All Things Equal: Social Networks as a Mechanism for Discrimination*

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Abstract

I study labor markets in which firms can hire via referrals. Despite full equality in the initial time period (e.g., equal ability, employment, wages, and network structure), *unequal* wages and employment still emerge over time between majority and minority workers, due to homophily—the well-documented tendency for people to associate more with others similar to themselves. This inequality can be mitigated by minority workers having more social ties or a "stronger-knit" network. Hence, this paper uncovers a mechanism for discriminatory outcomes that neither relies on past inequality nor on discriminatory motives (i.e., this form of discrimination is distinct from both dominant economic models of taste-based and statistical discrimination). These findings introduce multiple policy implications, including disproving a primary justification for "colorblind" approaches—namely disproving the position that such approaches are inherently merit-enhancing.

Keywords: discrimination, diversity, employment, homophily, inequality, labor market, social networks

JEL Codes: D63, D85, J15, J16, J31, J71, Z13

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1 Introduction

U.S. history has seen Black workers suffer significant labor market disadvantages, receiving lower average income and levels of employment. The vestiges of slavery and state-enforced racial segregation directly and indirectly have contributed to these disparities, confounding efforts to foster equality.

Suppose, however, that the circumstances of different demographic groups were equalized—that the slate were wiped clean. In a world without prejudice—and one beginning in a state of equality—would labor market disparities still arise between majority and minority workers? In this paper, I develop an employment model and assume equal ability, employment, and network structure between majority and minority workers in the initial time period. Despite these equalizing assumptions, I find that if majority and minority workers have (1) an equal chance of having a social tie (i.e., an equivalent network density) and (2) an equal bias in favor of forming same-group social ties (i.e., an equivalent type in-group bias), then the probability of a firm offering a job through referral to minority workers is *lower* than their share of the labor force. For minority workers to have a proportional chance of receiving job offers through referral, they must compensate with a stronger network density and/or type in-group bias. The estimated welfare gap increases in a convex way with the majority group share of the labor force, suggesting that the disadvantage of minority workers magnifies with the degree of their minority status in the labor market. Finally, this paper calibrates the model, determining under what conditions female doctors (who represent a minority of U.S. physicians) achieve gender parity in referrals.

While the model works for arbitrary group assignments, social science research suggests that race and ethnicity create the greatest divide socially, with gender also proving relevant (McPherson, Smith-Lovin, and Cook 2001).[1] Hence, this paper's findings impact the debates surrounding policies that explicitly support racial minorities and women. Many

1. According to this research, after race and ethnicity, the greatest social divides are created by age, religion, education, occupation, and gender, in approximately that order.

who debate such policies share a common assumption: that in a world without historical discrimination—without misogyny or Jim Crow or implicit biases—"no policy" would be the best policy. No policy would yield the most meritocratic outcome, with opportunities distributed according to corresponding ability or "merit."[2] Yet the findings in this paper suggest otherwise. Achieving equality of opportunity may remain elusive even in the absence of psychological prejudice, historical wrongs, and differences in ability or education among the population.[3] Advantages from being within a larger or more strongly-connected social network may persist, despite one's talent. All else equal, outcomes may remain unequal.

This paper makes five contributions. First, the paper presents a novel theoretical approach for uncovering discriminatory outcomes independent from discriminatory motives (i.e., independent from both prevailing models of discrimination in economics: taste-based and statistical)—these discriminatory outcomes arise even under an initial state of equality (unlike in previous work, like Calvo-Armengol and Jackson (2004)). In other words, not only does this paper develop a standard labor-market model that reveals the limitations of past economic models of discrimination, but also this work offers a direct rigorous account of "institutional discrimination" using mainstream economic theory.[4] Second, this paper performs equilibrium analysis of employment and wage differences caused by homophily along majority/minority status, which is a contribution to both the economic and sociological literatures on labor markets. Third, in doing so the paper isolates a potential underlying mechanism for inequality, adding to our understanding of labor market disparities that have been widely studied across the social sciences (e.g., ones relating to both race and gender). The paper

2. Various members of the Supreme Court have voiced this sentiment, along with some legal scholars, particularly in the context of "colorblind" policies. For economic analysis of the implications of various color-blind policies, see, e.g., Chan and Eyster (2003) and Ray and Sethi (2010).

3. Notwithstanding little change in recent support for redistribution despite rises in inequality (Ashok, Kuziemko, and Washington 2015), greater understanding of the fairness (or lack thereof) of the economic system might influence some people's preferences for redistribution (Alesina, Stantcheva, and Teso 2018).

4. Small and Pager (2020) defines "institutional discrimination" as differential treatment by race that is either perpetrated by organizations or codified into law. This article also provides a lengthy discussion on the divergence between economic and sociological approaches to discrimination, as well as propositions from the sociology of racial discrimination "worth noting by economists." Yet the discrimination in this paper does not rely on organizations beginning as racially homophilous, unlike the example presented in Small and Pager (2020).

accomplishes this while making predictions on when disparities might arise—as well as on when they might not. Fourth, this paper introduces a parameterization of homophily that refines one that has been used in the social network theory literature; the refinement is especially useful when evaluating equal bias in favor of forming same-group social ties between two groups of different sizes.[5] Fifth, the paper calibrates the model using 2008-2012 data on U.S. physician referrals and determines under what conditions gender parity would be achieved. In this calibration exercise, female physicians represent the minority group and male physicians the majority group, based on relative levels of representation in the industry. The type in-group bias required for female physicians to receive a proportional number of referrals is higher than both its initial value *and* the initial type in-group bias of male physicians. In order to rectify the labor market disadvantage, 1.3 million (out of 7.8 million)—or 18%—of female-initiated referrals that initially went to male physicians would instead need to go to female physicians.

Related Literature.— The model in this paper extends the one from Montgomery (1991) to incorporate two-dimensional heterogeneity: while the original model only groups workers by ability, this paper also groups them by majority/minority status. Doing so yields findings that go far beyond a mere application of the base model. The original model does not focus on outcomes when homophily (the well-documented tendency for people to associate more with others similar to themselves) exists along characteristics *uncorrelated* with ability—namely, on what effects emerge when social ties are formed along dimensions orthogonal to productivity. The original model does not incorporate demographic considerations.

Filling this gap is important. Within sociology, research has explored homophily in various contexts, including its causes (Wimmer and Lewis 2010; Leszczensky and Pink 2019) as well as how it influences friendships (Blau 1977; Syed and Juan 2012), interethnic marriages (Skvoretz 2013), and social inequality (DiMaggio and Garip 2012), among other areas.

5. See Equation 1 in the Equilibrium Section (after reading the simplified numerical example that immediately precedes it); also read the detailed exposition in the Discussion Section.

Within economics, there is an extant literature on the impact of referrals on inequality,[6] yet findings have relied on the existence of some degree of prior period inequality—namely, that if a demographic group has higher past employment, then that would yield an advantage in securing future jobs. This paper makes a new contribution combining insights from both fields, demonstrating that referral advantages may still unequally accrue over time even under initial equality, due to homophily. In particular, this paper adds a theoretical foundation for why homophily may contribute to inequality in referral markets, as well as predicts under what conditions such disparities will *not* surface.

There has been increasing focus on uncovering mechanisms behind racial disparities in labor market outcomes (Bayer and Charles 2018; Chetty et al. 2020), wage inequalities (Card and Lemieux 1994; Lemieux 2006), as well as the persistent gender-wage gap (Bertrand, Goldin, and Katz 2010; Blau and Kahn 2017). Evidence for racial and gender differences in networking outcomes exists (Korenman and Turner 1996; Lalanne and Seabright 2011; Mengel 2015; Lindenlaub and Prummer 2016). Zeltzer (2020) presents empirical evidence to suggest that gender homophily is a significant factor in explaining the gender-wage gap among medical professionals. Jackson (2009) discusses how homophily leads to segregation of groups, which leads to different equilibrium investment decisions in areas like education. Yet this paper introduces a more direct mechanism for inequality from homophily: that even given *equal* investments in human capital, homophily may still directly foster disparities through hiring dynamics.[7] Moreover, unlike in Calvo-Armengol and Jackson (2004), which provides a network-based mechanism for perpetuating pre-existing inequality, this paper uncovers inequality even without historical labor market disadvantages. In addition, this paper's findings are unlike those from sociology that link homophily to social inequality (see,

6. For some of the evidence, see Arrow and Borzekowski (2004), Ioannides and Datcher Loury (2004), Bayer, Ross, and Topa (2008), Hellerstein, McInerney, and Neumark (2011), Renneboog and Zhao (2011), Burks et al. (2015), and Pallais and Sands (2016). Relatedly, Cai and Szeidl (2018) studies the effect of business networks on firm performance.

7. Early versions of these findings can be found in the author's undergraduate honors thesis (see Okafor (2007)). Buhai and Leij (2020) explores how inequality can result from different choices in skill specialization, i.e., different investments in human capital.

e.g., DiMaggio and Garip 2012); in this paper, in contrast, homophily's impact on inequality does not operate through a mechanism that exacerbates individual level differences (recall our model assumes groups have *equal* ability and initial employment). Notably, this paper's findings are also fully distinct from past economics and sociology research on the influence of more traditional discrimination in hiring (see, e.g., Bertrand and Mullainathan 2004; Pager, Bonikowski, and Western 2009). Unlike those articles, this one uncovers disparities even in contexts in which discriminatory motives and implicit biases are not only absent but impossible, as the model in this paper does not allow firms to distinguish who is a majority and who is a minority worker. Hence, the inequality this paper uncovers may relate to—or exacerbate—broader disadvantages associated with structural racism (see, e.g., Williams, Lawrence, and Davis 2019).

This paper proceeds as follows: Section 2 introduces a formal setup of the model. Section 3 presents the model's key findings for majority and minority workers. Section 4 provides discussion. Section 5 performs a calibration of the model under simplifying assumptions. Section 6 concludes.

2 Model

Here, I extend the Montgomery (1991) two-period model to incorporate two-dimensional heterogeneity: while the original model only groups workers by ability, this one also groups them by majority/minority status.[8]

Workers: I consider a labor market with two time periods ($t = 1$ and $t = 2$) and many workers, with an equal measure in each period.[9] Each worker works one period, and is one of two types: majority or minority. Each worker's type is predetermined and assigned before the period in which he or she enters the market. The fraction of majority workers is $\delta > \frac{1}{2}$,

8. Most of the model's assumptions are standard in labor-market models of adverse selection, especially that of Greenwald (1986) (Montgomery 1991).

9. Similar to Montgomery (1991), I simplify the analysis by examining the model as the number of workers approaches infinity.

while $1-\delta < \frac{1}{2}$ are minority. Similar to Montgomery (1991), I assume that $\frac{1}{2}$ of the workers within each type are high-ability, while $\frac{1}{2}$ are low-ability. High-ability workers produce one unit of output, while low-ability workers produce zero units. Workers are observationally equivalent: firms neither know what ability workers possess (before production), nor whether workers are of the majority or minority type (at any time).[10]

Firms: Firms are free to enter the market in either period. At most, each firm may employ one worker. A firm's profit in each period is equal to the productivity of its worker minus the wage paid.[11] Each firm must set wages before it learns the productivity of its worker. There are no output-contingent contracts.[12]

Structure of Social Network: As the focus of the model is referrals, now I describe how the social network through which referrals occur is drawn. As described, there are four categories of workers: high-ability majority, high-ability minority, low-ability majority, and low-ability minority. Now let us represent each period-2 worker as an urn, and each social tie that a period-1 worker possesses as a ball.[13] The assignment of social ties is equivalent to a scenario where the balls are randomly dropped into the urns.[14] A period-1 worker possesses a social tie ("ball") with a probability equal to its majority/minority type's network density (denoted by τ_{maj} or τ_{min}).[15] A period-1 worker's sole social tie, if they have one, is dropped into an "urn" (period-2 worker) which is: (1) of the same ability with probability $\alpha \in (\frac{1}{2}, 1)$[16]; and (2) of the same majority/minority type with a probability determined by the period-1 worker's in-group bias.[17] The network structure is thus characterized by three parameters:

10. Later we see that period-1 workers' actions are nonstrategic; hence, no assumption needs to be made on their knowledge of their own or of period-2 workers' types.

11. Product price is exogenously determined and normalized to unity.

12. This assumption captures a significant rationale for screening of job applicants and the use of referrals: the inability to fully tie compensation to productivity. See Montgomery (1991) and Greenwald (1986) for further discussion of this assumption.

13. The "urn-ball" model is standard in probability theory and has been used in various economics models. For more background on the "urn-ball" model, see Shimer (2007).

14. Hence, period-2 workers can have zero, one or more than one social tie across period-1 workers.

15. If the period-1 worker is a majority worker, he or she possesses a social tie with probability $\tau_{maj} \in (0, 1)$; a minority worker possesses a social tie with probability $\tau_{min} \in (0, 1)$.

16. The matched period-2 worker is hence of a different ability with probability $1 - \alpha \in (0, \frac{1}{2})$.

17. The following subsection explains both the conceptualization and the parameterization of "in-group bias."

network density (τ_{maj} and τ_{min}), majority/minority type in-group bias (denoted by ψ_{maj} and ψ_{min}, respectively), and ability in-group bias (α).

Timing: Each firm hires a period-1 worker through the market and learns his or her ability. As period-1 workers are observationally equivalent (and cannot be referred for jobs since there is no previous time period), each firm hiring through the market receives a high-ability worker with probability $\frac{1}{2}$.

After learning the ability of its current worker, each firm may set a referral offer to be relayed to the worker's social tie. Whether the referral offer is relayed is conditional on the firm's worker holding a social tie. If he or she does hold one, then the firm will only attract the acquaintance if the referral offer exceeds both the period-2 market wage and all other referral offers received by the acquaintance. A firm not wishing to hire through referral will set no referral offer (or might just set a referral offer below the period-2 market wage, which has no probability of acceptance). Period-2 workers then compare all offers received, accepting the highest.

All period-2 workers who receive no referral offers must find employment through the general market.

In summary, the timing of the game is as follows.

1. Each firm hires period-1 workers through the market at a wage of w_{M1}.

2. Period-1 production occurs, after which each firm learns the productivity of its worker.

3. Social ties are determined.

4. If a firm wishes to hire through referral, it sets a referral offer. I denote firm i's referral offer by w_{Ri}. (Conditional on having a social tie, each period-1 worker then relays his or her firm's wage offer (w_{Ri}) to their period-2 acquaintance.)

5. Each period-2 worker compares all wage offers received. They either accept one or wait to find employment through the general market.

6. Any period-2 worker with no offers (or who refuses all offers) goes on the market. Wages in this market are denoted w_{M2}.

7. Period 2 production occurs.

2.1 Note on In-Group Bias

The "in-group bias" can be conceptualized as capturing the fact that there are shared attributes that simply make it easier for some workers to form social ties with each other than with others. For example, for a given chance encounter, one is more likely to form a social tie with another worker who shares a more similar background, because there are simply more elements in common to establish the foundation of a relationship. Hence, "in-group bias" does not represent favoritism toward a demographic group: in the model, any given worker views members of one's own majority/minority group and members of the other group equivalently, conditional on having a social tie. Similarly, conditional on *not* having a social tie, any given worker views members of the same majority/minority group and members of the other group with the same level of (dis)interest. Hence, although "in-group bias" deeply impacts network formation, it is fully distinct from (racial or group) animus, in-group favoritism, or traditional conceptions of taste-based preferences.[18]

18. Proposition 1 and the Discussion Section describe why the following specification for "in-group bias" is used: Pr{period-1 worker knows own majority/minority type} $= \frac{w \cdot \psi}{[w \cdot \psi]+[(1-w)(1-\psi)]}$, where w is the share of the labor force for the worker type (either δ or $1-\delta$) and ψ is the type in-group bias of the worker type (either ψ_{maj} or ψ_{min}).

Figure 1: Model Parameters

Parameter	Name	Description	Range
δ	Majority share	Share of the total labor force comprised of majority workers	$\delta \in (\frac{1}{2}, 1)$
$1-\delta$	Minority share	Share of the total labor force comprised of minority workers	$1-\delta \in (0, \frac{1}{2})$
α	Ability in-group bias	Probability a worker's social tie is with another worker of equal ability	$\alpha \in (\frac{1}{2}, 1)$
τ_{maj}	Network density (majority group)	Probability a majority worker has a social tie	$\tau_{maj} \in (0, 1)$
τ_{min}	Network density (minority group)	Probability a minority worker has a social tie	$\tau_{min} \in (0, 1)$
ψ_{maj}	Type in-group bias (majority group)	Bias a majority worker exhibits toward workers of the same group in favor of forming social ties. Value of 1/2 means no bias (probability of social tie with another majority worker = δ). Value of 1 means full bias (probability of social tie with another majority worker = 1).* I assume neither full nor no bias.	$\psi_{maj} \in (\frac{1}{2}, 1)$
ψ_{min}	Type in-group bias (minority group)	Bias a minority worker exhibits toward workers of the same group in favor of forming social ties. Value of 1/2 means no bias (probability of social tie with another minority worker = $1-\delta$). Value of 1 means full bias (probability of social tie with another minority worker = 1).* I assume neither full nor no bias.	$\psi_{min} \in (\frac{1}{2}, 1)$

*$\psi \in [0, \frac{1}{2})$ represents heterophily, a rare social network phenomenon outside the scope of this paper.

3 Equilibrium

I examine a competitive equilibrium of the economy in which firms seek to maximize profits.

The first subsection below presents basic equilibrium properties shared with Montgomery (1991). The second subsection presents new propositions that I have added to the model by incorporating two-dimensional heterogeneiy (i.e., categorization of workers by majority and minority type in addition to (instead of solely by) ability level). These new propositions relate to discrimination and labor market disparities.

3.1 Basic Equilibrium Properties

Basic equilibrium properties can be expressed via the following three lemmas, which establish that (1) referral wage offers are dispersed within an interval between the period-2 market wage and a maximum referral wage offer; (2) a firm will only hire through referral if it employs a high-ability worker in period 1; and (3) the period-1 market wage is greater than the expected period-1 productivity. More discussion on each of these points is included below. Omitted proofs are found in the Appendix.

Lemma 1 *Referral wage offers lie within the interval between w_{M2} and a maximum referral wage offer $\overline{w}_R$; hence, $w_R \in [w_{M2}, \overline{w}_R]$. The density of the referral wage offer distribution is positive across this entire range.*

Proof. Claim 4 in Burdett and Judd (1983) establishes the existence and uniqueness of an equilibrium, while Theorem 4 proves wage dispersion exists in the equilibrium (since the probability that a period-2 worker receives exactly one referral offer is strictly between 0 and 1). Given this wage dispersion, the market wage (w_{M2}) must coincide with the bottom of the referral wage distribution, as any referral offer below the market wage will necessarily be rejected by workers in favor of going to the general market. At the maximum referral wage offer (denoted $\overline{w}_R$ and derived in Appendix Equation A.7), the probability a worker accepts the referral offer is 1. Hence, firms will not offer a referral wage above this amount as it will necessarily reduce profits. Proposition 2.2 in Butters (1977) proves that there are no gaps in the wage distribution. If there were a gap between some w_1 and w_2, then a firm offering the higher wage could reduce its offer by ϵ without reducing the probability its offer is accepted, thereby increasing its profits. ■

Lemma 2 *A firm will attempt to hire through referral if and only if it employs a high-ability worker in period 1.*

This result follows from the ability in-group bias (α). Hiring through the market yields zero expected profit (due to the free entry of firms and the symmetric lack of information on the ability of workers). For firms employing high-ability workers in period 1, an accepted referral offer yields constant positive profit over the range of the referral offer distribution $[w_{M2}, \overline{w}_R]$. Higher wage offers yield a higher probability of attracting a period-2 worker. Firms employing low-ability workers in period 1 will not hire through the referral market, since the ability in-group bias means the referred worker will more likely also be low-ability.

As a result of this lemma, a disproportionately high number of low-ability workers find employment through the general market. This drives the market wage below the average productivity of the entire population. However, adverse selection does not eliminate the market. Since some high-ability workers are not "well-connected," they fail to receive referral wage offers, which leads them to find employment in the general market. Thus, the market wage remains above zero.

Lemma 3 *The period-1 market wage is greater than the expected period-1 productivity.*

If a firm obtains a high-ability worker in period 1, they expect positive period-2 profits. This fact drives the period-1 market wage higher than the productivity of the population. This wage can be viewed as comprising the average productivity of the worker plus an "option value" of a period-2 referral. This option will be exercised if the period-1 worker reveals themselves to be high-ability (which occurs after period-1 production concludes, if the worker is in fact high-ability).

3.2 Propositions on Discrimination

New propositions reflecting discrimination and inequality are detailed below. The propositions establish that minority workers receive a disproportionately low fraction of job offers through referral and a lower expected wage, all else equal. Recall that the market wage lies below the referral offer distribution. Hence, all these effects taken together yield a welfare gap between minority and majority workers in period 2 that did not exist in period 1. Omitted proofs are in the Appendix.

Proposition 1A *In an environment with equal magnitude of majority/minority network parameters ($\tau_{maj} = \tau_{min}$ and $\psi_{maj} = \psi_{min}$), the probability a high-ability minority worker in period 2 receives a referral offer is lower than their share of the labor force. The inverse holds for majority workers:*

$$\Pr\{\textit{period-2 high-ability minority worker receives referral offer}\} < \frac{1-\delta}{2}$$
$$\Pr\{\textit{period-2 high-ability majority worker receives referral offer}\} > \frac{\delta}{2}$$

Proposition 1B *The inequality in the distribution of referral job offers can be eliminated by minority workers having a sufficiently higher type in-group bias (ψ_{min}).*

Simplified Numerical Example: First I explore the intuition of Proposition 1A by using a simple numerical example that illustrates type in-group bias.[19] Suppose there are two majority workers in both periods, and one minority worker in both periods—i.e., $\delta = 2/3$. Also suppose there is type in-group bias: each worker has a bias in favor of forming social ties with workers of the same majority/minority type. To illustrate this bias, let us say that for encounters between majority period-1 and majority period-2 workers, there is a 2/3 chance of forming a tie, whereas encounters between a majority period-1 and minority period-2 worker has a 1/3 chance of forming a tie—i.e., $\psi = 2/3$. Suppose all three period-1 workers encounter all three period-2 workers. The expected number of ties period-1 majority workers form with their own type is thus 4/3 (while the expected number of ties with minority

19. This example is illustrative and does not incorporate ability heterogeneity (i.e., it assumes all workers are high-ability), so is simplified relative to the main model. Furthermore, the example includes a finite number of workers whereas the model includes a continuum.

workers is 1/3); this means the fraction of same-type social ties for majority workers is 4/5. Let $\phi_{maj} = 0.8$ denote this fraction of same-type social ties (note that ϕ is not a parameter of the model and can be calculated directly from ψ via Equation 1 below). Similarly, it is straightforward to calculate that the fraction of same-type social ties for the period-1 minority worker is $\phi_{min} = 0.5$.

Now let us apply these bias dynamics to a case in which period-1 workers each have exactly one social tie with probability 1 (i.e., $\tau_{maj} = \tau_{min} = 1$). The fraction of referral job offers going to majority workers is simply a weighted sum:

$$
\begin{aligned}
&\Pr\{\text{referral job offer goes to majority per-2 worker}\} \\
&= \Pr\{\text{per-1 worker is majority}\} \cdot \Pr\{\text{per-1 majority knows per-2 majority}\} \\
&\qquad + \Pr\{\text{per-1 worker is minority}\} \cdot \Pr\{\text{per-1 minority knows per-2 majority}\} \\
&= \delta \cdot \tau_{maj} \cdot \phi_{maj} + (1-\delta) \cdot \tau_{min} \cdot (1-\phi_{min}) \\
&= 2/3 \cdot 1 \cdot 0.8 + 1/3 \cdot 1 \cdot 0.5 \\
&= 0.7
\end{aligned}
$$

Hence, only 0.3 of referral job offers go to minority workers, even though they occupy 1/3 of the labor force. This simple example illustrates the distorting influence of having the same magnitude of bias operating on groups of different sizes. The bias in favor of a same-group social tie for a majority worker extends toward a greater fraction of the population than does the same magnitude of bias for a minority worker. Hence, equal magnitudes of bias *unequally* impact the respective chances of knowing workers of the same type. [**End of Example**]

I can generalize both the reasoning of the simplified numerical example above and the type-in group bias of the model as follows:

$$
(1) \qquad \Pr\{\text{period-1 worker knows own maj/min type}\} = \phi = \frac{w \cdot \psi}{[w \cdot \psi] + [(1-w)(1-\psi)]}
$$

where w is the share of the labor force for the worker type (either δ or $1-\delta$) and ψ is the in-group bias of the worker type (either ψ_{maj} or ψ_{min}). $\psi = 0.5$ reflects no bias—i.e., a proportional chance of a social tie being with another of the same type—and $\psi = 1$ reflects when all ties are with members of the same type. ϕ represents either ϕ_{maj} or ϕ_{min}, depending on whether the period-1 worker belongs to the majority or minority group, respectively. The Discussion section further explores the relationship between ψ and ϕ, and its implications on the results.

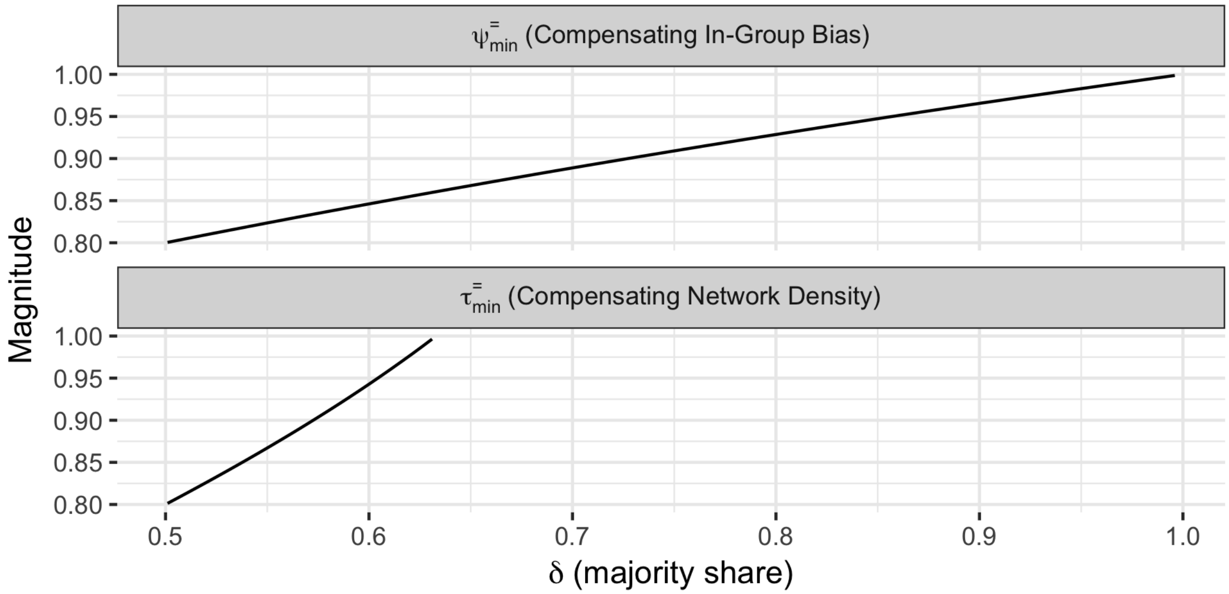
In the Appendix, I prove that parity in the distribution of job offers between high-ability majority and minority workers is accomplished only when:

$$(2) \quad (1-\delta)\left[\delta\tau_{maj}\phi_{maj} + (1-\delta)\tau_{min}(1-\phi_{min})\right] = \delta\left[\delta\tau_{maj}(1-\phi_{maj}) + (1-\delta)\tau_{min}(\phi_{min})\right]$$

where ϕ_{maj} and ϕ_{min} are calculated from Equation 1 above. From Equation 2, one can calculate what magnitude other parameters must be for parity. I denote such parameters for minority workers as $\tau^{=}_{min}$ (compensating network density) and $\psi^{=}_{min}$ (compensating in-group bias). Figure 2 illustrates that a sufficiently high network density or type in-group bias can mitigate the disproportionality in the distribution of job offers through referral. All else equal, minority workers can either have more social ties ($\tau^{=}_{min} > \tau_{maj}$) or a "stronger-knit" social network ($\psi^{=}_{min} > \psi_{maj}$).

The network density required to eliminate the inequality ($\tau^{=}_{min}$) increases in τ_{maj}, ψ_{maj}, and δ. It decreases in ψ_{min}, which can readily be understood intuitively. The greater the probability of majority workers having social ties (and/or the greater the degree of their homophily), the greater minority workers' compensating parameters ($\tau^{=}_{min}$ or $\psi^{=}_{min}$) must be to achieve a proportional amount of all job offers through referrals. In Figure 2, the plot of $\tau^{=}_{min}$ has no values when δ is greater than approximately 0.63. This is because there is no attainable magnitude of network density that will yield parity in the distribution of job offers when δ surpasses this threshold.

Figure 2: Magnitude of Minority Group Network Parameters
Required for Parity in Referral Job Offers*



*In each chart, all other relevant network parameters = 0.8.

Proposition 2 *In an environment with equal magnitude of majority/minority network parameters ($\tau_{maj} = \tau_{min}$ and $\psi_{maj} = \psi_{min}$), the period-2 market wage (w_{M2}) decreases as majority workers occupy a greater fraction of the labor force.*

Recall that workers who do not receive jobs through referral must find employment through the market. Proposition 1 shows that minority workers, all else equal, receive a disproportionately low fraction of job offers through referral, and thus disproportionately find employment through the market. Decreases in the market wage (w_{M2}) thereby hurt the average welfare of minority workers, relative to that of majority workers.

The Appendix includes the expression for w_{M2}. Given $\alpha > \frac{1}{2}$, w_{M2} is always less than $\frac{1}{2}$, the average productivity of the population. Analysis shows that w_{M2} is decreasing in α. For all $\psi_{maj} = \psi_{min}$ and $\tau_{maj} = \tau_{min}$, w_{M2} also decreases in δ.

Proposition 3 *In an environment with equal magnitude of majority/minority network parameters ($\tau_{maj} = \tau_{min}$ and $\psi_{maj} = \psi_{min}$), the welfare (i.e., average expected wage) for minority workers is lower than for majority workers.*

Much of the intuition behind this finding follows from Proposition 1 (that majority workers receive a disproportionately high number of job offers through referral). There are two margins to consider. First, the extensive margin: majority workers disproportionately get hired through the referral market (which provides higher wages than the general market), driving up expected welfare for the majority group. Second, the intensive margin: recall that workers accept the maximum referral wage offer received. Hence, by majority workers receiving a higher number of referral offers, their expected maximum offer increases, thereby also driving up their relative welfare.

Let $E\prod_H(w_R)$ denote the expected period-2 profit earned by a firm employing a high-ability worker and setting a referral wage. To maintain equilibrium wage dispersion, firms must earn the same expected profit on each referral wage offered:

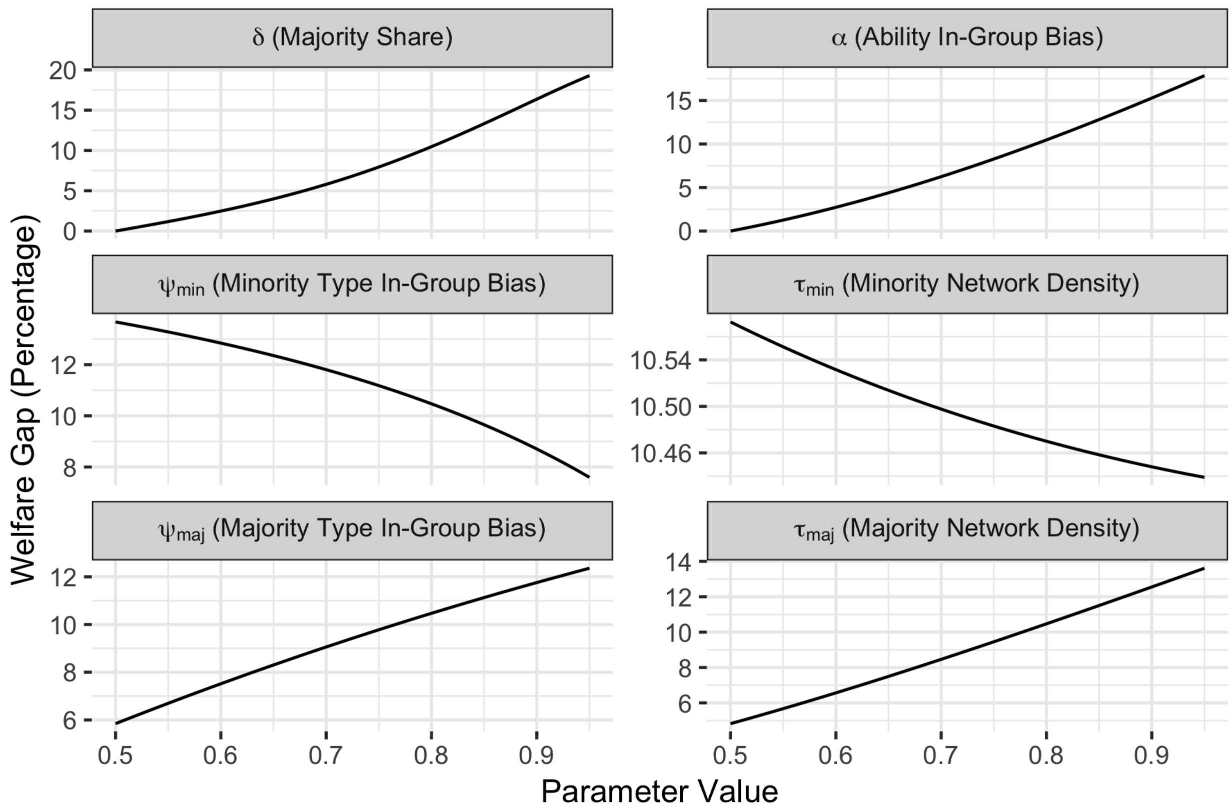
$$E\prod_H(w_R) = c \qquad \forall w_R \in [w_{M2}, \overline{w}_R]$$

Given the expression for c derived in the Appendix, firms with high-ability workers who have social ties earn positive expected profits as long as $\alpha > \frac{1}{2}$. Analysis shows that c is increasing in τ_{maj} and τ_{min}. Furthermore, the equilibrium referral-offer distribution $F(\bullet)$ may be determined by setting $E\prod_H(w_R)$ equal to c for all potential wage offers w_R.

Unfortunately, doing so does not yield a closed-form solution for $F(w_R)$. Given a continuum of firms, the equilibrium referral-offer distribution $F(\bullet)$ can be interpreted as either: 1) each firm randomizes over the entire distribution; or 2) a fraction $f(w_R)$ of firms offers each wage for sure. From the second interpretation, I denote these referral wages with w_{Rk}. One can then derive an expression for $w_{Rk}(\alpha, \delta, \tau_{maj}, \tau_{min}, \psi_{maj}, \psi_{min}, F(w_{Rk}))$ and calculate an average referral wage received by a majority worker (denoted $E(w_{R_{Hmaj}}))$ vs. a minority worker (denoted $E(w_{R_{Hmin}}))$, for any given δ, α, τ_{maj}, τ_{min}, ψ_{maj}, and ψ_{min}.

Analysis shows that when majority/minority network parameters are the same magnitude, if $\alpha > \frac{1}{2}$ and $\delta > \frac{1}{2}$, $E(w_{R_{Hmaj}}) > E(w_{R_{Hmin}})$. In other words, the expected referral wage for high-ability majority workers is greater than for high-ability minority workers.

Figure 3: Estimated Welfare Gap of Minority Workers*



*In each chart, the parameters not being varied all equal 0.8.

Welfare (i.e., expected wage) is calculated by summing the market wage (w_{M2}) and expected referral wage ($E(w_{R_{Hmaj}})$ or $E(w_{R_{Hmin}})$), weighted by the likelihood of the worker's type gaining employment through the market or through referrals, respectively. Figure 3 plots the estimated welfare gap between majority and minority workers as a function of various network parameters.[20] The welfare gap increases in δ, α, ψ_{maj}, and τ_{maj}. It decreases in ψ_{min} and τ_{min}. Of note, the welfare gap increases convexly as the majority group occupies a greater share of the labor force.

20. Estimation of wage gap normalizes to 1 the number of offers that referred high-ability minority workers receive and assumes a uniform distribution across the referral wage distribution ($w_R \sim U(w_{M2}, \overline{w}_R)$)

4 Discussion

This paper introduces a parameterization of homophily that refines one that has been used in other social network theory literature; the refinement is especially useful when evaluating equal bias in favor of forming same-group social ties between two groups of different sizes. For example, Coleman (1958) uses a specification that implies that the relationship between Pr{Period-1 worker knows own majority/minority type} and in-group bias would be linear.[21] The only justification the paper mentions for this parameterization of inbreeding is that it leads to a measure that "varies between zero and one" (pg. 34). Yet this linear scaling is plausibly less appropriate for modeling the bias individual workers exhibit across a population. To understand why, consider an individual worker from each group. For the majority worker, any given magnitude of bias in favor of same-group social ties operates on a greater share of the population than it does for a minority worker. Hence, one would expect the bias to have some multiplicative relationship with the share of the labor force, which the linear scaling does not have.[22] Furthermore, one would expect any given magnitude of bias to have an amplified effect on the likelihood of having a same-group social tie for a given majority worker compared to a given minority worker. A linear scaling does not account for the fact that the same magnitude of bias would plausibly have a disproportionately larger impact on the incidence of same-group social ties for the larger social group. The specification I use in this paper corrects for these shortcomings.

The relationship between this paper's parameterization of in-group bias and the incidence of social ties can be seen in Figure 4. Specifically, Figure 4 illustrates the sensitivity of same-group social ties (ϕ) to the in-group bias parameter (ψ). This relationship is based on the expression linking share of the labor force (w) to in-group bias described in Equation 1 of

21. Coleman's specification would be Pr(Period-1 worker knows own type) $= (1 - w) \cdot \psi + w$, with $\psi \in [0, 1]$. One recent paper that uses this specification is Currarini, Jackson, and Pin (2009).

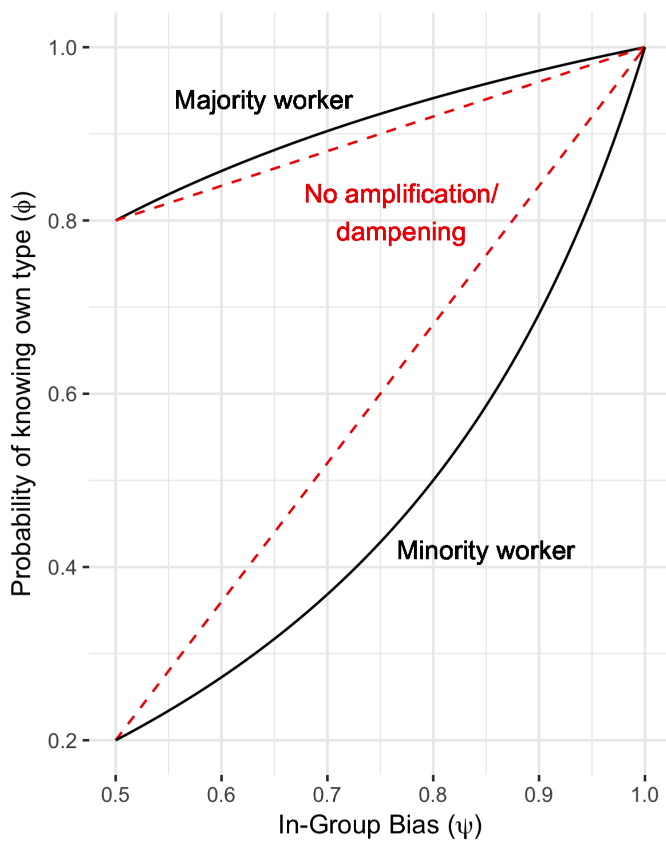
22. See the Conclusion for several examples of common real-world settings in which one would expect these social network dynamics to take place.

Proposition 1:

$$\Pr\{\text{period-1 worker knows own maj/min type}\} = \phi = \frac{w \cdot \psi}{[w \cdot \psi] + [(1-w)(1-\psi)]}$$

(The simplified numerical example immediately following Proposition 1 in the Equilibrium Section illustrates the logic of this expression.) When $\psi = 0.5$, there is no bias (i.e., probability of social ties with the same type $= w$); when $\psi = 1$, there is full bias (i.e., probability of social ties with the same type $= 1$). The dashed line represents a linear scaling, in which there is no amplification/dampening effect for majority and minority workers. Though not included in the graph below, the specification used in this article yields a linear relationship if both workers' groups occupy 50% of the labor force (i.e., when social groups are the same size there is no amplification/dampening effect).

Figure 4: Social Tie Sensitivity to In-Group Bias*



*where majority share of the labor force is 80%

The main findings in this paper are robust to specifications for bias—e.g., Equation 1 from Proposition 1—where the relationship between the probability of knowing one's own type (ϕ) and the bias (ψ) is more concave for majority workers than for minority workers. In other words, it is robust to specifications where there is a comparative amplification effect on the incidence of same-group social ties for majority workers (as seen in Figure 4). This relationship captures the fact that an equal magnitude of bias has a disproportionately larger impact on majority workers than on minority workers.

Recall that the predicted inequality between majority and minority workers is based on several assumptions, which include: (1) majority and minority workers have no labor market disparities in the initial time period; (2) the only distinguishable difference between groups is relative size (i.e., ability, network density, and in-group biases are all equivalent); (3) workers are more likely to know others with similar characteristics; and (4) there is no psychological prejudice. These assumptions are critical when considering historical examples where minority workers enjoy *greater* welfare than majority workers (e.g., white South Africans), or when particular demographic groups who comprise a majority of a local labor market face worse outcomes (e.g., Black workers in a variety of U.S. metropolitan areas). These cases do not undermine the accuracy of the model, not only because their circumstances clearly violate the model's assumptions (e.g., that there is full equality between groups in the initial time period), but also because these cases intimately involve the distorting influence or legacy of psychological prejudice, which the employment model explicitly and intentionally does not incorporate.

5 Calibration

This section calibrates the model to assess, in one application, the magnitudes of model parameters such as majority and minority type in-group bias, and to calculate the parameters that would be needed to restore parity in referrals. The setting for this calibration exercise is the physician referral market for Medicare beneficiaries between 2008-2012. In this context, male physicians represent the majority group (81% of referring physicians), while female physicians represent the minority group. I make some simplifying assumptions for the purpose of the calibration, but also relax the model assumption that there is no disadvantage for the minority group in the initial time period; gender disparities today persist from even starker historical disadvantages.

The data source is the Carrier database, which consists of panel data on all physician-billed services for a random sample of 20 percent of Medicare beneficiaries between 2008-2012. The data are managed by the Centers for Medicare and Medicaid Services (CMS) and encode the identities of the patients, physicians, and specialists. The sample contains information on patients with traditional fee-for-service Medicare (which comprises two-thirds of all Medicare beneficiaries) and represents a total of 35 million covered people and more than half a million physicians across the United States.

One simplifying assumption I use is that the referral behavior does not come from underlying unobserved ability differences. While this assumption may seem extreme, there is evidence that at least some element of the referral behavior of physicians is dependent on gender, independent of ability (Sarsons 2017; Zeltzer 2020). Furthermore, recall that our model assumes one social tie per period-1 worker; actual physicians undoubtedly have more. To account for this, I assume the sole social tie from the model simply corresponds with the strongest link (i.e., the physician who is actually referred). Other simplifying assumptions include: (1) that each referral corresponds to a distinct period-1 worker; (2) that each physician represents a high-ability worker; and (3) that the patient decision in selecting a physician is akin to the firm decision in hiring workers (in both cases, the referral presumably reduces uncertainty about the ability of the physician/worker being hired). Lastly, in actual U.S. Medicare referrals there is no wage dispersion: cost of care is set exogenously by Medicare policy and is equivalent across all referrals. Hence, this calibration primarily focuses on the chance of getting a job through referral (i.e., Proposition 1).

	To Female Physician (Period-2 minority)	To Male Physician (Period-2 majority)	Total
From Female Physician (Period-1 minority)	1.73	7.78	9.51
From Male Physician (Period-1 majority)	5.65	35.03	40.68
Total	7.38	42.81	50.19

Table 1: U.S. Physician Referrals (Millions), 2008-2012

I now determine the following unknown parameters: δ, $1-\delta$, τ_{maj}, and τ_{min}. Table 1 shows the breakdown of physician referrals by gender. Using these values, I can calculate that $\delta = 0.81$ (total male-initiated referrals divided by total referrals).[23] Similarly, $1-\delta = 0.19$. Finally, I benchmark male physician network density ($\tau_{maj} = 0.108$) and female physician network density ($\tau_{min} = 0.185$) to a previous study.[24]

The remaining unknown parameters I will calibrate are the type in-group biases ψ_{maj} and ψ_{min}.[25] I will be calculating the type in-group bias values using Equation 1 from Proposition 1. Adapting that equation to this particular context gives us the following two Expressions:

$$(1) \qquad \Pr\{\text{period-1 male refers male}\} = \phi_{maj} = \frac{\delta \cdot \psi_{maj}}{[\delta \cdot \psi_{maj}] + [(1-\delta)(1-\psi_{maj})]}$$

$$(2) \qquad \Pr\{\text{period-1 female refers female}\} = \phi_{min} = \frac{(1-\delta) \cdot \psi_{min}}{[(1-\delta) \cdot \psi_{min}] + [\delta \cdot (1-\psi_{min})]}$$

First, from Table 1 I can calculate in a straightforward manner Pr(period-1 male physician refers male physician) $= \phi_{maj} = 0.86$. Similarly, Pr(period-1 female physician refers female physician) $= \phi_{min} = 0.18$. Now using Expression 1, I can calculate the magnitude of the

23. This approximation for δ yields more conservative estimates of gender disparities than using estimates of the overall share of male physicians in the U.S. ($\delta = 0.67$ in 2012 according to https://stats.oecd.org)

24. These values represent the general definition of network density from the literature (i.e., the proportion of possible ties that are actualized among the members of a network). As no U.S. estimates were found, I benchmark network density values for male and female physicians to ones estimated from a healthcare study in another industrialized nation (Aguirre-Duarte, Carswell, and Kenealy 2020).

25. By construction in our model, the ability in-group bias, α, is symmetric between majority and minority workers, so key findings do not depend on its value.

type in-group bias for male physicians, which is $\psi_{maj} = 0.59$. Using Expression 2, I can do the same for female physicians, which is $\psi_{min} = 0.50$. Notice that ψ_{min} equals 0.5, which means under our simplifying assumptions female physicians do not demonstrate any bias in their referral behavior in favor of same-group social ties.

Now that I have calculated all relevant unknown parameters to our model, I can determine how much minority network parameters would need to shift in order to achieve gender parity. To do so, I will use Expression 3 below (which mirrors Equation 2 from Proposition 1). Expression 3 demonstrates the condition under which parity in the distribution of job offers between high-ability majority workers (male physicians) and high-ability minority workers (female physicians) is achieved. This outcome occurs only when:

$$(3)\quad (1-\delta)\left[\delta\tau_{maj}\phi_{maj} + (1-\delta)\tau_{min}(1-\phi_{min})\right] = \delta\left[\delta\tau_{maj}(1-\phi_{maj}) + (1-\delta)\tau_{min}(\phi_{min})\right]$$

First, let us try to calculate the compensating minority network density ($\tau_{min}^{=}$), which is the minority network density that would achieve gender parity in the distribution of referral job offers. Using Expression 3, I discover that no value of $\tau_{min} \in [0,1]$ can achieve such parity. This result is in line with the prediction from Figure 2 of the Results section (i.e., that under certain social network conditions δ can be too large for parity to be achievable by simply increasing minority network density alone). In short, this exercise suggests the parameters of this physician social network are too imbalanced for female physicians to achieve gender parity simply by increasing their likelihood of having social ties compared with male physicians.

Next, let us calculate the compensating minority type in-group bias ($\psi_{min}^{=}$), which is the minority type in-group bias that would achieve gender parity in the distribution of referral job offers. Again I use Expression 3, this time first solving for ϕ_{min} (holding other parameters fixed at their calibrated values). Then, I use Expression 2 on this calculated value of ϕ_{min} to determine $\psi_{min}^{=}$. By doing so, one finds that $\psi_{min}^{=} = 0.66$. Not only is this value higher than the female baseline bias of 0.50, but also it is higher than the calibrated bias of male physicians (at 0.59). This supports Proposition 1B, which asserts that minority workers must have a sufficiently higher type in-group bias to reduce labor market disparities sprouting from homophily along majority/minority status. In this case, despite female physicians having a higher network density, they still need to be better connected to achieve gender parity. All else equal, minority workers must be more "strongly-knit." It is straightforward to calculate that at this magnitude of the compensating in-group bias (when $\psi_{min}^{=} = 0.66$), 1.27 million of the 2008-2012 referrals that went to male physicians would need to have instead gone to female physicians.

If the 1.27 million change in referral behavior were shouldered solely by women, the female-to-female referrals would be larger by 73.3% and the female-to-male referrals would be lower by 16.3%. Instead, if this change in the incidence of referrals were shouldered solely by men, the male-to-female referrals would be larger by 22.4% and the male-to-male referrals would be lower by 3.6%. Under this final scenario where men alter their referral behavior, the in-group bias of male physicians would be lower: ψ_{maj} would now be 0.50 (as opposed to the initial value of 0.59)—this means male physicians would no longer be demonstrating any same-group bias in their referral behavior.

6 Conclusion

The setting described in this model is relevant to a wide variety of real-world contexts. For example, important social ties are formed in postsecondary schooling (e.g., colleges and universities), in professional schools (e.g., business schools, law schools, and medical schools), and in workplaces. The social ties formed in these arenas may prove instrumental for future referral opportunities, with the formation of such ties influenced by social network phenomena like homophily (which is captured by the "in-group bias" parameter of this paper's model). Yet the opportunity for minority agents to form same-group social ties may be constrained due to the relative size of the demographic groups. The relevance of social ties formed in these settings for future personal and professional opportunities plausibly allows for the social network discrimination uncovered in this paper to arise—a realization which not only may help explain substantial inequalities that persist between demographic groups today, but also may serve to better inform solutions. Accomplishing this latter objective I leave for future research.

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Appendix

Lemma 2 *A firm will attempt to hire through referral if and only if it employs a high-ability worker in period 1.*

Proof. Firms employing high-ability workers in period 1 will make referral offers for two reasons. First, hiring through the market yields zero expected profit (given the assumption of free entry of firms). Second, an accepted referral offer yields constant positive profit over the range of the referral offer wage distribution $[w_{M2}, \overline{w}_R]$. (If it did not yield constant positive profits, it would be impossible to maintain equilibrium wage dispersion; the profit-maximizing firms would offer only a subset of the distribution—i.e., those wages that maximized profits.) An offer below w_{M2} will never be accepted, while an offer above $\overline{w}_R$ increases the wage without increasing the probability of attracting a worker. To complete the proof of this lemma, I show that firms employing low-ability workers in period 1 will hire through the market (i.e., not rely on referrals).

If a firm employing a low-ability worker did deviate from this Lemma and made a referral offer w_R, its expected profit (denoted $E\prod_L(w_R)$) would be represented by a slight modification to the expression for the expected profit for a firm employing a high-ability worker (denoted $E\prod_H(w_R)$ and derived in Equation A.6 of Proposition 3). The expression for $E\prod_L(w_R)$ differs from that of $E\prod_H(w_R)$ in that the incidences of α in the first p^{HMAJ}, p^{HMIN}, p^{LMAJ}, and p^{LMIN} terms are replaced with $(1-\alpha)$, and vice versa.

In this scenario, as long as $\alpha > \frac{1}{2}$, $E\prod_L(w_R) < 0$. Hence, expected profits are less than those from hiring in the general market, which equals zero due to free entry of firms. The lemma is thus proved: a firm employing a low-ability worker in period-1 prefers to hire through the market, maximizing expected profit. ∎

Lemma 3 *The period-1 market wage is greater than the expected period-1 productivity.*

Proof. Firms hiring in the period-1 market earn an expected period-2 profit equal to the probability of obtaining a high-ability period-1 worker times the expected profit (denoted c) from a referral. Free entry thus drives the wage above expected period-1 productivity:

$$\begin{aligned} & w_{M1}(\alpha, \delta, \tau_{maj}, \tau_{min}, \psi_{maj}, \psi_{min}) \\ & = \frac{1}{2} + \frac{1}{2} c(\alpha, \delta, \tau_{maj}, \tau_{min}, \psi_{maj}, \psi_{min}) \\ & = \frac{1}{2}[1 + c(\alpha, \delta, \tau_{maj}, \tau_{min}, \psi_{maj}, \psi_{min})] \end{aligned}$$

The expression for c is derived in Proposition 3. Given comparative-statics results on c, w_{M1} is increasing in τ_{maj} and τ_{min}. When both $\tau_{maj} = \tau_{min}$ and $\psi_{maj} = \psi_{min}$, w_{M1} is decreasing in δ. ∎

Proposition 1A *In an environment with equal magnitude of majority/minority network parameters ($\tau_{maj} = \tau_{min}$ and $\psi_{maj} = \psi_{min}$), the probability a high-ability minority worker in period 2 receives a referral offer is lower than their share of the labor force. The inverse* *holds for majority workers:*

$$\Pr\{\textit{period-2 high-ability minority worker receives referral offer}\} < \frac{1-\delta}{2}$$
$$\Pr\{\textit{period-2 high-ability majority worker receives referral offer}\} > \frac{\delta}{2}$$

Proposition 1B *The inequality in the distribution of referral job offers can be eliminated by minority workers having a sufficiently higher type in-group bias (ψ_{min}).*

Proof. Let us first consider a given high-ability period-2 worker (H). Since all referral wage offers are above the period-2 market wage, the probability that H would accept a referral wage offer w_{Ri} from firm i can be expressed:

$$\Pr\{H \text{ accepts } w_{Ri}\} = \Pr\{H \text{ receives no higher offer } w_{Rj} \ \forall \text{ firm } j \neq i\}$$

Since referral offers are allocated independently,

$$\begin{aligned}\Pr\{H \text{ accepts } w_{Ri}\} &= \prod_{j\neq i} \Pr\{H \text{ receives no higher offer } w_{Rj}\} \\ &= \prod_{j\neq i} [\, 1 - \Pr\{H \text{ receives an offer } w_{Rj} > w_{Ri}\}]\end{aligned}$$

The probability that firm j offers a wage $w_{Rj} > w_{Ri}$ to H is the product of two independent probabilities:

$$\Pr\{H \text{ receives an offer } w_{Rj} > w_{Ri}\} = \Pr\{\text{firm } j \text{ makes offer to } H\} \cdot \Pr\{w_{Rj} > w_{Ri}\}$$

If $2N$ workers were in period-1, free entry implies that N firms employ high-ability workers. Now I will analyze both parts of the expression from the perspective of a high-ability majority worker (H_{maj}) and high-ability minority worker (H_{min}).

The probability that firm j offers a referral to H_{maj} is a weighted average of whether firm j hired a majority or minority worker in period 1. Denote ϕ_{maj} as the probability a majority worker knows another majority worker, and ϕ_{min} as the probability a minority worker knows another minority worker, where:

$$\phi_{maj} = \frac{(\delta \cdot \psi_{maj})}{(\delta \cdot \psi_{maj}) + [(1-\delta)\cdot(1-\psi_{maj})]} \text{ , and}$$

$$\phi_{min} = \frac{(1-\delta)\cdot \psi_{min}}{[(1-\delta)\cdot \psi_{min}] + [\delta \cdot (1-\ \psi_{min})]}$$

Then:

$$\text{(A.1)} \qquad \Pr\{\text{firm } j \text{ makes offer to } H_{maj}\} = \delta\left(\frac{\alpha\tau_{maj}\phi_{maj}}{N}\right) + (1-\delta)\left(\frac{\alpha\tau_{min}(1-\phi_{min})}{N}\right)$$

Likewise, for a period-2 minority high-ability worker:

$$\Pr\{\text{firm } j \text{ makes offer to } H_{min}\} = \delta\left(\frac{\alpha\tau_{maj}(1-\phi_{maj})}{N}\right) + (1-\delta)\left(\frac{\alpha\tau_{min}\phi_{min}}{N}\right)$$

Based on these expressions, for minority workers to have a proportional chance of receiving job offers through referral, the following must hold:

$$\Pr\{\text{firms make offer to } H_{maj}\} \propto \Pr\{\text{firms make offer to } H_{min}\}$$

only when

$$(1-\delta)\left[\delta\tau_{maj}\phi_{maj} + (1-\delta)\tau_{min}(1-\phi_{min})\right] = \delta\left[\delta\tau_{maj}(1-\phi_{maj}) + (1-\delta)\tau_{min}(\phi_{min})\right]$$

Both the minority network density required to compensate for the inequality (denoted $\tau^{=}_{min}$) and the minority type in-group bias required to compensate (denoted $\psi^{=}_{min}$) increase in τ_{maj}, ψ_{maj}, and δ. The greater the probability of majority workers having social ties (or the greater the degree of their type in-group bias or likelihood of possessing social ties), the greater minority workers' compensating parameters ($\tau^{=}_{min}$ or $\psi^{=}_{min}$) must be to achieve a proportional amount of all job offers through referrals.

Though higher minority network density and type in-group bias can both reduce the disproportionality in the distribution of job offers through referral, analysis shows that between these two parameters only $\psi^{=}_{min}$ is attainable (i.e., below 1) under all possible combinations of social network parameters. ■

Proposition 2 *In an environment with equal magnitude of majority/minority network parameters ($\tau_{maj} = \tau_{min}$ and $\psi_{maj} = \psi_{min}$), the period-2 market wage (w_{M2}) decreases as majority workers occupy a greater fraction of the labor force.*

Proof. First I derive an expression for period-2 market wage. I build on the analysis from Proposition 1. If there were $2N$ workers in period 1, free entry implies that N firms employ high-ability workers. If firms select their referral wage offer by randomizing over the equilibrium wage distribution $F(\bullet)$ (to be derived below),

$$\Pr\{H_{maj} \text{ receives an offer } w_{Rj} > w_{Ri}\} = \Pr\{\text{firm } j \text{ makes offer to } H_{maj}\} \cdot [1 - F(w_{Ri})]$$

for all firms j who employ a high-ability worker in period 1. I have already shown that:

$$\begin{aligned}\Pr\{H \text{ accepts } w_{Ri}\} &= \prod_{j\neq i}\Pr\{H \text{ receives no higher offer } w_{Rj}\}\\ &= \prod_{j\neq i}[1 - \Pr\{H \text{ receives an offer } w_{Rj} > w_{Ri}\}]\end{aligned}$$

Substitution yields:

$$\begin{aligned}&\Pr\{H_{maj} \text{ accepts } w_{Ri}\}\\&= \{1 - [\frac{1}{N}(\delta\alpha\tau_{maj}\phi_{maj} + (1-\delta)\alpha\tau_{min}(1-\phi_{min}))] \cdot [1 - F(w_{Ri})]\}^{N-1}\end{aligned}$$

Since the model assumes a large number of workers, as N approaches ∞,

$$\text{(A.2)} \quad \Pr\{H_{maj} \text{ accepts } w_{Ri}\} = \exp\{-[\delta\alpha\tau_{maj}\phi_{maj} + (1-\delta)\alpha\tau_{min}(1-\phi_{min})][1 - F(w_{Ri})]\}$$

Details on this step can be found in Rapoport (1963) and Montgomery (1991). One can use similar steps to obtain the probability that firm i's offer is accepted by a given high-ability majority worker (H_{maj}), low-ability majority worker(L_{maj}), and low-ability minority worker(L_{min}).

As high-ability workers tend to receive more offers, they are less likely to accept any given offer $w_{Ri} < \overline{w}_R$. Since a period-2 worker finds employment through the market only if he receives no offers (or rejects all referral offers):

$$\Pr\{\text{market} \mid H_{maj}\} = \Pr\{H_{maj} \text{ accept } w_{M2}\}$$

The market wage coincides with the bottom of the referral wage distribution, $F(\bullet)$, because any referral wage below the market wage will be rejected by period-2 workers, to gain employment through the market. Thus, given that $F(w_{M2}) = 0$:

$$\Pr\{\text{market} \mid H_{maj}\} = \exp\{-[\delta\alpha\tau_{maj}\phi_{maj} + (1-\delta)\alpha\tau_{min}(1-\phi_{min})]\}$$

I can derive similar expressions for H_{min}, L_{maj}, and L_{min}. Let:

$$\text{(A.3)} \quad \begin{aligned}e^{HMAJ} &= \exp\{-[\delta\alpha\tau_{maj}\phi_{maj} + (1-\delta)\alpha\tau_{min}(1-\phi_{min})]\}\\e^{HMIN} &= \exp\{-[\delta\alpha\tau_{maj}(1-\phi_{maj}) + (1-\delta)\alpha\tau_{min}\phi_{min}]\}\\e^{LMAJ} &= \exp\{-[\delta(1-\alpha)\tau_{maj}\phi_{maj} + (1-\delta)(1-\alpha)\tau_{min}(1-\phi_{min})]\}\\e^{LMIN} &= \exp\{-[\delta(1-\alpha)\tau_{maj}(1-\phi_{maj}) + (1-\delta)(1-\alpha)\tau_{min}\phi_{min}]\}\end{aligned}$$

I now use Bayes's rule to calculate the period-2 market wage:

$$\text{(A.4)} \quad \begin{aligned}&w_{M2}(\alpha, \delta, \tau_{maj}, \tau_{min}, \psi_{maj}, \psi_{min})\\&= E(\text{productivity} \mid \text{market})\\&= \frac{\Pr(\text{market} \mid H_{maj}) \cdot \Pr(H_{maj}) + \Pr(\text{market} \mid H_{min}) \cdot \Pr(H_{min})}{\Pr(\text{market} \mid H) \cdot \Pr(H) + \Pr(\text{market} \mid L) \cdot \Pr(L)}\\&= \frac{(e^{HMAJ} \cdot \delta) + (e^{HMIN} \cdot (1-\delta))}{(e^{HMAJ} + e^{LMAJ}) \cdot \delta + (e^{HMIN} + e^{LMIN}) \cdot (1-\delta)}\end{aligned}$$

Given $\alpha > \frac{1}{2}$ and both network densities (τ_{maj} and τ_{min}) greater than zero, w_{M2} is always less than $\frac{1}{2}$, the average productivity of the population. Analysis shows that w_{M2} is decreasing in α. Furthermore, for all $\psi_{maj} = \psi_{min}$ and $\tau_{maj} = \tau_{min}$, w_{M2} is also decreasing in δ. ■

Proposition 3 *In an environment with equal magnitude of majority/minority network parameters ($\tau_{maj} = \tau_{min}$ and $\psi_{maj} = \psi_{min}$), the welfare (i.e., average expected wage) for minority workers is lower than for majority workers.*

Proof. Consider the expected period-2 profit earned by a firm employing a high-ability worker and setting a referral wage (recall the productivity of high-ability workers equals one, while that of low-ability workers equals zero):

$$
\begin{aligned}
&E\prod\nolimits_H(w_R)\\
&\quad = \Pr\{\text{high-ability majority period-2 referral hired} \mid w_R\} \cdot (1 - w_R)\\
&\qquad + \Pr\{\text{high-ability minority period-2 referral hired} \mid w_R\} \cdot (1 - w_R)\\
&\qquad + \Pr\{\text{low-ability majority period-2 referral hired} \mid w_R\} \cdot (-w_R)\\
&\qquad + \Pr\{\text{low-ability minority period-2 referral hired} \mid w_R\} \cdot (-w_R)
\end{aligned}
$$

(If no referred worker is hired, perhaps because the period-1 worker possesses no social tie or because the referred acquaintance receives a better offer, the firm hires through the market and earns zero expected profit.)

The probability of hiring a high-ability majority period-2 referred worker is the product of two independent probabilities (substituting from Equations A.1 and A.2 from Propositions 1 and 2):

$$
\begin{aligned}
&\Pr\{\text{high-ability period-2 majority referral hired} \mid w_R\}\\
&= \Pr\{\text{offer made to a high-ability majority referral}\} \cdot \Pr\{H_{maj} \text{ accepts } w_R\}\\
&= \delta\alpha\tau_{maj}\phi_{maj} \ + (1-\delta)\alpha\tau_{min}(1-\phi_{min})\\
&\quad \cdot \exp\{-[\delta\alpha\tau_{maj}\phi_{maj} + (1-\delta)\alpha\tau_{min}(1-\phi_{min})][1-F(w_R)]\}
\end{aligned}
$$

Similar steps can be followed to derive the respective conditional probability for high-ability minority, low-ability majority, and low-ability minority workers.

Let:

$$
\begin{aligned}
p^{HMAJ} &= \delta\alpha\tau_{maj}\phi_{maj} + (1-\delta)\alpha\tau_{min}(1-\phi_{min})\\
p^{HMIN} &= \delta\alpha\tau_{maj}(1-\phi_{maj}) + (1-\delta)\alpha\tau_{min}\phi_{min}\\
p^{LMAJ} &= \delta(1-\alpha)\tau_{maj}\phi_{maj} + (1-\delta)(1-\alpha)\tau_{min}(1-\phi_{min})\\
p^{LMIN} &= \delta(1-\alpha)\tau_{maj}(1-\phi_{maj}) + (1-\delta)(1-\alpha)\tau_{min}\phi_{min}
\end{aligned} \tag{A.5}
$$

So, to simplify:

$$
\begin{aligned}
&E\prod\nolimits_H(w_R)\\
&\quad = p^{HMAJ} \cdot \exp\{-[p^{HMAJ}][1-F(w_{Ri})]\} \cdot (1-w_R)\\
&\qquad + p^{HMIN} \cdot \exp\{-[p^{HMIN}][1-F(w_{Ri})]\} \cdot (1-w_R)\\
&\qquad + p^{LMAJ} \cdot \exp\{-[p^{LMAJ}][1-F(w_{Ri})]\} \cdot (-w_R)\\
&\qquad + p^{LMIN} \cdot \exp\{-[p^{LMIN}][1-F(w_{Ri})]\} \cdot (-w_R)
\end{aligned} \tag{A.6}
$$

To maintain equilibrium wage dispersion, firms must earn the same expected profit on each referral wage offered:

$$E\prod_{H}(w_R) = c \qquad \forall w_R \in [w_{M2}, \overline{w}_R]$$

To calculate this profit constant, note that the firm could deviate from the specified strategy and offer a wage of w_{M2}; in this case, the referred worker accepts the firm's offer only if they receive no other offers.

Recall that $F(w_{M2}) = 0$. The firm's expected profit is therefore given by (using terms defined in Equations A.3 and A.5):

$$\begin{aligned}
&E\prod_{H}(w_{M2}) \\
&= (p^{HMAJ})(e^{HMAJ})(1 - w_{M2}) + (p^{HMIN})(e^{HMIN})(1 - w_{M2}) \\
&\quad + (p^{LMAJ})(e^{LMAJ})(-w_{M2}) + (p^{LMIN})(e^{LMIN})(-w_{M2}) \\
&= c
\end{aligned}$$

Substituting for w_{M2} (Equation A.4), I can determine $c(\alpha, \delta, \tau_{maj}, \tau_{min}, \psi_{maj}, \psi_{min})$. Given $\alpha > \frac{1}{2}$, firms with high-ability workers who possess social ties earn positive expected profits. Analysis shows that c is increasing in α, τ_{maj}, and τ_{min}. When both $\psi_{maj} = \psi_{min}$ and $\tau_{maj} = \tau_{min}$, c is decreasing in δ.

Given the expression for $c(\alpha, \delta, \tau_{maj}, \tau_{min}, \psi_{maj}, \psi_{min})$, the equilibrium referral-offer distribution $F(\bullet)$ can be determined by setting $E\prod_H(w_R)$ (Equation A.6) equal to c for all potential wage offers w_R.

Unfortunately, doing so does not yield a closed-form solution for $F(w_R)$. Given a continuum of firms, the equilibrium referral-offer distribution $F(\bullet)$ can be interpreted as either: (1) each firm randomizes over the entire distribution; or (2) a fraction $f(w_R)$ of firms offers each wage for sure.

From the second interpretation, one can denote these referral wages with w_{Rk} and estimate the average referral wage received by a high-ability majority worker (denoted $E(w_{R_{Hmaj}})$) vs. a high-ability minority worker (denoted $E(w_{R_{Hmin}})$), for any given δ, α, τ_{maj}, τ_{min}, ψ_{maj}, and ψ_{min}. Analysis shows that in an environment with equal magnitude of majority/minority network parameters, if $\alpha > \frac{1}{2}$ and $\delta > \frac{1}{2}$, $E(w_{R_{Hmaj}}) > E(w_{R_{Hmin}})$.

Proposition 1 shows that, all else equal, minority workers receive a smaller proportion of jobs through referral than their fraction of the population. As a result, minority workers more frequently gain employment through the market, receiving the (lower) market wage. In this Proposition, I showed that even when offered a job through referral, minority workers have lower expected referral wages than majority workers. ■

To conclude, one can derive an expression for the maximum referral wage offered $\overline{w}_R$ (where $F(\overline{w}_R) = 1$, by definition):

$$\overline{w}_R(\alpha, \delta, \tau_{maj}, \tau_{min}, \psi_{maj}, \psi_{min}) = \frac{p^{HMAJ} + p^{HMIN} - c}{p^{HMAJ} + p^{HMIN} + p^{LMAJ} + p^{LMIN}} \tag{A.7}$$

A firm that offers a referral wage of $\overline{w}_R$ attracts a referred worker with probability 1 (conditional on its period-1 worker possessing a social tie). The firm's expected profit, c, is thus equal to $p^{HMAJ} + p^{HMIN} - \overline{w}_R(p^{HMAJ} + p^{HMIN} + p^{LMAJ} + p^{LMIN})$. $\overline{w}_R$ is increasing in α, τ_{maj}, and τ_{min}.