This Appendix covers many additional simulations from “Optimal Taxation in Overlapping Generations Economies with Aggregate Risk.” The main text presents selected simulations to build intuition for how the model behaves in numerical applications. This Appendix provides a more comprehensive view of how the results vary with the IES, social welfare function, market completeness assumption, and application.

Stochastic Government Expenditure

Complete Markets. Figures 1 and 2 respectively show the behavior of several variables across state transitions, and the autocorrelation structure of labor taxes, if the IES is 1/4 and markets are complete. The concave planner has $\zeta = -8$. Likewise, Figures 3 and 4 respectively show the behavior of several variables across state transitions, and the autocorrelation structure of labor taxes, if the IES is 4 and markets are complete. The concave planner has $\zeta = 8$. The results are qualitatively extremely similar to each other and the log-log case in the main text. The primary difference is that the planner chooses tax rates that depend more strongly on the current state and less on the previous state when the IES is low; this reflects the strong lack of Ricardian equivalence and incentive to run a balanced budget.

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†More examples to be added soon. Please check back if there’s a particular simulation you were hoping to see.

Unfortunately, switching to a recursive representation that does not place all policies affecting a given generation into the same static optimization problem means that certain intertemporal aspects of policy are less precisely measured, especially when the recursive representation features four state variables and is therefore solved imprecisely. This is the case with the incomplete markets results below. In particular, it is always suboptimal to grant lump-sum transfers in each state of the world at date $t + 1$ but levy a positive labor income tax at date $t$, though that may not be precisely enforced in these results. Additionally, capital taxes are likely to be measured with substantial error. Furthermore, the more concave the planner, the more likely intergenerational concerns are to massively overshadow intragenerational policies, and therefore the more likely these errors are to be large.
It is worth noting that comparing concave planners in different models is not straightforward; one must take a stand on whether the planner actually cares about inequality in cardinal utility or inequality in consumption equivalents. I will not attempt that exercise here.

Incomplete Markets. Figures 5 and 6 respectively show the behavior of several variables across state transitions, and the autocorrelation structure of labor taxes, if the IES is 1/4 and markets are incomplete. The concave planner has $\zeta = -8$. Likewise, Figures 7 and 8 respectively show the behavior of several variables across state transitions, and the autocorrelation structure of labor taxes, if the IES is 4 and markets are complete. The concave planner has $\zeta = 8$. Again, these are quite qualitatively similar to their log-log equivalents, with just a couple of changes. Lump-sum transfers to the old do play a somewhat larger role, even in percentage terms and even for the utilitarian planner—especially for low IES. This stems from the issue mentioned above: Utilitarian (or others) planners’ preference for equality in dollar terms is not constant, but depends on households’ risk aversion. Since these higher $\sigma$ households have higher risk aversion, even a utilitarian planner will have a stronger motive for redistribution and therefore compel the old to hold a larger share of risk. The other alteration is that, for the first time, we see that capital is non-negligibly taxed (or subsidized) by the concave planner. This occurs with the opposite sign that would be predicted by the hedging term, and therefore suggests that income effects dominate the capital tax discussion.

Productivity Shocks

I limit discussion to complete markets models. As mentioned in the text, this seems the more natural assumption when discussing productivity shocks. Figures 9, 10, and 11 respectively show the behavior of several variables across state transitions, the autocorrelation structure of labor taxes, and the autocorrelation structure of SS payments if the IES is 1/4 and markets are complete. The concave planner has $\zeta = -2$. Likewise, Figures 12, 13, and 14 show the same when the IES is 4. The concave planner has $\zeta = 8$. The primary difference was highlighted in the text: The fact that income effects are so strong when $\sigma = 4$ means that taxes are actually higher on less well-off generations. Policies are more variable when $\sigma = 1/4$, since the economy is closer to Ricardian and thus redistributive policy is more palatable.

2Additionally, the caveats mentioned in the first footnote to this Appendix apply especially strongly here, making incomplete markets results highly dubious.
Figure 1: This figure captures the salient features of state transitions in the complete markets model of stochastic government expenditure when utility is isoelastic with $\sigma = 4$. Shading marks periods of high government spending (“wars”). The concave planner has $\zeta = -8$. The model was first allowed to run for 100 periods in state 1 to arrive at a steady state. The Ramsey model was started with debt equal to -1.7% of nonstochastic steady state GDP, which corresponds to the level of government debt in the OLG model after 100 periods of low government spending; initial conditions are irrelevant for the OLG models.
Figure 2: This figure presents the results of a random simulation of the complete markets model of stochastic government expenditure when utility is isoelastic with $\sigma = 4$. The model is simulated over 10000 periods (after a 100-period startup period that is excluded). The concave planner has $\zeta = -8$. The lagged labor tax rate is on the horizontal axis, and the current labor tax rate is on the vertical axis. The different shades of gray represent different combinations of the state $s$ and lagged state $s_{-1}$. The Ramsey model was started with debt equal to -1.7% of nonstochastic steady state GDP, which corresponds to the level of government debt in the OLG model after 100 periods of low government spending; initial conditions are irrelevant for the OLG models.
Figure 3: This figure captures the salient features of state transitions in the complete markets model of stochastic government expenditure when utility is isoelastic with $\sigma = 1/4$. Shading marks periods of high government spending ("wars"). The concave planner has $\zeta = 8$. The model was first allowed to run for 100 periods in state 1 to arrive at a steady state. The Ramsey model was started with debt equal to -3.5% of nonstochastic steady state GDP, which corresponds to the level of government debt in the OLG model after 100 periods of low government spending; initial conditions are irrelevant for the OLG models.
Figure 4: This figure presents the results of a random simulation of the complete markets model of stochastic government expenditure when utility is isoelastic with $\sigma = 1/4$. The model is simulated over 10000 periods (after a 100-period startup period that is excluded). The concave planner has $\zeta = -8$. The lagged labor tax rate is on the horizontal axis, and the current labor tax rate is on the vertical axis. The different shades of gray represent different combinations of the state $s$ and lagged state $s_{-1}$. The Ramsey model was started with debt equal to -3.5% of nonstochastic steady state GDP, which corresponds to the level of government debt in the OLG model after 100 periods of low government spending; initial conditions are irrelevant for the OLG models.
Figure 5: This figure captures the salient features of state transitions in the incomplete markets model of stochastic government expenditure when utility is isoelastic with $\sigma = 4$. Shading marks periods of high government spending ("wars"). The concave planner has $\zeta = -8$. The model was first allowed to run for 100 periods in state 1 to arrive at a steady state.
Figure 6: This figure presents the results of a random simulation of the incomplete markets model of stochastic government expenditure when utility is isoelastic with $\sigma = 4$. The model is simulated over 10000 periods (after a 100-period startup period that is excluded). The concave planner has $\zeta = -8$. The lagged labor tax rate is on the horizontal axis, and the current labor tax rate is on the vertical axis. The different shades of gray represent different combinations of the state $s$ and lagged state $s_{-1}$.
Figure 7: This figure captures the salient features of state transitions in the incomplete markets model of stochastic government expenditure when utility is isoelastic with $\sigma = 1/4$. Shading marks periods of high government spending ("wars"). The concave planner has $\zeta = 8$. The model was first allowed to run for 100 periods in state 1 to arrive at a steady state.
Figure 8: This figure presents the results of a random simulation of the incomplete markets model of stochastic government expenditure when utility is isoelastic with $\sigma = 1/4$. The model is simulated over 10000 periods (after a 100-period startup period that is excluded). The concave planner has $\zeta = -8$. The lagged labor tax rate is on the horizontal axis, and the current labor tax rate is on the vertical axis. The different shades of gray represent different combinations of the state $s$ and lagged state $s_{-1}$. 
Figure 9: This figure captures the salient features of state transitions in the complete markets model of stochastic productivity when utility is isoelastic with $\sigma = 4$. Shading marks periods of low productivity. The concave planner has $\zeta = -2$. The model was first allowed to run for 100 periods in state 1 to arrive at a steady state.
Figure 10: This figure presents the results of a random simulation of the complete markets model of stochastic productivity when utility is isoelastic with \( \sigma = 4 \). The model is simulated over 10000 periods (after a 100-period startup period that is excluded). The concave planner has \( \zeta = -2 \). The lagged labor tax rate is on the horizontal axis, and the current labor tax rate is on the vertical axis. The different shades of gray represent different combinations of the state \( s \) and lagged state \( s_{-1} \).
Figure 11: This figure presents the results of a random simulation of the complete markets model of stochastic productivity when utility is isoelastic with $\sigma = 4$. The model is simulated over 10000 periods (after a 100-period startup period that is excluded). The concave planner has $\zeta = -2$. The lagged debt (SS) payment is on the horizontal axis, and the current debt (SS) payment is on the vertical axis. The different shades of gray represent different combinations of the state $s$ and lagged state $s_{-1}$. 
Figure 12: This figure captures the salient features of state transitions in the complete markets model of stochastic productivity when utility is isoelastic with $\sigma = 1/4$. Shading marks periods of low productivity. The concave planner has $\zeta = 8$. The model was first allowed to run for 100 periods in state 1 to arrive at a steady state.
Figure 13: This figure presents the results of a random simulation of the complete markets model of stochastic productivity when utility is isoelastic with $\sigma = 1/4$. The model is simulated over 10000 periods (after a 100-period startup period that is excluded). The concave planner has $\zeta = 8$. The lagged labor tax rate is on the horizontal axis, and the current labor tax rate is on the vertical axis. The different shades of gray represent different combinations of the state $s$ and lagged state $s_{-1}$. 
Figure 14: This figure presents the results of a random simulation of the complete markets model of stochastic productivity when utility is isoelastic with $\sigma = 1/4$. The model is simulated over 10000 periods (after a 100-period startup period that is excluded). The concave planner has $\zeta = 8$. The lagged debt (SS) payment is on the horizontal axis, and the current debt (SS) payment is on the vertical axis. The different shades of gray represent different combinations of the state $s$ and lagged state $s_{-1}$. 