ECON3102-005

Chapter 5: A Closed-Economy One-Period Macroeconomic Model (Part 1)

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3. All markets clear (supply = demand for each market).

4. The government satisfies its budget constraint:

   $$G = T$$
Exogenous and Endogenous Variables

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- In this closed-economy one-period model, the exogenous variables are $G, z, K$, and the endogenous variables are $c, N^d, N^s, T, Y, w$.

- Making use of the model is running experiments to see how changes in the exogenous variables change the endogenous variables.
Production Function

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• Note that the maximum output that can be produced is $Y^*$, where

$$Y^* = zF(K, h)$$
Recall that $N = h - l$. Then it makes sense to put $Y = zF(K, h - l)$, so we can express output as a function of leisure.
Output as a Function of Leisure

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- If $l = 0$, then $N = h$ and $Y^*$ is produced.
- If $l = h$, then the consumer takes all his time as leisure, and nothing is produced.
We now want to express the previous graph not as an output-leisure relationship, but rather as a consumption-leisure relationship (the two goods which the consumer cares about).

- Since in equilibrium $Y = C + G$ (because of the income-expenditure identity (aka the market clearing condition for consumption goods), then

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- This means that we can take the previous graph, shift it down by some amount \( G \), and then get the production possibilities frontier (PPF).
Output as a Function of Leisure

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- Points on segment BD are feasible.
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Marginal Rate of Transformation

- The negative of the slope of the PPF is also called the **marginal rate of transformation**; this is the rate at which one good can be converted technologically into another.

- Call this rate the $MRT_{I,c}$. In particular, note that

$$MRT_{I,c} = MP_N = -(\text{slope of PPF})$$
Competitive Equilibrium on Graph

- The PPF is curve FH.
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Given $w$, the firm chooses $N$ to maximize $\pi$ by setting $MP_N = w$. Hence,

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In equilibrium, minus the slope of the PPF equals $w$; line AD is tangent to the PPF at J; here $MP_N = w$. 
Now let’s consider the consumers preferences.

- Distance $DB = \pi - G$, which equals $\pi - T$ by equilibrium properties.
- $ADB$ is the consumers budget constraint.
- Point $J$ is the competitive equilibrium. By consistency, $c^*$ is the desired consumption and $h^*$ is the desired labor supply.
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A Necessary Condition for Competitive Equilibrium

\[ MRS_{I,c} = MRT_{I,c} = MP_N \]
We now have our equilibrium concept where the agents are all price-takers and the market does all the work. But how “good” is this market outcome?
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To answer this, the (almost) universal benchmark is that of Pareto optimality:

**Definition** A competitive equilibrium is Pareto optimal if there is no way to rearrange production or reallocate goods so that someone is better off without making someone else worse off.
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  1. Order the firm to hire $N^d = N$ hours of labor and produce $Y$ units of output.

  2. Order the consumer to work $N^s = N$ hours.

  3. Take an amount $G$ of output and give the remainder to the consumer.
Hence the planners problem is to choose $c$ and $l$ that, given technological constraints, maximize the utility of the consumer.

- Formally, he solves:

$$\max_{c,l} U(c, l)$$

subject to

$$c = zF(K, h - l) - G$$

$$c \geq 0$$

$$0 \leq l \leq h$$
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This is similar to our previous problem, but we don’t get to worry about the budget constraint.
**Pareto Optimal and Social Planner’s Problem**

From the figure note that

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- Pareto optimality satisfies $MRS_{l,c} = MRT_{l,c} = MP_N$. 
CE and Pareto Optimum

- In this model, the competitive equilibrium and the Pareto optimum are identical, as both satisfy

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  - **Theorem (The Second Fundamental Theorem of Welfare Economics)** Under certain conditions, a Pareto optimal allocation can be established as a competitive equilibrium.

  - Free market economies tend to produce socially efficient economic outcomes.
Sources of Social Inefficiency

- The previous theorems sound nice, but do they always hold? More importantly, when do they not hold?

1. Externalities
2. The presence of market power
3. Distorting taxes
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- Negative externalities cause overproduction of the good.

- Positive externalities cause underproduction of the good.

- Hence, the socially efficient outcome is not reached. A competitive equilibrium is not Pareto optimal.
As you remember from 1101, monopoly power leads to underproduction relative to the social optimum:
Market Power

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• But the firm optimizes when \( MP_N = w \), so

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• And the equivalence condition breaks down.
Solving the CE

- To avoid dealing with prices we will use the equivalence between competitive equilibrium and Pareto optimal allocations. Thus, the solution to the planners problem is our competitive equilibrium.