education costs and public subsidies in China over time through simple approximations. In order to test the sensitivity of our results to these modeling choices, we simplify our modeling of the education system. More specifically, we keep the initial (1960) steady state the same as the benchmark model but do not incorporate any changes in returns to schooling or the total tuition cost over time. We also let all three education levels be completely financed by public subsidies thereby relaxing the budget constraint of the poor households. Note that even though we make education tuition free in this experiment, there is still a time cost of attending school.

Figure 17 shows the percentage difference in key macroeconomic variables between the two fertility paths for the benchmark model and the education sensitivity experiment. Even under the simplifying assumptions on education, higher fertility results in lower levels of human capital in the long run (refer to figure A.4 in appendix A.3). However, without private tuition, the impact of fertility on education shares is smaller and effective labor supply in the high fertility path is largely able to catch up with
the low fertility path over time. Since households do not have to save for the future education of their children, the experiment in the simplified model sees a slightly larger initial reduction in capital stock compared to the benchmark model. However, over time physical capital and output also exhibit smaller (though still negative) declines relative to the benchmark model. Overall, even when muting the human capital channel by simplifying the education sector, we still find negative effects of higher fertility in both the short and long run.

8.2 Health Insurance and Social Security

In our model calibration, we assume a constant health insurance reimbursement rate of 70% and a constant social security replacement rate of 35% throughout the transition. However, the social insurance programs in China have changed dramatically since the 1960’s (see Liu (2004) for details). In order to test the sensitivity of our transition results to these changes in health insurance and pension programs, we allow for dif-
ferent medical reimbursement and pension replacement rates in transition to mimic the actual evolution of these programs over time. More specifically, we assume health insurance reimbursement rates decline linearly from 70% to 16% and pension replacement rates from 35% to 5% between 1980 and 2000 to approximate the significant decline in program coverage after decentralization reforms.\(^{35}\) We then keep these rates fixed at 16% and 5% from 2000 to 2010 to capture the last decade in China where the rural sector had practically no medical coverage or pensions. Following the new rural social insurance programs in China that started in the mid 2000’s, we assume the reimbursement rate increases linearly from 16% to 70% and the replacement rate from 5% to 35% between 2010 and 2050. In doing this exercise, both the initial and final steady states remain the same but the transition results differ from our benchmark model. We run two separate experiments incorporating the health insurance and pension changes separately to isolate their relative impact on results.

There are only small increases in effective labor supply and physical capital stock under the new health insurance and pension regimes as compared to the benchmark model (refer to figures A.5 and A.6 in appendix A.3). Reduction in medical coverage and pensions results in an increase in investment in both physical and human capital to insure against idiosyncratic medical expenditure shocks and provide for the consumption of the elderly. As a result, both output and consumption per capita are slightly higher in the new model along both high (1.8) and low (1.5) fertility paths. Reducing social insurance coverage also reduces the payroll and consumption taxes which further results in small increases in consumption and labor earnings. However, when comparing the relative effect of increased fertility across models, the changes in health insurance and pensions schemes have little effects on our results. As figure 18 shows, for key macroeconomic variables — effective labor, capital and output per capita — the percentage difference between high and low fertility paths remain the same as in the benchmark model. As such these experiments do not change our fertility comparisons in spite of small changes along the transition for each fertility path.

### 8.3 Mortality

In our benchmark model, we assume that mortality risk declines linearly along the transition path between 1970 and 2100 to capture improvements in age-specific mortality in China over time. In order to test the sensitivity of results to these mortality changes we conduct an experiment where we keep the age-specific mortality profile fixed at the initial steady state level. Figure 19 shows the results of this experiment for select macroeconomic variables. We again report percentage difference in each variable between the two fertility paths for the benchmark model and the model without mortality changes. A salient feature in these graphs is that the negative effect of higher fertility on macroeconomic variables is similar in the short run but exacerbated in the

\(^{35}\)National health insurance coverage was an estimated 23% in 2003 (Barber and Yao, 2010) so we set the reimbursement rate to 16% = 23% × 70%. About 14% of the total population was participating in the basic pension scheme in 2006 (China, 2014), so we set the replacement rate to 5% = 14% × 35%.
long run under fixed mortality. For instance, the long run decline in output per capita between the low and high fertility path is roughly 6% as compared to 2% in the benchmark model. The long run difference in capital and effective labor per capita is also roughly 2-3 times bigger.

This result is not surprising given that one of the main benefits of higher fertility in our benchmark model is through the increase in working-age share. However, fixing mortality schedules at the 1960’s levels results in much lower old age dependency ratio and a much higher working-age share all along the transition (refer to figure A.7 in the appendix). Unlike the benchmark model, a small bump in fertility here is not able to increase working-age share in the long run. On the contrary, it results in a small decline in the working population through increase in the child dependency ratio.\textsuperscript{36}

\textsuperscript{36}Figure A.7c in appendix shows the relationship between fertility rate and the working-age share for a given mortality schedule in steady state. Higher mortality levels results in a peak in working-age share at lower levels of fertility (1.39 in the case of mortality levels of 1960’s and 2.7 for the mortality levels projected for 2100 in China). Beyond this peak level working-age share, increasing fertility further only reduces the working-age share through increase in the child-dependency ratio. Also refer
we find bigger macroeconomic declines going from low to high fertility in the economy with higher mortality as compared to the benchmark. On the other hand, while short run results seem less sensitive to mortality assumptions, if projected mortality declines are underestimated by the UN, the negative long run impact of higher fertility may be overestimated in our benchmark results.

9 Conclusion

After having implemented one of the most stringent fertility policies in the late 1970’s to slow down population growth, China recently relaxed its one child policy in hopes of promoting economic growth and social welfare. In this paper, we develop a dynamic general equilibrium model for understanding the short and long run macroeconomic effects of feasibly induced changes in aggregate fertility rates. We incorporate endoge-
nous education decisions and intra-household transfers to capture the effect of changing demographics on human and physical capital accumulation.

Our results indicate that relaxing the one child policy is unlikely to offset the negative macroeconomic effects of population aging in China — a fertility bump results in lower short-run income per capita in our benchmark model economy. These results may persist in the long-run as well where a small bump in fertility results in big declines in human capital investments in the long run. More generally, we find that maximizing the working-age share of the population is not the same as maximizing income per capita. Moreover, the relationship between fertility and long-run macroeconomic outcomes is not monotonic — at very low levels of fertility, the increase in working-age share from a marginal fertility increase dominates the negative effects on human and physical capital, raising long-run income per capita. As such, even though our focus in this paper has been on China, our findings are generalizable to other countries which are experiencing population aging. Faster aging economies with high levels of human capital, such as Japan, Germany, Italy, Finland and Portugal (projected to have old-age dependency ratios greater than 40% by 2030), might benefit from a bump in fertility in the long-run.

Our study is not without limitations. Most notable being the treatment of fertility as an exogenous change which is homogeneous across households in the model. While this reduces the computational burden of solving our model, it may have important implications for our results. For instance, we implicitly assume that the urban and rural sectors in China will respond in a similar way to the relaxation of the one child policy. However, empirical estimates suggest that the urban sector would see a bigger response to the policy change as compared to the rural sector. If fertility changes are concentrated within the urban sector — composed of wealthier families on average — then higher fertility may have smaller effects on human capital through household budget constraints. We also make several simplifying assumptions about human capital investments and the evolution of labor productivity. We abstract away from labor supply decisions, health dynamics and model medical expenditure uncertainty only for the very old. We are also limited to analyzing the economic impact of fertility changes and stress that a welfare analysis is outside the scope of our current framework. Addressing some these limitations leaves room for important future research in this direction.

\[37\] Zeng and Wang (2014) estimate that the rural TFR would increase from 2.01 to 2.15 between 2015 and 2030 while the urban TFR would increase from 1.24 to 1.67.
A Appendix

A.1 Savings Data

We use data from the China Statistical Yearbook (2014) to compute net national savings rates and investment output ratios from 1978-2013. All nominal values are deflated using the GDP deflator from the World Bank, World Development Indicators. The net national savings rate is defined as:

\[ \frac{Y_t - C_t - \delta K_t}{Y_t - \delta K_t}, \]

where \( K \) is capital stock, \( Y \) is aggregate output (GDP by expenditure approach), \( C \) is aggregate consumption inclusive of government spending (Final Consumption Expenditures), and \( \delta \) is the depreciation rate (assumed 10%). We use the perpetual inventory method to compute capital stock according to:

\[ K_{t+1} = (1 - \delta) K_t + I_t, \]

where investment \( I \) is “Gross Capital Formation” from the statistical yearbook. The initial capital stock is given by:

\[ K_0 = I_0 / (\delta + g), \]

where \( g \) is the growth rate of GDP reported in the World Development Indicators averaged from 1962 to 1977 (7.05%).

A.2 Alternate Model Specifications

In terms of calibration, we simply set the parameter \( \theta_f = 0 \) for eliminating the time cost of children. For removing the endogenous human capital margin, we eliminate education choice from our benchmark model (i.e. \( e = 1 \) for all individuals). In both cases, we re-estimate the parameters \( \beta, \rho \) and \( \sigma_\eta \) to match the same three data targets as our benchmark model. However, we keep the first stage calibration (parameters set/estimated outside the model) unchanged across all three models. Table A.1 provides the calibration results for both models without the human capital investment channel. Exogenous human capital increases income inequality and reduces inter-generational persistence in labor earnings compared to our benchmark model. As a result, we find that the calibration of these alternate models without the human capital choice results in a higher persistence and a lower variance in the labor productivity shock when matching the same targeted moments. It should be noted that consumption taxes only finance health care expenditures in the first two models but also finance education expenditures in the benchmark model.
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Moment</th>
<th>Data</th>
<th>Exg. HC</th>
<th>Exg. HC + TC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
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<td>K/Y</td>
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<td>3.16</td>
<td>3.18</td>
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<td>$\rho$</td>
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<td>Income mobility</td>
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<tr>
<td>$\sigma_\eta$</td>
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<td>Income Gini</td>
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Notes: Exg. HC refers to the exogenous human capital model and Exg. HC + TC is the model with exogenous human capital and time cost of children.

### A.3 Additional Figures and Tables

#### Table A.2: Initial Steady State Results

<table>
<thead>
<tr>
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<th>Exg. HC</th>
<th>Exg. HC + TC</th>
<th>Benchmark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Working-age share (%)</td>
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<td>52.58</td>
<td>52.58</td>
</tr>
<tr>
<td>Output per capita</td>
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<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Consumption per capita</td>
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<td>0.46</td>
<td>0.44</td>
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<td>Consumption per ad equi</td>
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<td>0.61</td>
<td>0.59</td>
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<tr>
<td>Capital per capita</td>
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<td>3.18</td>
<td>3.16</td>
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<td>Average labor supply</td>
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<td>0.56</td>
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<td>Effective labor per capita</td>
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<td>Investment per capita</td>
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<tr>
<td>Savings rate</td>
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<tr>
<td>Consumption tax (%)</td>
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<td>Private Medical expenditures (% of GDP)</td>
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<td>0.11</td>
</tr>
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<td>Public Medical expenditures (% of GDP)</td>
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<td>0.26</td>
</tr>
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<td>SS payments (% of GDP)</td>
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<td>1.95</td>
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<td>Primary share (%)</td>
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<td>93.89</td>
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<tr>
<td>Secondary share (%)</td>
<td>-</td>
<td>-</td>
<td>5.97</td>
</tr>
<tr>
<td>College share (%)</td>
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<td>-</td>
<td>0.14</td>
</tr>
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</table>

Notes: Per-capita outcomes have been normalized to output for each model specification.
Figure A.1: Benchmark Transition Dynamics: Prices

(a) Interest Rate

(b) After-Tax Wage

(c) Social Security Tax Rate

(d) Consumption Tax Rate

(e) Private Tuition
Figure A.2: Transition Dynamics: Exogenous Human Capital

(a) Working-age share

(b) Effective labor supply per capita

(c) Capital per capita

(d) Output per capita
Figure A.3: Transition Dynamics: Exogenous Human Capital & Time Cost of Children

(a) Working-age share

(b) Effective labor supply per capita

(c) Capital per capita

(d) Output per capita
Figure A.4: Sensitivity: Education

(a) After Tax Wage

(b) College Share

(c) Secondary Share

(d) Primary Share
Figure A.5: Sensitivity: Health Insurance

(a) Effective Labor Supply

(b) Capital

(c) Output

(d) Consumption
Figure A.6: Sensitivity: Social Security

(a) Effective labor per capita

(b) Capital per capita

(c) Output per capita

(d) Consumption per capita
Figure A.7: Sensitivity: Mortality

(a) Old-age dependency ratio

(b) Working-age share

(c) Long-term working-age share and fertility
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<th>Variables</th>
<th>Exg. HC</th>
<th>Exg. HC + TC</th>
<th>Benchmark</th>
</tr>
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<tbody>
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<td></td>
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<td>1.5</td>
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<td>Working-age share (%)</td>
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<td>54.69</td>
<td>51.89</td>
</tr>
<tr>
<td>Output per capita</td>
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<td>1.00</td>
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<tr>
<td>% change</td>
<td>-</td>
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<tr>
<td>Consumption per capita</td>
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<td>0.61</td>
<td>0.60</td>
</tr>
<tr>
<td>% change</td>
<td>-</td>
<td>0.59</td>
<td>-</td>
</tr>
<tr>
<td>Consumption per ad equ</td>
<td>0.65</td>
<td>0.67</td>
<td>0.64</td>
</tr>
<tr>
<td>% change</td>
<td>-</td>
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<td>-</td>
</tr>
<tr>
<td>Capital per capita</td>
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<td>3.17</td>
<td>3.12</td>
</tr>
<tr>
<td>% change</td>
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<td>1.84</td>
<td>-</td>
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<tr>
<td>Average labor supply</td>
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<td>0.60</td>
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<tr>
<td>% change</td>
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<td>Effective labor per capita</td>
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<td>% change</td>
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<td>% change</td>
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<td>Secondary share (%)</td>
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<td>College share (%)</td>
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<td>Private college cost</td>
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*Notes:* Per capita outcomes have been normalized to output in the 1.5 fertility case for each model specification.
References


