March 2, 2005 Edition > Section: Arts and Letters

Kurt Godel's Astounding Achievement

Books

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Out in the remotest regions of mathematics, far from the bustling and long-populated center, out where this great thriving empire adjoins the windswept badlands of philosophy, is the topic called Foundations. Here mathematicians use the techniques of their discipline to inquire into the nature of that discipline itself, into the very fundamentals of math: number, set, proof, and contradiction. The ideas that underlie Foundations can be traced back to Leibniz and Descartes, but as a coherent subject of study it is quite new, having properly begun only with the work of Giuseppe Peano in the 1890s.

To this day the most sensational achievement in Foundations remains the 1931 paper "On Formally Undecidable Propositions of 'Principia Mathematica' and Related Systems" by the Austrian Kurt Godel. In that paper Godel (1906-78) brought to an abrupt end the program of strict formalism in Foundations - the program, that is, to reduce mathematics to a content-free game in which arbitrary symbols are combined according to arbitrary rules. "Principia Mathematica" was one of the great early texts of Foundations. Bertrand Russell, one of its authors, had declared that: "Mathematics is that subject in which we do not know what we are talking about, nor whether what we say is true." That was the keynote of formalism; that was the conception of mathematics shattered by Godel's great paper. Thanks to Godel we now know - with mathematical certainty! - that any attempt to reduce math to pure formalism must fail, just as the attempt to comb down a sphere covered with hair must always leave a "whorl point" where the combing collapses into uncertainty.

In fact, as Rebecca Goldstein says in "Incompleteness" (Atlas Books, 304 pages, $22.95), Godel's paper " is the third leg, together with Heisenberg's uncertainty principle and Einstein's relativity, of that tripod of theoretical cataclysms that have been felt to force disturbances deep down in the foundations of the 'exact sciences'" during the 20th century. The results in that paper, she goes on to say, "range far beyond their narrow formal domain, addressing such vast and messy issues as the nature of truth and knowledge and certainty."

That last assertion is open to question. There are philosophers who will deny that Godel's results have any consequences at all for epistemology (that is, the philosophical theory of knowledge). If the point is arguable, though, Rebecca Goldstein is just the person to argue it. Professor of philosophy at Trinity College, Hartford, and the author of several novels and a short-story collection, Ms. Goldstein has had a long-standing interest, apparent in her fiction, in the nature of truth and in the connections between thought, language, and the external world. In this new book she offers a discursive account of Godel, his influences, his life, and the contents of that stunning 1931 paper.

"Incompleteness" is a difficult book to categorize. It is not really a biography, though it includes a sufficient account of Godel's rather eventless life and highly peculiar personality. (The standard biography, published in 1997, is by John W. Dawson.)

Nor is it merely a popularized account of Godel's 1931 result, though a sketch of that result and the method Godel used to prove it is included. Ms. Goldstein, rather, uses Godel as a frame on which to hang some commentaries on epistemology and related matters.

Her central concern is the nature of mathematical truth. What does a mathematician actually mean by saying that such and such a proposition is true? The strict formalists of the early 20th century would have replied: He means only that the proposition can be derived from agreed axioms by agreed rules of deduction. This was, to borrow a phrase from logician Martin Davis, the applecart that Godel overturned.

Godel was a firm Platonist, who believed that mathematical objects, while of course they do not belong to the physical world, none the less have a reality that we can "trust," in the same sense that we "trust" what our senses tell us about physical objects. He wrote: "I don't see any reason why we should have any less confidence in this kind of perception, i.e. in mathematical intuition, than in sense perception, which induces us to build up physical theories and to expect that future sense perceptions will agree with them and, moreover, to believe that a question not decidable now has meaning and may be decided in the future."

This clear Platonism put Godel at odds with the foremost mathematician of his time, David Hilbert, who had written that: "Mathematics is a game played according to certain simple rules with meaningless marks on paper." In a landmark address in 1900, Hilbert had proposed 23 problems for mathematicians to concentrate on in the new century. Problem no. 2 was to prove that the rules of arithmetic, properly stated, would suffice to prove every conceivable arithmetic statement either true or false. Hilbert had then gone on to launch a formalist program, devising a means to investigate the nature of mathematical proof by dint of techniques he named "metamathematics." Godel turned these same techniques against the formalist program and exposed its inherent limitations.

Godel's Platonism in fact went against all the main currents of thought in his time, not only in mathematics but also in philosophy and in physics. From 1926 to 1928 Godel was a regular at the weekly meetings of the Vienna Circle, a discussion group led by the philosopher Moritz Schlick. Already a Platonist, Godel must have found the radical empiricism of the Circle uncongenial. As Ms. Goldstein says: "He could not have been more at odds with their metaconvictions." It is something of a mystery why he stuck with them for so long. Ludwig Wittgenstein was a hero to the Circle (though he declined an invitation to join), and Ms. Goldstein has some very interesting things to say about the parallels between Wittgenstein's "Unsayable" and Godel's "Undecidable."

The connection with physics is even more intriguing. It would have been striking enough at any time to say, as Godel did, that mathematical objects possess real existence independent of human minds. It was doubly so to say this when the physical world of solid objects was itself evaporating away into a cloud of quantumtheoretical abstractions. Prominent physicists of Godel's time - notably Werner Heisenberg - denied the existence of any real-world independent of observers. Godel's first intention, on entering the University of Vienna in 1924, had been to study physics, and he maintained a keen interest in the subject all his life.

Albert Einstein was Godel's dearest friend, and the two of them spent many hours together in the 1940s and early 1950s, strolling the lanes and pathways around Princeton's Institute for Advanced Study, where they were both employed. Ms. Goldstein's book is particularly illuminating on the friendship between these two men of genius, and on their common aversion to the denial of objectivity, both physical and mathematical, so characteristic of their time - that cast of mind that later seeped into the humanities and fine arts, poisoning them with the absurdities of "postmodernism."

"Incompleteness" is a difficult book, but not unnecessarily so. This is difficult material, at the borders of what we understand about human knowledge. The author has skillfully humanized it by showing us Godel, Wittgenstein, and Einstein in their work, their friendships, and their disagreements. Perhaps only a novelist could have done this. Rebecca Goldstein has, in any case, done it superbly well.

Mr. Derbyshire last wrote in these pages on Pol Pot.

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