# Methodology for Analyzing Market Dynamics.

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- Keynote Address: Society for Applied Dynamic Games, Amsterdam, July.
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by

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## Background: Methodological Developments in IO.

- We have been developing tools that enable us to better analyze market outcomes.

- Common thread: emphasis on incorporating the institutional background needed to make sense of the data used in analyzing the likely causes of historical events, or the likely responses to environmental and policy changes.

- Focus. **Incorporate**
  (i) **heterogeneity** (in plant productivity, products demanded, bidders and/or consumers) and where possible,
  (ii) **equilibrium conditions** (Nash in prices or quantities, and extensions designed to analyze allocations in network, platform, and vertical markets).

We largely relied on earlier work by our game theory colleagues for the analytic frameworks.

- Each agent's actions affect all agents' payoffs, and

- At the "equilibrium" or "rest point"
  (i) agents have correct perceptions, and
  (ii) the system is in some form of Nash equilibrium (policies such that no agent has an incentive to deviate).

- Our contribution is the development of an ability to adapt the analysis to the richness of different real world institutions.

**Claim 1.** The tools developed for the analysis of market allocations conditional on the "state variables" of the problem (characteristics of products marketed, cost determinants, ...) pass a market test for success as:
(i) They have been incorporated into applied work in virtually all of economics that deals with market allocations (especially where productivity and/or demand is needed),
(ii) They are used by public agencies, consultancies and to some extent by firms and
(iii) They do surprisingly well, both in fit and in providing a deeper understanding of empirical phenomena.

**Note.** There are improvements still being done, and important new work in analyzing equilibrium allocations in markets where Nash in prices or quantities seems inappropriate; e.g. vertical markets, platform markets,...

*E.g. of Fit: Pricing Behavior.* Wollman's dissertation (commercial trucks). Estimate BLP demand, regress Nash markup on instruments to get $\widehat{markup}$ ($R^2$=.44 or .46 with time dummies; sophisticated IV would do better). Look to fit & whether coefficient of $\widehat{markup} \approx 1$?

**Table 1: Fit of Pricing Equilibrium.**

| | Price | (S.E.) | Price | (S.E.) |
|---|---|---|---|---|
| Gross Weight | .36 | (0.01) | .36 | (.003) |
| Cab-over | .13 | (0.01) | .13 | (0.01) |
| Compact front | -.19 | (0.04) | 0.21 | (0.03) |
| long cab | -.01 | (0.04) | 0.03 | (0.03) |
| Wage | .08 | (.003) | 0.08 | (.003) |
| $\widehat{Markup}$ | .92 | (0.31) | 1.12 | (0.22) |
| Time dummies? | No | n.r. | Yes | n.r. |
| $R^2$ | 0.86 | n.r. | 0.94 | n.r. |

Nobs=1,777; firms=16; t=1992-2012; Heter-cons s.e.

**Note.** Level shifts (time dummies) are 8% of the 14% of unexplained variance.

**What About "Dynamics"?** Use textbook distinction: (i) static models solve for profits conditional on state variables, (ii) dynamics analyzes the evolution of those state variables.

The initial frameworks by our theory colleagues made assumptions which insurred that the

1. state variables evolve as a Markov process

2. and the equililbrium is some form of Markov Perfection (no agent has an incentive to deviate at any value of the state variables).

E.g. Maskin and Tirole (1988) and Ericson and Pakes (1995). We now consider each of these in turn.

**On (1); the Markov Assumption.** Except in situations involving active experimentation and learning (where policies are transient), applied work is likely to stick with the assumption that states evolve as a time homogenous Markov process of finite order. There are at least three reasons for this:

- It is a convenient and fits the data well.
- Realism suggests information access and retention conditions limit the memory used.
- We can bound unilateral deviations (similar to Weintraub, 2014), and have conditions which insure those deviations can be made arbitrarily small by letting the length of the kept history grow (White and Scherer, 1994).

**On 2: Perfection.** The type of rationality built into Markov Perfection is more questionnable; even though it has been useful in the simple models used by our theory colleagues to explore possible outcomes in a structured way. We come back to this below.

**Empirical work on dynamics** proceeded in a similar way to what we did in static analysis; we took the Markov Perfect framework and tried to incorporate the institutions that seemed necessary to analyze actual markets.

**The Result.** Though the MP framework was useful in guiding analysis of several issues (e.g. productivity) it became unweildly when confronted with the task of analyzing market dynamics. This because of the complexity of the institutions we were trying to model. The difficulties became evident when we tried to use the Markov Perfect notions to structure

- the estimation of parameters, or to

- compute the fixed points that defined the equilibria or rest points of the system.

**Our response.** Keep the equilibrium notion and develop techniques to make it easier to circumvent the estimation and computational problems. Useful contribution in this regard:

- The development of estimation techniques that circumvent the problem of repeatedly computing equilibria (that do not require a nested fixed point algorithm).

- the use of approximations and functional forms for primitives which enabled us to compute equilibria quicker and/or with less memory requirements.

The underlying ideas: (i) are useful under other equilibirum assumptions, and (ii) enabled an expansion of computational dynamic theory. However they were not powerful enough to allow us to incorporate sufficient realism into empirical work based on Markov Perfection.

This leads me to my second claim.

**Claim 2.** Empirical work on dynamic models have not passed beyond the hands of a few diligent I.O. researchers.

As a result dynamic issues are analyzed in a much less rigorous way than static issues even when dynamic computational results indicate that they are essential to understanding the implications of the phenomena of interest (e.g.'s: mergers or collusion). Moreover the complexity of Markov Perfection not only limits our ability to do dynamic analysis of market outcomes it also

- leads to a question of whether some other notion of equilibria will better approximate agents' behavior.

I want to focus on the last point. The fact that Markov Perfect framework becomes unwieldily when confronted by the complexity of real world institutions, not only limits our ability to do empirical analysis of market dynamics

- it also raises the question of whether some other notion of equilibria will better approximate agents' behavior.

**I.e.** If we abandon Markov Perfection can we both

- better approximate agents' behavior and,

- enlarge the set of dynamic questions we are able to analyze.

**The complexity issue.** When we try to incorporate what seems to be essential institutional background we find

- That the agent is required to; (i) access a large amount of information (all state variables), and (ii) either compute or learn an unrealistic number of strategies (one for each information set).

**How demanding is this?** Consider markets where consumer, as well as producer, choices are dynamic (e.g.'s; durable, experience, or network goods); need the distribution of; current stocks $\times$ household characteristics, production costs, .... In a symmetric information MPE an agent would have to access all state variables, and then either compute a doubly nested fixed point, or learn and retain, policies from each distinct information set.

**Theory Fix:** Assume agents only have access to a subset of the state variables.

- Since agents presumably know their own characteristics and these tend to be persistent, we would need to allow for assymetric information: the "perfectness" notion would then lead us to a "Bayesian" Markov Perfect solution.

**Problem.** The burden of computing its strategies insures that they will not be directly computed by either agents or the analyst for even the simplest (realistic) applied problem. The additional burden results from the need to compute posteriors, as well as optimal policies; and the requirement that they be consistent with one another.

Could agents learn these policies, or at least policies which maintain some of the logical features of Bayesian Perfect policies, from combining data on past behavior with market outcomes? They would have to learn about;

- primitives (some empirical work on this),
- the likely behavior of their competitors, and
- market outcomes given primitives, competitor behavior, and their own policies.

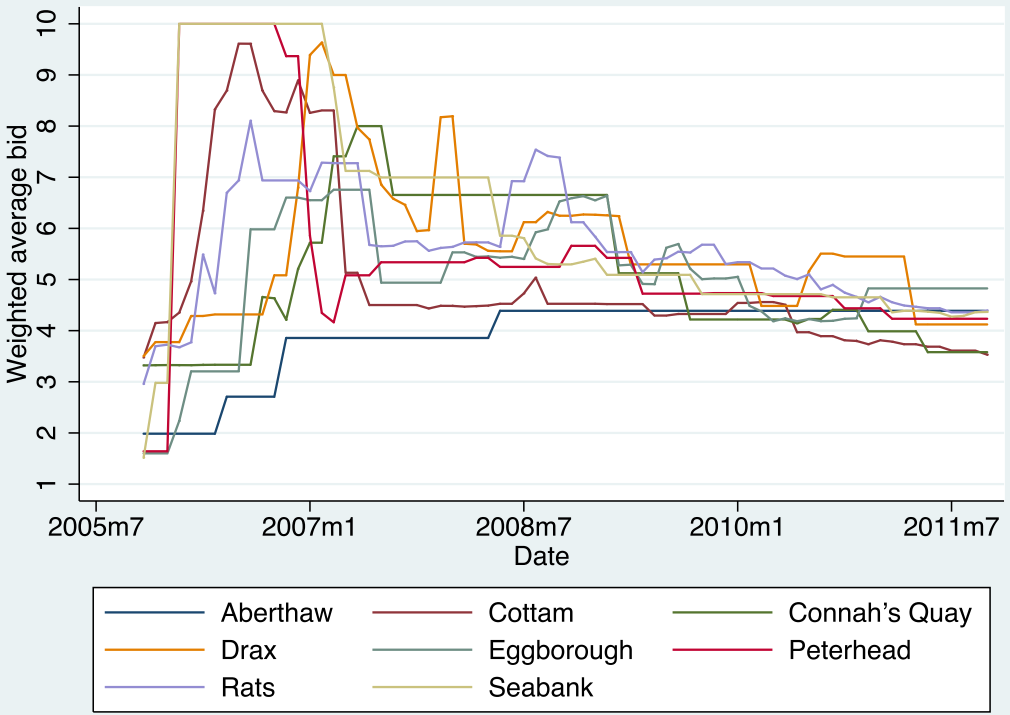
There is surprising little empirical evidence on how firms formulate their perceptions about either other firms' behavior, or on the impact of their own strategies given primitives and the actions of competitors.

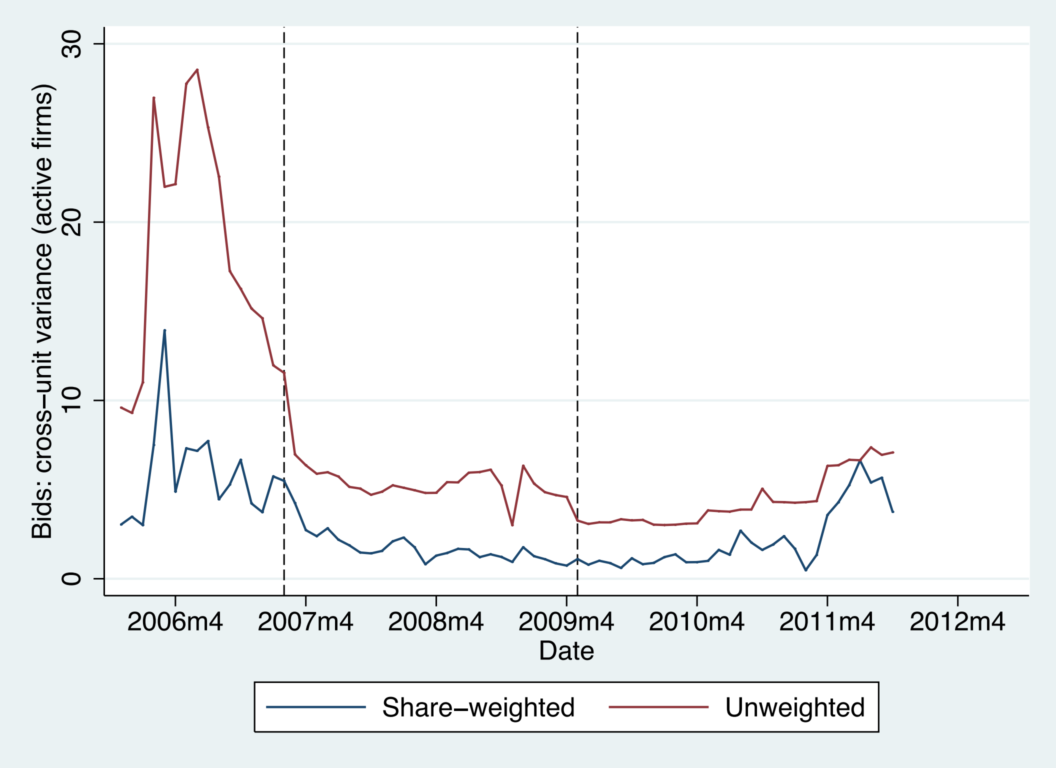
An ongoing study by U. Doraszelski, G. Lewis and myself of bids from the date the British Electric Utility market for frequency response opened, addresses this question. The conclusions we are reasonably confident on to date are (see the figures)

- The bids do eventually converge, and they converge to what looks like a Nash equilibrium (in 2009), and

- In its initial stages, the learning process is complex, involves experimentation, and differs among firms.

- The smaller changes that occured in the first half of 2010 (Drax signs long-term contract with NG) seems more structurred and proceeds to an equilibrium much quicker.


Weighted average bid
10
9
8
7
6
5
4
3
2
1
2005m7
2007m1
2008m7
2010m1
2011m7
Date
Aberthaw
Cottam
Connah's Quay
Drax
Eggborough
Peterhead
Rats
Seabank



Bids: cross−unit variance (active firms)
0
10
20
30
2006m4
2007m4
2008m4
2009m4
2010m4
2011m4
2012m4
Date
Share−weighted
Unweighted

**Rest of Talk.**

- Unfortunately, I have little to say on "active" experimentation periods; on modeling beliefs on the value of different experiments.

- For more stable environments I introduce (i) a notion of equilibrium that is less demanding than Markov Perfect for both the agents, and the analyst, and show how to (ii) compute the equilibrium and (iii) estimate off of equilibrium conditions.

- Consider restrictions that mitigate multiple equilibria.

- Provide a computed example of this equilibrium (electric utility generation).

I start with strategies that are "rest points" to a dynamical system. Later I will consider institutional change, but only changes where it is reasonable to model responses to the change with a simple reinforcement learning process (I do not consider changes that lead to active experimentation). This makes my job much easier because:

- Strategies at the rest point likely satisfy a Nash condition of some sort; else someone has an incentive to deviate.

- However it still leaves opens the question: What is the form of the Nash Condition?

## What Conditions Can We Assume for the Rest Point at States that are Visited Repeatedly?

We expect (and I believe should integrate into our modelling) that

1. Agents perceive that they are doing the best they can at each of these points, and that

2. These perceptions are at least consistent with what they observe.

**Note.** It might be reasonable to assume more than this: that agents (i) know and/or (ii) explore, properties of outcomes of states not visited repeatedly. I come back to this below.

## Formalization of Assumptions.

• Denote the information set of firm $i$ in period $t$ by $J_{i,t}$. $J_{i,t}$ will contain both public ($\xi_t$) and private ($\omega_{i,t}$) information, so $J_{i,t} = \{\xi_t, \omega_{i,t}\}$.

• Assume $J_{i,t}$ evolves as a (controlled) finite state Markov process on $\mathcal{J}$ (or can be adequately approximated by one); and only a finite number of firms are ever simultaneously active.

• Policies, say $m_{i,t} \in \mathcal{M}$, will be functions of $J_{i,t}$. For simplicity assume $\#\mathcal{M}$ is finite, and that it is a simple capital accumulation game, i.e. $\forall (m_i, m_{-i}) \in \mathcal{M}^n$, & $\forall\, \omega \in \Omega$

$$P_\omega(\cdot|m_i, m_{-i}, \omega) = P_\omega(\cdot|m_i, \omega).$$

where the public information, $\xi$, is used to predict competitor behavior and common demand and cost conditions which evolve as an exogenous Markov process.

• So a "state" of the system, is

$$s_t = \{J_{1,t}, \ldots, J_{n_t,t}\} \in \mathcal{S},$$

$\#\mathcal{S}$ is finite. $\Rightarrow$ any set of policies will insure that $s_t$ will wander into a recurrent subset of $\mathcal{S}$, say $\mathcal{R} \subset \mathcal{S}$, in finite time, and after that $s_{t+\tau} \in \mathcal{R}$ w.p.1 forever.

• Agents do not: (i) know $s_t$, or (ii) calculate policies for its components.

• $\Rightarrow$ If $J_{i,t} \in \mathcal{J}$ and $\#\mathcal{J} = K$, while the # firms is $N$, the number of states changes from $K^N$ to either:
(i) $K$ (symmetric agents), or
(ii) $K \times N$ (otherwise).

• Let the agent's perception of the expected discounted value of current and future net cash flow were it to chose $m$ at state $J_i$, be

$$W(m|J_i), \ \ \forall m \in \mathcal{M} \ \ \& \ \ \forall J_i \in \mathcal{J},$$

• and of expected profits be

$$\pi^E(m|J_i).$$

**Our assumptions imply**:

- Each agent choses an action which maximizes its perception of its expected discounted value, and

- For those states that are visited repeatedly (are in $\mathcal{R}$) these perceptions are consistent with observed outcomes.

**Formally**

**A**. $W(m^*|J_i) \geq W(m|J_i), \;\; \forall m \in \mathcal{M} \;\; \& \;\; \forall J_i \in \mathcal{J}$,

**B**. &, $\forall J_i$ which is a component of an $s \in \mathcal{R}$

$$W(m(J_i)|J_i) = \pi^E(m|J_i) + \beta \sum_{J_i'} W(m^*(J_i')|J_i')p^e(J_i'|J_i),$$

where, if $p^e(\cdot)$ provides the empirical probability (the fraction of periods the event occurs)

$$\pi^E(m|J_i) \equiv \sum_{J_{-i}} E[\pi(\cdot)|J_i, J_{-i}]p^e(J_{-i}|J_i),$$

and

$$\Big\{p^e(J_{-i}|J_i) \equiv \frac{p^e(J_{-i}, J_i)}{p^e(J_i)}\Big\}_{J_{-i},J_i},$$

while

$$\Big\{p^e(J_i'|J_i) \equiv \frac{p^e(J_i', J_i)}{p^e(J_i)}\Big\}_{J_i',J_i}. \;\; \spadesuit$$

## "Experience Based Equilibrium"

These are the conditions of a (restricted) EBE (Fershtman and Pakes, 2012; for related earlier work see Fudenberg and Levine, 1993). Bayesian Perfect satisfy them, but so do weaker notions. We now turn to its :
(i) computational and estimation properties,
(ii) overcoming multiplicity issues,
(iii) and then to an example.

**Computational Algorithm.** "Reinforcement learning" algorithm (Pakes and McGuire, 2001).
• Can be viewed as a learning process. Makes it a candidate to: (i) analyze (small) perturbations to the environment, as well as (ii) to compute equilibrium.
• Does not generate a curse of dimensionality in either: (i) the number of states or (ii) the computation of continuation values.

**Iterative Algorithm: Iterations defined by**

• A location, say $L^k = (J^k_1, \ldots J^k_{n(k)}) \in \mathcal{S}$: is the information sets of agents active.

• Objects in memory (i.e. $M^k$):

(i) perceived evaluations, $W^k$,

(ii) No. of visits to each point, $h^k$.

**So algorithm must update** $(L^k, W^k, h^k)$**.** Computational burden determined by; memory constraint, and compute time. I use a simple (not neccesarily) optimal structure to memory.

## Update Location.

• Calculate "greedy" policies for each agent

$$m^*_{i,k} = arg \max_{m \in \mathcal{M}} W^k(m|J_{i,k})$$

• Take random draws on outcomes conditional on $m^*_{i,k}$: i.e. if we invest in "payoff relevant"

$\omega_{i,k} \in J_{i,k}$, draw $\omega_{i,k+1}$ conditional on $(\omega_{i,k}, m^*_{i,k})$.
• Update $\{J_{i,k}\}_i$.

## Update $W^k$.

• "Learning" interpretation: Assume agent observes $b(m_{-i})$ and knows the primitives $(\pi(\cdot), p(\cdot|\omega, m))$ conditional on $(b(m_{-i}), \omega, m)$.

• Its ex poste perception of what its value would have been had it chosen $m$ is

$$V^{k+1}(J_{i,k}, m) =$$

$$\pi(\omega_{i,k}, m, b(m_{-i,k}), d_k) + \max_{\tilde{m} \in M} \beta W^k(\tilde{m}|J_{i,k+1}(m)),$$

where $J_i^{k+1}(m)$ is what the $k+1$ information would have been given $m$ and *competitors actual play*.

Treat $V^{k+1}(J_{i,k})$ as a random draw from the possible realizations of $W(m|J_{i,k})$, and update $W^k$ as in stochastic integration (Robbins and Monroe,1956)

$$W^{k+1}(m|J_{i,k}) - W^k(m|J_{i,k}) =$$

$$\frac{1}{h^k(J_{i,k})}[V^{k+1}(J_{i,k}, m) - W^k(m|J_{i,k})].$$

or

$$W^{k+1}(m|J_{i,k}) =$$

$$\frac{1}{h^k(J_{i,k})}V^{k+1}(J_{i,k}, m) + \frac{(h^k(J_{i,k}) - 1)}{h^k(J_{i,k})}W^k(m|J_{i,k}),$$

(other weights might be more efficient).

**Notes.**

• If we have equilibrium valuations we tend to stay their, i.e. if * designates equilibrium

$$E[V^*(J_i, m^*)|W^*] = W^*(m^*|J_i).$$

• To learn equilibrium values we need to visit points repeatedly; only likely for states in $\mathcal{R}$.

• Agents (not only the analyst) could use the algorithm to find equilibrium policies or adjust to perturbations in the environment.

• Algorithm has no curse of dimensionality.
(i) Computing continuation values: integration is replaced by averaging two numbers.
(ii) States: algorithm eventually wanders into $\mathcal{R}$ and stays their, and $\#\mathcal{R} \leq \#\mathcal{J}$.

•The algorithm uses the stochastic approximation literature and as in that literature it can be augmented to use functional form approximations where needed (“TD learning”; Sutton and Barto,1998).

## Computational Properties.

• Testing. The algorithm does not necessarily converge, but a test for convergence exists and does not involve a curse of dimensionality (Fershtman and Pakes, 2012).

• The test is based on simulation. It produces a consistent estimate of an $L^2(P(\mathcal{R}))$ norm of the percentage bias in the implied estimates of $V(m, J_i)$; where $P(\mathcal{R})$ is the invariant measure on the recurrent class.

## Estimation.

• Need a candidate for $J_i$. Either: (i) empirically investigate determinants of controls, or (ii) ask actual participants.

• Does not require nested fixed point algorithm. Use estimation advances designed for MP equilibria (POB or BBL), or a perturbation (or "Euler" like) condition (below).

**Euler-Like Condition.**

• With assymetric information the equilibrium condition

$$W(m^*|J_i) \geq W(m|J_i)$$

is an inequality which can generate (set) estimators of parameters.

• $J_i$ contains both public and private information. Let $J^1$ have the same public, but differnt private, information then $J^2$. If a firm is at $J^1$ it knows it could have played $m^*(J^2)$ and its competitors would respond by playing *on the equilbrium path* from $J^2$.

• If $m^*(J^2)$ results in outcomes in $\mathcal{R}$, we can simulate a sample path from $J^2$ using only observed equilibrium play. The Markov property insures it would intersect the sample path from the DGP at a random stopping time with probability one and from that time forward the two paths would generate the same profits.

- The conditional (on $J_i$) expectation of the difference in discounted profits between the simulated and actual path from the period of the deviation to the random stopping time, should, when evaluated at the true parameter vector, be positive. This yields moment inequalities for estimation as in Pakes, Porter, Ho and Ishii (forthcoming), Pakes, (2010).

## Multiplicity.

• $\mathcal{R}$ contains both "interior" and "boundary" points. Points at which there are feasible strategies which can lead outside of $\mathcal{R}$ are boundary points. Interior points are points that can only transit to other points in $\mathcal{R}$ no matter which (feasible) policy is chosen.

• Our conditions only insure that perceptions of outcomes are consistent with the results from actual play at interior points. Perceptions of outcomes for some feasible (but inoptimal) policy at boundary points are not tied down by actual outcomes.

• MPBE are a special case of (restricted) EBE and they have multiplicity. Here differing perceptions at boundary points can support a (possibly much) wider range of equilibria.

## Narrowing the Set of Equilibria.

• In any empirical appliction the data will rule out equilibria. $m^*$ is observable, at least for states in $\mathcal{R}$, and this implies inequalities on $W(m|\cdot)$. With enough data $W(m^*|\cdot)$ will also be observable up to a mean zero error.

• Use external information to constrain perceptions of the value of outcomes outside of $\mathcal{R}$. If available use it.

• Allow firms to experiment with $m_i \neq m_i^*$ at boundary points (as in Asker, Fershtman, Jihye, and Pakes, 2014). Leads to a stronger notion of, and test for, equilibrium. We insure that perceptions are consistent with the results from **actual play** for each **feasible** action at boundary points (and hence on $\mathcal{R}$).

## Boundary Consistency.

Let $B(J_i|\mathcal{W})$ be the set of actions at $J_i \in s \in \mathcal{R}$ which could generate outcomes which are not in the recurrent class (so $J_i$ is a boundary point) and $B(\mathcal{W}) = \cup_{J_i \in \mathcal{R}} B(J_i|\mathcal{W})$. Then the extra condition needed to insure "Boundary Consistency" is:

**Extra Condition.** Let $\tau$ index future periods, then $\forall (m, J_i) \in B(\mathcal{W})$

$$E\Big[\sum_{\tau=0}^{\infty} \delta^{\tau} \pi(m(J_{i,\tau}), m(J_{-i,\tau}))|J_i = J_{i,0}, \mathcal{W}\Big] \leq W(m^*|J_i),$$

where $E[\cdot|J_i, \mathcal{W}]$ takes expectations over future states starting at $J_i$ using the policies generated by $\mathcal{W}$. ♠

**Testing for Boundary Consistency.**

From each $(m, J_i) \in B(\mathcal{W})$ simulate independent sample paths. Index the periods of a path by $\tau$ & terminate it the first time $J_i \in s \in \mathcal{R}$ (or if that does not occur at some large number), say $\tau^*$. The path's estimate of $W(m|J_i)$ is

$$\tilde{W}(m|J_i) \equiv \sum_{\tau=0}^{\tau^*} \delta^\tau \pi(m(J_{i,\tau}), m(J_{-i,\tau})) + \delta^{\tau^*} W(m^*|J_{i,\tau^*}),$$

with mean and variance; $\overline{W}(m|J_i)$, $Var[\overline{W}(m|J_i)]$.

Let $f(x)_+ = max[0, f(x)]$, and

$$T(m|J_i) \equiv \frac{[\overline{W}(m|J_i) - W(m^*|J_i)]_+}{W(m^*|J_i)}$$

Now use a one-sided test of

$$H_0 : \frac{\sum_{(m,J_i)\in B(\mathcal{W})} T(m|J_i)}{\sqrt{\sum_{(m,J_i)\in B(\mathcal{W})} Var[T(m|J_i)]}} = 0,$$

where $Var[T(m|J_i)]$ is the variance of $T(m|J_i)$.

## Simple Electric Utility Eg.

*Two firms*: each has a vector of generators. *Firm's decisions*: bid or not each generator. If not bid, do maintenance or not.
*ISO*: sum bid functions, intersect with demand (varies by day of the week), pay a uniform price to accepted electricity.

- $\omega \in \Omega$. Cost of producing electricy on each firm's generators. Cost increases stochastically with use, but reverts to a starting value if the firm goes down for maintenance.

- $m_i \in M_i$. Vector of $m_{i,r} \in \{0, 1, 2\}$; $0 \Rightarrow$ shutdown without maintenance, $1 \Rightarrow$ shutdown with maintenance, $2 \Rightarrow$ bid into market.

- $b(m_i) : m_i \to \{0, b_i\}^{n_i}$ where $b_i$ is the fixed bid schedule of firm $i$. $b$ observed. $m$ not observed.

- $d$ is demand on that day, $f$ is maintenance cost ( " investment" ), $p = p(b(m_i), b(m_{-i}), d)$ is price, $q = q(b(m_i), b(m_{-i}), d)$ is allocated quantity vector, so realized profits are

$$\pi_{i,t} = \sum_r p_t q_{i,r,t} - \sum_r c_i(\omega_{i,r,t}, q_{i,r,t}) - f_i \sum_r \{m_{i,r,t} = 1\}$$

$$\equiv \pi_i(\omega_i, m_i, b(m_{-i}), d)$$

$m_{i,r,t} = 0 \Rightarrow \omega_{i,r,t+1} = \omega_{i,r,t}$,
$m_{i,r,t} = 1 \Rightarrow \omega_{i,r,t+1} = \overline{\omega}_{i,r}$ ($\overline{\omega}$=restart state),
$m_{i,r,t} = 2 \Rightarrow \omega_{i,r,t+1} = \omega_{i,r,t} - \eta_{i,r,t}$
$with\ P(\eta) > 0\ for\ \ \eta \in \{0, 1\}$.

**Note** $b(m)$ is the only signal sent in each period. $b(m_{-i,t-1})$ is a signal on $\omega_{-i,t-1}$ which is unobserved to $i$ and is a determinant of $b(m_{-i,t})$ (and so $\pi_{i,t}$).

**State of the game.** $s_{i,t} = (J_{1,t}, \ldots J_{n_t,t}) \in \mathcal{S}$, and

$$J_{i,t} = (\xi_t, \omega_{i,t}) \in (\Omega(\xi), \Omega)$$

where

- $\omega_{i,t}$ represents private information
- and $\xi_t$ is public information (shared by all). Example $\xi_t = \{b(m_{1,\tau}), b(m_{2,\tau}), d_\tau\}_{\tau \leq t}$, and knowledge of $\omega_{-i,t}$ the last period of revelation (happens every $T$ periods).

# Model Details.

| Parameter | Firm B | Firm S |
|---|---|---|
| Number of Generators | 2 | 3 |
| Range of $\omega$ | 0-4 | 0-4 |
| MC @ $\omega = (0, 1, 2, 3)^*$ | (20,60,80,100) | (50,100,130,170) |
| Capacity at Const MC | 25 | 15 |
| Costs of Maintenance | 5,000 | 2,000 |

*MC is constant at this cost until capacity and then goes up linearly. At $\omega = 4$ the generator shuts down.

Firm S: small (gas fired) generators with high MC but low start up costs.
Firm B: large (coal fired) generators lower MC and higher start up costs.
Constant, small, elasticity of demand.

**Computational Details**.

- High initial conditions "insures" we try all strategies (induces a lot of experimentation).
- Convergence test is in terms of $\mathcal{L}^2(\mathcal{P}(\mathcal{R}))$ norm of percentage bias in estimates of $W$. 300 million iterations $\mathcal{L}^2(\mathcal{P}(\mathcal{R})) \approx .00005$.

## The Economics of Alternative Environments: Planner vs AsI.

**Base Case: Planner Strategy.** Constrain planner to use the same bid function (compare just investment strategies). Never shuts down without doing maintenance. Weekdays: operates at almost full capacity. Maintenance done on weekend. Maintenance done about 15% of the periods for both B and S generators.

**Base Case: AsI Equilibrium.** Shuts down about 20% of the periods. However about half the time generators are shutdown they are not doing maintenance. Only does maintenance in about 10% of the periods. $\Rightarrow$ 25-30% *more* shutdown but 30% *less* maintenance than the social planner. Most (but not all) shutdown on weekends (just as social planner).

**Base Case: Costs.** Planner does more maintenance and can optimize maintenance jointly over large and small generators. $\Rightarrow$ much lower production costs and lower total costs per unit quantity.

• I.e. the planner produces more and has lower average total costs in a model in which marginal costs increase in quantity. Effect of increased maintenance.

**Base Case: Prices and Quantities.** Planners 2% more output on weekdays, with inelastic demand $\Rightarrow$ price fall of $\approx 10\%$.

• Planner's extra maintenance makes it optimal for it to bid in more and therefore keep price down, and it internalizes the extra CS. AsI firms do not.

• Even the social planner has weekday prices that are 20% higher than weekend prices (the AsI difference is larger). With these primitives large weekend/weekday price differences are "optimal".

| | Base Case | | |
|---|---|---|---|
| | SP | AsI | FI |
| **Panel A: Strategies.** | | | |
| Firm B: Shutdown and Maintenance. | | | |
| Shutdown % | 14.52 | 19.96 | 12.31 |
| Maintenance % | 14.52 | 10.1 | 10.9 |
| Firm S: Shutdown and Maintenance. | | | |
| Shutdown % | 16.85 | 21.48 | 20.74 |
| Maintenance % | 16.85 | 9.83 | 9.91 |
| Firm B: Operating Generators (by day). | | | |
| Saturday | 1,41 | 1.08 | 1.72 |
| Sunday | .88 | 1.21 | 1.65 |
| Weekday Ave. | 1.93 | 1.78 | 1.78 |
| Firm S: Operating Generators (by day). | | | |
| Saturday | 1.55 | 1,56 | 2.03 |
| Sunday | 1.89 | 1.75 | 1.86 |
| Weekday Ave. | 2.80 | 2.64 | 2.55 |
| **Panel B: Costs** ( $\times 10^{-3}$). | | | |
| Maint. B | 29 | 20.2 | 21.95 |
| Maint. S | 20.2 | 11.8 | 11.9 |
| Var. B | 211.1 | 235.1 | 240.4 |
| Var. S | 174.8 | 228.1 | 215.9 |
| Total/Quantity | 0.389 | 0.452 | 0.444 |
| **Panel C: Quantities and Prices.** | | | |
| Ave. Q Wkend | 93.5 | 92.0 | 98.6 |
| Ave. P Wkend | 303 | 325 | 260 |
| Ave. Q Wkday | 185.7 | 181.8 | 181.2 |
| Ave. P Wkday | 374 | 401 | 411 |

## Base Case vs Excess Capacity: AsI & FI

• Maintenance and Shutown.
*Base case:* the FI equlibrium generates less shutdown and more maintenance.
*"Excess" Capacity* (more capacity relative to demand) the AsI equilibrium generates less shutdown and more maintenance.

• Weekday vs Weekend.
*Base case:* AsI vs FI strategies: weekends the AsI equilibrium shuts down more genrators. This enables the firms to signal that their generators will be bid in on the high-priced weekdays.
*Excess Capacity:* Now the ASI firm no longer distinguishes much between weekend and weekday.

• Prices.
With excess capacity the difference between weekday and weekend prices drops dramatically (to 5.4% in the AsI and 1% in the FI equilibrium) and AsI operation increases on weekend.

- Costs.

Increasing capacity relative to demand the average cost is over 30% lower. Raises questions of what are the capital costs and incentives for private generator construction?

- *Total Surplus:*

Increase in capacity/demand ratio generates a large increase in consumer surplus, and a somewhat smaller total surplus increases. Does increased surplus cover social cost of generator construction? And if so how do we induce the investment?

| | Base Case | | Excess Capacity | |
|---|---|---|---|---|
| | AsI | FI | AsI | FI |
| **Panel A: Strategies.** | | | | |
| Firm B: Shutdown and Maintenance. | | | | |
| Shutdown % | 19.96 | 12.31 | 41.97 | 43.75 |
| Maintenance % | 10.1 | 10.9 | 6.47 | 6.25 |
| Firm S: Shutdwon and Maintenance. | | | | |
| Shutdown % | 21.48 | 20.74 | 53.1 | 56.4 |
| Maintenance % | 9.83 | 9.91 | 5.22 | 4.84 |
| Firm B: Operating Generators (by day). | | | | |
| Saturday | 1.08 | 1.72 | 1.03 | 1.0 |
| Sunday | 1.21 | 1.65 | 1.03 | 1.0 |
| Weekday Ave. | 1.78 | 1.78 | 1.03 | 1.0 |
| Firm S: Operating Generators (by day). | | | | |
| Saturday | 1,56 | 2.03 | 1.21 | 0.48 |
| Sunday | 1.75 | 1.86 | 1.20 | 0.44 |
| Weekday Ave. | 2.64 | 2.55 | 1.25 | 1.44 |
| **Panel B: Quantities and Prices.** | | | | |
| Ave. Q Wkend | 92.0 | 98.6 | 33.6 | 33.1 |
| Ave. P Wkend | 325 | 260 | 168 | 175.6 |
| Ave. Q Wkday | 181.8 | 181.2 | 42.50 | 42.43 |
| Ave. P Wkday | 401 | 411 | 177 | 177 |

## Costs, Consumer Surplus and Total Surplus ($\times 10^{-3}$) .

| | Base Case | | Excess Capacity | |
|---|---|---|---|---|
| | AsI | FI | AsI | FI |
| Average Cost | .452 | .444 | .290 | .282 |
| CS* | 581.5 | 595.0 | 1,316 | 1,311 |
| Total Surplus | 288.9 | 301.4 | 1,374 | 1,373 |

* CS= these numbers plus 58,000.

** Total Surplus = these numbers plus 59,000.

## Conclusions.

- There is a need for increased research on the dynamics of market outcomes.

- The framework used for this analysis ought probably to require less of both the agent and the analyst then does "Bayesian Perfect" notions of equilibria.

- Ultimately, that framework will have to integrate the analysis of the reactions to changes in institutions with an analysis of policies for states that are observed repeatedly. "Adaptation" processes, like reinforcement learning, might be adequate for reactions to changes that do not induce calculated experimentation.

- A start for equilibirum conditions at situations that are observed repeatedly are those of "Experience Based Equilibrium". If more stringent equilibrium conditions are justified they should be imposed as they will result in a more precise analysis.