Multiproduct Pricing: Theory and Evidence From Large Retailers
– Online Appendix*

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*The views expressed herein are those of the authors and not necessarily those of the Bank of Canada, or the Bank of Israel.
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A Data Appendix

A.1 Price discounts, final prices and price synchronization

Do synchronization patterns we uncover for regular price adjustments also apply to final prices, which incorporate various types of price discounts? Although our main dataset contains only regular prices, we collected information on price discounts for 10 stores of the largest retail chain in Israel, Shufersal, from January 2016 until mid-2019. We constructed the final price based on the available regular price and the discount code, indicating, for example, a buy-one-get-one-free discount, a third-product-free discount, or two products for 10 NIS (Israeli New Shekel).

Price discounts, or “sales,” are common in the data, accounting for 26% of all price observations (Table A.1). A typical price discount is a large and temporary reduction in price (Klenow and Kryvtsov, 2008; Nakamura and Steinsson, 2008). A sale is associated with a discounted price that is on average 24% lower than the corresponding regular price, and it lasts around 49 days. Since final prices incorporate discounts, they change more frequently and by a larger magnitude than regular prices. The mean fraction of final price changes is 25.7% per month (10.0% for regular price changes), and the mean absolute size of those changes is 23.2% (19.2% for regular price changes).

We find that our measures of synchronization yield similar results for final and regular prices. Only 4.8% (1.9%) of days in the Shufersal sample account for 50% (25%) of all final price changes, which is close to 3.3% and 1.1% of days for regular price changes (Table A.2). The mean Fisher-Konieczny index values for final prices, 0.355, are only a bit higher than 0.262 for regular prices, possibly reflecting higher average fraction of adjusting final prices (Table A.3). These results suggest that retailer’s decisions to post price discounts are largely independent of decisions to change regular prices. In particular, there are no clear peak days for changing price discounts in the store like we observe for regular prices.
Table A.1: Price discounts at Shufersal.

Note: The data cover 10 Shufersal retail stores from January 2016 until mid-2019. Shaded areas outline different store types.

<table>
<thead>
<tr>
<th>Store</th>
<th>Sub-chain</th>
<th>Fraction of discounts</th>
<th>Mean abs size of discounts, %</th>
<th>Duration of discounts, days</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Complete spells</td>
<td>All spells</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Premium stores</td>
<td>0.288</td>
<td>26.4</td>
<td>56</td>
</tr>
<tr>
<td>23</td>
<td>Premium stores</td>
<td>0.289</td>
<td>26.3</td>
<td>50</td>
</tr>
<tr>
<td>39</td>
<td>Premium stores</td>
<td>0.243</td>
<td>26.3</td>
<td>51</td>
</tr>
<tr>
<td>168</td>
<td>Premium stores</td>
<td>0.295</td>
<td>26.4</td>
<td>53</td>
</tr>
<tr>
<td>55</td>
<td>Discount stores</td>
<td>0.302</td>
<td>22.4</td>
<td>41</td>
</tr>
<tr>
<td>113</td>
<td>Discount stores</td>
<td>0.305</td>
<td>22.2</td>
<td>46</td>
</tr>
<tr>
<td>188</td>
<td>Discount stores</td>
<td>0.316</td>
<td>22.4</td>
<td>46</td>
</tr>
<tr>
<td>217</td>
<td>Express (convenience)</td>
<td>0.166</td>
<td>26.4</td>
<td>49</td>
</tr>
<tr>
<td>296</td>
<td>Express (convenience)</td>
<td>0.158</td>
<td>26.2</td>
<td>49</td>
</tr>
<tr>
<td>606</td>
<td>Ultra-discount stores</td>
<td>0.218</td>
<td>22.0</td>
<td>56</td>
</tr>
<tr>
<td>Mean (stores)</td>
<td></td>
<td>0.258</td>
<td>24.7</td>
<td>50</td>
</tr>
<tr>
<td>Mean (pooled)</td>
<td></td>
<td>0.259</td>
<td>24.4</td>
<td>49</td>
</tr>
</tbody>
</table>

Table A.2: Re-pricing peaks for final and regular prices for Shufersal.

Note: The data cover 10 Shufersal retail stores from January 2016 until mid-2019. Column entries compare mean frequency of daily final price changes with regular price changes for peak and off-peak days. Peaks are the days with the highest fraction of price changes that jointly account for half of all price changes in the store. Shaded areas outline different store types.

<table>
<thead>
<tr>
<th>Store</th>
<th>Sub-chain</th>
<th>Peaks (Regular)</th>
<th>Off-peaks (Regular)</th>
<th>Peaks (Final)</th>
<th>Off-peaks (Final)</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td># days</td>
<td>Freq</td>
<td># days</td>
<td>Freq</td>
<td># days</td>
</tr>
<tr>
<td>6</td>
<td>Premium stores</td>
<td>22</td>
<td>0.156</td>
<td>898</td>
<td>0.004</td>
<td>42</td>
</tr>
<tr>
<td>23</td>
<td>Premium stores</td>
<td>25</td>
<td>0.135</td>
<td>880</td>
<td>0.004</td>
<td>44</td>
</tr>
<tr>
<td>39</td>
<td>Premium stores</td>
<td>18</td>
<td>0.155</td>
<td>888</td>
<td>0.003</td>
<td>38</td>
</tr>
<tr>
<td>168</td>
<td>Premium stores</td>
<td>19</td>
<td>0.177</td>
<td>897</td>
<td>0.004</td>
<td>42</td>
</tr>
<tr>
<td>55</td>
<td>Discount stores</td>
<td>19</td>
<td>0.179</td>
<td>890</td>
<td>0.004</td>
<td>51</td>
</tr>
<tr>
<td>113</td>
<td>Discount stores</td>
<td>30</td>
<td>0.109</td>
<td>886</td>
<td>0.004</td>
<td>43</td>
</tr>
<tr>
<td>188</td>
<td>Discount stores</td>
<td>22</td>
<td>0.155</td>
<td>900</td>
<td>0.004</td>
<td>50</td>
</tr>
<tr>
<td>217</td>
<td>Express (convenience)</td>
<td>51</td>
<td>0.033</td>
<td>747</td>
<td>0.002</td>
<td>50</td>
</tr>
<tr>
<td>296</td>
<td>Express (convenience)</td>
<td>31</td>
<td>0.070</td>
<td>903</td>
<td>0.002</td>
<td>46</td>
</tr>
<tr>
<td>606</td>
<td>Ultra-discount stores</td>
<td>67</td>
<td>0.012</td>
<td>894</td>
<td>0.001</td>
<td>31</td>
</tr>
<tr>
<td>Mean (stores)</td>
<td></td>
<td>30</td>
<td>0.118</td>
<td>878</td>
<td>0.003</td>
<td>44</td>
</tr>
<tr>
<td>Mean (pooled)</td>
<td></td>
<td>0.091</td>
<td></td>
<td>0.003</td>
<td></td>
<td>0.232</td>
</tr>
<tr>
<td>Store</td>
<td>Sub-chain</td>
<td>Fisher-Konieczny index</td>
<td>Gini index</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-------</td>
<td>-------------------</td>
<td>------------------------</td>
<td>------------</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Regular prices</td>
<td>Final prices</td>
<td>Regular prices</td>
<td>Final prices</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Premium stores</td>
<td>0.298</td>
<td>0.380</td>
<td>0.810</td>
<td>0.717</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>Premium stores</td>
<td>0.282</td>
<td>0.365</td>
<td>0.798</td>
<td>0.687</td>
<td></td>
</tr>
<tr>
<td>39</td>
<td>Premium stores</td>
<td>0.304</td>
<td>0.364</td>
<td>0.838</td>
<td>0.757</td>
<td></td>
</tr>
<tr>
<td>168</td>
<td>Premium stores</td>
<td>0.319</td>
<td>0.380</td>
<td>0.826</td>
<td>0.720</td>
<td></td>
</tr>
<tr>
<td>55</td>
<td>Discount stores</td>
<td>0.319</td>
<td>0.380</td>
<td>0.827</td>
<td>0.706</td>
<td></td>
</tr>
<tr>
<td>113</td>
<td>Discount stores</td>
<td>0.276</td>
<td>0.369</td>
<td>0.814</td>
<td>0.698</td>
<td></td>
</tr>
<tr>
<td>188</td>
<td>Discount stores</td>
<td>0.305</td>
<td>0.368</td>
<td>0.827</td>
<td>0.688</td>
<td></td>
</tr>
<tr>
<td>217</td>
<td>Express (convenience)</td>
<td>0.177</td>
<td>0.290</td>
<td>0.694</td>
<td>0.630</td>
<td></td>
</tr>
<tr>
<td>296</td>
<td>Express (convenience)</td>
<td>0.212</td>
<td>0.298</td>
<td>0.790</td>
<td>0.698</td>
<td></td>
</tr>
<tr>
<td>606</td>
<td>Ultra-discount stores</td>
<td>0.124</td>
<td>0.353</td>
<td>0.751</td>
<td>0.778</td>
<td></td>
</tr>
<tr>
<td>Mean (stores)</td>
<td></td>
<td>0.262</td>
<td>0.355</td>
<td>0.797</td>
<td>0.708</td>
<td></td>
</tr>
<tr>
<td>Mean (pooled)</td>
<td></td>
<td>0.274</td>
<td>0.363</td>
<td>0.798</td>
<td>0.709</td>
<td></td>
</tr>
</tbody>
</table>

Table A.3: Synchronization of regular and final price changes for Shufersal.

Note: The data cover 10 Shufersal retail stores from January 2016 until mid-2019. Column entries compare synchronization statistics (Fisher-Konieczny and Gini) for daily final and regular price changes. Shaded areas outline different store types.
A.2 Comparisons of synchronization statistics

Probit regressions

We studied two alternative measures of synchronization. Perhaps, the most widely used measure is based on estimated logit or probit regressions. For each store in the sample, we estimate a probit regression, where the dependent variable is the indicator of a price change for a given product and date, and the right-hand side is the fraction of price adjustments for all other products in that store on that date. Table A.4 reports the weighted mean across stores of the estimated average marginal effects (all estimated coefficients are highly significant, with \( p \)-values well below 0.01). At daily (weekly) frequency, the average marginal effect is 0.22 (0.61), i.e., for a 10 percentage point change in the fraction of price changes (for all other products in the store on the same day), the probability of a price change for a given product goes up by 2.2 (6.1) percentage points. This degree of synchronization is somewhat higher than in other studies. For example, Midrigan (2011) reports the range of 0.09 to 0.27 in the weekly scanner data. The difference can be explain, to some degree, by a larger number of products in the cross-section, which is found to be associated with higher degree of synchronization (Bhattarai and Schoenle (2014); Dedola, Kristoffersenz, and Züllig, 2019).

More importantly, however, we can demonstrate that synchronization measures derived from probit regressions are no longer reliable when the distribution of the fraction of price changes places a lot of weight on extremes, as is the case in our analysis. This can be illustrated analytically as follows.

Consider the standard probit regression based on data of a single store:

\[
P(I_{i,t} = 1) = \Phi(\beta_0 + \beta_1 F_{-i,t})
\]

Above, \( \Phi(\cdot) \) is the normal c.d.f., \( I_{i,t} \) is a dummy variable that assumes the value 1 when the price of product \( i \) changes in period \( t \), and \( F_{-i,t} = \frac{1}{N-1} \sum_{j \neq i} I_{j,t} \) is the fraction of price changes at date \( t \) excluding product \( i \), where \( N \) is the total number of products. We consider here a case in which the number of products sold by a store is large, in such a way that results are essentially the same if \( F_{-i,t} \) is replaced by the total fraction of adjustments at date \( t \), which is independent of \( i \).
<table>
<thead>
<tr>
<th>Sample</th>
<th>Frequency of observations</th>
<th>Synchronization statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. By store</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>daily</td>
<td>0.259</td>
<td>0.747</td>
</tr>
<tr>
<td>weekly</td>
<td>0.241</td>
<td>0.534</td>
</tr>
<tr>
<td>monthly</td>
<td>0.201</td>
<td>0.315</td>
</tr>
<tr>
<td>Price increases</td>
<td></td>
<td></td>
</tr>
<tr>
<td>daily</td>
<td>0.218</td>
<td>0.784</td>
</tr>
<tr>
<td>Price decreases</td>
<td></td>
<td></td>
</tr>
<tr>
<td>daily</td>
<td>0.201</td>
<td>0.767</td>
</tr>
<tr>
<td>Same product category*</td>
<td>daily</td>
<td>0.436</td>
</tr>
<tr>
<td>Flexible price goods</td>
<td>daily</td>
<td>0.366</td>
</tr>
<tr>
<td>Larger stores</td>
<td>daily</td>
<td>0.264</td>
</tr>
<tr>
<td>Offline stores**</td>
<td>daily</td>
<td>0.280</td>
</tr>
<tr>
<td>Online stores**</td>
<td>daily</td>
<td>0.343</td>
</tr>
<tr>
<td><strong>B. By chain</strong></td>
<td>daily</td>
<td>0.235</td>
</tr>
<tr>
<td><strong>C. All observations</strong></td>
<td>daily</td>
<td>0.206</td>
</tr>
<tr>
<td><strong>D. IRI data</strong></td>
<td>weekly</td>
<td>0.098</td>
</tr>
<tr>
<td>monthly</td>
<td>0.133</td>
<td>0.138</td>
</tr>
</tbody>
</table>

Table A.4: Comparison of synchronization statistics.  
*Note: The table provides weighted means for synchronization indices across stores (Panels A, D), chains (Panel B), or unweighted means (Panel C). Weights are the average number of products in a store per day. Synchronization of price changes is measured by Fisher and Konieczny (2000) index defined in Section 2.2 (main text), and by Gini and probit indices defined in this Appendix. Same product category*: daily statistics computed for subsets of products belonging to the same product category (* Shufersal stores only). “Flexible price goods”: statistics computed for subsets of products with the daily fraction of price changes in the top quartile in the store. “Larger stores”: daily statistics computed for subsets of products in stores larger than the median store (by the number of products per day). ** We select only brick-and-mortar stores belonging to the same chains as the four online stores in our dataset. † 10% random sample is used for probit regressions.

Now assume, as in the model with free adjustments in Appendix B.6, that there are peak days in which the fraction of adjustments is $M$ and non-peak days in which the firm adjusts a small share of its prices $m < M$. The probit coefficients would then converge to the solution of the system of equations below:

\[
m = \Phi(\beta_0 + \beta_1 m)
\]

\[
M = \Phi(\beta_0 + \beta_1 M)
\]

This implies

\[
\beta_1 = \left(\frac{\Phi^{-1}(M) - \Phi^{-1}(m)}{M - m}\right)
\]
$$\beta_0 = \frac{M\Phi^{-1}(m) - m\Phi^{-1}(M)}{M - m}$$

If $p$ is the fraction of peak days, we can then compute the average marginal effects implied by the probit regression as

$$\text{avg. marginal effects} = p\beta_1\phi(\beta_0 + \beta_1 m) + (1 - p)\beta_1\phi(\beta_0 + \beta_1 m)$$

Figure A.1 shows the relationship between synchronization, measured both as probit marginal effects and the Fischer-Konieczny index, and frequency of peaks. Results are generated as follows. We fix the mass of off-peak adjustments at $m = 0.05\%$ and the total daily frequency of adjustments $f = 0.87\%$, as in our data. For a given value of peak frequency $p$, the mass of peak adjustments $M$ is then given by $pM + (1 - p)m = f$. It is possible to see that, counterintuitively, the probit suggests a lower degree of synchronization for models with less frequent, but large peaks. This motivates our choice of the FK index as our main synchronization measure.

![Figure A.1: Fisher-Konieczny index probit marginal effects.](image)

**Gini index**

The second synchronization index is akin to the Gini inequality index. It is based on the Lorenz curve that depicts the distribution of repricing activity by plotting the percentile of days by the number of price changes on the horizontal axis and cumulative fraction of price
changes on the vertical axis. Figure A.2 shows that re-pricing activity is very unequal across
days, with around 6% of days accounting for half of all price changes. The Gini statistic is
the size of the area between the Lorenz curve and the 45° line divided by a half. Table A.4
shows a high degree of synchronization (i.e., high inequality of repricing across days), 0.747
on average, and which varies substantially across stores (Panel D, Figure A.5).

Although both FK and Gini measures are derived from the distribution of the fraction of
adjusting prices over time, they are expressed against different quantitative scales. Nonethe-
less, Table A.4 shows that two synchronization measures are qualitatively in line with each
other across different dimensions of the data.

Figure A.2: Lorenz curve for inequality of re-pricing activity across days, selected stores.

*Note: Figure A.2 shows the Lorenz curve (cumulative distribution of price changes across days) for four
selected stores.*
A.3 Synchronization and calendar effects

A.3.1 Frequency of price changes and holidays in Israel

We study the overlap of repricing peaks with holidays or holiday eves in Israel. Out of 1598 days in the sample, 138 are national holidays. On some of the holidays (e.g., religious holidays) stores are closed. The overlap between the peaks in store and the holidays is small: on average, only 6.5% of peak days are holidays and, in turn, only 4.1% of holidays are peaks (Table A.5). These frequencies are similar if we aggregate store observations by chain or the entire sample.

During the week before a holiday, the likelihood of a peak day falls by about one-quarter. Using the subset of the Israeli data for the largest food retailer, Shufersal, we find that during weeks before the holidays, retailers increase the frequency and size of price discounts, especially before two major holidays—Rosh Hashana (New Year) and Pessach (similar to Easter). The effect of weeks prior to holidays on the final price (the sum of the regular price and the discount) is not significantly different from zero. This evidence suggests that prior to holidays, retailers shift their pricing activity toward price discounts. This is consistent with evidence in Warner and Barsky (1995) that retailers time their markdowns to periods of high demand and fierce competition between chains.

<table>
<thead>
<tr>
<th>Store</th>
<th>Fraction of daily observations</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Not-Peak / Not-Holiday</td>
<td>Peak / Not-Holiday</td>
</tr>
<tr>
<td>By store</td>
<td>85.07%</td>
<td>5.52%</td>
</tr>
<tr>
<td>By chain</td>
<td>85.92%</td>
<td>4.78%</td>
</tr>
<tr>
<td>All observations</td>
<td>85.89%</td>
<td>4.88%</td>
</tr>
</tbody>
</table>

Table A.5: Peaks and holidays.

*Note: Peaks are the days with the highest fraction of regular price changes that jointly account for half of all price changes in the store. For each store we compute the fraction of days that are peaks (not peaks) and fall (do not fall) on a holiday. Entries are weighted means of these fractions across stores (first row), chains (second row), or unweighted means (last row). Weights are the average number of products in a store per day.*
A.3.2 Frequency of price changes and weekly/monthly fixed effects

In this section, we analyze the prevalence of peaks by day of the week and by day of the month. Figures A.3 and A.4 provide distributions of the daily price-change observations and frequency of price changes by day of the week (month). Peaks tend to fall more frequently early in the week (Mondays and Tuesdays) and are the least frequent during the weekend (Fridays and Saturdays). This pattern is somewhat offset by the higher fraction of price adjustments during peaks that fall on Fridays (11%) and Saturdays (14%) relative to working days (around 8%). Peaks are twice as likely to occur during the first week of the month than on any other week. The number of adjustments during a peak day does not depend on the day of the month.

This evidence is in line with earlier work by Levy et al. (1997), Levy et al. (1998), Dutta et al. (1999) who found some evidence of time-dependent pricing using weekly price data for U.S. supermarket and drugstore chains. For example, Levy et al. (1997) reports that most prices in a store are changed on Sundays and Mondays. Since our data is daily, we can provide a more accurate account of how much calendar events overlap with re-pricing peaks. We find that re-pricing peaks are specific to each store, and price changes show little synchronization across retailers. In all, only some of the partial synchronization of price changes by stores can be associated with holidays and week/month fixed effects.

Figure A.3: Fraction of observations, by day of the week and by day of the month.

Note: Figure A.3 shows the (weighted) fraction of price observations, by day of the week (left plot) and by day of the month (right plot), for all days and only peak days. Weights are the average number of products in a store per day. Peaks are the days with the highest fraction of regular price changes that jointly account for half of all price changes in the store. Shaded (empty) bars correspond to changes for all days (for the subset of peak days).
Figure A.4: Fraction of price changes, by day of the week and by day of the month.
Note: Figure A.4 shows the (weighted) fraction of price changes, by day of the week (left plot) and by day of the month (right plot), for all days and only peak days. Weights are the average number of products in a store per day. Peaks are the days with the highest fraction of regular price changes that jointly account for half of all price changes in the store. Shaded (empty) bars correspond to changes for all days (for the subset of peak days).
A.4 Synchronization of price changes across stores and across products in a store

Figure A.5 provides distribution of the number of products across stores, average fraction of adjusting prices across stores, and two measures of synchronization across stores. The number of products sold in a given store is large: on average 7,217 products are sold on a given day, 1,311 (31,847) products in the smallest (largest) store in the dataset. Due to the relatively short time span of the data and stable near-zero inflation during the sample period, a large proportion of products, around 38%, do not register a regular price change in our sample.

Figure A.5: Store-level number of products, frequency and synchronization of price changes. 
Note: Panel A provides histograms for the average number of price observations per store per day (all products and products with at least one price change over the sample period). Panel B shows the histogram of the fraction of price changes across stores. Panel C gives the histogram of the Fisher-Konieczny synchronization index across stores, and Panel D provides the histogram of the Gini index values across stores (Gini index is defined in Appendix A.2).

We observe, in particular, that synchronization within stores is higher than across stores. When we pool observations across stores, the corresponding FK synchronization index values are substantially lower than their store-level averages: 0.206 vs 0.259 (row C in Table 1, main text). This finding is in line with findings from previous studies of various datasets: Lach and Tsiddon (1996) using selected food prices in Israel, Midrigan (2011) using U.S. Dominick’s

To highlight the variation of price-change synchronization across stores, we study how it depends on similarities across goods, price flexibility, store size and type, and chain effects.

First, we ask whether prices are more synchronized for products within the same broad category than across categories. Previous empirical studies of retail micro data found that prices for similar products tend to be more synchronized (Levy et al., 1997; Dutta et al., 1999). The Bank of Israel classified 50 broad product categories for Shufersal stores. Panel A in Figure A.6 compares the distribution of the FK synchronization index across stores computed for all products with the distribution conditional on products in the same category. Indeed, price changes are more synchronized within, rather than across, categories. On average, the FK index is 0.285 for products in the same category, versus 0.248 for all products in Shufersal retail chain (Table A.6).

<table>
<thead>
<tr>
<th>Sample</th>
<th>Mean number of products</th>
<th>Mean fraction of price changes</th>
<th>Mean absolute size of price changes</th>
<th>Synchronization of price changes</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Daily obs</td>
<td>7,217</td>
<td>0.87%</td>
<td>20.8%</td>
<td>0.259</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Price increases</td>
<td>7,217</td>
<td>0.45%</td>
<td>19.9%</td>
<td>0.218</td>
</tr>
<tr>
<td>Price decreases</td>
<td>7,217</td>
<td>0.42%</td>
<td>21.4%</td>
<td>0.201</td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shufersal stores</td>
<td>6,989</td>
<td>0.41%</td>
<td>20.4%</td>
<td>0.248</td>
</tr>
<tr>
<td>Same product category</td>
<td>6,989</td>
<td>0.52%</td>
<td>19.4%</td>
<td>0.285</td>
</tr>
<tr>
<td>Food categories</td>
<td>6,711</td>
<td>0.37%</td>
<td>20.3%</td>
<td>0.228</td>
</tr>
<tr>
<td>Non-food categories</td>
<td>837</td>
<td>0.52%</td>
<td>22.1%</td>
<td>0.283</td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flexible price goods</td>
<td>2,092</td>
<td>2.30%</td>
<td>21.2%</td>
<td>0.366</td>
</tr>
<tr>
<td>Larger stores</td>
<td>10,267</td>
<td>0.89%</td>
<td>21.0%</td>
<td>0.264</td>
</tr>
<tr>
<td>E</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Offline stores</td>
<td>7,897</td>
<td>1.01%</td>
<td>20.7%</td>
<td>0.280</td>
</tr>
<tr>
<td>Online stores</td>
<td>10,003</td>
<td>0.99%</td>
<td>19.4%</td>
<td>0.343</td>
</tr>
<tr>
<td>F</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IRI Symphony (weekly)</td>
<td>1,854</td>
<td>15.96%</td>
<td>26.2%</td>
<td>0.098</td>
</tr>
<tr>
<td>IRI Symphony (monthly)</td>
<td>2,376</td>
<td>28.56%</td>
<td>23.9%</td>
<td>0.133</td>
</tr>
</tbody>
</table>

Table A.6: Store-level synchronization of prices across stores and products.

Note: The table provides weighted means for statistic in columns (1)–(4) across stores. Panel B reports statistics for price increases and decreases. Panel C provides statistics for Shufersal stores. “Shufersal stores”:
all Shufersal stores. “Same product category”: daily statistics computed for subsets of products belonging to the same product category (weighted by the number of items in a category). “Food categories”: statistics for food products, “Non-food categories”: statistics for non-food products. Panel D, “Flexible price goods”: statistics computed for subsets of products with the daily fraction of price changes in the top quartile in the store; “Larger stores”: daily statistics computed for subsets of products in stores larger than the median store (by the number of products per day). Panel E compares the results for four online stores with four corresponding brick-and-mortar stores from the same chains. Panel F provides the results using IRI Symphony weekly or monthly data. Weights are the average number of products in a store per day. Synchronization of price changes in column (4) is measured by Fisher and Konieczny (2000) index defined in Section 2.2 in the main text.
Figure A.6: Fisher-Konieczny synchronization across stores.

Note: Panels A and B provide histograms for the Fisher-Konieczny (FK) index across stores. Panel A: all products and those within the same product category (Shufersal stores only). Panel B: all products and those for the products with mean frequency of price changes in the top quartile for that store. Panel C: scatter plot of store size (number of products per day) versus corresponding store FK index values, with the (unweighted) fitted OLS line. Panel D: scatter plot of FK index values for the store (x-axis) and its chain (y-axis), with 45° line and fitted OLS line.

We also document that 0.228 synchronization of price changes for food products—the dominant product category in these data—is similar to 0.283 synchronization for non-food products, such as personal care products, cleaning supplies, and small household appliances. Slightly higher synchronization for non-food products can be attributed to their higher frequency of price changes, 0.52% vs 0.37% for food products. This evidence suggests partial synchronization applies to a broader set of products, not just food products.

Similarly, Panel B in Figure A.6 compares the distribution for all products with the one for only flexible price products, which we define as products in the top quartile of the frequency of price changes in the store. Flexible price changes are more synchronized, with an FK index of 0.366. This finding is consistent with Goldberg and Hellerstein (2009) and Bhattarai and Schoenle (2014), who use producer price data from the Bureau of Labor Statistics to document that firms selling more products adjust prices more frequently.

In our data, larger stores tend to have more synchronized price changes, as evidenced in Panel C in Figure A.6. The FK index for the subset of stores above the median is 0.264,
versus 0.259 for all stores. Using Danish PPI micro data, Dedola, Kristoffersenz, and Züllig (2019) find that synchronization of price changes is stronger within firms than across firms in the same industry, and that within-firm synchronization increases with the number of goods per firm.

We also compared synchronization of prices in online and brick-and-mortar stores. For cleaner comparison, we only looked at brick-and-mortar stores belonging to the same chains as the four online stores in our dataset. The frequency and size of price changes are very similar for online and offline prices, but online prices are more synchronized, with an FK index of 0.343 versus 0.280 for prices in the brick-and-mortar stores in the same chains. These findings are consistent with Cavallo (2017), who studies price adjustments of large retailers that sell both online and offline. He finds that the frequency and size of online and offline price changes are similar, and that there is little synchronization between online and offline price changes. Gorodnichenko and Talavera (2017) and Gorodnichenko, Sheremirov, and Talavera (2018) use online price data from price-comparison websites and an online shopping platform. They find that relative to prices in brick-and-mortar stores, online prices change more frequently, but otherwise they exhibit similar behavior; in particular, prices posted by the same seller are similarly synchronized. Online stores in our dataset have a much larger scale, offering 10,003 products per day, which may explain the higher synchronization of price changes relative to prices in conventional stores. We also compare only stores of the same chain, which helps us control for chain effects.

We compare FK index values for each store with corresponding FK index values for the chain to which that store belongs. The scatter plot in Panel D of Figure A.6 shows that synchronization of price changes across products of the entire chain is somewhat lower than at a store level. The weighted mean FK index goes down to 0.235. This suggests that retailers actively synchronize price changes across their stores. This is consistent with recent findings of uniform pricing by DellaVigna and Gentzkow (2019) and Hitsch, Hortaçsu, and Lin (2019) for U.S. retail chains.

Finally, we investigate whether peaks of price adjustment are characteristic only for a subset of products within a store, as opposed to being a store-specific feature. In short, we do not find that peaks are driven by adjustments of a subset of products. Figure A.7 contains two scatter plots. Each point on a scatter plot represents a pair of two frequencies for a product (or product category) in a store: frequency of adjustment on off-peak days (x-axis) and a frequency of adjustments on peak days (y-axis). These scatter plots are convenient for gauging the balance between adjustment frequencies on peak versus off-peak days at a product (or product category) level. Products for which these two frequencies are equal would show as a dot on the 45° line. Products for which adjustments fall predominantly on
peak days are above the 45° line. If a store sampled prices completely randomly across its products each day, product-level price-change frequencies would be the same for the subsets of peak and off-peak days.

The left plot is for Shufersal stores only, where each point of the scatter plot is for products in the same product category (50 categories). The right plot is for all individual products in the dataset, with at least 10 price-change observations. Both plots show that the mass of dots is spread above and below the 45° line: there are products that adjust prices mostly during peak days, but there are also products that adjust mostly during off-peak days. The share of products whose adjustments are more likely during peak days than during off-peak days is 0.82 for Shufersal and 0.76 for all stores. Similarly, the share of products whose adjustments are twice as likely during peak days than during off-peak days is 0.65 for Shufersal and 0.52 for all stores. Conversely, the share of products whose adjustments are twice as likely during off-peaks days than during peak days is 0.12 for Shufersal and 0.07 for all stores. Hence, on peak re-pricing days, stores increase (decrease) the likelihood of changing prices for some of their products relative to other days.

Figure A.7: Fraction of price changes during peak and off-peak days, by product.
A.5 Comparisons with weekly IRI scanner data

We highlighted that comprehensive coverage of products in a store is a distinguishing feature of this dataset which reveals striking patterns of price adjustments by large retailers. The second distinctive feature of the dataset is its detailed coverage over time—providing daily price quotes for each of the store’s very many products. Most micro data on prices provide less frequent price quotes, typically at weekly or monthly frequency. Time aggregation of daily frequency of observations, however, filters out high-frequency movement in product prices. For example, mean duration of price spells implied by the weekly and monthly frequencies of price changes reported in Table 1 (main text) are 8.4 months and 5 months (23 weeks) respectively—which is longer than 3.5 months (104 days) implied by daily frequency.  

To underscore the implications of time aggregation of price observations, we compare our results for Israel with those from the Information Resources, Inc (IRI) data for the United States. The IRI marketing and market research dataset contains weekly transactions (“scanner”) data for 31 grocery products, such as milk, beer, coffee, razors, laundry detergent, and frozen pizza. The data provide weekly revenues and quantities for products sold in grocery stores across 50 U.S. metropolitan areas. In a given week, unit prices for each product are constructed by dividing weekly revenue by the quantity sold. The share of nominal revenues over the sample period in total revenues over the sample period is used as a store weight.

For better comparisons with the Israeli data, we focus on stores that belong to large retail chains, and so we exclude independent stores from the IRI sample. The dataset that we use contains around 448 million weekly observations in 518 stores and 75 retail chains, and covers the span of 132 months, from January 2001 to December 2011. Unlike the data for Israel, the IRI dataset includes small stores. Nonetheless, the average number of products sold in a given week is 1,854, reflecting a sufficiently large number of large stores. Another reason for a smaller number of products per week is that the IRI dataset does not provide prices for products that did not register a sale within a week. This implies a higher frequency of weekly price changes in the IRI dataset, 16% (Panel D, Table 1, main text), than the frequency of weekly posted prices in Israel, 4.3%. Conditional on a price change, however, the absolute magnitudes of price changes are not so far apart: 26.2% in the IRI data versus 20.2% in the Israeli data.

Measured synchronization of weekly price changes in IRI data is much lower than syn-

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1Duration implied by frequency \( f r t \) is given by the continuous time formula \(-1/\ln(1 - fr_t)\). Measuring duration directly from observed spells leads to similar conclusions.

2More details are provided in Bronnenberg, Kruger, and Mela (2008).

3We define a unique product identifier by matching UPC codes for that product with the product description (e.g., Budweiser lager 355ml). We exclude products that belong to a store’s private label (their coding was changed by IRI in 2007 and 2008), and products that have fewer than two observations per week, and we exclude observations with a unit price less than $0.10.
chronization of weekly price changes in the Israeli data, 0.098 versus 0.241. When we further aggregate Israeli price changes to a monthly frequency of observations, synchronization decreases even more, to 0.201. We do not obtain the same aggregation result when we aggregate weekly IRI price changes to a monthly frequency, rather, the FK index increases to 0.133.

Higher frequency of price changes in IRI data than in the Israeli data can be attributed to mandatory item price tags in Israel. Levy et al. (1997) examine the process for changing prices in five U.S. supermarket chains. They show that supermarkets situated in states that require a separate price tag on each item face two-and-a-half times higher menu cost and adjust their prices twice less frequently than supermarkets not facing such requirements. At the same time, lower synchronization of price changes in the U.S. data suggests that store-specific cost of price adjustments—costs of collecting information, making and implementing pricing decisions—are also lower in the U.S. relative to Israel.\(^4\) The differences in the results may also reflect time aggregation of the IRI dataset, which only provides average price changes on a weekly basis. Cavallo (2018) argues that even datasets that provide weekly average prices can produce biased results for price change statistics, such as the frequency and size of price changes.

To gauge the extent to which our quantitative results depend on time aggregation, we compare IRI and Israeli price changes along two additional sets of statistics that indirectly depend on the degree of price change synchronization.

**Spectral analysis of store-level price changes.** A useful way of circumventing the time aggregation problem is to examine the spectral densities of store-level time series of the frequency of price changes.\(^5\) We apply spectral analysis of three store samples: regular price changes in all retail stores in the Israeli data, regular and discounted price changes in Shufersal stores in the Israeli data, and regular price changes in IRI data. For each store in the sample, we compute spectral density of its daily (weekly, monthly) fraction of price changes. We then compute the weighted mean of the store-level spectra, where weights are the average number of products in a store per day.

There are two main findings (Figure A.8). First, for the Israeli stores, spectral density is high for short frequencies of less than 30 days. This remains the case as we aggregate daily price changes into weekly and monthly, and then construct corresponding weekly and monthly spectra. Heavy short-frequency spectra are consistent with the time series patterns

\(^4\)The price-setting model we introduce in Section 3 (main text) predicts that raising item-specific menu cost \textit{ceteris paribus} increases the average fraction of adjusting prices and increases the synchronization of price changes. Synchronization of price changes in IRI data is \textit{lower} than in the Israeli data (Table 1, main text). Hence, to account for the differences in the average fraction and synchronization of price changes in the U.S. and Israel, the model requires lower store-specific cost of price adjustment in the U.S. relative to Israel.

\(^5\)We thank Fernando Alvarez for this insight.
we document for daily price changes—a bulk of them occur during peak days, which occur more frequently than once a month. For the most part, peaks do not occur at regular time intervals, which elevates the whole short-frequency part of the spectrum. Second, including price discounts does not alter the shape of the spectra, which is consistent with our conclusion that peaks are not driven by sales.

Figure A.8: Spectral density of regular prices changes.

Note: Figures provide average spectral density of store-level regular price changes for all retail stores in the Israeli data (“BoI”); regular and discounted price changes in Shufersal stores in the Israeli data (“BoI, Shufersal”); and regular price changes in IRI data. For each store in the sample, we compute spectral density of its daily (weekly, monthly) fraction of price changes. We then compute the weighted mean of the store-level spectra, where weights are the average number of products in a store per day. Spectrum frequencies are daily (top left), weekly (top right), and monthly (bottom).

Finally, the weekly IRI spectrum is also heavier on the short end, suggesting high-frequency movements in price changes similar to those we find in the Israeli data. This is no longer the case when we aggregate IRI data from weekly to monthly frequency. The monthly IRI spectrum is now tilted upward, unlike the Israeli data, which are still sloping downward. This suggests that time averaging in IRI data washes out some of the important
high-frequency variation in the daily frequency of price changes.

**Distribution of standardized price changes.** First, the literature has emphasized that quantitative success of menu-cost models is associated with their ability to match the distribution of price changes in the data. In particular, the mass of small price changes (Klenow and Kryvtsov, 2008) and the thickness of the tails of the distribution (Midrigan, 2011; Alvarez, Le Bihan, and Lippi, 2016) have been related to menu-cost models’ predictions for the degree of monetary non-neutrality.

Following the literature, we filter out heterogeneity across stores by constructing “standardized” log price changes for product $i$, store $s$, and time $t$, $z_{ist}$, by subtracting from log price change $p_{ist}$ the mean $\mu_s$ of non-zero log price changes and dividing by their standard deviation $\sigma_s$ at a store level: $z_{ist} \equiv \frac{p_{ist} - \mu_s}{\mu_s}$ $p_{ist} \neq 0$. The distribution of standardized regular price changes in Israel is provided in Figure A.9. We then compute four statistics characterizing the shape of the distribution of standardized price changes that are used in Table II in Alvarez and Lippi (2014): the ratio of the standard deviation of price changes to the mean absolute price change, kurtosis of price changes, and the share of price changes that, in absolute value, are smaller than half (quarter) of the mean absolute price change. To control for heterogeneity of price changes within a store, we use three alternative definitions of standardized changes, where for prices within a store we control for differences in price levels, product’s frequency of price changes, or product category. For comparison with IRI data, we also report these statistics at weekly and monthly observation frequencies.

Our results show important influence of time averaging on the distribution of price changes. Time aggregation of Israeli statistics from daily to weekly or monthly frequency roughly doubles the mass of small price changes. Furthermore, at weekly or monthly frequency, kurtosis and the mass of small price changes are slightly lower in Israeli data than in scanner data, which could be in part due to price averaging in the weekly scanner data. The results are not driven by differences across datasets or heterogeneity across products (see Table A.7). These results are in line with Cavallo (2018) who shows that time aggregation can materially affect the distribution of price changes.

In all, these findings underscore the benefits of using daily price observations for fully exploring the implications of synchronization of price changes.

We compare our results for Israel with those from the Information Resources, Inc (IRI) data for the United States. The IRI marketing and market research dataset contains weekly transactions (“scanner”) data for 31 grocery products, such as milk, beer, coffee, razors, laundry detergent, and frozen pizza, across 50 U.S. metropolitan areas.\[^{6}\]

Measured synchronization of weekly regular price changes in IRI data is much lower than

\[^{6}\text{More details are provided in Bronnenberg, Kruger, and Mela (2008).}\]
synchronization of weekly regular price changes in the Israeli data, 0.098 versus 0.241. In addition to differences between datasets,\textsuperscript{7} some of the difference could be due to time aggregation of daily price changes to a weekly frequency. To better account for time aggregation in dataset comparisons, we produce two additional sets of statistics: the spectrum of store-level price changes and the fraction of small price changes.

Figure A.9: Distribution of standardized price changes.

\textit{Note: Figure A.9 shows the unweighted distribution of standardized non-zero price changes across stores in Israeli data. Define standardized log price changes for product i, store s and time t, }$z_{ist}$, \textit{by subtracting from log price change }$p_{ist}$\textit{ the mean }$\mu_s$\textit{ of non-zero log price change and dividing by their standard deviation }$\sigma_s$\textit{ of log price changes at a store level: }$z_{ist} \equiv \frac{p_{ist} - \mu_s}{\mu_s} \left|_{p_{ist} \neq 0}.}$

\textsuperscript{7}Higher frequency of weekly regular price changes in IRI data, 16.0\%, than in the Israeli data, 4.3\%, can be attributed to mandatory item price tags in Israel. Levy et al. (1997) examine the process for changing prices in five U.S. supermarket chains. They show that supermarkets situated in states that require a separate price tag on each item face two-and-a-half times higher menu cost and adjust their prices twice less frequently than supermarkets not facing such requirements. At the same time, lower synchronization of price changes in the U.S. data suggests that store-specific cost of pricing adjustments—costs of collecting information, making and implementing pricing decisions—are also lower in the U.S. relative to Israel.
| Source                  | Country | Data | $\text{std}(|z_{ist}|)/\text{mean}(|z_{ist}|)$ | $\text{kurtosis}(z_{ist})$ | Fraction $|z_{ist}| < 0.5 \times \text{mean}(|z_{ist}|)$ | Fraction $|z_{ist}| < 0.25 \times \text{mean}(|z_{ist}|)$ |
|------------------------|---------|------|-----------------------------------------------|-----------------------------|-------------------------------------------------|--------------------------------------------------|
| A Standardized by store| Israel  | Daily | 0.68                                          | 3.5                         | 0.14                                            | 0.06                                             |
|                        |         | Weekly| 0.67                                          | 3.3                         | 0.27                                            | 0.10                                             |
|                        |         | Monthly| 0.69                                         | 3.4                         | 0.28                                            | 0.11                                             |
| B By store & price level| Israel  | Daily | 0.69                                          | 3.6                         | 0.15                                            | 0.07                                             |
|                        |         | Weekly| 0.67                                          | 3.4                         | 0.26                                            | 0.11                                             |
|                        |         | Monthly| 0.69                                         | 3.4                         | 0.28                                            | 0.11                                             |
| C By store & freq of pch| Israel  | Daily | 0.68                                          | 3.8                         | 0.14                                            | 0.05                                             |
|                        |         | Weekly| 0.67                                          | 3.6                         | 0.27                                            | 0.10                                             |
|                        |         | Monthly| 0.70                                         | 3.7                         | 0.28                                            | 0.11                                             |
| D By product (Shufersal)| Israel  | Daily | 0.67                                          | 4.1                         | 0.15                                            | 0.08                                             |
|                        |         | Weekly| 0.67                                          | 4.1                         | 0.26                                            | 0.13                                             |
|                        |         | Monthly| 0.72                                         | 4.0                         | 0.28                                            | 0.15                                             |
| E IRI Symphony         | United States| Weekly| 0.85                                          | 3.4                         | 0.37                                            | 0.17                                             |
|                        |         | Monthly| 0.91                                          | 4.7                         | 0.38                                            | 0.20                                             |
| F Midrigan AC Nielsen  | United States| Weekly| 0.72                                          | 3.6                         | 0.25                                            | 0.10                                             |
|                        |         | Monthly| 0.81                                          | 4.5                         | 0.31                                            | 0.14                                             |
|                        | Bhattarai and Schoenle| Monthly| 1.55                                          | 17                          | 0.50                                            | 0.38                                             |

Table A.7: Standardized price changes, Israel and the United States.

Note: We compute four statistics characterizing the shape of the distribution of standardized price changes that are used in Table II in Alvarez and Lippi (2014). Define “standardized” log price changes for product $i$, store $s$ and time $t$, $z_{ist}$, by subtracting from log price $p_{ist}$ the mean $\mu_s$ of non-zero log price and dividing by their standard deviation $\sigma_s$ changes at a store level: $z_{ist} = \frac{p_{ist} - \mu_s}{\sigma_s}_{p_{ist} \neq 0}$. Panel A provides statistics for daily, weekly and monthly frequency of observations. To control for heterogeneity of price changes within a store, we also use three alternative definitions of standardized changes, where for prices within a store we control for (i) differences in price levels (Panel B), (ii) product’s frequency of price changes (Panel C), or (iii) product category (Panel D). In cases (i) and (ii), we define standardized price changes by store and decile of price level (or frequency). In case (iii), for Israel we use the data for 10 Shufersal retail stores from January 2016 until mid-2019, which includes information on product categories. In cases (ii) and (iii), we define standardized price changes by store and product category. In IRI Symphony data we use 31 broad product categories (Panel E). In Panel F, entries for Bhattarai and Schoenle (2014) (10 goods) and Midrigan (2011) (regular prices) are taken from Table II in Alvarez and Lippi (2014), who note, “For the Bhattarai and Schoenle (2011) data, the number of products $n$ is the mean of the categories considered based on the information in Table 1, the ratio $\text{Std}(|\Delta p_{i}|)/E(|\Delta p_{i}|)$ is from Table 2 (firm-based), the fraction of $|\Delta p_{i}|$ that is small is from Table 14, and the kurtosis is from Figure 7. The data from Midrigan (2009) are taken from the distribution of standardized prices in Table 2b.”
B Model Appendix

B.1 General equilibrium model

This appendix provides the microeconomic foundations for our general equilibrium model, and is based on Alvarez and Lippi (2014) and Bonomo et al. (2021).

B.1.1 Consumers

The economy is populated by a representative agent that has preferences over paths of a consumption bundle $C_t$ and hours worked $N_t$, given by

$$U = \int_0^\infty e^{-\rho t} u(C_t, N_t) \, dt.$$

The budget constraint of the representative consumer is

$$\dot{B}_t = W_t N_t + D_t + i_t B_t - \int k \int_i P_{k,i,t} C_{k,i,t} \, di \, dk,$$

where $B_t$ denotes bond holdings of the representative consumer, which satisfy $B_t = 0$ in equilibrium, $i_t$ is the nominal interest rate, $D_t$ is the flow of dividends paid by firms and $W_t$ is the nominal wage. There is a continuum of retailers, indexed by $k$, each one selling a continuum of varieties, indexed by $i$. Variety $i$, purchased from retailer $k$, has price $P_{k,i,t}$ and its consumption is denoted by $C_{k,i,t}$. The consumption bundle $C_t$, from which the consumer derives utility, is given by a Dixit-Stiglitz aggregator of consumption purchased from each retailer $k$, as follows:

$$C_t = \left[ \int C_{k,t}^{1-1/\theta} \, dk \right]^{\theta/\theta}.$$

Each $C_{k,t}$, in turn, is a basket composed by all varieties sold by retailer $k$:

$$C_{k,t} = \left[ \int \left( \frac{C_{k,i,t}}{A_{k,i,t}} \right)^{1-1/\xi} \, dk \right]^{\xi/\xi}.$$

Each $A_{k,i,t}$ is a preference shifter for variety $i$, sold by retailer $k$. We allow, for now, the elasticity of substitution of between retailers, $\theta$, to be different from the elasticity of substitution between goods sold by the same retailer, $\xi$.

In what follows, we drop time subscripts for conciseness. Define the aggregate price level,
Expenditure minimization allows us to relate the optimal choice of $C_{i,k}$ to aggregate $C$ via

$$C_{k,i} = A_{k,i}^{1-\xi} \left( \frac{P_k}{P} \right)^{-\theta} \left( \frac{P_{k,i}}{P_k} \right)^{-\xi} C. \quad (A.1)$$

### B.1.2 Firms

Firms is the model are large retailers, each one producing a continuum of varieties according to the production function

$$Y_{k,i} = A_{k,i} N_{k,i}$$

The productivity level $A_{k,i}$ is the same as the preference shifter. The reason for this assumption is that it keeps the problem stationary even when $A_{k,i}$ follows a non-stationary stochastic process, such as a Brownian motion, as we assume in the main text. $N_{k,i}$ is the amount of labor employed.

Given the production function, the cost of producing $C_{k,i}$ units is $WC_{k,i}/A_{k,i}$. The flow of real profits that arises from selling good $i$ are, therefore, given by

$$\Pi = \frac{P_{k,i}}{P} C_{k,i} - \frac{1}{A_{k,i}} \frac{W}{P} C_{k,i}$$

We omit for now the arguments of the function $\Pi$. Using equation (A.1), we can express it as

$$\Pi = C A_{k,i}^{1-\xi} \left( \frac{P_k}{P} \right)^{-\theta} \left( \frac{P_{k,i}}{P_k} \right)^{-\xi} P_{k,i} - C \frac{W}{P} A_{k,i}^{1-\xi} \left( \frac{P_k}{P} \right)^{-\theta} \left( \frac{P_{k,i}}{P_k} \right)^{-\xi}.$$  

Absent frictions, the optimal choice of $P_{k,i}$, which we call the frictionless optimal price (FOP), is given by the usual markup over marginal cost:

$$P_{k,i}^* = \frac{\xi}{\xi - 1} A_{k,i}.$$
By defining the price gap as $x_{k,i} = \log P_{k,i} - \log P^*_k$, we can rewrite real profits as

$$\Pi = \left( \frac{\xi}{\xi - 1} \right)^{-\xi} C \left( \frac{W}{P} \right)^{1-\xi} \left( \frac{P_k}{P} \right)^{\xi-\theta} \left[ \frac{\xi}{\xi - 1} e^{-(\xi-1)x} - e^{-\xi x} \right].$$

Here, we make the additional assumption that $\varepsilon = \theta$, so the profit function becomes

$$\Pi \left( x_{k,i}, C, \frac{W}{P} \right) = \left( \frac{\xi}{\xi - 1} \right)^{-\xi} C \left( \frac{W}{P} \right)^{1-\xi} \left[ \frac{\xi}{\xi - 1} e^{-(\xi-1)x} - e^{-\xi x} \right].$$

Let the steady state level of a given variable $Z$ be $\bar{Z}$. Since $x_{i,k}$ is the deviation of log price from log FOP, we have

$$0 = \left. \frac{\partial \Pi}{\partial x} \right|_{x=0} = \left. \frac{\partial^2 \Pi}{\partial x \partial C} \right|_{x=0} = \left. \frac{\partial^2 \Pi}{\partial x \partial (W/P)} \right|_{x=0} = \left. \frac{\partial^2 \Pi}{\partial x \partial (P_k/P)} \right|_{x=0}.$$

We approximate $\Pi$ by a second order Taylor expansion around the point $(0, C, W/P)$ to obtain

$$\Pi \left( x_{k,i}, C, \frac{W}{P}, \frac{P_k}{P} \right) = \Pi + \frac{1}{2} \xi \left( \frac{\xi}{\xi - 1} \right)^{-\xi} C \left( \frac{W}{P} \right) x_{k,i}^2 + \text{terms independent of } x_{k,i}.$$

Therefore, the losses incurred for not charging the FOP are proportional to $x_{k,i}^2$ and, integrating over all products, we obtain equation (1, main text). We need the assumption $\xi = \theta$, otherwise the loss function would feature a term $\log P_k - \log P$, which would generate strategic complementarity between firms actions and, consequently, a general equilibrium feedback that would make the problem less tractable. As is common in the literature, we abstract away from such complementarities in order to focus exclusively on the effects of synchronization on monetary non-neutrality. It is important to notice that this approximation does not rely on any other specific parameter values, or on the utility function $u(C, N)$.

### B.1.3 Interpretation of aggregate shocks

With the model we developed so far, we can study the response of the price level to changes in the aggregate component of the marginal cost (or price gaps), which is the nominal wage $W$. In order to interpret these as aggregate demand shocks, as we do in the main text, we make the simplifying assumption that the representative consumer has utility $u(C, N) = \log C - N$. Consequently, the optimal choice between consumption and labor, characterized by $u_n = -(W/P) u_c$, boils down to

$$W = CP$$
Moreover, all output is consumed, so we have $C = Y$ and, consequently, $W = PY$, which means that the nominal wage is equal to nominal output. Therefore, we can interpret shocks to $W$ as shocks to nominal aggregate demand.

**B.2 Proofs**

**B.2.1 Lemma 1**

Substitute (4, main text) into (3, main text) and use $\phi''(x) = -x\phi'(x) - \phi(x)$.

**B.2.2 Lemma 2**

From (4, main text), it follows that

$$\int_{-\infty}^{+\infty} x^2 g_t(x) \, dx = \int_{-\infty}^{+\infty} \left\{ \int_{-\infty}^{+\infty} x^2 \frac{1}{\sqrt{\sigma_t}} \phi \left( \frac{x-y}{\sqrt{\sigma_t}} \right) \, dx \right\} \, g_0(y) \, dy$$

$$= \int_{-\infty}^{+\infty} \left\{ y^2 + \sigma^2 t \right\} g_0(y) \, dy$$

$$= \int_{-\infty}^{+\infty} y^2 g_0(y) \, dy + \sigma^2 t.$$

The second line follows because we have the p.d.f. of a Normal distribution with mean $y$ and variance $\sigma^2 t$, and the third line follows from the fact that $g_0(x)$ integrates to one.

**B.2.3 Proposition 1**

Let $\bar{\rho}_s^\varepsilon(t)$, for $t \geq 0$, be the average price of a firm whose last price adjustment before the shock was at date $-s$ (i.e., $s$ periods before the shock hit) for $0 \leq s < \tau^*$. Since firms are uniformly distributed according to the time elapsed between the last adjustment date and $t = 0$, it follows that

$$P_{\varepsilon}(t) = \int_{0}^{\tau^*} \frac{1}{\tau^*} \bar{\rho}_s^\varepsilon(t) \, ds.$$

Therefore,

$$\lim_{\varepsilon \to 0} \frac{P_{\varepsilon}(t)}{\varepsilon} = \lim_{\varepsilon \to 0} \int_{0}^{\tau^*} \frac{1}{\tau^*} \frac{\bar{\rho}_s^\varepsilon(t)}{\varepsilon} \, ds.$$
Let \( \{ T_k^s(\varepsilon) \}_{k=1}^{\infty} \) be the optimal adjustment dates for firm \( s \), and \( \Delta^s_k \) be the change in the firm’s average price after the \( k \)-th adjustment episode following the shock. Of course, \( \Delta^s_k \) depends on \( \varepsilon \) and on the optimal policy, but we omit this dependence to ease notation. \( \Delta^s_k \) will be studied in more detail in the next proof. We have:

\[
\bar{p}^s_\varepsilon(t) = \begin{cases} 
0 & t \in [0, T^s_1(\varepsilon)) \\
\sum_{j=1}^{k-1} \Delta^s_j & t \in [T^s_{k-1}(\varepsilon), T^s_k(\varepsilon)) \text{ and } k \geq 2.
\end{cases}
\]

If adjustment dates are continuous in \( \varepsilon \) at point \( \varepsilon = 0 \), we have \( T^s_k(\varepsilon) \to T^s_k(0) = k\tau^* - s \) as \( \varepsilon \to 0 \). As a consequence,

\[
\frac{\bar{p}^s_\varepsilon(t)}{\varepsilon} \to \begin{cases} 
0 & t \in (0, \tau^* - s) \\
\sum_{j=1}^{k-1} \delta_j & t \in ((k-1)\tau^* - s, k\tau^* - s) \text{ and } k \geq 2.
\end{cases}
\]

Two observations are important here. First, the limit in points of the form \( k\tau^* - s \) may be undefined. It could be either \( \sum_{j=1}^{k-1} \delta_j \) if \( T^s_k(\varepsilon) \) converges to \( k\tau^* - s \) from below, \( \sum_{j=1}^{k} \delta_j \) if it converges from above, or otherwise undefined. Nevertheless, this is irrelevant since it is a set of measure zero. Second, \( \delta_k \) does not depend on \( s \). This happens because firms with higher \( s \) do exactly the same as firms with lower \( s \), only with a delay. This could fail for large shocks if, for example, two firms characterized by different \( s \) respond immediately to a large shock, but this is irrelevant as the shock size goes to zero and \( T^s_k(\varepsilon) \to k\tau^* - s \).

Moreover, we have \( \frac{\bar{p}^s_\varepsilon(t)}{\varepsilon} \in [0, 1] \), i.e., the firm’s average price level does not overshoot the increase in demand from the aggregate shock. We can, therefore, exchange the order of integration and limit operators in \( (A.2) \), which gives us

\[
\lim_{\varepsilon \to 0} \frac{P^s(\varepsilon)}{\varepsilon} = \begin{cases} 
\delta_1 \frac{t}{\tau^*} & t \in [0, \tau^*) \\
\sum_{j=1}^{k-1} \frac{\delta_j}{\tau^*} + \delta_k \left[ \frac{t}{\tau^*} - (k-1) \right] & t \in [(k-1)\tau^*, k\tau^*) \text{ and } k \geq 2.
\end{cases}
\]

**B.2.4 Lemma 3**

For any given sequence of adjustment dates \( \{ T_j \}_{j=1}^{\infty} \) and thresholds \( \{ \bar{x}_j \}_{j=1}^{\infty} \), not necessarily optimal, let \( g_k(x; \varepsilon, \{ T_j \}_{j=1}^{k}, \{ \bar{x}_j \}_{j=1}^{k-1}) \) be the distribution of price gaps that emerges immediately before the \( k \)-th adjustment episode. Note that \( g_k \) is determined by the first \( k \) values of the \( \{ T_j \}_{j=1}^{\infty} \), but only by the first \( k - 1 \) values of \( \{ \bar{x}_j \}_{j=1}^{\infty} \), since is it the distribution at date
$T_k$ before adjustments take place. We can then express $\Delta_k$ as

$$
\Delta_k(\varepsilon, \{T_j\}_{j=1}^k, \{\bar{x}_j\}_{j=1}^k) = - \int_{|x| > \bar{x}_k} x g_k(x, \varepsilon, \{T_j\}_{j=1}^k, \{\bar{x}_j\}_{j=1}^{k-1}) \, dx.
$$

(A.3)

For a small aggregate shock, we can use the chain rule to obtain the following approximation:

$$
\Delta_k(\varepsilon, \{T_j(\varepsilon)\}_{j=1}^k, \{\bar{x}_j(\varepsilon)\}_{j=1}^k) \approx \Delta_k(0, \{T_j(0)\}_{j=1}^k, \{\bar{x}_j(0)\}_{j=1}^k)
$$

$$
+ \left[ \frac{\partial \Delta_k}{\partial \varepsilon} + \sum_{j=1}^k \frac{\partial \Delta_k}{\partial T_j} T_j' \bigg|_{T_j=0} + \sum_{j=1}^k \frac{\partial \Delta_k}{\partial \bar{x}_j} \bar{x}_j' \bigg|_{\bar{x}_j=0} \right] \varepsilon.
$$

(A.4)

In the above, all the partial derivatives are evaluated at point $(0, \{T_j(0)\}_{j=1}^k, \{\bar{x}_j(0)\}_{j=1}^k)$, but this argument is omitted for conciseness. Note, moreover, that in the absence of any innovation ($\varepsilon = 0$), we are in a steady state with a constant price level, so $\Delta_k(0, \{T_j(0)\}_{j=1}^k, \{\bar{x}_j(0)\}_{j=1}^k) = 0$, and the approximation becomes

$$
\Delta_k(\varepsilon, \{T_j(\varepsilon)\}_{j=1}^k, \{\bar{x}_j(\varepsilon)\}_{j=1}^k) \approx \left[ \frac{\partial \Delta_k}{\partial \varepsilon} + \sum_{j=1}^k \frac{\partial \Delta_k}{\partial T_j} T_j' \bigg|_{T_j=0} + \sum_{j=1}^k \frac{\partial \Delta_k}{\partial \bar{x}_j} \bar{x}_j' \bigg|_{\bar{x}_j=0} \right] \varepsilon.
$$

Now observe that if we set $\varepsilon = 0$, the distribution $g_k$ will be symmetric for all $k$ regardless of the sequences $\{T_j\}_{j=1}^\infty$ and $\{\bar{x}_j\}_{j=1}^\infty$. Therefore, the integral (A.3) will always be zero and is consequently independent of $T_j$ and $\bar{x}_j$ for any $j$, so the partial derivatives with respect to policy variables are zero and our first order approximation becomes

$$
\Delta_k(\varepsilon, \{T_j(\varepsilon)\}_{j=1}^k, \{\bar{x}_j(\varepsilon)\}_{j=1}^k) \approx \frac{\partial \Delta_k}{\partial \varepsilon} \varepsilon.
$$

### B.2.5 Slope of the first segment of the aggregate price response

In steady state, we have:

$$
\frac{\partial \Delta_1}{\partial \varepsilon} = \frac{\partial}{\partial \varepsilon} \left( - \int_{|x| > \bar{x}^*} x g^*(x + \varepsilon) \, dx \right)
$$

$$
= - \int_{|x| > \bar{x}^*} x g''(x) \, dx.
$$

Integrating by parts, we have

$$
\frac{\partial \Delta_1}{\partial \varepsilon} = 2\bar{x}^* g(\bar{x}^*) + m
$$

(A.5)
\[ m = \int_{|x| > \bar{x}^*} g^*(x) \, dx. \]

The result follows from rearranging (A.5) and using \( F = m/\tau^* \) and

\[ f(x) = \frac{2g(x)1(x \geq \bar{x}^*)}{m}. \]

### B.3 Numerical method for solving the price-setting problem

The strategy we adopt to solve the recursive problem (6, main text) is to approximate the distribution \( g \) by a member of some parametric family of probability distributions. What motivates our choice is the following. Given an initial condition \( g_0 \), equation (4, main text) tells us that the solution to the KFE \( g_t \) can be written as the p.d.f. of a sum of two independent random variables: one with p.d.f. \( g_0 \) and the other normally distributed as \( N(0, \sigma^2 t) \). As a consequence, it is possible to show that after properly scaling and shifting \( g_t \) so that it becomes the distribution of a zero-mean, unit-variance random variable, the resulting function converges to the p.d.f. of a standard normal distribution as \( t \to \infty \).

Interestingly, for our numerical purposes the convergence happens fast enough that \( g_T \) in (4, main text) can be well approximated by a normal distribution. The fit is better for higher values of \( T \) and \( \sigma \), but even for \( T \) as low as 0.02, as the approximation is good in our calibration. We choose our state variable in the recursive formulation of the problem to be the distribution of prices gaps immediately after the payment of the fixed cost because, at that instant, the time elapsed since the last adjustment episode is maximal and so the distribution of discrepancies is as close as possible to a Gaussian curve.

If we approximate our infinite dimensional state variable by a normal distribution, we are left with a two-dimensional problem, since normally distributed variables may be characterized by two parameters. In fact, since in steady state the mean of the price gap distribution is zero, our problem becomes unidimensional. Finally, to show the goodness of fit, Figure B.1 compares the steady-state distribution that would arise from following the optimal policy for calibrated parameter values, given by equation (7, main text), with the corresponding normal approximation. It also shows the difference between the c.d.f. of the approximating normal \( \Phi(x) \) and the one obtained numerically \( G(x) \). The right-hand panel of Figure B.1 shows that for no interval \( [a, b] \), the approximation predicts a mass of price gaps that is more than 0.0206 away from the true value, which happens for the interval \( [-0.088, 0.091] \).
B.4 Derivation of slope of the first segment of the aggregate price response

Here we extend Caballero and Engel (2007)’s flexibility index for our multiproduct setting. It will provide an analytical formula for the slope of the first segment of the aggregate price response characterized in Section 4.1. We first establish another important property of the impulse response.

Let $F$ be the steady-state instantaneous frequency of price adjustments, defined as

$$F = \lim_{\Delta t \to 0} \frac{\text{Fraction of prices that change in } (t, t + \Delta t)}{\Delta t}.$$

Note that since $F$ is a time density, it can take any positive value, including values greater than one. Also, define $f(x)$ as the density of the distribution of absolute size of price changes, which is simply the part of the curves shown in Figure 2 (main text) associated with $x \geq 0$, and scaled to integrate to 1. To characterize the slope of the first line segment, we make use of the following lemma, whose proof and necessary definitions are presented in Appendix B.2.

Lemma. Changes in policies in response to an aggregate shock do not have first-order effects on $\Delta_k$ around steady state. More precisely, for any positive integers $j$ and $k$ we have

$$\frac{\partial \Delta_k}{\partial T_j} = \frac{\partial \Delta_k}{\partial \bar{x}_j} = 0. \quad (A.6)$$

Proof. See Appendix B.2. \hfill \square

Lemma 3 is reminiscent of Proposition 1 in Alvarez and Lippi (2014), and earlier insights
from Caballero and Engel (1993, 2007). Alvarez and Lippi show that a firm’s optimal policy after a monetary shock differs from the steady-state policy only up to second-order terms, because aggregate variables do not interact with price gaps up to second order. In turn, Lemma 3 establishes that in partial equilibrium, the effect of the shock on the optimal policy is second order.

As a consequence of Lemma 3, we have the following result.

**Proposition 1.** The slope of the first line segment in the impulse response function is

\[
\frac{\delta_1}{\tau^*} = F \times [1 + \bar{x}^* f(\bar{x}^*)].
\]

(A.7)

**Proof.** See Appendix B.2.

The intuition for this result is the following. The immediate response of the price level is the product of an extensive margin component \( F \) and an intensive margin, or selection component, \( 1 + \bar{x}^* f(\bar{x}^*) \), which depends on the size of the marginal adjustment \( \bar{x}^* \) multiplied by its density \( f(\bar{x}^*) \). Caballero and Engel (2007) prove a similar result for a single-product model.

### B.5 Interpretation of price-adjustment frictions

Suppose we have an economy in which profit-maximizing prices are unobservable and economies of scope come not from the price-adjustment technology, but from the information-acquiring process. More precisely, imagine that \( K \) now is a cost whose payment is required to observe frictionless optimal prices for all products simultaneously, while \( c \) remains a standard, product-specific menu cost. One could ask how this economy would differ from the one we have studied so far.

First, observe that, since the profit-maximizing price of a given product is a martingale (i.e., it satisfies \( \mathbb{E}_t p^*_i,t+h = p^*_i,t \) for \( h > 0 \)), no adjustments will be made without new information. Intuitively, a firm would not adjust the price of a product if the frictionless optimal price does not change in expectation. Adjustments are only made when the fixed cost for acquiring information is paid, as in Alvarez, Lippi, and Paciello (2011) and, in the limiting case of infinitely many products, in Alvarez and Lippi (2014). However, since the firm sells a continuum of products, the distribution of price gaps is perfectly predictable and given by the KFE (3, main text), as long as there are no aggregate shocks. In other words, the law of large numbers makes the distribution of price gaps perfectly predictable, even though each individual price gap is unobserved. Therefore, the steady-state optimal policy and stationary distributions are the same as in our baseline menu-cost framework.
The menu cost and the informational friction economies would differ, however, in response to an aggregate shock. In the menu-cost economy, when the shock hits at $t = 0$, firms are allowed to instantaneously recalculate the optimal policy, whereas in the costly information economy, firms would only learn about the shock after collecting information. Nonetheless, we could follow the same steps for proving (8, main text) and (9, main text) to obtain exactly analogous results to this case. As a consequence, even though the two economies could respond differently to an aggregate shock, there is no difference up to the first order.

This result offers an important insight into the nature of the frictions in product markets and their implications for monetary non-neutrality. When monetary shocks are sufficiently small, the responses of aggregate price and output of an industry dominated by large multi-product retailers are not likely to be sensitive to whether price stickiness is due to physical menu cost or the cost of acquiring and processing information.

Notice that if the profit maximizing price process has an aggregate stochastic component and $K$ is an information cost, our solution algorithm would still be valid. Although the aggregate effect of a monetary shock would depend on the realization of the aggregate shock process, the effects we found would correspond to the average effect over those realizations.
B.6 A model with random free adjustments

We consider here an extension of the partial synchronization model (with Brownian shocks) in which free adjustment opportunities arrive independently for each product at rate $\lambda$. Since free adjustments arrive for all prices, regardless of the size of the price gap, it generates a mass of small price changes. We consequently choose the fraction of small price adjustments as the extra empirical moment we need to calibrate the additional parameter $\lambda$.

Tables B.1 and B.2 show moments and calibrated parameter values. A very small $\lambda$ is enough to match the fraction of small adjustments; this is because, in our sample, they are not too frequent. The calibrated value $\lambda = 0.47$ corresponds roughly to one free adjustment per product every two years, on average. Consequently, the distribution of price changes, shown in figure B.2 is not too different from the original one, except for the mass of small adjustments. Thus, impulse response functions for monetary shocks, shown in figure B.3, are almost indistinguishable.

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Partial sync.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Targeted moments</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Daily fraction of price changes</td>
<td>0.0087</td>
<td>0.0087</td>
</tr>
<tr>
<td>Average $</td>
<td>\Delta p</td>
<td>$</td>
</tr>
<tr>
<td>Fisher-Konieczny index</td>
<td>0.259</td>
<td>0.259</td>
</tr>
<tr>
<td>Fraction of small adjustments</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>Additional moments</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard deviation of $\Delta p$</td>
<td>0.248</td>
<td>0.218</td>
</tr>
<tr>
<td>Kurtosis of $\Delta p$</td>
<td>3.53</td>
<td>1.22</td>
</tr>
</tbody>
</table>

Table B.1: Moments from data and model with random free adjustments.

Note: Values in the data are weighted means across stores in the data, provided in Table 1 (main text), first row, columns (2)-(4). Small adjustments are defined as those smaller than one-quarter of the average size.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Partial sync.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>0.390</td>
</tr>
<tr>
<td>$K$</td>
<td>5.04e-05</td>
</tr>
<tr>
<td>$c$</td>
<td>0.0028</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.477</td>
</tr>
</tbody>
</table>

Table B.2: Calibrated parameter values for model with random free adjustments.
Figure B.2: Price-change distributions.

Figure B.3: Impulse response functions.
References


