ECON3102-005

Chapter 8: Two-Period Model: The Consumption-Savings Decision and Credit Markets

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Сonsumer’s consumption-savings decision: responses of consumers to changes in income and interest rates.
Outline

- Consumer’s consumption-savings decision: responses of consumers to changes in income and interest rates.

- Government budget deficits and the Ricardian Equivalence Theorem.
• Consumer’s consumption-savings decision: responses of consumers to changes in income and interest rates.

• Government budget deficits and the Ricardian Equivalence Theorem.
  • This theorem states that the size of government deficit is irrelevant as it does not affect macro variables of importance to economic welfare.
Things to keep in mind

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• In a multi-period model, saving-borrowing and the interest rate are key elements. Saving-borrowing allows the consumer to smooth consumption over time.
Model: assumptions

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- Each consumer leaves after 2 periods.
- Consumers receive an exogenous income (they do not make a work-leisure decision).
- Specifically, consumers receive income \( y \) in the first period, and \( y' \) in the second period.
**Notations**

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• BC (budget constraints), IC (indifference curves).
Borrowing/Lending (1/2)

- Borrowing/lending is done through one single instrument: a one-period bond that yields interest rate $r$. 

- Consumers' budget constraint in the first period is: $c + s = y - t$,
  - $s > 0$ implies that the consumer is saving (buying the bond),
  - $s < 0$ implies that the consumer is borrowing (selling the bond),
  - $y - t$ is the consumer's disposable income after tax.

- A bond issued with face value $s$ yields a return of $(1 + r)s$ in the following period. Note that the unit here is consumption goods.
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where

\( c \) is consumption, \( s \) is saving (or borrowing), \( y \) is income, and \( t \) is taxes.
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Borrowing/Lending (2/2)

- Consumers’ BC in the first period is $c + s = y - t$.

- Consumer’s BC in the second period is $c' = (1 + r)s + y' - t'$

If $s < 0$, then the consumer pays back both interest and principal in the second period.

If $s > 0$, then the consumer receives the promised return on her savings in the second period.
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Consumer’s Problem

The consumer’s problem is given by

\[ \max_{c, c', s} V(c, c') \]  \hspace{1cm} (1) 

subject to

\[ c + s = y - t \]  \hspace{1cm} (2) 

\[ c' = (1 + r)s + y' - t' \]  \hspace{1cm} (3)
Simply the Problem

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- However, we can substitute $s$ in the second equation by the first one:

\[
\begin{align*}
\text{From BC(1):} \\
& s = y - t - c \\
\text{Replace } s \text{ in BC(2) by the equation above:} \\
& c' = y' - t' + (1 + r)(y - t - c) \\
\text{After rearranging the equation, we have} \\
& c + c' + \frac{1}{1 + r} = y - t + y' - t' + \frac{1}{1 + r}(PVBC) \\
\text{This is the consumer’s present value budget constraint (PVBC).} \\
\text{Note that now we have just one PVBC and two variables to solve for the consumer’s problem. We can conduct the same graphical analysis as we did for the static problem.}
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Present value means the value in terms of the consumption goods in period 1.

That is, \( \frac{1}{1+r} \) is the relative price of future consumption in terms of current consumption:

- One unit of consumption today is equivalent to \( 1 + r \) units of consumption tomorrow.
Intuition in the PVBC (2/2)

\[ c + \frac{c'}{1 + r} = y + \frac{y'}{1 + r} - t - \frac{t'}{1 + r} \]

 lifetime wealth
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\[ \text{lifetime wealth} \]

- Denote the lifetime wealth by \( we \equiv y + \frac{y'}{1+r} - t - \frac{t'}{1+r} \), which is the lifetime resource a consumer has for consumption today and tomorrow.
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• We can rewrite the PVBC as

\[ c + \frac{c'}{1 + r} = we \]

\[ c' = we(1 + r) - (1 + r) c \]

y-intercept  slope
At point $E$, the consumer neither borrows or lends ($s = 0$). Note that $E$ is the endowment.
PVBC (LTBC) on Graph

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PVBC (LTBC) on Graph

- At point $E$, the consumer neither borrows or lends ($s = 0$). Note that $E$ is the endowment.
- To the NW of $E$, the consumer is a lender with positive savings.
- To the SE of $E$, the consumer is a borrower with negative savings.
Consumer’s Preferences

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- (monotonicity): consumers prefer more to less.

- (convexity): consumers prefer combinations to extremes.
  - This assumption implies that consumers will prefer to smooth their consumption over time. They do not like consume everything today and nothing tomorrow (or everything tomorrow and nothing today).

- (normal goods): current and future consumptions are normal goods. As the LTBC increases, both current and future consumptions will increase.
Indifference Curves

- The slope of the blue line is the $-MRS_{c,c'}$ at point A, which means the consumer is willing to give up a lot of consumption today to get a little consumption tomorrow.
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• At point A, the consumer has a lot of consumption today and very little consumption tomorrow.
Consumer’s Optimization

- As in chapter 4, the consumer optimizes where an IC is tangent to the BC:

\[ MRS_{c,c'} = 1 + r \]
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- As in chapter 4, the consumer optimizes where an IC is tangent to the BC:
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- \( MRS_{c,c'} \) is how much future consumption the consumer needs to stay on the same IC if she gives up one unit of current consumption.
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- If \( MRS_{c,c'} < 1 + r \), for one unit of current consumption, the consumer gets more future consumption than she needs to stay on the same indifference curve. So the consumer is better off trading away current consumption.
A Consumer Who is a Lender

Savings is \( y - t - c^* = DB \).
A Consumer Who is a Borrower

Savings is \( y - t - c^* = DB \).
An Increase in Current Income $y$

- Holding everything else constant, suppose current income $y$ increases by $\Delta y$.

- Then, $we$ increases by $\Delta y$.

Predictions:

- Consumptions in both periods increase.
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Predictions:

- Consumptions in both periods increase.

- Savings increase.

- Consumers act to smooth their consumptions over time.
An Increase in Current Income $y$

- The LTBD moves from $we_1$ to $we_2$, and the slope remains unchanged.

Note that $\Delta y > \Delta c^*$, so savings must have increased.
An Increase in Current Income $y$

- The LTBD moves from $we_1$ to $we_2$, and the slope remains unchanged.

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Remember in chapter 3, we observed that consumption is less volatile than RGDP. Our prediction is consistent with this observation.
Is this consistent with data?

• Remember in chapter 3, we observed that consumption is less volatile than RGDP. Our prediction is consistent with this observation.

• The observation is evidence that in practice, people do smooth their consumptions.
Durable, Non-durable goods and RGDP

- Aggregate consumption of non-durable goods is smooth relative to RGDP, but aggregate consumption of durable goods is more volatile than RGDP.
Aggregate consumption of non-durable goods is smooth relative to RGDP, but aggregate consumption of durable goods is more volatile than RGDP.

This is because economically consumption of durable goods are more like investment.
An Increase in Future Income

• Holding everything else constant, suppose future income $y'$ increases by $\Delta y'$.

• Then, we increases by $\frac{\Delta y'}{1+r}$.

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• Again, these results are explained by consumers’ actions to smooth their consumptions over time.
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- The OCB moves from $A$ to $B$.
- Consumption in both periods increase.
- $\Delta y' = AD$, $\Delta c'^* = AF$.
- Note that $\Delta y' > \Delta c'^*$, so savings must have decreased.
- With the increase in future income, the consumer wants to smooth consumption by saving less today.
AN INCREASE IN FUTURE INCOME

- The LTBD moves from $w_{e1}$ to $w_{e2}$, and the slope remains unchanged.

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The Permanent Income Hypothesis

It stipulates that:

- As a permanent increase in income will have a larger effect on lifetime wealth than a temporary increase.
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- As a permanent increase in income will have a larger effect on lifetime wealth than a temporary increase.

- This will in turn create a larger effect on current consumption.

- In other words, the consumer will tend to save most of a purely temporary income increase.
An Increase in Permanent Income

- The initial budget constraint is AB.
AN INCREASE IN PERMANENT INCOME

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- When only current income increases, the new budget constraint is ED (OCB moves from H to J, saving increases).

- In the second case, the effect on current consumption is much larger.

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An Increase in Permanent Income

- The initial budget constraint is AB.
- When only current income increases, the new budget constraint is ED (OCB moves from H to J, saving increases).
- When both y and \( y' \) increase simultaneously, the new budget constraint is GF (OCB moves from H to K.).

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- With the increase in future income, the consumer wants to smooth consumption by saving.
An Increase in the Real Interest Rate

- the slope changes and the budget constraint pivots around the endowment point.

![Graph showing the effect of an increase in the real interest rate on consumption and future consumption. The budget constraint pivots around the endowment point.](image)
An Increase in the Real Interest Rate

- the slope changes and the budget constraint pivots around the endowment point.

- This is because the consumer should be able to afford his endowment point, no matter what prices are.

\[ c' = \text{Future Consumption} \]

\[ w_{e2}(1 + r_2) \]

\[ w_{e1}(1 + r_1) \]

\[ c = \text{Current Consumption} \]

\[ w_{e2} \]

\[ w_{e1} \]
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We will see that the income and substitution effects depend on if the consumer is a borrower or a lender.
An Increase in the Real Interest Rate for a Lender

- the substitution effect for a lender:
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An Increase in the Real Interest Rate for a Lender

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  - From A to D is the substitution effect: how the consumer substitutes to remain equally happy after the price change.
  - The direction of the substitution effect is clear: as $r \uparrow$, $c'$ becomes cheaper compared to $c$, so $c' \uparrow$ and $c \downarrow$. 
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- the income effect for a lender:
  - Since $r \uparrow \Rightarrow \uparrow$ in lifetime wealth,
  - The income effect is the parallel upward shift from fictive budget line (FG) to new budget constraint (BE).
  - Since both goods are normal, both current and future consumptions increase from D to B.
An Increase in the Real Interest Rate for a Lender

- The substitution effect $\Rightarrow c \downarrow$ and $c' \uparrow$
- The income effect $\Rightarrow c \uparrow$ and $c' \uparrow$

Hence, $c$ may $\uparrow$ or $\downarrow$ depending on which effect is larger, and $c'$ will always $\uparrow$. 
An Increase in the Real Interest Rate for a Lender

- The substitution effect ⇒ $c \downarrow$ and $c' \uparrow$
- The income effect ⇒ $c \uparrow$ and $c' \uparrow$
- Hence, $c$ may $\uparrow$ or $\downarrow$ depending on which effect is larger,
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- The income effect $\Rightarrow c \uparrow$ and $c' \uparrow$
- Hence, $c$ may $\uparrow$ or $\downarrow$ depending on which effect is larger,
- and $c'$ will always $\uparrow$. 
An Increase in the Real Interest Rate for a Borrower

- the substitution effect for a lender:
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- the substitution effect for a lender:
  - (FG) is an artificial budget line that has the slope of the new budget constraint (reflecting the new price ratio) and is tangent to the first IC.
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- the income effect for a lender:
  - The borrower is hurt by an increase in the interest rate. Hence, we need to increase the consumer’s wealth until he is as happy as he was before the rise in the interest rate.
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- The income effect for a lender:
  - The borrower is hurt by an increase in the interest rate. Hence, we need to increase the consumer’s wealth until he is as happy as he was before the rise in the interest rate.

- Therefore, for a borrower, the income effect is negative (shift from (FG) to (EB)) and creates a decrease in the consumption of both goods.
An Increase in the Real Interest Rate for a Borrower

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- Hence, $c'$ may $\uparrow$ or $\downarrow$ depending on which effect is larger,
- and $c$ will always $\downarrow$, and $s$ necessarily $\uparrow$
The government consumes $G$ today and $G'$ tomorrow.
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- Recall there are $N$ consumers, each paying taxes $t$ today and $t'$ tomorrow.

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- Let $B$ denote the quantity of government's issued bond. $B < 0 \Rightarrow$ the govn't is lending.
The Government Budget Constraint

\[ G = T + B \] (period 1)
\[ G' + (1 + r)B = T' \] (period 2)

• Solving for \( B = \frac{T' - G'}{1 + r} \) in the second equation and replacing \( B \) in the first one yields:

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- This is equivalent to saying all government debt has to be paid with taxes.
The CE with Government

The CE with government is a set of prices and quantities such that

- The consumer optimally chooses $c$, $c'$ and $s$, taking $r$ as given.
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- The consumer optimally chooses $c$, $c'$ and $s$, taking $r$ as given.
- The government’s present value budget constraint (PVBC) holds.
- The credit market clears:

$$S^p = B$$

That is, private savings = quantity of debt issued by the government.
Theorem: \( S^p = B \iff Y = C + G \)

Sketch of the proof:

• First, observe from the consumer’s budget constraint that:

\[ S^p = Y - C - T. \]

• Now, solve for \( B \) in \( B = G - T \) and substitute both \( S^p \) and \( B \) to get:

\[ Y - C - T = G - T, \] which is,

\[ Y = C + G. \]

• This result is important because it makes it simpler to solve for the competitive equilibrium:

• Instead of checking that \( S^p = B \), we now only have to check that \( Y = C + G \).
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Sketch of the proof:

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2. Now, solve for $B$ in $B = G - T$ and substitute both $S^p$ and $B$ to get: $Y - C - T = G - T$, which is, $Y = C + G$.

3. This result is important because it makes it simpler to solve for the competitive equilibrium:
   - Instead of checking that $S^p = B$, we now only have to check that $Y = C + G$. 

Ricardian Equivalence Theorem

Everything else equal, two scheme of taxes that yield the same present value, but are different in their timings, will affect the economy in an identical fashion: both the interest rate and the path of individual consumption will remain identical.
Proof of the Ricardian Equivalence Theorem

- Substitute $T = Nt$ and $T' = Nt'$ into the govn’t PVBC to get:

$$G + \frac{G'}{1 + r} = Nt + \frac{Nt'}{1 + r}$$

- Rearrange the equation above and it gives:

$$t + \frac{t'}{1 + r} = \frac{1}{N} \left[ G + \frac{G'}{1 + r} \right]$$

- Substitute into the consumer’s PVBC:

$$c + \frac{c'}{1 + r} = y + \frac{y'}{1 + r} - \frac{1}{N} \left[ G + \frac{G'}{1 + r} \right]$$

- Suppose there is a change in the tax schedule such that

$$\Delta t + \frac{\Delta t'}{1 + r} = 0$$
Proof of the Ricardian Equivalence Theorem (cont’d)

- Because there is no change in the wage and since the consumer takes $r$ as given, the consumer’s choices as a function of $r$ will remain the same.

- Now, since $Y = C + G$ still holds, the interest rate $r$ remains the same.

- Hence both the interest rate and the consumer’s choices are unchanged as a result of the change in the tax scheme.
Assumptions in Ricardian Equivalence

- Tax changes are the same for all consumers in both present and future (no redistribution).
- Debt issued by the government is paid off during the lifetimes of the people alive when the debt was issued.
- Taxes are lump-sum (non-distortionary).
- Perfect Credit Markets.
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