Labor Market Frictions in a Monetary Union

Rohan Kekre*

May 13, 2016

Abstract

I ask how frictional labor markets affect stabilization trade-offs and optimal monetary policy in a two-country monetary union. With sticky prices in each country, I obtain two main results. First, given a commonly-used set of preferences and technologies, the union’s central bank can achieve the constrained efficient allocation if and only if productivity shocks affecting each country are symmetric — as in the frictionless Mundellian benchmark — regardless of the level of or heterogeneity in labor market frictions across countries. Second, for a given set of asymmetric shocks that necessitates distortions in the union, optimal policy targets smaller inflation and output gaps in the country with a more sclerotic labor market. I discuss the implications of my analysis for the sustainability of the Euro and policy choices of the ECB, given the richness and diversity in labor markets across Eurozone members.

*Department of Economics, Harvard University. Email: rkekre@fas.harvard.edu. For invaluable guidance on this project, I sincerely thank Mark Aguiar, Alberto Alesina, Pol Antras, Robert Barro, Gabriel Chodorow-Reich, Emmanuel Farhi, Mark Gertler, Gita Gopinath, Elhanan Helpman, Marc Melitz, Maurice Obstfeld, Ken Rogoff, and Harald Uhlig. For many helpful discussions, I also thank Ben Hebert, Mikkel Plagborg-Moller, and Jesse Schreger.
1 Introduction

The prolonged slump in Southern European economies in recent years has brought a spotlight back on labor market frictions in Europe. Several decades ago, diverging labor market performance on either side of the Atlantic led to a wave of research asking if European labor market institutions might be responsible. In the last few years, and now with many European countries bound by the Euro, unemployment rates exceeding 15% in Greece, Spain, and Portugal have again renewed academic and policy interest in whether differing institutional environments in these countries might be partly to blame. On measures such as mandated severance payments, depicted in Figure 1, these countries are indeed considerably more generous than other Eurozone members.

Existing analyses do not provide a sufficiently rich framework to think through the interplay between the functioning of a particular country’s labor market and its membership in a monetary union. While many researchers have accounted for stickiness in nominal wages, virtually none have explored whether the search, matching, or bargaining features of real-world labor markets matter for macroeconomic outcomes across the union.\(^1\) This paper takes a first step to fill that gap.

Within a standard model of a two-country monetary union subject to country-specific TFP shocks, I embed labor market frictions in the tradition of Diamond [1981], Mortensen [1982], and Pissarides [1984] (hereafter, DMP). In this setting, I revisit two of the central questions in the theory of monetary unions in the presence of non-trivial and heterogeneous labor market frictions across countries. First, when will such a monetary union face distortions? Is it possible that Mundell [1961]'s classic business cycle synchronicity criterion may be modified — that symmetric shocks may interact with asymmetric labor markets to generate inefficiencies in a monetary union? Second, how should second-best optimal monetary policy be conducted? Should it target smaller employment fluctuations in the more sclerotic, or the more fluid, labor market in the union?

Under benchmark preferences and technologies in the literature, I obtain two sharp results: institutional irrelevance for the question of when a monetary union will face distortions, and relative accommodation of the more sclerotic member of the union at the second-best optimum. Institutional irrelevance is a consequence of invariant employment to TFP shocks in the flexible price and wage allocation, rendering the terms of trade between countries just a function of relative TFP regardless of the level of or cross-country heterogeneity in DMP frictions. Given asymmetric shocks, relative accommodation of the more sclerotic union member — that is, the member with greater hiring costs and lower labor market flows\(^2\) — is a consequence of both greater welfare losses and greater inflationary/deflationary pressure from output fluctuations in that member. Together, these results suggest that while labor market frictions may not bear on the optimality of the Eurozone as a currency area, they should inform stabilization policy by union-wide institutions such as the ECB.

\(^1\)Among recent analyses of monetary unions, Gali and Monacelli [2008], Kehoe and Pastorino [2014], and Chari et al. [2015] assume a Walrasian labor market, while Benigno [2004], Schmitt-Grohe and Uribe [2012, 2013], Farhi et al. [2013], and Farhi and Werning [2014] allow for sticky wages but otherwise assume frictionless labor markets. An emerging literature studies monetary unions in which members’ labor markets are characterized by DMP frictions, which my paper joins. I situate my paper within this literature later in this section.

\(^2\)This definition of sclerotic vs. fluid labor markets follows in the tradition of Blanchard and Gali [2010].
Figure 1: mandated severance pay in the Eurozone

I begin by building a static, two-country model of a currency union with DMP frictions. The model merges standard elements from the international macro and macro/labor traditions. From the first, production of distinct intermediate goods uses domestic labor and country-specific TFP, international trade is in differentiated final goods, and domestic money prices of imported varieties are mediated by the nominal exchange rate which is assumed fixed. From the second, recruiting workers is costly, workers and vacancies in each country are brought together by a matching function, and wages split the bilateral surplus from a match. Importantly, I specify preferences and technologies consistent with widely-used benchmarks in each literature: utility from consumption which is separable from labor, log over aggregate consumption, and Cobb-Douglas over goods produced in each country (as in Cole and Obstfeld [1991]), and recruiting costs expressed in forgone incumbent worker time in production, thus scaling with domestic TFP (as in Shimer [2010]).

I first study the equilibrium with flexible prices and wages in each member of union — the natural allocation — and find that it is characterized by three properties important to understanding the stabilization problem under sticky prices. First, equilibrium employment in each country is invariant to domestic TFP. This generalizes the benchmark results of Blanchard and Gali [2010] and Shimer [2010] to the open economy, and relies on the assumption of Cole and Obstfeld [1991] preferences and recruiting costs which scale with domestic TFP. Second, equilibrium employment in each country is invariant to all foreign economic conditions. This generalizes the benchmark result in Clarida et al. [2002] to a labor market characterized by DMP frictions, and relies again on the assumed preferences and fact that recruiting costs use only domestic resources. Third and finally,

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1I model a monetary union as characterized by a fixed exchange rate between two countries, nesting the case of a common currency (where the exchange rate is one).
the natural allocation is constrained efficient if, in each country, the distortion from monopolistic competition in the retail sector is offset with a production subsidy and surplus sharing in the labor market satisfies the Hosios [1990] condition. The latter offsets search externalities in each labor market just as in the closed economy because of my maintained assumption of no labor mobility across union members — a better approximation for Eurozone countries than it is for U.S. states.

Given nominal rigidity in prices and TFP shocks across the union, I can then tractably study the stabilization problem facing a union-wide central bank by studying a linear-quadratic approximation to the Ramsey policy problem. Suppose that only some retailers in each country can set prices after the realization of TFP shocks; as the other retailers accommodate demand at preset prices, union-wide monetary policy is non-neutral. Under the important assumption that the natural allocation is constrained efficient as described above, a linear-quadratic approximation to the Ramsey policy problem suffices to characterize the optimal allocation up to first-order from steady-state. The linear constraints of this problem consist of relations linking producer-price inflation and output in each country — the Phillips Curve in this one-period environment — as well as a relation linking inflation and output across countries owing to the fixed exchange rate. Approximated in this way, the coefficients on policymakers’ objective, each country’s Phillips Curve, and the constraint imposed by the fixed exchange rate encode all the necessary information to characterize the effect of labor market frictions on stabilization challenges facing a monetary union.

With this formulation of the stabilization problem in hand, I first obtain a sharp institutional irrelevance result: the constrained efficient allocation is achievable if and only if TFP shocks are symmetric across countries, regardless of the level of or heterogeneity in DMP frictions. Intuitively, a sufficient statistic for inefficiency in a monetary union is movement in the terms of trade in the natural allocation. If the natural terms of trade adjust given a set of TFP shocks, a monetary union with sticky prices will be unable to replicate this movement in relative prices without some distortions in prices, output, or both. Under the benchmark preference and technology assumptions described above, employment in each country is invariant to domestic and foreign TFP, and thus relative production is simply a function of relative TFP, in the natural allocation. It immediately follows that the natural terms of trade will adjust if and only if TFP shocks are asymmetric across countries. As such, Mundell [1961]’s business cycle synchronicity criterion for optimal currency areas is robust to the presence of rich and diverse labor market frictions across countries.

In the presence of asymmetric shocks necessitating some distortions in a monetary union, I then find that optimal policy is characterized by relative accommodation of the more sclerotic labor market. In particular, greater hiring costs or lower labor market flows in a particular union member tend to raise the deadweight costs of inefficient hiring in that country. This increases the welfare cost of output distortions and the slope of the Phillips Curve in that country — which, in fact, are tightly linked when the natural allocation is constrained efficient, reflecting the connection

\[\text{The constrained efficient allocation is that chosen by a planner subject only to the global economy’s technological constraints in producing and recruiting.}\]

\[\text{As a result, a competitive search equilibrium will continue to induce efficiency in the labor market (generalizing Moen [1997] to the open economy). In the body of the paper I instead assume random search and Nash bargaining.}\]
between the curvature of social welfare and the sensitivity of firms’ real marginal costs to output in that case. Since the more sclerotic labor market features greater welfare losses and greater price pressure from a given distortion in output, the optimal policy accommodates that market by targeting both a smaller output gap and smaller inflation/deflation at the second-best optimum.

I conclude by tracing out tentative implications of my results for the Eurozone, motivating fruitful directions for future work. My institutional irrelevance result clarifies that the existence and heterogeneity of labor market frictions across member states need not have anything to do with the optimality of the Eurozone as a currency area. But my relative accommodation result suggests that the fluidity of labor markets should indeed inform the conduct of stabilization policy by the ECB when perfect stabilization is unattainable — and may in fact mean that the historical conduct of policy has been suboptimal. To more thoroughly explore these results and their consequences for the Eurozone, future work should extend the model to an infinite horizon and perform a quantitative calibration, taking advantage of the rich dynamics introduced by the DMP modeling of the labor market and a long tradition in connecting such models to the data.

This paper marries two active literatures: from international macro, the literature on monetary unions, and from macro/labor, the literature connecting technological and institutional features of labor markets with the level and cyclical behavior of unemployment. The questions of this paper are inspired by the seminal contributions of Mundell [1961], McKinnon [1963], and Kenen [1969] on Optimal Currency Areas, and Ljungqvist and Sargent [1998], Nickell and Layard [1999], and Blanchard and Wolfers [2000] on European institutions and unemployment. The model I develop combines elements of Obstfeld and Rogoff [1995] and Gali and Monacelli [2008]’s more modern analyses of fixed exchange rates and monetary unions, and Blanchard and Gali [2010] and Shimer [2010]’s analysis of labor market dynamics in the presence of DMP frictions.

In focusing on the consequences of cross-country heterogeneity on stabilization policy in a monetary union, this paper is distinct from, but complementary with, well-known work by Benigno [2004]. Benigno [2004] studies stabilization of a monetary union in which the degree of nominal rigidity differs across countries, whereas I focus on heterogeneity in the search and matching features of labor markets. These affect the stabilization problem through different channels — the cost of inflation in the first case, versus the cost of output fluctuations in the second. They also admit distinct empirical counterparts — for instance, the observed duration of nominal contracts in the first case, versus the institutional and technological costs involved in hiring and firing workers in the second. Our analyses are complementary, however, in that both imply the optimality of targeting policy to accommodate the more rigid member of the monetary union. And indeed, the key insight of Benigno [2004] is nested and holds within my model: optimal policy converges to targeting zero inflation in one union member as prices converge to full flexibility in the other.

Motivated by policy interest in Eurozone labor markets in recent years, several other authors have recently studied the consequences of DMP frictions in a monetary union; relative to these, my analysis is distinguished by revisiting the classic normative insights of the Optimal Currency Area literature in such an environment. Campolmi and Faia [2011] and Abbritti and Mueller
[2013] instead focus on the positive features of such a union, obtaining empirical and quantitative results consistent with the log-linearized equilibrium relations of this paper. Eggertsson et al. [2014] analyze Ramsey optimal policy in a similar environment, but focus on the constraint posed by a union-wide zero lower bound rather than adjustment in the terms of trade. Cacciatore et al. [2016] also analyze optimal policy, but focus instead on the policy trade-offs in using union-wide monetary policy to undo steady-state distortions from inefficient labor or product markets among members.

In section 2 I outline the economic environment. In section 3 I study the natural allocation, and in section 4 I develop and characterize the stabilization problem under partially sticky prices, obtaining the main results of the paper. In section 5 I discuss the implications of my analysis for the Eurozone, as well as directions for future work. Finally, in section 6 I conclude.

2 Environment: a monetary union with DMP frictions

In this section I characterize a global economy consisting of two countries bound by a fixed exchange rate (as implicitly the case in a monetary union), each subject to country-specific TFP shocks, and each featuring a labor market characterized by DMP frictions. The model ingredients merge standard elements of the international macro and macro/labor literatures.

2.1 Primitives

I first specify tastes, technologies, endowments, and market structures in a one-period global economy comprised of Home and Foreign. Throughout the paper, I use asterisks to denote variables chosen by or endowed to Foreign agents.

Each country is comprised of measure one agents organized into measure one households. Owing to complete domestic asset markets and ex-ante symmetry across households, I follow Merz [1995] and Andolfatto [1996] in focusing on a representative household in each country with preferences

\[ u(c, n) = \log c - \chi \frac{n^{1+\varphi}}{1+\varphi}, \quad u^*(c^*, n^*) = \log c^* - \chi^* \frac{n^*^{1+\varphi^*}}{1+\varphi^*}, \]

where \( c \) and \( c^* \) denote consumption for each member of the household, and \( n \in [0, 1] \) and \( n^* \in [0, 1] \) denote the fraction of the household which is employed. Consumption, in turn, is a Cobb-Douglas aggregator of consumption of goods produced in each country (with home bias \( \gamma > \frac{1}{2} \)), each of which is a CES aggregator over that country’s measure one of varieties (with common elasticity of substitution \( \varepsilon \) only for expository simplicity):

\[ c = (c_H)^\gamma (c_F)^{1-\gamma}, \quad c^* = (c_H^*)^{1-\gamma} (c_F^*)^\gamma, \]

\[ c_H = \left[ \int_0^1 c_H(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad c_H^* = \left[ \int_0^1 c_H^*(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}, \]

\[ c_F = \left[ \int_0^1 c_F(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad c_F^* = \left[ \int_0^1 c_F^*(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}. \]
I consider two classes of firms in each country: perfectly competitive intermediate good producers and monopolistically competitive final good retailers. Producers recruit workers in their frictional domestic labor market, produce an intermediate good with linear technology and TFP $a$ and $a^*$, and sell only to domestic retailers. Retailers, in turn, transform their country’s intermediate good into differentiated varieties with a pure-pass through technology and sell them globally.

The labor market in each country is the novel element of my framework relative to the monetary union literature. Following Shimer [2010], producers must use $k$ and $k^*$ incumbent workers to manage the recruiting process for a single vacancy. Under my maintained assumption that labor is immobile across borders, with $u$ and $u^*$ unemployed workers searching for employment and $v$ and $v^*$ vacancies posted by producers, the Home and Foreign economies see

$$m(u, v) = \bar{m}(u)^{1-\eta}(v)^{\eta}, \quad m^*(u^*, v^*) = \bar{m}^*(u^*)^{1-\eta^*}(v^*)^{\eta^*}$$

aggregate matches, respectively. Workers search randomly across posted vacancies, and wages are Nash bargained ex-post with worker bargaining shares $\beta$ and $\beta^*$ in Home and Foreign, respectively.\(^6\)

Finally, I specify endowments and international asset markets. I assume that fractions $(1-\delta)n_0$ and $(1-\delta^*)n^*_0$ of each country’s representative household have already matched with firms at the start of the period, where I interpret $n_0$ and $n^*_0$ as the fictitious prior period’s employment rate, and $\delta$ and $\delta^*$ as each economy’s separation rate.\(^7\) I assume that households start with equal ownership of their domestic firms but hold zero net foreign assets because international asset markets are non-existent. The latter will be without loss of generality, given that the seminal result of Cole and Obstfeld [1991] holds when vacancy-posting requires domestic resources only.

2.2 Defining the equilibrium

Having specified the global economy’s tastes, technologies, endowments, and market structures, I now define a competitive equilibrium.

I begin by defining prices facing agents in the global economy. Let starred prices denote nominal quantities in Foreign’s unit of account. Then $P_H(j)$ and $P^*_H(j)$ are the prices faced by residents of Home and Foreign for variety $j$ produced in Home; $P_F(j^*)$ and $P^*_F(j^*)$ are the prices faced by these same residents for variety $j^*$ produced in Foreign; $P^I$ and $P^{I*}$ are the price of intermediate goods in each country; and $W$ and $W^*$ are the prevailing wages in each country. Given the nested

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\(^6\)All of the stabilization results in section 4 would be unchanged if firms instead posted wages and workers direct their search, since these results will assume that the Hosios [1990] condition holds.

\(^7\)The distinction between \{$n_0, n^*_0\}$ and \{$\delta, \delta^*\}$ will become meaningful when, in my analysis of macroeconomic stabilization in section 4, I assume that the steady-state realization of TFP shocks \{$a, a^*$\} is consistent with \{$n = n_0, n^* = n^*_0\}$. Then, $\delta$ and $\delta^*$ will capture the magnitude of flows in each labor market even in this one-period model.
CES structure of household preferences, it will prove useful to define the following price indices:

\[ P_H = \left[ \int_0^1 P_H(j)^{1-\varepsilon} \, dj \right]^{\frac{1}{1-\varepsilon}}, \quad P_F = \left[ \int_0^1 P_F(j^*)^{1-\varepsilon} \, dj^* \right]^{\frac{1}{1-\varepsilon}} \tag{4} \]

\[ P_H^* = \left[ \int_0^1 P_H^*(j)^{1-\varepsilon} \, dj \right]^{\frac{1}{1-\varepsilon}}, \quad P_F^* = \left[ \int_0^1 P_F^*(j^*)^{1-\varepsilon} \, dj^* \right]^{\frac{1}{1-\varepsilon}}. \tag{5} \]

In addition to market prices, labor market tightness will play an equilibrating role in each country’s frictional labor market. Tightness in each country is defined as

\[ \theta = \frac{v}{u}, \quad \theta^* = \frac{v^*}{u^*}, \]

where \( \{u, u^*, v, v^*\} \) are aggregate unemployment and vacancies, giving rise to vacancy-filling probabilities per vacancy posted

\[ q(\theta) = \frac{m(u, v)}{v}, \quad q^*(\theta^*) = \frac{m^*(u^*, v^*)}{v^*}, \]

and job-finding probabilities for each unemployed agent

\[ p(\theta) = \frac{m(u, v)}{u}, \quad p^*(\theta^*) = \frac{m^*(u^*, v^*)}{u^*}. \]

Finally, I define the policy instruments assumed available to a union-wide policymaker throughout the paper. In terms of fiscal instruments, I assume that policymakers can assess ad-valorem taxes \( \tau_r \) and \( \tau_r^* \) on retailers’ purchases of intermediate goods, and assess lump-sum taxes \( T \) and \( T^* \) on domestic residents. In terms of monetary instruments, I assume that policymakers can set the monetary bases \( \bar{M} \) and \( \bar{M}^* \) in each country. Money will serve as a unit of account in each country, and I will complement agents’ optimization problems described below with ad-hoc money market clearing conditions in equilibrium.

We are now in a position to specify agents’ optimization. Recalling that representative households have \((1-\delta)n_0\) and \((1-\delta^*)n_0^*\) workers already matched at the start of the period, and assuming for simplicity that incumbent and new workers are paid the same wage, the households will choose

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8This is innocuous; given risk-sharing within the household, the wage paid to incumbent workers has no effect on the equilibrium allocation, as is well known in the macro/labor literature.
Finally, retailer consumption and labor force participation among their unemployed members according to

\[ v((1 - \delta)n_0) = \max_{\{c_H(j), c_F(j^*)\}_{j^*, u^* \in [0, 1]} (1 - \delta)n_0 \log c - \frac{\chi n^{1+\varphi} H}{1 + \varphi}} \] s.t.\n
\[ (RC): \int_0^1 P_H(j)c_H(j)\, dj + \int_0^1 P_F(j^*)c_F(j^*)\, dj^* \leq Wn + \Pi + \int_0^1 \Pi'(j)\, dj - T, \] (6)\n
\[ (Evol): n = (1 - \delta)n_0 + p(\theta)u, \] (7)\n
\[ v^*((1 - \delta)n_0^*) = \max_{\{c_H(j), c_F(j^*)\}_{j^*, u^* \in [0, 1]} (1 - \delta)n_0^* \log c^* - \frac{\chi^* (n^*)^{1+\varphi^*}}{1 + \varphi^*}} \] s.t.\n
\[ (RC)^*: \int_0^1 P_H^*(j)c_H^*(j)\, dj + \int_0^1 P_F^*(j^*)c_F^*(j^*)\, dj^* \leq W^*n^* + \Pi^* + \int_0^1 \Pi'^*(j^*)\, dj^* - T^*, \] (8)\n
\[ (Evol)^*: n^* = (1 - \delta)n_0^* + p^*(\theta^*)u^*, \] (9)\n
given the consumption aggregators defined in (1) - (3) and firm profits \{\Pi, \{\Pi'(j)\}_{j^*}, \Pi^* \{\Pi'^*(j^*)\}_{j^*}\} characterized below. Assuming ex-ante symmetry across producers, constant returns to scale in production and recruiting means that we can focus on representative producers facing

\[ \Pi((1 - \delta)n_0) = \max_{\Pi} P^I a [(1 - \delta)n_0 + q(\theta) v - kv] - W [(1 - \delta)n_0 + q(\theta) v], \] (10)\n
\[ \Pi^*((1 - \delta)n_0^*) = \max_{\Pi^*} P^I a^* [(1 - \delta)n_0^* + q^*(\theta) v^* - k^* v^*] - W^* [(1 - \delta)n_0^* + q^*(\theta) v^*]. \] (11)\n
Finally, retailer \( j \in [0, 1] \) in Home and \( j^* \in [0, 1] \) in Foreign face

\[ \Pi'(j) = \max_{y(j), x(j), P_H(j)} P_H(j) y(j) - (1 + \tau') P^I x(j) \] s.t.\n
\[ (Tech)(j) : y(j) = x(j), \] (12)\n
\[ (Demand)(j) : y(j) = \left( \frac{P_H(j)}{P_H^*} \right)^{-\varepsilon} c_H + \left( \frac{P_H^*(j)}{P_H} \right)^{-\varepsilon} c_H^*; \] (13)\n
\[ (PCP)(j) : P_H^*(j) = \frac{P_H(j)}{E}, \] (14)\n
\[ \Pi'^*(j^*) = \max_{y(j^*), x(j^*), P_F(j^*)} P_F^*(j^*) y^*(j^*) - (1 + \tau'^*) P^I x^*(j^*) \] s.t.\n
\[ (Tech)^*(j^*) : y^*(j^*) = x^*(j^*), \] (15)\n
\[ (Demand)^*(j^*) : y^*(j^*) = \left( \frac{P_F^*(j^*)}{P_F^*} \right)^{-\varepsilon} c_F^* + \left( \frac{P_F(j^*)}{P_F} \right)^{-\varepsilon} c_F; \] (16)\n
\[ (PCP)^*(j^*) : P_F(j^*) = E P_F^*(j^*), \] (17)\n
respectively, where the (Demand) constraints reflect the standard solution of households’ lower-stage optimization problem in (6) and (7) given CES price indices defined in (4) and (5), the nominal exchange rate \( E \) is units of Home currency per unit of Foreign’s currency, and the (PCP) constraints reflect the assumption of producer-currency pricing in the global economy (which is also already baked into the specification of retailers’ profit functions).
I now turn to market clearing and budget balance. For intermediate goods, it must be that
\[ \int_0^1 x(j) \, dj = a \left[ (1 - \delta) n_0 + q(\theta) v - kv \right], \] \hspace{1cm} (12)
\[ \int_0^1 x^*(j^*) \, dj^* = a^* \left[ (1 - \delta^*) n_0^* + q^* (\theta^*) v^* - k^* v^* \right]. \] \hspace{1cm} (13)

For final goods, it must be that
\[ c_H(j) + c_H^*(j^*) = y(j), \] \hspace{1cm} (14)
\[ c_F(j^*) + c_F^*(j^*) = y^*(j^*) \] \hspace{1cm} (15)
for each \( j \in [0, 1] \) and \( j^* \in [0, 1] \). Each country’s fiscal budget balance requires
\[ \int_0^1 \tau^r P^I x(j) \, dj + T = 0, \] \hspace{1cm} (16)
\[ \int_0^1 \tau^r^* P^I^* x^*(j^*) \, dj^* + T^* = 0. \] \hspace{1cm} (17)

Finally, I characterize the money market. I assume ad-hoc money market equilibrium conditions
\[ \bar{M} = \frac{\psi}{\gamma} P_H c_H, \] \hspace{1cm} (18)
\[ \bar{M}^* = \frac{\psi^*}{\gamma} P_F^* c_F^*, \] \hspace{1cm} (19)
which could be microfounded by giving agents separable utility from real money balances.\(^9\) I capture the monetary union between Home and Foreign by assuming that policymakers maintain a fixed exchange rate
\[ E = \bar{E}. \] \hspace{1cm} (20)

We are now ready to define a flexible price and wage equilibrium.

**Definition 1.** A flexible price and wage equilibrium is an allocation \( \{\{c_H(j)\}_j, \{c_F(j^*)\}_j^*, u, v, \theta, n, \{x(j)\}_j, \{y(j)\}_j\}, \{\{c_H^*(j)\}_j, \{c_F^*(j^*)\}_j^*, u^*, v^*, \theta^*, n^*, \{x^*(j^*)\}_j^*, \{y^*(j^*)\}_j^*\} \) and set of nominal wages, prices, and profits \( \{\{W, P^I, \{P_H(j)\}_j, \{P_H^*(j)\}_j, \Pi, \{\Pi^*(j)\}_j\}, \{W^*, P^I^*, \{P_F(j)\}_j^*, \{P_F^*(j^*)\}_j^*, \Pi^*, \{\Pi^{*(j^*)}\}_j^*\}\} \) such that, given policy \( \{\{\tau^r, T, \bar{M}\}, \{\tau^{r^*}, T^*, \bar{M}^*\}, E\} \):

1. households solve (6) and (7);
2. producers solve (8) and (9);
3. retailers solve (10) and (11);

\(^9\)Ad-hoc money market equilibrium is commonly assumed in the background of dynamic monetary models at the “cashless limit”, as in Woodford [2003] and Gali [2008]. I adopt it in the present static setting so that my results on optimal stabilization policy are unaffected by utility fluctuations from real money balances.
4. Wages are Nash bargained (as detailed in the appendix);

5. Tightness is consistent with aggregate vacancies and job-seekers ($\theta = \frac{v}{u}, \theta = \frac{v^*}{u^*}$);

6. Intermediate and final goods markets clear according to (12)-(15);

7. Fiscal budgets are balanced according to (16) and (17);

8. And the money market is assumed to “clear” and be consistent with fixed exchange rate $\bar{E}$ according to (18)-(20).

In addition to the flexible price and wage equilibrium, this paper is concerned with the equilibrium when not all retailers are able to update their prices in response to TFP shocks. In particular, suppose that only measure $\iota$ and $\iota^*$ of retailers in Home and Foreign can freely set prices according to (10) and (11), respectively (without loss of generality, index retailers such that these are the “first” $\iota$ and $\iota^*$ retailers). The remaining retailers must remain committed to preset prices

$$P_H(j) = \bar{P}_H, \quad P_F^*(j^*) = \bar{P}_F^*.$$  \hfill (21)

Continuing to assume producer-currency pricing, these retailers will accommodate consumption demand at posted prices provided they earn non-negative profits:

$$x(j) = y(j) = \begin{cases} \left( \frac{P_H}{\bar{P}_H} \right)^{-\varepsilon} c_H + \left( \frac{P_H/E}{P_H} \right)^{-\varepsilon} c_H^* & \text{provided } \bar{P}_H \geq (1 + \tau^*) P^I, \\ 0 & \text{otherwise,} \end{cases}$$  \hfill (22)

$$x^*(j^*) = y^*(j^*) = \begin{cases} \left( \frac{P_F^*}{\bar{P}_F^*} \right)^{-\varepsilon} c_F^* + \left( \frac{E P_F^*}{P_F^*} \right)^{-\varepsilon} c_F & \text{provided } \bar{P}_F^* \geq (1 + \tau^{**}) P^{I*}, \\ 0 & \text{otherwise.} \end{cases}$$  \hfill (23)

This motivates the following definition of a sticky price equilibrium.

**Definition 2.** A sticky price equilibrium is an allocation and set of nominal wages, prices, and profits such that, given policy, conditions 1-2 and 4-8 of Definition 1 are satisfied; the first $\iota$ and $\iota^*$ of retailers solve (10) and (11); and the remaining $1 - \iota$ and $1 - \iota^*$ of retailers solve (21)-(23).

**2.3 Characterizing equilibrium**

I now derive equilibrium conditions for general final goods prices $\{P_H(j)\}_j$, $\{P_H^*(j)\}_j$, $\{P_F(j^*)\}_{j^*}$, and $\{P_F^*(j^*)\}_{j^*}$, provided that these are consistent with producer-currency pricing. The resulting conditions hold both in the flexible price and wage equilibrium as well as the sticky price equilibrium. In the next two sections of the paper, I complete the description of each by characterizing equilibrium final-goods prices in each case. A reader uninterested in the derivation of equilibrium can proceed directly to the next two sections.
First consider trade balance in equilibrium absent international risk-sharing:\(^1\)
\[
\int_0^1 P_F(j^*) c_F(j^*) \, dj^* = \int_0^1 P_H(j) c_H(j) \, dj. 
\]
Now consider households' consumption allocation problems in (6) and (7). The standard solution given the nested CES structure of preferences implies
\[
c_H(j) = \left( \frac{P_H(j)}{P_H} \right)^{-\varepsilon} c_H, \quad c_F(j^*) = \left( \frac{P_F(j^*)}{P_F} \right)^{-\varepsilon} c_F, \tag{24}
\]
\[
c_H^*(j) = \left( \frac{P_H^*(j)}{P_H} \right)^{-\varepsilon} c_H^*, \quad c_F^*(j^*) = \left( \frac{P_F^*(j^*)}{P_F} \right)^{-\varepsilon} c_F^*, \tag{25}
\]
which implies that trade balance can be written
\[
P_F c_F = P_H c_H^*. 
\]
Defining the terms of trade
\[
s = \frac{P_H}{P_F} = \frac{P_H^*}{P_F^*},
\]
where the equality follows from producer-currency pricing, we have that the equilibrium terms of trade satisfy
\[
s = \frac{c_F}{c_H^*}.
\]
Since the nested CES structure of preferences further implies that
\[
c_F = \left( \frac{1 - \gamma}{\gamma} \right) \frac{P_H}{P_F} c_H, \quad c_H^* = \left( \frac{1 - \gamma}{\gamma} \right) \frac{P_F^*}{P_H^*} c_F^*, \tag{26}
\]
we can equivalently say that the equilibrium terms of trade satisfy
\[
s = \frac{c_F^*}{c_H^*}. \tag{27}
\]

Turn now to equilibrium in the labor market. Producers' optimization problems (8) and (9)\(^\text{10}\)Formally, this can be derived as follows. Given Home households' budget constraint in (6), substitute on the right-hand side for domestic firm profits, the consistency of tightness with aggregate vacancies and job-seekers, fiscal budget balance (16), and intermediate goods market clearing (12), obtaining
\[
\int_0^1 P_H(j) c_H(j) \, dj + \int_0^1 P_F(j^*) c_F(j^*) \, dj^* = \int_0^1 P_H(j) y(j) \, dj.
\]
This simply reflects the identity that aggregate consumption must equal aggregate income, absent international risk-sharing. Substituting on the right-hand side for final goods market clearing for each domestic variety \(j\) in (14), and cancelling terms involving each \(c_H(j)\), yields the desired result.

\(^{12}\)
require that
\[ P^I a - W = P^I \frac{ka}{q(\theta)}, \]
\[ P^{I*} a^* - W^* = P^{I*} \frac{k^* a^*}{q^*(\theta^*)} \]
for an interior optimum in vacancy posting to exist. As shown in the appendix, the Nash bargained wage in each country is
\[ W = -P \frac{u_n}{u_c} + \frac{\beta}{1 - \beta} (P^I a - W), \]
\[ W^* = -P^* \frac{u_n^*}{u_c^*} + \frac{\beta^*}{1 - \beta^*} (P^{I*} a^* - W^*), \]
where \( P \) and \( P^* \) are the standard upper-level price indices
\[ P = \frac{1}{\gamma(1 - \gamma)^{\gamma - \gamma}(P_H)^{\gamma}(P_F)^{1 - \gamma}}, \quad P^* = \frac{1}{\gamma^*(1 - \gamma^*)^{\gamma^*}(P^*_H)^{\gamma}}. \]
Combining these with interiority in vacancy posting, we obtain labor market equilibrium conditions
\[ P^I a = -P \frac{u_n}{u_c} + \frac{1}{1 - \beta} P^I \frac{ka}{q(\theta)}, \]
\[ P^{I*} a^* = -P^* \frac{u_n^*}{u_c^*} + \frac{1}{1 - \beta^*} P^{I*} \frac{k^* a^*}{q^*(\theta^*)}. \]
Given preferences as in Cole and Obstfeld [1991], we can re-express the marginal rate of substitution in each country in terms of that between labor and domestic consumption:
\[ -P \frac{u_n}{u_c} = \frac{\chi}{\gamma} P_{HcH}(n)^{\phi}, \]
\[ -P^* \frac{u_n^*}{u_c^*} = \frac{\chi^*}{\gamma} P^*_{FcF}(n^*)^{\phi^*}. \]
Hence, labor market equilibrium can be summarized by
\[ P^I a = \frac{\chi}{\gamma} P_{HcH}(n)^{\phi} + \frac{\beta}{1 - \beta} P^I \frac{ka}{q(\theta)}, \tag{28} \]
\[ P^{I*} a^* = \frac{\chi^*}{\gamma} P^*_{FcF}(n^*)^{\phi^*} + \frac{\beta^*}{1 - \beta^*} P^{I*} \frac{k^* a^*}{q^*(\theta^*)}. \tag{29} \]
Moreover, since the equilibrium wage exceeds the marginal rate of substitution between labor and consumption in each country, is is straightforward to show that households will optimally ensure
all initially unemployed agents participate in the labor market:

\[ u = 1 - (1 - \delta)n_0, \]
\[ u^* = 1 - (1 - \delta^*)n_0^*. \]

Equilibrium employment in each market is thus

\[ n = (1 - \delta)n_0 + p(\theta)(1 - (1 - \delta)n_0), \quad (30) \]
\[ n^* = (1 - \delta^*)n_0^* + p^*(\theta^*)(1 - (1 - \delta^*)n_0^*), \quad (31) \]

Lastly, consider final good market clearing (14) and (15) for each variety produced by each country. Integrating over varieties, we obtain

\[ \int_0^1 c_H(j) dj + \int_0^1 c_H^*(j) dj = \int_0^1 y(j) dj, \]
\[ \int_0^1 c_F(j^*) dj^* + \int_0^1 c_F^*(j^*) dj^* = \int_0^1 y^*(j^*) dj^*. \]

Substituting in households’ optimal consumption of each variety in (24) and (25), and making use of retailers’ production technologies \( y(j) = x(j) \) and \( y^*(j^*) = x^*(j^*) \), intermediate good market clearing in (12) and (13), and the consistency of tightness with aggregate vacancies and job-seekers, we obtain

\[ \left( \int_0^1 \left( \frac{P_H(j)}{P_H} \right)^{-\epsilon} dj \right) c_H + \left( \int_0^1 \left( \frac{P_H^*(j)}{P_H^*} \right)^{-\epsilon} dj \right) c_H^* = an - kv, \]
\[ \left( \int_0^1 \left( \frac{P_F(j^*)}{P_F} \right)^{-\epsilon} dj^* \right) c_F + \left( \int_0^1 \left( \frac{P_F^*(j^*)}{P_F^*} \right)^{-\epsilon} dj^* \right) c_F^* = a*n^* - k^*a^*v^*. \]

Defining the price dispersion indices

\[ D_H \equiv \int_0^1 \left( \frac{P_H(j)}{P_H} \right)^{-\epsilon} dj = \int_0^1 \left( \frac{P_H^*(j)}{P_H^*} \right)^{-\epsilon} dj, \quad (32) \]
\[ D_F^* \equiv \int_0^1 \left( \frac{P_F(j^*)}{P_F} \right)^{-\epsilon} dj^* = \int_0^1 \left( \frac{P_F^*(j^*)}{P_F^*} \right)^{-\epsilon} dj^*, \quad (33) \]

where the equalities make use of producer-currency pricing, we have

\[ c_H + c_H^* = \frac{1}{D_H} a (n - kv), \]
\[ c_F + c_F^* = \frac{1}{D_F^*} a^* (n^* - k^*v^*). \]

Substituting in for import consumption \( c_H^* \) and \( c_F \) using (26) and (27), and using the optimal full participation of unemployed agents in the labor market coupled with tightness consistent with
aggregate vacancies and job-seekers, we obtain
\[
c_H = \gamma \frac{1}{D_H} a (n - k(1 - (1 - \delta)n_0)\theta), \quad \text{(34)}
\]
\[
c^*_F = \gamma \frac{1}{D^*_F} a^* (n^* - k^*(1 - (1 - \delta^*)n^*_0)\theta^*). \quad \text{(35)}
\]

In the next two sections, I will complement equilibrium conditions (24)-(35) with conditions on final good prices to characterize the flexible price/wage and sticky price equilibria, respectively.

3 Natural allocation: equilibrium with flexible prices and wages

In this section I characterize the natural allocation, the quantities and relative prices in the equilibrium with flexible prices and wages. I first summarize the welfare-relevant components of the natural allocation by a system of nine, transparent equations in nine unknowns. I then study this system to outline three key properties of the natural allocation, and define a notion of sclerotic vs. fluid labor markets in steady-state, setting the stage for the stabilization results in the next section.

3.1 Summarizing the natural allocation

When retailers are free to set prices after the realization of TFP shocks in each country, the solutions to (10) and (11) imply standard optimal price-setting conditions
\[
P_H(j) = \frac{\varepsilon}{\varepsilon - 1} (1 + \tau^r)p, \quad P^*_F(j^*) = \frac{\varepsilon}{\varepsilon - 1} (1 + \tau^{r^*})p^{l^*}. \quad \text{(36)}
\]

Given identical final goods prices, workers’ optimally consume identical amounts of varieties produced by a given country (as seen in (24) and (25)), so we can focus on characterizing the aggregators \( \{c_H, c_F, c^*_H, c^*_F\} \). Plugging in (36) into the goods market equilibrium conditions in (34) and (35) and the labor market equilibrium conditions in (28) and (29), and re-considering equilibrium conditions (26), (27), (30), and (31), we obtain the following characterization of the natural allocation (denoted with \( n \) superscripts).

**Lemma 1.** In the natural allocation, \( \{\{c^n_H, c^n_F, \theta^n, n^n\}, \{c^{s^n}_H, c^{s^n}_F, \theta^{s^n}, n^{s^n}\}, s^n\} \) satisfy
\[
\begin{align*}
\frac{1}{\varepsilon - 1(1 + \tau^r)} a = \frac{\gamma}{\varepsilon - 1} c^n_H(n^n)^2 + \frac{1}{\varepsilon - 1(1 + \tau^r)} \left( \frac{1}{1 - \beta} \right) \frac{k a n}{q(\theta^n)}, \\
n^n = (1 - \delta)n_0 + p(\theta^n)(1 - (1 - \delta)n_0), \\
c^n_H = \gamma a [n^n - k(1 - (1 - \delta)n_0)\theta^n],
\end{align*}
\]
\[
\begin{align*}
\frac{1}{\varepsilon - 1(1 + \tau^{r^*})} a^* = \frac{\gamma}{\varepsilon - 1} c^{s^n}_F(n^{s^n})^2 + \frac{1}{\varepsilon - 1(1 + \tau^{r^*})} \left( \frac{1}{1 - \beta^*} \right) \frac{k a^*}{q^*(\theta^{s^n})}, \\
n^{s^n} = (1 - \delta^*)n^*_0 + p^*(\theta^{s^n})(1 - (1 - \delta^*)n^*_0), \\
c^{s^n}_F = \gamma a^* [n^{s^n} - k^*(1 - (1 - \delta^*)n^*_0)\theta^{s^n}],
\end{align*}
\]

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Lemma 1 makes evident two immediate features of the natural allocation. First, it is fully determined without reference to nominal prices and wages in the global economy, or the (fixed) exchange rate between countries, reflecting a standard real/nominal dichotomy. Second, it is block recursive: domestic consumption and labor market conditions in each country are fully determined on their own, after which the terms of trade and consumption of imported goods are determined. I discuss the implications of this result, and other key properties of the system (37)-(39), next.

3.2 Three properties of the natural allocation

An analysis of (37)-(39) reveals three key properties of the natural allocation which are important in understanding the stabilization results of the next section. I present them here as the first proposition of the paper, and then discuss each in further detail.

Proposition 1. Three properties characterize the natural allocation:

1. Employment in each country is invariant to domestic TFP.

2. Employment in each country is invariant to foreign structural parameters and foreign TFP.

3. Define the constrained efficient allocation as that chosen by a planner maximizing union-wide utilitarian social welfare subject to the global economy’s technological constraints in producing and recruiting. Then the natural allocation is constrained efficient if and only if in each country retailers are subsidized to offset the distortion from monopolistic competition ($\tau^r = \tau^r = -\frac{1}{\xi}$) and the Hosios [1990] condition holds ($\beta = 1 - \eta$, $\beta^* = 1 - \eta^*$).

Property 1 generalizes the benchmark results of Blanchard and Gali [2010] and Shimer [2010] in the macro/labor literature to the open economy. These authors demonstrate that in a real business cycle framework with DMP labor market frictions, log preferences over consumption and recruiting costs which scale with TFP mean that equilibrium employment is invariant to TFP. This itself generalizes the standard result from a frictionless RBC model in which, under log preferences, income and substitution effects from changes in TFP exactly offset to leave equilibrium hours unchanged. Along with the earlier work of Shimer [2005], these authors’ neutrality result has given rise to a large literature on the “unemployment volatility puzzle”, seeking other ways to explain the observed cyclical fluctuations in unemployment. My result implies that their neutrality result further generalizes to the open economy in the presence of Cole and Obstfeld [1991] preferences.

Property 2 generalizes a benchmark result of Clarida et al. [2002] in the international macro literature to a frictional labor market. These authors demonstrate that in a two-country global economy with trade in intermediate goods, domestic real marginal cost is invariant to foreign output in the presence of Cole and Obstfeld [1991] preferences. In this case, greater foreign output
has offsetting impacts on the terms of trade and domestic consumption, rendering domestic real marginal cost unchanged. They demonstrate that this invariance result has important implications — for instance, only away from this case are there gains from international monetary policy coordination. My result implies that their separability result generalizes to a frictional labor market provided that recruiting costs only use domestic resources and scale with domestic TFP.

Finally, property 3 implies that standard efficiency results from the monetary, international macro, and macro/labor literatures continue to induce efficiency in this richer environment which combines all three. Relative to the constrained efficient allocation, there are three sets of possible distortions in the competitive equilibrium: monopoly power among retailers in each country, missing international risk-sharing markets, and search externalities imposed on others in a frictional labor market. A subsidy provided to retailers can undo the first, a standard result in the monetary literature. As shown by Cole and Obstfeld [1991], the assumed preferences undo the second since relative price movements in goods markets provide perfect cross-country insurance; this result generalizes to the frictional labor market when recruiting requires only domestic resources. Finally, participants in the labor market only impose search externalities on others in the domestic market in view of the assumed lack of cross-country labor mobility, implying that the Hosios [1990] condition ensures that these externalities are internalized just as in the closed economy.

3.3 Steady-state and sclerotic vs. fluid labor markets

Before turning to sticky prices and the union’s stabilization problem, I define the union’s steady-state. Suppose that the random process governing TFP \( \{a, a^*\} \) means that it fluctuates around some \( \{\bar{a}, \bar{a}^*\} \). I denote the equilibrium allocation in this steady-state with bars, and assume its distinctive characteristic is that employment is unchanged from the fictional prior period:

**Assumption 1.** \( \{n_0, n_0^*\} \) are such that \( \bar{n} = n_0, \bar{n}^* = n_0^* \).

Given the equilibrium evolution of employment in (30) and (31), this assumption means that in steady-state

\[
\delta \bar{n} = p(\bar{\theta})(1 - (1 - \delta)n), \quad \delta^* \bar{n}^* = p^*(\bar{\theta}^*)(1 - (1 - \delta^*)n^*),
\]

allowing me to interpret the separation rates \( \{\delta, \delta^*\} \) as the magnitude of steady-state flows in each labor market even in this one-period environment.

Following Blanchard and Gali [2010], in what follows I will focus on the distinction between a sclerotic and a fluid labor market, where the former is characterized as having greater hiring costs (higher \( k \)) and lower labor market flows (lower \( \delta \)). Using comparative statics on the natural allocation characterized in Lemma 1, it can be shown that a more sclerotic labor market will tend to exhibit lower steady-state employment than a more fluid labor market:

**Lemma 2.** Steady-state employment in a given country falls as:

- hiring costs in that country rise \( \left( \frac{dn}{dk} < 0, \frac{dn^*}{dk^*} < 0 \right) \);
labor market flows in that country fall \((d\bar{n}/dn > 0, d\bar{n}*/dn > 0)\), at least in the neighborhood of small steady-state unemployment \((1 - \bar{n}, 1 - \bar{n}^* \text{ close to zero})\).

In a dynamic environment, \(\{k, k^*, \delta, \delta^*\}\) will also mediate the continuation value of a match in each country, a channel which is missing here and which underscores the importance of advancing the present framework to an infinite horizon in future work. Nonetheless, even in this one-period environment, \(\{k, k^*, \delta, \delta^*\}\) will affect the equilibrium cost of output fluctuations and thus my results on macroeconomic stabilization in a direction which, I conjecture, is robust to an infinite horizon.

4 Stabilization: sticky prices, trade-offs, and optimal policy

In this section I study the equilibrium with partially sticky prices, obtaining the main results of the paper on macroeconomic trade-offs and optimal second-best monetary policy. I first characterize a linear-quadratic approximation to the Ramsey policy problem. I then solve this problem to obtain two main results: (i) institutional irrelevance for the question of when the constrained efficient allocation is achievable, and (ii) relative accommodation of the more sclerotic labor market — the one with greater hiring costs or smaller labor market flows — at the second-best optimum.

4.1 Sticky price equilibrium and the Ramsey policy problem

Let \(P_H\) and \(P_F^*\) be the final goods prices chosen by the \(i\) Home and \(i^*\) Foreign retailers who can set prices after the realization of TFP shocks, respectively. In all of the analysis which follows, I will assume that retailer subsidies are fixed at \(\tau_r = \tau_r^* = -\frac{1}{\xi}\). Following (36), these retailers will uniformly choose

\[
\begin{align*}
P_H &= \frac{\varepsilon}{\varepsilon - 1} (1 + \tau) P^I = P^I, \\
P_F^* &= \frac{\varepsilon}{\varepsilon - 1} (1 + \tau^*) P^I^* = P^I^*.
\end{align*}
\]

The remaining \(1 - i\) and \(1 - i^*\) retailers in each country will accommodate demand at posted prices \(\bar{P}_H\) and \(\bar{P}_F^*\), respectively. Per Definition 2 and the other conditions derived in section 2.3, it is straightforward to summarize the sticky price equilibrium with a system of conditions in which the real and nominal sides of the global economy are now jointly determined, given a particular choice of union-wide monetary policy \(\{\bar{M}, \bar{M}^*\}\) consistent with \(E = \bar{E}\) in a monetary union.

Here, I proceed directly to characterize the set of implementable real allocations given the possible choices of \(\{\bar{M}, \bar{M}^*\}\) consistent with \(E = \bar{E}\). In addition to summarizing the essence of the equilibrium, these conditions serve as constraints in the optimal policy problem described below.

Lemma 3. The real allocation \(\{\{c_H, c_F, \theta, n\}, \{c_H^*, c_F^*, \theta^*, n^*\}, s\}\) forms part of a sticky price equilibrium if and only if there exist nominal prices/indices \(\{\{\bar{P}_H, \bar{P}_H, D_H\}, \{\bar{P}_F^*, \bar{P}_F^*, D_F^*\}\}\) such that:

\[
\begin{align*}
\bar{P}_H^a &= \frac{\xi}{\xi - 1} c_H(n) + \frac{\bar{P}_H}{q(\theta)} \left(1 - \frac{1}{\xi}\right) \frac{k}{q(\theta)}, \\
n &= (1 - \delta) n_0 + p(\theta)(1 - (1 - \delta)n_0), \\
c_H &= \gamma \bar{P}_H^a [n - k(1 - (1 - \delta)n_0)\theta],
\end{align*}
\]

(40)
\[
\begin{align*}
    \frac{P_F}{(P_F)^*} a^* &= \frac{\chi}{\gamma} c_F^*(n^*) \phi^* + \frac{1}{(1-\beta)} \left( \frac{k^* a^*}{q^*(\theta^*)} \right), \\
    n^* &= (1-\delta^*)n_0^* + p^*(\theta^*) (1 - (1-\delta^*) n_0^*), \\
    c_F^* &= \gamma \frac{1}{(P_F)^*} a^* \left[ n^* - k^* (1 - (1-\delta^*) n_0^*) \theta^* \right], \\
    P_H &= \left[ (\bar{P}_H)_{1-\varepsilon} + (1 - \varepsilon) \left( \bar{P}_H \right)^{1-\varepsilon} \right] \frac{1}{1-\varepsilon}, \\
    D_H &= \left[ (\bar{P}_H)_{1-\varepsilon} + (1 - \varepsilon) \left( \bar{P}_H \right)^{1-\varepsilon} \right] \frac{1}{1-\varepsilon}, \\
    P_F^* &= \left[ \varepsilon (P_F^*)_{1-\varepsilon} + (1 - \varepsilon) \left( \bar{P}_F^* \right)^{1-\varepsilon} \right] \frac{1}{1-\varepsilon}, \\
    D_F^* &= \left[ \varepsilon (P_F^*)_{1-\varepsilon} + (1 - \varepsilon) \left( \bar{P}_F^* \right)^{1-\varepsilon} \right] \frac{1}{1-\varepsilon}, \\
    \frac{P_H}{\bar{P}_F} &= s.
\end{align*}
\]

I organize these constraints into six blocks to facilitate comparison with the natural allocation described in Lemma 1. (40) and (41) clearly generalize (37) and (38) to the case with potential dispersion in retailer prices (given \( \tau_r = \tau_r^* = -\frac{1}{\varepsilon} \)). (42) is identical to (39). (43) and (44) summarize prices in each country in the sticky price equilibrium, and (45) is the fundamental constraint on implementable allocations imposed by a fixed exchange rate.

Now consider the problem faced by the union’s central bank seeking to choose \( \{\bar{M}, \bar{M}^*\} \) to maximize union-wide utilitarian social welfare in response to TFP shocks. Given Lemma 3, Ramsey optimal monetary policy will implement the real allocation maximizing

\[
U \equiv u(c,n) + u^*(c^*,n^*)
\]

subject to implementability constraints (40)-(45).

### 4.2 Linear-quadratic approximation to the Ramsey problem

I follow the monetary economics literature in adopting the following notation for any endogenous variable \( z \): let \( \bar{z} \) denote its value under the steady-state realization of TFP shocks, \( \hat{z} \equiv \log z - \log \bar{z} \) denote the log deviation from steady-state, \( \hat{z}^n \equiv \log z^n - \log \bar{z} \) denote the log deviation of \( z \) under the natural allocation from steady-state, and \( \tilde{z} \equiv \hat{z} - \hat{z}^n \) denote the log deviation of \( z \) from its value under the natural allocation. I will refer to \( \tilde{z} \) as the “gap” in \( z \).

I now specify definitions of inflation, production, and output which will substantially ease the analysis of the stabilization problem in the present environment. Define (producer-price) inflation
in each country
\[
\pi_H \equiv \log P_H - \log \bar{P}_H, \\
\pi^*_F \equiv \log P^*_F - \log \bar{P}^*_F.
\]

This definition captures the idea that the sticky prices \(\bar{P}_H\) and \(\bar{P}^*_F\) reflect retailer prices in the fictional “prior” period to the one-period model under consideration. Define the effective units of labor engaged in production (inclusive of equilibrium recruiting activity)
\[
f(n) \equiv n - k(1 - \delta)n_0)p^{-1} \left(\frac{n - (1 - \delta)n_0}{1 - (1 - \delta)n_0}\right),
\]
\[
f^*(n^*) \equiv n^* - k^*(1 - (1 - \delta^*)n_0^*)p^{*-1} \left(\frac{n^* - (1 - \delta^*)n_0^*}{1 - (1 - \delta^*)n_0^*}\right),
\]
such that, given the evolution of employment in (30) and (31) and consistency of tightness with aggregate vacancies and job search, in equilibrium the aggregate output of intermediate goods in each country is characterized by the technological relation
\[
x \equiv a[(1 - \delta)n_0 + q(\theta)v - kv] = af(n),
\]
\[
x^* \equiv a^*[(1 - \delta^*)n_0^* + q^*(\theta^*)v^* - k^*v^*] = a^*f^*(n^*).
\]

I now make the following assumptions:

**Assumption 2.** The natural allocation is constrained efficient (implemented as described in property 3 of Proposition 1).

**Assumption 3.** The TFP shocks \(\{a, a^*\}\) are small around the steady-state \(\{\bar{a}, \bar{a}^*\}\).

**Assumption 4.** The preset prices \(\bar{P}_H\) and \(\bar{P}^*_F\) are consistent with a zero-inflation steady-state under the Ramsey optimal policy characterized below.

Following Woodford [2003], assumptions 2-4 ensure that the Ramsey optimal allocation can be approximated up to first order in deviations from steady-state by studying a linear approximation to the implementability constraints (40)-(45) and quadratic approximation to social welfare (46).

I first discuss the quadratic approximation to utilitarian social welfare.

**Lemma 4.** Up to second order around steady-state, (46), making use of (40)-(44), implies
\[
U - \bar{U} = -\frac{1}{2} \left[(\lambda_\pi \pi_H^2 + \lambda_\pi \tilde{\pi}^2) + (\lambda_\pi^* (\pi^*_F)^2 + \lambda_\pi^* \tilde{\pi}^* (\tilde{\pi}^*)^2)\right] + \text{tips} 
\] (47)
where tips denotes terms independent of policy,

\[
\lambda_{\pi} \equiv \left( \frac{1 - \iota}{\iota} \right) \epsilon, \\
\lambda_{\pi}^* \equiv \left( \frac{1 - \iota^*}{\iota^*} \right) \epsilon^*, \\
\lambda_x \equiv 1 + \frac{\varphi - \epsilon'_{n}}{\epsilon_{n}}, \\
\lambda_x^* \equiv 1 + \frac{\varphi^* - \epsilon'_{n^*}}{\epsilon_{n^*}},
\]

and the \( \epsilon \) terms denote elasticities of the technologies \( f \) and \( f^* \):

\[
\epsilon_{n}^f \equiv \frac{f'(n)n}{f(n)}, \quad \epsilon_{n}''^f \equiv \frac{f''(n)n}{f'(n)}, \\
\epsilon_{n}^{f*} \equiv \frac{f^{*'}(n^*)n^*}{f^{*}(n^*)}, \quad \epsilon_{n^*}''^f \equiv \frac{f^{*''}(n^*)n^*}{f^{*'}(n^*)}.
\]

As is standard in the monetary economics literature, the welfare loss from relative price dispersion in each country is rising in the stickiness of prices and the elasticity of substitution across varieties \( \epsilon \). The welfare loss from output distortions in each country is rising in the inverse Frisch elasticity of labor supply \( \{\varphi, \varphi^*\} \), as this controls the disutility of incremental work; is falling in the returns to scale in production \( \{\epsilon_{n}, \epsilon_{n'}^f\} \), as this informs how many more workers are needed to produce the incremental unit; and is rising in the (absolute value of the) elasticity of marginal production with respect to employment \( \{-\epsilon_{n}, -\epsilon_{n'}^f\} \), as this governs how much the productivity of the last worker falls with higher employment.

I now turn to the linear approximation to the implementability constraints.

**Lemma 5.** Up to first order around steady-state, (40)-(45) imply

\[
\pi_H = \mu \tilde{x}, \quad (48) \\
\pi_F^* = \mu^* \tilde{x}^*, \quad (49) \\
\tilde{s}^n = (\pi_H + \tilde{x}) - (\pi_F^* + \tilde{x}^*), \quad (50)
\]

where

\[
\mu \equiv \frac{\iota}{1 - \iota} \left[ 1 + \frac{\varphi - \epsilon'_{n}}{\epsilon_{n}} \right], \\
\mu^* \equiv \frac{\iota^*}{1 - \iota^*} \left[ 1 + \frac{\varphi^* - \epsilon'_{n^*}}{\epsilon_{n^*}} \right].
\]

(48) and (49) relate producer-price inflation with the output gap in each economy, and thus constitute the Phillips Curves in this one-period environment. The slopes \( \{\mu, \mu^*\} \) are rising in the
degree of price flexibility \(\{\iota, \iota^*\}\), as well as a term summarizing the increase in firms’ real marginal costs in response to an increase in output. We see that the latter term in fact is identical to the welfare cost of output fluctuations \(\{\lambda_x, \lambda^*_x\}\), just as it is in the benchmark without DMP frictions (e.g., Woodford [2003] and Gali [2008]). This follows from the constrained efficiency of the natural allocation in assumption 2 (in particular, the assumption that the Hosios [1990] condition holds), which creates a tight link between the first derivative of the social welfare function with respect to output and the real marginal cost faced by firms in the competitive equilibrium.

(50) summarizes the key constraint imposed on countries by being part of a monetary union. The combination of sticky prices and a fixed exchange rate hinders adjustment in the terms of trade in the competitive equilibrium. If the natural terms of trade \(\hat{s}^n\) would adjust in response to macroeconomic shocks, this leads to costly inflation/deflation, output distortions, or both.

Taken together, (47)-(50) imply the linear-quadratic approximation to the Ramsey problem:

\[
\min_{\pi_H, \pi_F, \pi^*_F, \pi^*_H} (\lambda_x \pi_H^2 + \lambda^* x^2) + (\lambda^*_x (\pi_F^*)^2 + \lambda^*_x (\hat{x}^*)^2) \text{ s.t.}
\]

\[
\pi_H = \mu \hat{x}, \\
\pi^*_F = \mu^* \hat{x}^*, \\
\hat{s}^n = (\pi_H + \hat{x}) - (\pi^*_F + \hat{x}^*).
\]

Relative to the exact Ramsey problem, there are two advantages of this approximation. First, it lays bare that a sufficient statistic for inefficiency in a monetary union is movement in the natural terms of trade \(\hat{s}^n\). Second, the coefficients on policymakers’ objective and each country’s Phillips Curve encode all the necessary information to characterize the effect of frictional and heterogeneous labor markets on the second-best optimal policy. In fact, in view of the tight link between the welfare cost of output fluctuations \(\{\lambda_x, \lambda^*_x\}\) and the slopes of the Phillips Curves \(\{\mu, \mu^*\}\), the effect of frictional labor markets on the second-best policy will operate through the single statistic in each country

\[
\phi \equiv 1 + \frac{\varphi - \epsilon'_n}{\epsilon_n}, \quad \phi^* \equiv 1 + \frac{\varphi^* - \epsilon'_n^*}{\epsilon_n^*},
\]

summarizing both the curvature in social welfare and the curvature of the cost function faced by firms as a function of domestic output. I will explore each of these points in more depth in the next two sub-sections, obtaining the paper’s main results on institutional irrelevance and relative accommodation in a monetary union.

4.3 Inefficiency in a monetary union: institutional irrelevance

I first ask whether frictional and heterogeneous labor markets affect when a monetary union will face distortions.\(^{11}\)

---

\(^{11}\)By studying the linear-quadratic approximation, I can only assess the existence of first-order distortions in prices and output. But an examination of the Ramsey problem demonstrates that Proposition 2 in fact holds exactly.
(51) makes evident that a nonzero deviation in the natural terms of trade from steady-state is a sufficient statistic for the existence of distortions in a monetary union. The following institutional irrelevance result finds that DMP frictions have no effect on movements in the natural terms of trade, and thus the circumstances under which such a union will face distortions.

**Proposition 2.** In the natural allocation,
\[ s^n = \hat{a}^* - \hat{a}. \]
Thus, the constrained efficient allocation can be achieved iff \( \hat{a} = \hat{a}^* \) regardless of the values of \( \{k, \delta, \beta = \eta\} \) and \( \{k^*, \delta^*, \beta^* = \eta^*\} \).

Thus, under the benchmark set of preferences and technologies used in this paper, Mundell [1961]'s business cycle synchronicity criterion for optimal currency areas is unchanged despite arbitrary levels of and heterogeneity in DMP frictions across countries. Intuitively, recall that properties 1 and 2 in Proposition 1 mean that in the natural allocation, employment in each country is invariant to TFP shocks originating at home and from abroad. Hence, relative production across countries is simply a function of relative TFP, so the relative price of goods produced by each country — the terms of trade — is also just a function of relative TFP. There is thus no interaction between shocks and institutions determining relative prices in the natural allocation.

The arguments underlying Proposition 2 are useful in clarifying when institutions will interact with shocks to generate distortions in a monetary union. In particular, departures from the underlying assumptions giving rise to properties 1 and 2 of the natural allocation — Cole and Obstfeld [1991] preferences and the specification of recruiting costs as scaling with domestic TFP and invariant to foreign TFP — would break this benchmark result.

### 4.4 Second-best monetary policy: relative accommodation

I now ask whether frictional and heterogeneous labor markets affect how second-best optimal monetary policy should be conducted in the presence of asymmetric shocks.

To answer this question, it is useful to first solve the stabilization problem (51):

**Lemma 6.** Up to first order deviations from steady-state, the Ramsey optimal allocation is characterized by
\[
\begin{align*}
\tilde{x} &= \omega s^n, \\
\tilde{x}^* &= -\omega^* s^n, \\
\pi_H &= \mu \omega s^n, \\
\pi_F^* &= -\mu^* \omega^* s^n,
\end{align*}
\]
where

\[
\omega \equiv \frac{(\lambda^\ast_\pi (\mu^\ast)^2 + \lambda^\ast_x) (\mu + 1)}{(\lambda_\pi (\mu)^2 + \lambda_x) (\mu + 1)^2 + (\lambda^\ast_\pi (\mu^\ast)^2 + \lambda^\ast_x) (\mu + 1)^2} > 0,
\]

\[
\omega^\ast \equiv \frac{(\lambda_\pi (\mu)^2 + \lambda_x) (\mu + 1)}{(\lambda_\pi (\mu)^2 + \lambda_x) (\mu^* + 1)^2 + (\lambda^\ast_\pi (\mu^\ast)^2 + \lambda^\ast_x) (\mu + 1)^2} > 0.
\]

Intuitively, given an appreciation in the natural terms of trade ($\hat{s}^H > 0$), sluggish price adjustment and the (implicitly) fixed exchange rate in the monetary union means that Home-produced goods will be “too cheap” relative to Foreign-produced goods. At the second-best optimum, the union must accept an inefficient boom at Home ($\hat{x}, \pi^*_H > 0$) and recession in Foreign ($\hat{x}, \pi^*_F < 0$).

In the present static environment, the Ramsey optimal allocation can be implemented equivalently by a rule targeting a weighted average of inflation rates or output gaps across the union. I focus here on the former representation, given that the ECB in fact only has a mandate for price stability, and (largely for that same reason) inflation targeting rules have been a focus of prior work on monetary unions such as Benigno [2004].

**Lemma 7.** The Ramsey optimal allocation can be implemented by an inflation targeting rule

\[ \xi \pi_H + (1 - \xi) \pi_F^* = 0 \]

with weight on Home producer-price inflation

\[ \xi \equiv \frac{\mu^\ast \omega^\ast}{\mu \omega + \mu^\ast \omega^\ast} = \frac{\lambda_\pi (\mu)^2 + \lambda_x}{\mu (\mu + 1)} + \frac{\lambda^\ast_\pi (\mu^\ast)^2 + \lambda^\ast_x}{\mu^*(\mu^* + 1)}. \]

I now ask how heterogeneity in labor market frictions affects this optimal policy rule. As derived and discussed in section 4.2, the effect of frictional labor markets will operate through the single curvature parameter for each country

\[ \phi \equiv 1 + \frac{\varphi - \epsilon^n_{n^*}}{\epsilon^n_{n}}, \quad \phi^\ast \equiv 1 + \frac{\varphi^\ast - \epsilon^{n^*}_{n^*}}{\epsilon^{n^*}_{n^*}} \]

which governs both the welfare cost of output fluctuations (the curvature of social welfare in output) and the slope of the Phillips Curve (the curvature of firms’ cost function in output) in each country:

\[ \lambda_x = \phi, \quad \lambda^\ast_x = \phi^\ast, \]

\[ \mu = \left( \frac{l}{1 - l} \right) \phi, \quad \mu^* = \left( \frac{l^*}{1 - l^*} \right) \phi^*. \]

This considerably simplifies our ability to characterize, and gain intuition behind, the effect of labor market frictions on the optimal policy rule. First, it is straightforward to show that the weight on a given country in the optimal inflation targeting rule unambiguously falls in its curvature parameter:
Lemma 8. The optimal inflation targeting rule is characterized by

\[
\frac{d\xi}{d\phi} > 0, \quad \frac{d\xi}{d\phi^*} < 0.
\]

The weight on Home in the equivalent optimal output-gap targeting rule is characterized by the same comparative statics. In this sense, optimal policy relatively accommodates the economy with a greater welfare cost of employment fluctuations and steeper Phillips Curve.

Now we need only characterize how the labor market frictions of interest — hiring costs \(\{k, k^*\}\) and magnitude of flows \(\{\delta, \delta^*\}\) — affect these curvature parameters. The next lemma expresses the curvature parameters in closed form.

Lemma 9. The curvature parameters defined in (52), governing both the welfare cost of output fluctuations and the slope of the Phillips Curve in each country, are given by

\[
\phi \equiv 1 + \frac{\varphi - \epsilon_n' \varphi}{\epsilon_n}, \quad \phi^* \equiv 1 + \frac{\varphi^* - \epsilon_n^*' \varphi^*}{\epsilon_n^*}.
\]

To gain intuition behind these expressions, note that as hiring costs disappear \((k, k^* \to 0)\), it is straightforward to show that

\[
1 + \frac{\varphi - \epsilon_n' \varphi}{\epsilon_n} \to 1 + \varphi, \quad 1 + \frac{\varphi^* - \epsilon_n^*' \varphi^*}{\epsilon_n^*} \to 1 + \varphi^*.
\]

That is, the welfare cost of output fluctuations and the slope of the Phillips Curves converge to those in the frictionless benchmarks of Woodford [2003] and Gali [2008] with log utility over consumption, constant returns to scale in production, and a zero discount factor (in view of the fact that the present model is a static one). In these benchmarks, the curvature in welfare and firms’ cost function arises purely from the increase in the marginal rate of substitution between labor and consumption given an increase in output.

Comparing these limiting cases to the expressions in Lemma 9, we see that recruiting costs in a frictional labor market have two consequences. First, they reduce the returns to scale in the economy’s production function (lower \(\epsilon_n^f\) and \(\epsilon_n^f^*\), below one). Second, they raise the extent to which an increase in production reduces the marginal productivity of labor (raise \(\epsilon_n^f\) and \(\epsilon_n^f^*\), above zero). Both of these effects follow from the fact that recruiting costs siphon resources away from production, and thus generate deadweight costs of adjusting output.

Given property 2 of Proposition 1, the curvature parameter for one country is invariant to primitives in the other. Hence, we only need to characterize the comparative statics of each country’s curvature parameter with respect to its own magnitude of hiring costs and labor market flows. Doing so, we obtain the paper’s second main result on relative accommodation in a monetary union.
Proposition 3. The welfare cost of output fluctuations and the slope of the Phillips Curve in a given country rises as

- that country’s hiring costs rise \( \left( \frac{d\phi}{dk} > 0, \frac{d\phi^*}{dk^*} > 0 \right) \),
- that country’s labor market flows get smaller \( \left( \frac{d\phi}{d\delta} < 0, \frac{d\phi^*}{d\delta^*} < 0 \right) \), at least for the case of small hiring costs \( \{k, k^*\} \) and steady-state unemployment \( \{1 - \bar{n}, 1 - \bar{n}^*\} \).

Hence, the weight on Home in the optimal inflation targeting rule \( \xi \) rises as its labor market grows more sclerotic (\( k \) rises or \( \delta \) falls) or Foreign's labor market grows more fluid (\( k^* \) falls or \( \delta^* \) rises).

Intuitively, higher hiring costs or smaller flows in a given country amplify the deadweight cost of inefficient hiring decisions, raising the welfare cost of output distortions and steepening the slope of the Phillips Curve. Facing greater welfare losses and greater inflationary/deflationary pressure from a given distortion in output, policy then targets smaller distortions in the more sclerotic union member at the second-best optimum.

This relative accommodation result is distinct from, but complementary with, the results in Benigno [2004]. Benigno [2004] focuses on cross-country heterogeneity in the degree of nominal rigidity, whereas Proposition 3 focuses on heterogeneity in the costs with which firms and workers search and match. These affect the stabilization problem through distinct theoretical channels: the cost of relative price dispersion in the first case, versus the cost of output fluctuations in the second. They also suggest distinct empirical counterparts, as I discuss in the next section. At the same time, the results are complementary in that both imply the optimality of targeting policy to accommodate the more rigid member of the union. And indeed, the key insight of Benigno [2004] is nested and holds within my model despite the presence of rich and diverse labor markets across countries: holding fixed the degree of Foreign price stickiness (\( \iota^* \)), as Home prices approach perfect flexibility (\( \iota \to 1 \)), optimal monetary policy fully seeks to stabilize inflation in Foreign (\( \xi \to 0 \)).

5 Implications for the Eurozone and future directions for research

In this section I trace out the implications of my results for macroeconomic stabilization in the Eurozone, motivating fruitful directions for future work building on the framework here.

Broadly, the current analysis suggests that while labor market frictions may not directly bear on the optimality of the Eurozone as a currency area, they should be incorporated in the design of stabilization policy by union-wide institutions such as the ECB. In particular, the institutional irrelevance result clarifies that the existence and heterogeneity of labor market frictions across member states need not have anything to do with the ECB’s ability to stabilize the union. However, the relative accommodation result implies that the institutional and technological features of labor markets across member states should inform the conduct of stabilization policy by the ECB when perfect stabilization is unattainable.

The theoretical mechanisms underlying my result on relative accommodation of the more sclerotic union member find support in European data and in other models. In particular, Campolmi
and Faia [2011] report a positive correlation between employment protection laws and inflation volatility across Eurozone members, consistent with the model’s key prediction that the sensitivity of marginal cost rises with hiring costs and falls with the magnitude of labor market flows. In a richer model studied numerically, Abbritti and Mueller [2013] also find that greater restrictions on flows into and out of unemployment steepen the Phillips curve for members of a monetary union.

This begs the provocative question: has the ECB, and more broadly Brussels, been getting stabilization policy wrong? In recent years, many commentators have argued that the policy of union-wide institutions has tended to favor the interests of core, Northern members at the expense of peripheral, Southern members. Yet on some labor market measures, such as the magnitude of legislated severance payments described in the Introduction, peripheral countries appear to have greater frictions in the matching process between workers and firms. A cursory application of the ideas in this paper would suggest that in this respect, Eurozone policy in recent years has been insufficiently accommodative of these countries.

To more fully address this question, future work should focus on extending the framework developed here to an infinite horizon and calibrating it to match salient labor market features of Eurozone economies. A dynamic extension will make full use of the conceptual advantages of a search and matching modeling of the labor market, where worker flows into and out of unemployment can be tractably studied. A quantitative calibration will allow a more formal evaluation of whether heterogeneity in the fluidity of Eurozone labor markets has a first-order effect on stabilization trade-offs and optimal policy — particularly as compared to other dimensions of heterogeneity studied elsewhere in the literature, such as the duration of prices or wages (Benigno [2004]) or debt levels across countries (Aguirar et al. [2015]).

Two other lines of inquiry would also be particularly interesting to expand beyond the specific mechanisms and trade-offs explored in this paper. First, it would be useful to study an enriched economic environment with endogenous layoffs and incomplete markets. This would permit a deeper analysis of heterogeneity in specific institutions such as severance payments and unemployment insurance across a monetary union, building on the seminal contributions of Acemoglu and Shimer [1999] and Blanchard and Tirole [2008], as well as some of my own work (Kekre [2016]). Second, it would be interesting to reverse the question asked by the present paper, and assess how the stabilization choices of union-wide bodies might affect the design of national labor market policies themselves. Might it be possible that by accommodating more sclerotic members of a monetary union, a union-wide central bank will blunt incentives by these national legislatures to reform?

6 Conclusion

In this paper I have analyzed the consequences of cross-country heterogeneity in labor market frictions for macroeconomic trade-offs and stabilization policy in a monetary union. I embedded DMP frictions into an otherwise standard model of a two-country union with sticky prices subject to TFP shocks, and used a linear-quadratic approximation to the Ramsey optimal monetary policy
problem to establish two key results: institutional irrelevance for the question of when a monetary union will face distortions, and relative accommodation of the more sclerotic member of the union at the second-best optimum. Applied to the Eurozone, my results suggest that labor market frictions may not bear on the optimality of the single currency, but may be quite important in guiding optimal stabilization policy by union-wide institutions such as the ECB.

References


Proofs†

**Proposition 1: three properties of the natural allocation**

*Proof.* Properties 1 and 2 are clear by examination of the equilibrium conditions defining the natural allocation in Lemma 1. Here I prove property 3 regarding constrained efficiency.

The constrained planner with a utilitarian objective faces

\[
\max_{c_H, c_F, c_H^*, c_F^*} u(c, n) + u^*(c^*, n^*) \quad \text{s.t.}
\]

\[
(Evol) : n = (1 - \delta)n_0 + p(\theta)u,
\]

\[
(Evol)^*: n^* = (1 - \delta^*)n_0^* + p^*(\theta^*)u^*,
\]

\[
(RC)_H : c_H + c_H^* = a[n - ku\theta],
\]

\[
(RC)_F : c_F + c_F^* = a^*[n^* - k^*u^*\theta^*],
\]

where I already account for the fact that the planner will choose identical production levels across varieties from a particular country, and identical consumption of those varieties from residents of a particular country, given the symmetry assumptions made on tastes and technologies.

It is straightforward to use \( FOC(u) \), \( FOC(u^*) \), \( FOC(\theta) \), and \( FOC(\theta^*) \) to show that

\[ u = 1 - (1 - \delta)n_0, \quad u^* = 1 - (1 - \delta^*)n_0^*, \]

since raising participation in the labor force will reduce the social cost of employing the same number of workers in each country.

It is also straightforward to use \( FOC(c_H) \), \( FOC(c_H^*) \), \( FOC(c_F) \), and \( FOC(c_F^*) \) to show that

\[ c_H^* = \left(1 - \frac{\gamma}{\gamma'}\right)c_H, \quad c_F^* = \left(1 - \frac{\gamma}{\gamma'}\right)c_F^* \]

(55)


Finally, we can use \( FOC(c_H) \), \( FOC(c_F^*) \), \( FOC(\theta) \), \( FOC(\theta^*) \), \( FOC(n) \), and \( FOC(n^*) \) to obtain

\[
a = \frac{\chi}{\gamma}c_H n^\phi + \frac{1}{\eta q(\theta)}\, k a,
\]

(56)

\[
a^* = \frac{\chi}{\gamma}c_F^* n^*\phi + \frac{1}{\eta^* q^*(\theta^*)}k^* a^*.
\]

(57)

(54)-(57), coupled with the constraints of (53), define the constrained efficient allocation. Comparing these to the conditions defining the natural allocation in Lemma 1, it is clear that inducing

†I provide proofs of results for which the steps are not fully obvious from the main text. Formal proofs of results not provided here are available on request.
the efficient levels of domestic consumption and production at arbitrary \( a \) in Home requires
\[
\tau^r = -\frac{1}{\varepsilon}, \quad \beta = 1 - \eta,
\]
and inducing the efficient levels of domestic consumption and production at arbitrary \( a^* \) in Foreign requires
\[
\tau^{r*} = -\frac{1}{\varepsilon}, \quad \beta^* = 1 - \eta^*.
\]
Then, imported consumption levels \( \{c_F, c_H^*\} \) will be efficient according to (55) at the relative price \( s = \frac{c_F}{c_H} \), where the right-hand side is evaluated at the constrained efficient allocation.

\begin{proof}

\end{proof}

Lemma 2: steady-state employment in sclerotic vs. fluid labor markets

Proof. Following Lemma 1 and Assumption 1, the steady-state \( \{\bar{c}_H, \bar{n}, \bar{\theta}\} \) are characterized by
\[
a = \frac{\chi}{\gamma} \bar{c}_H \bar{n}^\varphi + \frac{1}{1 - \beta} \frac{ka}{q(\bar{\theta})},
\]
\[
\bar{n} = (1 - \delta) \bar{n} + p(\bar{\theta})(1 - (1 - \delta)\bar{n}),
\]
\[
\bar{c}_H = \gamma a \left[ \bar{n} - k(1 - (1 - \delta)\bar{n})\bar{\theta} \right].
\]

It will prove useful later (in the proof of Proposition 3) to begin by working with
\[
\kappa \equiv \frac{k}{q(\bar{\theta})},
\]
which has an economic interpretation as the equilibrium hiring costs per hired worker. Combining the above steady-state conditions, \( \bar{\kappa} \) solves
\[
0 = v(\bar{\kappa}; k, \delta) \equiv 1 - \chi (1 - \delta \bar{\kappa}) \bar{n}(\bar{\kappa}; k, \delta)^{\varphi+1} - \frac{1}{1 - \beta} \bar{\kappa},
\]
where
\[
n = \bar{n}(\bar{\kappa}; k, \delta) = \frac{p(q^{-1}(k/\bar{\kappa}))}{\delta + (1 - \delta)p(q^{-1}(k/\bar{\kappa}))}.
\]
in steady-state. We can then show that
\[
v_\kappa = \frac{1}{\bar{\kappa}} \chi (1 - \delta \bar{\kappa}) \bar{n}^{\varphi+1} \left[ \frac{\delta \bar{\kappa}}{1 - \delta \bar{\kappa}} - (\varphi + 1) \frac{\bar{n}}{\bar{\kappa}} \frac{\partial \bar{n}}{\partial \bar{\kappa}} - \frac{1}{1 - \beta} \frac{\bar{\kappa}}{1 - \beta \bar{\kappa}} \right] < 0,
\]
\[
v_k = \frac{1}{\bar{k}} \chi (1 - \delta \bar{\kappa}) \bar{n}^{\varphi+1} \left[ 0 \right] > 0,
\]
\[
v_\delta = \frac{1}{\delta} \chi (1 - \delta \bar{\kappa}) \bar{n}^{\varphi+1} \left[ \frac{\delta \bar{\kappa}}{1 - \delta \bar{\kappa}} - (\varphi + 1) \frac{\delta \bar{n}}{\bar{n} \partial \delta} \right] > 0,
\]

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where
\[
\frac{\partial \bar{n}}{\partial \kappa} = \frac{\bar{n}}{\kappa} \left( \frac{\delta}{\delta + (1-\delta)p(\theta)} \right) \frac{\eta}{1-\eta} > 0,
\]
\[
\frac{\partial \bar{n}}{\partial k} = -\frac{\bar{n}}{k} \left( \frac{\delta}{\delta + (1-\delta)p(\theta)} \right) \frac{\eta}{1-\eta} < 0,
\]
\[
\frac{\partial \bar{n}}{\partial \delta} = -\frac{\bar{n}}{\delta} \left( \frac{\delta(1-p(\theta))}{\delta + (1-\delta)p(\theta)} \right) < 0.
\]

By the implicit function theorem, we then have
\[
\frac{d \bar{\kappa}}{dk} = -\frac{v_k}{v_{\kappa}} > 0,
\]
\[
\frac{d \bar{\kappa}}{d\delta} = -\frac{v_k}{v_{\kappa}} > 0.
\]

Now consider the comparative statics of \( \bar{n} \) with respect to \( k \) and \( \delta \). First we have that
\[
\frac{d \bar{n}}{dk} = \frac{\partial \bar{n}}{\partial k} + \frac{\partial \bar{n}}{\partial \kappa} \frac{d \bar{\kappa}}{dk},
\]
\[
= \frac{\chi (1-\delta_{\kappa}) \bar{n}^{\varphi+1}}{v_k \bar{\kappa}} \left[ \frac{\delta_{\kappa}}{1-\delta_{\kappa}} - \frac{1}{1-\frac{1}{1-\beta_{\kappa}}} \right] \frac{\partial \bar{n}}{\partial k},
\]
\[
< 0
\]
as claimed. Then we have that
\[
\frac{d \bar{n}}{d\delta} = \frac{\partial \bar{n}}{\partial \delta} + \frac{\partial \bar{n}}{\partial \kappa} \frac{d \bar{\kappa}}{d\delta},
\]
\[
= \frac{\chi (1-\delta_{\kappa}) \bar{n}^{\varphi+1}}{v_k \bar{\kappa}} \left[ \left( 1 + \frac{1}{1-p(\theta) \frac{\eta}{1-\eta}} \right) \frac{\delta_{\kappa}}{1-\delta_{\kappa}} - \frac{1}{1-\frac{1}{1-\beta_{\kappa}}} \right] \frac{\partial \bar{n}}{\partial \delta}.
\]
The term in brackets here is of ambiguous sign. However, when \( \bar{n} \) is close to 1 and thus \( p(\theta) \) is also close to 1, we have that
\[
\frac{d \bar{n}}{d\delta} > 0
\]
as claimed.

\textbf{Lemma 4: second-order approximation to welfare}

\textit{Proof.} Plugging in the implementability constraints
\[
c_F = \frac{1-\gamma}{\gamma} sc_H, \quad c_H^* = \frac{1-\gamma}{\gamma} \frac{1}{s} c_F^*.
\]
into utilitarian social welfare defined in (46), the terms of trade drop out and

\[ U = \left( \log c_H - \chi \frac{n^{1+\phi}}{1 + \phi} \right) + \left( \log c_F^* - \chi^* \frac{n^{1+\phi^*}}{1 + \phi^*} \right) + \text{const.} \]

Further plugging in the implementability constraints

\[ c_H = \frac{\gamma}{D_H} x, \quad c_F^* = \frac{\gamma}{D_F^*} x^*, \]

\[ n = f^{-1}(x/a), \quad n^* = f^*{-1}(x^*/a^*) \]

implied by the definitions of output \( \{x, x^*\} \) and technologies \( \{f, f^*\} \) in section 4.2, we can write social welfare as

\[ U(D_H, x, D_F^*, x^*, a, a^*) = \left( -\log D_H + \log \gamma x - \chi \frac{(f^{-1}(x/a))^{1+\phi}}{1 + \phi} \right) + \left( -\log D_F^* + \log \gamma x^* - \chi^* \frac{(f^*{-1}(x^*/a^*))^{1+\phi^*}}{1 + \phi^*} \right) + \text{const.} \]

In view of the separability between terms given the assumed preferences and technologies, up to second order we then have

\[ U = \bar{U} + \bar{U}_{D_H} \tilde{D}_H + \frac{1}{2} \bar{U}_{D_H D_H} \tilde{D}_H^2 + \bar{U}_{x \tilde{x} \tilde{x}} + \frac{1}{2} \bar{U}_{x x \tilde{x} \tilde{x}} + \bar{U}_{x a \tilde{x} \tilde{a}} \]

\[ + \frac{1}{2} \bar{U}_{D_F^*} \tilde{D}_F^* + \frac{1}{2} \bar{U}_{D_F^* D_F^*} \tilde{D}_F^2 + \bar{U}_{x^* \tilde{x}^* \tilde{x}^*} + \frac{1}{2} \bar{U}_{x^* x^* \tilde{x}^* \tilde{x}^*} + \bar{U}_{x^* a^* \tilde{x}^* \tilde{a}^* \tilde{a}^*} + \text{tips}, \quad (58) \]

where a bar over the partial derivative means that it is evaluated at the zero-inflation steady-state, and \( \text{tips} \) denotes terms independent of policy.

First consider the price dispersion terms. Plugging in the price index \( P_H \) into the dispersion index \( D_H \) and expanding \( D_H \) up to second order in \( P_H \) around the zero-inflation steady-state yields

\[ \hat{D}_H = \frac{1}{2} \nu (1 - \nu) P_H^2, \]

so that \( \hat{D}_H^2 \) will be zero up to second order terms. Moreover, using (71) to plug in for domestic (producer-price) inflation, we obtain

\[ \tilde{D}_H = \frac{1}{2} \left( \frac{1 - \nu}{\nu} \right) \tilde{\pi}_H^2. \quad (59) \]

Finally, it is straightforward to show that

\[ \bar{U}_{D_H} \tilde{D}_H = -1. \quad (60) \]
Analogous steps in Foreign then imply
\[
\hat{D}_F = \frac{1}{2} \left( \frac{1 - \iota^*}{\iota^*} \right) \varepsilon^* \pi_{F2}^*, \quad (61)
\]
\[
\bar{U}_D F \hat{D}_F = -1. \quad (62)
\]

Now consider the terms involving output. It is straightforward to differentiate the objective and obtain
\[
U_x = \frac{1}{x} - \frac{1}{af'(f^{-1}(x/a))} \chi (f^{-1}(x/a))^\varphi,
\]
\[
U_{x^*} = \frac{1}{x^*} - \frac{1}{a^* f'^*(f^{*-1}(x^*/a^*)))} \chi (f^{*-1}(x^*/a^*)))^{\varphi^*},
\]
using the inverse function theorem. Since
\[
f'(n) = 1 - \frac{1}{\eta q(\theta)},
\]
\[
f'^*(n) = 1 - \frac{1}{\eta^* q^*(\theta^*)},
\]
and the steady-state is constrained efficient per Assumption 2, the efficiency conditions (56) and (57) mean that
\[
\bar{U}_x = 0, \quad (63)
\]
\[
\bar{U}_{x^*} = 0, \quad (64)
\]
so the first-order terms involving output in (58) drop out (justifying linear approximations of the implementability constraints as sufficient to characterize the Ramsey optimal allocation up to first order, following Woodford [2003]). Again differentiating the objective, making use of the inverse function theorem, and evaluating it in steady-state yields
\[
\bar{U}_{xx} x^2 = - \left[ 1 + \frac{\varphi - \epsilon_n}{\epsilon_n} \right], \quad (65)
\]
\[
\bar{U}_{x^* x^*} x^{*2} = - \left[ 1 + \frac{\varphi^* - \epsilon_n^*}{\epsilon_n^*} \right], \quad (66)
\]
given the definitions of the elasticities \( \epsilon \) in the statement of the lemma. Finally, since the natural allocation is constrained efficient per Assumption 2, standard arguments mean that we can complete
the square and collect second-order terms so that
\[ \frac{1}{2} \bar{U}_{xx} \bar{x}^2 + \bar{U}_{xa} \bar{x} \bar{a} = \frac{1}{2} \tilde{U}_{xx} \tilde{x}^2 + \text{tips}, \] (67)
\[ \frac{1}{2} \bar{U}_{x*a} \bar{x} \bar{a} + \bar{U}_{x*a} \bar{a} \bar{a} = \frac{1}{2} \tilde{U}_{x*a} \tilde{x} \tilde{a} + \text{tips}. \] (68)

Plugging in (59)-(68) into (58), we obtain the desired approximation (47) given the definitions of \( \lambda_\pi, \lambda^{*}_\pi, \lambda_x, \) and \( \lambda^{*}_x. \)

Lemma 5: first-order approximation to implementability

Proof. First note that the implementability constraints (40) can be combined to give
\[ \frac{\mathcal{P}_H}{\hat{\mathcal{P}}_H} = \frac{\frac{\lambda}{\gamma} \xi_H n^\phi}{a \left(1 - \frac{1}{1-\beta} \frac{k}{q(\theta)}\right)}, \]
\[ = \frac{1}{D_H} \frac{\chi a f(n) n^\phi}{af'(n)}, \]
\[ = \frac{1}{D_H} \frac{\chi x(f^{-1}(x/a))^{\phi}}{af'(f^{-1}(x/a))}, \]
where the second and third lines use the definition of technology \( f(n), \) output \( x, \) and the constrained efficiency of the natural allocation per Assumption 2 (implying that the Hosios [1990] condition \( 1 - \beta = \eta \) is satisfied). Log-linearizing this condition around the zero-inflation steady-state, we obtain
\[ (\hat{\mathcal{P}}_H - \hat{\mathcal{P}}_H) = -\hat{D}_H + \left[ 1 + \frac{\phi - \epsilon_f}{\epsilon_a} \right] (\hat{x} - \hat{a}), \] (69)
for the elasticities \( \epsilon \) defined in the statement of Lemma 4, while in the natural allocation the same steps imply
\[ \hat{x}^n = \hat{a}. \] (70)

Log-linearizing (43) around the zero-inflation steady-state, we obtain
\[ \pi_H = \hat{\mathcal{P}}_H = \iota \hat{\mathcal{P}}_H, \]
\[ \hat{D}_H = 0. \] (72)

Combining (69)-(72), we obtain the Phillips Curve at Home
\[ \pi_H = \frac{\iota}{1 - \iota} \left[ 1 + \frac{\phi - \epsilon_f}{\epsilon_a} \right] \tilde{x}. \]
With exactly analogous steps, we can combine and log-linearize the implementability constraints (41) and (44), making use of Assumption 2, to obtain Foreign’s Phillips Curve

$$\pi_F^* = \frac{\theta^*}{1 - \theta^*} \left[ 1 + \frac{\phi^* - \epsilon_n^{*'}}{\epsilon_n^{*'}} \right] \tilde{x}^*.$$  

Finally, log-linearizing the implementability constraint

$$s = \frac{c_F^*}{c_H}$$

both in the sticky price equilibrium and in the natural allocation, in gap notation we have

$$s^n = \hat{s} + \tilde{c}_H - \tilde{c}_F^*.$$  

(73)

Log-linearizing implementability constraint (45) around the zero-inflation steady-state yields

$$\hat{s} = \pi_H - \pi_F^*,$$  

(74)

while log-linearizing the implementability constraints

$$c_H = \frac{\gamma}{D} x, \quad c_F^* = \frac{\gamma}{D_F^*} x^*.$$  

around the zero-inflation steady-state and using (72), the analog for Foreign, and gap notation yields

$$\tilde{c}_H = \tilde{x}, \quad \tilde{c}_F^* = \tilde{x}^*.$$  

(75)

Combining (73)-(75) yields

$$s^n = (\pi_H + \tilde{x}) - (\pi_F^* + \tilde{x}^*),$$

the constraint posed by membership in a monetary union.

**Proposition 2: institutional irrelevance**

*Proof.* It is apparent from the implementability constraints in Lemma 5 that

$$\pi_H = 0, \quad \tilde{x} = 0, \quad \pi_F^* = 0, \quad \tilde{x}^* = 0$$

can be achieved if and only if \(s^n = 0\). Since the natural allocation is constrained efficient by Assumption 2, it follows that the constrained efficient allocation can be achieved if and only if
\( \tilde{s}^n = 0 \). Finally, note that

\[
\begin{align*}
\hat{s}^n &= \hat{c}_F^n - \hat{c}_H^n, \\
&= \hat{x}^n - \hat{x}^*, \\
&= \hat{a}^* - \hat{a},
\end{align*}
\]

where the third line uses (70) and the analog for Foreign.

Lemma 8: impact of \( \{\phi, \phi^*\} \) on the optimal policy rule

Proof. It is immediate that

\[
\begin{align*}
d\xi &= \frac{\lambda_\pi(\mu)^2 + \lambda_x}{\mu(\mu + 1)} > 0, \\
d\xi^* &= \frac{\lambda_\pi^*(\mu^*)^2 + \lambda_x^*}{\mu^*(\mu^* + 1)} < 0.
\end{align*}
\]

Using the definitions of \( \{\lambda_\pi, \lambda_x, \mu\} \) in Lemmas 4 and 5,

\[
\frac{\lambda_\pi(\mu)^2 + \lambda_x}{\mu(\mu + 1)} = \left( \frac{1 - \epsilon}{\epsilon} \right) \left( \frac{\epsilon}{\epsilon - 1} \right) \phi^2 + \phi.
\]

It is straightforward to then show

\[
\frac{d}{d\phi} \left[ \frac{\lambda_\pi(\mu)^2 + \lambda_x}{\mu(\mu + 1)} \right] \propto \epsilon - 1 > 0.
\]

Analogous steps imply

\[
\frac{d}{d\phi^*} \left[ \frac{\lambda_\pi^*(\mu^*)^2 + \lambda_x^*}{\mu^*(\mu^* + 1)} \right] \propto \epsilon - 1 > 0.
\]

The result immediately follows.

Lemma 9: \( \{\phi, \phi^*\} \) in closed form

Proof. Given the effective units of labor engaged in production

\[
f(n) = \left[ n - k(1 - (1 - \delta)n_0)p^{-1} \left( \frac{n - (1 - \delta)n_0}{1 - (1 - \delta)n_0} \right) \right],
\]

38
it is straightforward to compute that in steady-state

\[
\epsilon_f^\prime_n = \frac{1 - \frac{1}{\eta q(\theta)}}{1 - \frac{1}{\eta q(\theta)}} \frac{k}{\eta q(\theta)},
\]

\[
\epsilon_f''_n = -\frac{1}{1 - \frac{1}{\eta q(\theta)}} \eta \frac{k}{q(\theta)} \left( \frac{1 - \eta}{\delta \eta} \right),
\]

and analogously for \( f^*(n^*) \), \( \epsilon_{n^*}', \) and \( \epsilon_{n^*}'' \).

Proposition 3: relative accommodation

Proof. First note that the only endogenous object in

\[
\phi \equiv 1 + \frac{\varphi - \epsilon_f'}{\epsilon_f} = 1 + \frac{\varphi + \frac{1}{1 - \frac{1}{\eta q(\theta)}} \frac{k}{\eta q(\theta)} \left( \frac{1 - \eta}{\delta \eta} \right)}{1 - \frac{1}{\eta q(\theta)}}
\]

is the steady-state hiring costs per hired worker

\[
\bar{\kappa} = \frac{k}{q(\theta)},
\]

which was studied extensively in the proof of Lemma 2.

So first considering the desired comparative static with respect to hiring costs \( k \), we have that

\[
\frac{d\phi}{dk} = \frac{d\phi}{d\bar{\kappa}} \frac{d\bar{\kappa}}{dk}.
\]

Since

\[
\frac{d\phi}{d\bar{\kappa}} \propto \epsilon_f \left( \frac{d - \epsilon_f'}{d\bar{\kappa}} \right) - (\varphi - \epsilon_f') \left( \frac{de_f}{d\bar{\kappa}} \right),
\]

\[
= \epsilon_f \left( \frac{1}{(1 - \frac{1}{\eta \bar{\kappa}})^2} \frac{1 - \eta}{\delta \eta^2} \right) + (\varphi - \epsilon_f') \left( \frac{1}{(1 - \delta \bar{\kappa})^2} \frac{1 - \delta}{\eta^2} \right),
\]

\[
> 0,
\]

(76)

and we know from the proof of Lemma 2 that \( \frac{d\bar{\kappa}}{dk > 0} \), we can conclude

\[
\frac{d\phi}{dk} > 0
\]

as claimed.
Now turn to the comparative static with respect to magnitude of flows $\delta$, where

$$
\frac{d\phi}{d\delta} = \frac{\partial \phi}{\partial \delta} + \frac{\partial \phi}{\partial \bar{\kappa}} \frac{d\bar{\kappa}}{d\delta}.
$$

The challenge here is that while

$$
\frac{\partial \phi}{\partial \delta} \propto \epsilon_f \left( \frac{\partial - \epsilon'_n}{\partial \delta} \right) - (\varphi - \epsilon'_n) \left( \frac{\partial \epsilon'_n}{\partial \delta} \right),
$$

$$
= \epsilon_n \left( - \frac{1}{\eta} \bar{\kappa} \left( \frac{1 - \eta}{\delta^2} \right) \right) - (\varphi - \epsilon'_n) \left( \frac{1}{(1 - \delta \bar{\kappa})^2} \right),
$$

$$< 0,$$

we know from (76) and the proof of Lemma 2 that

$$
\frac{\partial \phi}{\partial \bar{\kappa}} \frac{d\bar{\kappa}}{d\delta} > 0.
$$

In economic terms, while a lower magnitude of flows directly raises the welfare cost of output fluctuations ($\frac{\partial \phi}{\partial \delta} < 0$), in equilibrium it also reduces hiring costs per hire $\bar{\kappa}$ in the present framework ($\frac{d\bar{\kappa}}{d\delta} > 0$), which has an offsetting effect on the welfare cost of output fluctuations (since $\frac{\partial \phi}{\partial \bar{\kappa}} > 0$). The fact that lower flows reduce hiring costs per hire contradicts the standard result from classic DMP models in which, by raising the continuation value of a match, a lower separation rate incentivizes vacancy posting and thus raise hiring costs per hire. The reason is twofold: the present setting is a one-period model with no continuation value of a match, and (more importantly) unlike classic DMP models I assume risk averse agents with a convex disutility of labor. Lower flows tend to raise the marginal rate of substitution between labor and consumption, and thus dis-incentivize vacancy posting in the present model.

In the reasonable benchmark case where $k$ (and thus $\bar{\kappa}$ — see the note below) is close to zero and steady-state unemployment $1 - \bar{n}$ is also close to zero, it is straightforward to show that

$$
\frac{\partial \phi}{\partial \bar{\kappa}} \frac{d\bar{\kappa}}{d\delta} < 0
$$

is an order of magnitude smaller than

$$
\frac{\partial \phi}{\partial \delta}
$$

and thus

$$
\frac{d\phi}{d\delta} < 0
$$

as claimed.

Note that the above argument implicitly uses the fact that as $k \to 0$, $\bar{\kappa} \to 0$. This can be seen
as follows: as $k \to 0$, the steady-state equilibrium conditions defining $\{\bar{c}_H, \bar{n}, \bar{\theta}\}$

$$a = \frac{X}{\gamma} \bar{c}_H \bar{n}^\varphi + \frac{1}{\eta q(\bar{\theta})} \frac{k a}{\eta},$$

$$\bar{n} = (1 - \delta)\bar{n} + p(\bar{\theta})(1 - (1 - \delta)\bar{n}),$$

$$\bar{c}_H = \gamma a \left[\bar{n} - k(1 - (1 - \delta)\bar{n})\bar{\theta}\right],$$

make evident that (there exists a path in which) the equilibrium $\bar{\theta} \to \bar{\theta}$, where

$$\bar{\theta} := \frac{p(\bar{\theta})}{\delta + (1 - \delta)p(\bar{\theta})} = \bar{n},$$

$$\bar{n} := \chi \bar{n}^{\varphi + 1} = 1.$$

That is, the steady-state equilibrium converges to the frictionless benchmark. This means that

$$\bar{\kappa} \equiv \frac{k}{q(\theta)} \to 0$$

as used above.

Analogous arguments imply that for Foreign,

$$\frac{d\phi^*}{dk^*} > 0, \quad \frac{d\phi^*}{d\delta^*} < 0,$$

where the latter is true when $k^*$ (and thus $\bar{\kappa}^*$) is close to zero and steady-state unemployment $1 - \bar{n}^*$ is also close to zero.

The result on relative accommodation then is immediately implied by Lemmas 8 and 9. □
Appendix: derivation of Nash bargained wages

Following Shimer [2010]’s approach to the Nash bargain, at Home let \( \tilde{v}_n(\hat{W}) \) denote the marginal value to the representative household of employing an additional worker at wage \( \hat{W} \), let \( \tilde{\Pi}_n(\hat{W}) \) denote the marginal profit for the representative producer of employing an additional worker at wage \( \hat{W} \), and let \( \tilde{v}_n^*(\hat{W}^*) \) and \( \tilde{\Pi}_n^*(\hat{W}^*) \) be analogs in Foreign. Then Nash bargained wages solve

\[
W = \arg \max_{\hat{W}} \left[ \tilde{v}_n(\hat{W}) \right]^\beta \left[ \tilde{\Pi}_n(\hat{W}) \right]^{1-\beta}, \tag{77}
\]

\[
W^* = \arg \max_{\hat{W}^*} \left[ \tilde{v}_n^*(\hat{W}^*) \right]^\beta \left[ \tilde{\Pi}_n^*(\hat{W}^*) \right]^{1-\beta^*}. \tag{78}
\]

To compute \( \tilde{v}_n(\hat{W}) \) at Home, let \( \hat{v}(n,W;\epsilon,\hat{W}) \) be the value to the representative household of \( n \) workers employed at wage \( W \) and \( \epsilon \) workers employed at wage \( \hat{W} \). Building on (6),

\[
\hat{v}(n,W;\epsilon,\hat{W}) = \max_{\{c_H(j)\}_j,\{c_F(j^*)\}_{j^*}} \log c - \frac{\chi (n + \epsilon)^{1+\varphi}}{1 + \varphi} \text{ s.t.}
\]

\[
\int_0^1 P_H(j)c_H(j) dj + \int_0^1 P_F(j^*)c_F(j^*) dj^* \leq Wn + \hat{W}\epsilon + \Pi + \int_0^1 \Pi'(j) dj - T.
\]

Then we have

\[
\tilde{v}_n(\hat{W}) \equiv \frac{\partial}{\partial \epsilon} \hat{v}(n,W;\epsilon,\hat{W})|_{\epsilon=0}. \tag{79}
\]

To compute \( \tilde{\Pi}_n(\hat{W}) \) at Home, let \( \hat{\Pi}(n,W;\epsilon,\hat{W}) \) be the profit for the representative producer of \( n \) workers employed at \( W \) and \( \epsilon \) workers employed at \( \hat{W} \) after vacancy posting costs have been sunk. Building on (8),

\[
\hat{\Pi}(n,W;\epsilon,\hat{W}) = (P^Ia - W)n + (P^Ia - \hat{W})\epsilon
\]

Then we have

\[
\tilde{\Pi}_n(\hat{W}) \equiv \frac{\partial}{\partial \epsilon} \hat{\Pi}(n,W;\epsilon,\hat{W})|_{\epsilon=0}. \tag{80}
\]

In Foreign we similarly have

\[
\tilde{v}_n^*(\hat{W}^*) \equiv \frac{\partial}{\partial \epsilon^*} \hat{v}^*(n^*,W^*;\epsilon^*,\hat{W}^*)|_{\epsilon^*=0}, \tag{81}
\]

\[
\tilde{\Pi}_n^*(\hat{W}^*) \equiv \frac{\partial}{\partial \epsilon^*} \hat{\Pi}^*(n^*,W^*;\epsilon^*,\hat{W}^*)|_{\epsilon^*=0} \tag{82}
\]

for \( \hat{v}^*(n^*,W^*;\epsilon^*,\hat{W}^*) \) and \( \hat{\Pi}^*(n^*,W^*;\epsilon^*,\hat{W}^*) \) defined analogously to those objects in Home.

Now, evaluating the right-hand side of (79) and (81) using the Envelope Theorem, the marginal
value to the representative household of an additional worker employed at $\hat{W}$ or $\hat{W}^*$ is

$$\tilde{\nu}_n(\hat{W}) = \frac{\hat{W}}{P} u_c - u_n,$$
$$\tilde{\nu}_n^*(\hat{W}^*) = \frac{\hat{W}^*}{P^*} u_c^* - u_n^*,$$

Evaluating the right-hand side of (80) and (82), the marginal value to the representative producer of an additional worker employed at $\hat{W}$ or $\hat{W}^*$ is

$$\tilde{\Pi}_n(\hat{W}) = P^I a - \hat{W},$$
$$\tilde{\Pi}_n^*(\hat{W}^*) = P^I^* a^* - \hat{W}^*,$$

It is then straightforward to establish that the solutions of (77) and (78) yield

$$W - P \frac{u_n}{u_c} = \frac{\beta}{1 - \beta} (P^I a - W),$$
$$W^* - P^* \frac{u_n^*}{u_c^*} = \frac{\beta^*}{1 - \beta^*} (P^I^* a^* - W^*),$$

as described in section 2.3 of the main text.