THE INFORMATION CONTENT OF THE INTEREST RATE AND OPTIMAL MONETARY POLICY*

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Optimal monetary policy rules are derived in a rational expectations cum contracting framework. Monetary policy is redundant if wage setters exploit the incomplete current information embodied in today's nominal interest rate. However, the monetary authorities can save wage setters the costs of "indexing" to the interest rate. A contemporaneous money supply feedback rule is as effective as wage indexation. A lagged rule, relevant under a regime of money supply targeting, is also as effective if investors use the interest rate. Both rules have the same implications for the real interest rate as Poole's combination policy. However, the two rules have strikingly different implications for the nominal interest rate.

I. INTRODUCTION

Today's nominal interest rate provides incomplete current information about aggregate disturbances in the money and goods markets. Employing a standard IS-LM model, Poole [1970] showed how the monetary authorities can best use this incomplete information in conducting a "combination policy" that reduces, but does not eliminate, the variance of output. Viewed in light of a decade of further research, Poole's static expectations analysis is open to an important criticism: it makes no allowance for the fact that private agents can also, if they choose, make rational use of the same incomplete information. In the present paper we update Poole's analysis by characterizing monetary policy under alternative assumptions about which

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of three groups of agents—wage setters, investors, and the monetary authorities—"use" the current nominal interest rate.

We demonstrate that if wage setters optimally "index" nominal wages to the interest rate, monetary policy is redundant. Symmetrically, monetary policy can make wage setters' use of the interest rate redundant. If the costs of indexation of wages to the interest rate are significant (as suggested by the absence of provisions linking wages and current financial variables in actual contracts), monetary policy can play an important stabilizing role.

The optimal contemporaneous money supply rule is the same whether or not investors use the interest rate in forming their inflation predictions. But investors' use of the interest rate is crucial for policy effectiveness when the monetary authorities respond to interest rate innovations with a lag. (A lagged response may be the only one possible when the monetary authorities, either to enhance their credibility or to satisfy externally imposed constraints, commit themselves to achieving money supply targets that are adjusted infrequently.)

Lagged and contemporaneous feedback rules can be equally effective in our model; both work through the real interest rate to make investment offset the goods market disturbance predicted on the basis of lagged information and the current interest rate. So, for example, when the variance of the disturbance to the goods market is relatively small, the monetary authorities seek to stabilize the real interest rate. Or, when the variance of the disturbance to the goods market is relatively large, the monetary authorities seek large offsetting changes in the real interest rate. Thus, our rational expectations cum contracting result is fundamentally the same as Poole's static expectations result.

However, lagged and contemporaneous feedback rules have

1. Sargent and Wallace [1975] show that monetary policy is redundant when private agents and the monetary authorities make rational use of the same lagged information.

2. Woglom [1979] derives a contemporaneous feedback rule under the assumption that wage setters do not use the interest rate. The question of whether investors use the interest rate to predict the inflation rate does not arise in Woglom's model because he assumes that investors base their decisions on the nominal rather than the real interest rate.

3. Recently, Weiss [1980], King [1983], and Waldo [1982] have demonstrated that a monetary policy based on lagged information can be effective when there are certain kinds of differences in information utilization among private agents. These authors find that lagged feedback rules can reduce the variance of output around its "natural rate" to zero. They obtain this result because they consider only one type of aggregate disturbance. Turnovsky [1980] analyzes a lagged feedback rule in a model with more than one aggregate disturbance. However, he too finds that the variance of output can be reduced to zero. He obtains this result even in the presence of more than one aggregate disturbance because he allows investors to have complete current information.
strikingly different implications for the nominal interest rate. An optimal contemporaneous feedback rule implies a low variance for the nominal interest rate when money market disturbances predominate. But an optimal lagged feedback rule generates a low variance for the nominal interest rate when goods market disturbances predominate. This sharp difference arises because under the optimal contemporaneous feedback rule changes in the real interest rate are caused solely by changes in the nominal interest rate, while under the optimal lagged feedback rule changes in the real interest rate are caused by changes in both the nominal interest rate and in investors' inflation expectations.

II. THE MODEL

The equations of the model are

\begin{align}
(1a) \quad y_t &= \theta(p_t - p_{t-1}^{t+1}), \\
(1b) \quad y_t &= -\delta[r_t - (p_{t+1}^{t+1} - p_{t-1}^{t+1})] + u_t, \\
(1c) \quad m_t - p_t &= \eta y_t - \lambda r_t + v_t,
\end{align}

where \(y, p,\) and \(m\) are (the deviations from trend values of the logarithms of) output, the price level, and the money supply; \(r\) is (the deviation from the trend value of) the nominal interest rate; and \(u\) and \(v\) are independent, normally distributed, and serially uncorrelated random variables with zero means and variances \(\sigma_u^2\) and \(\sigma_v^2\).

There is an important difference between our model and the usual rational expectations IS-LM model. In making their "rational" predictions of \(p_t\) and \(p_{t+1}\), some or all agents may use not only the information available at the end of period \(t-1\) but also the current interest rate \(r_t\).\(^5\) We denote predictions based on the lagged-only information set by \(p_{t-1}^{t} \) and \(p_{t+1}^{t} \) and predictions based on the

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4. See the Appendix for a discussion of how the nonstochastic trend values are determined.

5. We assume that all agents, official and private, have indirect utility functions that are quadratic in the price prediction errors for the prices that are unknown to them. Under this assumption agents maximize their expected utility by setting their price predictions equal to the conditional expectations of the unknown prices whether or not the disturbance terms, and therefore prices, are normally distributed. However, assuming normality makes the calculation of conditional expectations straightforward as shown, for example, in Meditch [1969, Ch. 3]. We also assume that agents' subjective expectations are identical to the mathematical expectations derived from the model. If it is assumed instead that agents' utility functions are general concave functions of price prediction errors, then the normality assumption is sufficient to insure that agents maximize their expected utility by choosing price predictions equal to their conditional expectations.
current-interest-rate-augmented information set by $p_{t\mid t-1}^+$ and $p_{t+1\mid t-1}^+$. The introduction of some additional notation allows us to represent all the cases we want to consider with equations (1). We denote predictions based either on one information set or on the other by $p_{t\mid t-1}^{(t)}$ and $p_{t+1\mid t-1}^{(t)}$. Because none, some, or all three sets of agents (wage setters, investors, and the monetary authorities) may use the interest rate, equations (1) actually subsume eight different cases.

Equation (1a) is an aggregate supply function that incorporates the "natural rate" hypothesis. Output depends positively on the difference between the price level and (the deviation from the trend value of the logarithm of) the nominal wage $w$:

\begin{equation}
(2) \quad y_t = \theta(p_t - w_t).
\end{equation}

Wage setters set the nominal wage so that the expected value of output, based on their information set, is equal to the trend (natural) rate of output:

\begin{equation}
(3) \quad y_{t\mid t-1}^{\uparrow} = 0 = \theta(p_{t\mid t-1}^{\uparrow} - w_t).
\end{equation}

Subtracting the right-hand equality in (3) from equation (2) yields the aggregate supply function (1a). Deviations of output from its natural rate are due to wage setters' price prediction errors.\(^6\)

Equation (1b) is the aggregate demand function; aggregate demand depends negatively on investors' expected real rate of interest. Some investors may also participate in the wage-setting decision. However, we assume that the investment decision may take place after the nominal wage is set. Thus, the price predictions that affect the investment decision may be based on the current interest rate when the price prediction that affects the wage decision is not.

Equation (1c) is the money market equilibrium condition. The demand for real balances depends positively on output and negatively on the nominal interest rate. As we explain in more detail below, the monetary authorities set the money supply on the basis of an information set that always includes all variables dated $t - 1$ and earlier and may in some cases include the current nominal interest rate as well. The monetary authorities have as their objective the minimization of the variance of output about its trend, which is equivalent

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6. We assume that there is no productivity disturbance (no disturbance in the aggregate supply function). Such a disturbance would not affect our redundancy results and would cloud the comparison of our results with those of Poole [1970]. Some implications of introducing a productivity disturbance are discussed in footnotes 7, 15, and 20.
to the minimization of the variance of wage setters' price prediction error.7

III. THE PRICE PREDICTION ERROR OF AGENTS WHO USE THE INTEREST RATE

In this section we isolate the information content of the current nominal interest rate for those agents, private or official, who use the interest rate in predicting the current price level. We employ a general approach to calculating conditional price predictions in models of the type employed here.8 This approach highlights the information filtering problem faced by agents with incomplete current aggregate information. It also facilitates the discussion in later sections of how this information can be optimally exploited by wage setters or the monetary authorities in their attempts to minimize wage setters' price prediction error.

We begin by noting that agents who use the current interest rate in making their price predictions need predict only the aggregate demand disturbance \( u_t \). This observation can be confirmed by inspecting the goods market equilibrium condition. Equating the right-hand sides of equations (1a) and (1b) and rearranging yields an expression for the equilibrium price:

\[
(4) \quad p_t = p_{t+1}^{t+1} - (\delta/\theta)[r_t - (p_{t+1}^{t+1} - p_t^{t+1})] + (1/\theta)u_t.
\]

The point is that agents who use the current interest rate \( r_t \) know or

7. The variance of output around its natural rate is a measure of the economic inefficiency that both the monetary authorities and wage setters are trying to eliminate. We use a "contracting" approach, like the one employed by Fischer [1977] and Gray [1976], in deriving the aggregate supply function (1a), but there are other ways to motivate this supply function; Taylor [1980] provides a good survey. We assume that the fundamental objective of both wage setters and the monetary authorities is minimization of the variance of employment around its full information value. This objective and the alternative objectives stated in the text are all equivalent if there is neither a productivity disturbance nor a labor supply disturbance. Under these assumptions, full information employment is always equal to trend employment, and there are one-to-one relationships between output and employment and between employment and the real wage. Gray [1976] shows how this equivalence breaks down when there is a productivity disturbance. In the model of the text, an interesting special case arises when there is a productivity disturbance and the labor supply schedule is both vertical and nonstochastic. Minimization of the variance of employment around its full information value is then equivalent to the stabilization of employment about its trend, and it is possible to show that both of these objectives imply the stabilization of nominal output.

8. Kareken, Muench, and Wallace [1973] and Leroy and Waud [1977] show how the monetary authorities should use this general approach to form conditional predictions of their target variable. In our model private agents as well as the monetary authorities follow this approach in calculating their conditional price predictions.
can calculate everything on the right-hand side of (4) except \( u_t \). They have enough information to calculate wage setters' predictions and investors' predictions whether or not those predictions are based on the current interest rate. The price prediction of agents who use the interest rate is

\[
(5) \quad p_{t+1|t-1}^+ = p_{t+1|t-1}^+ - (\delta/\theta)[r_t - (p_{t+1|t-1}^+ - p_{t+1|t-1}^+)] + (1/\theta)u_{t+1|t-1}^+,
\]

where \( u_{t+1|t-1}^+ \) is the expected value of \( u_t \) conditioned on lagged information and the current interest rate. Equation (5) is one of two key equations that are used repeatedly throughout this paper.

Next we determine what information about \( u_t \) agents can extract using their knowledge of \( r_t \). Movements in the current interest rate are induced by disturbances in both the goods and money markets. In fact, since our model is linear, \( r_t \) depends upon a linear combination of the disturbances in the goods and money markets. The information content of \( r_t \) is this linear combination of \( u_t \) and \( v_t \). Agents who use the interest rate can find this linear combination by using (1a) and (1b) to eliminate the unobservables \( y_t \) and \( p_t \) from (1c) and by rearranging to obtain

\[
(6) \quad (1 + \eta \theta)u_t + \theta v_t = \theta m_t + (\theta \lambda + \alpha) r_t - \alpha (p_{t+1|t-1}^+ - p_{t+1|t-1}^+) - \theta p_{t+1|t-1}^+,
\]

where \( \alpha = \delta(1 + \eta \theta) \). These agents can infer the linear combination of disturbances on the left-hand side of equation (6) because they either know or are capable of calculating everything on the right-hand side of the equation. In particular, they can calculate the money supply \( m_t \), since by assumption all the information available to the monetary authorities is exploited by agents who use the interest rate.9 Equation (6) is the second of two key equations that are used repeatedly below.

And how is the information content of the interest rate used to predict \( u_t \)? The expected value of \( u_t \) conditional on \( (1 + \eta \theta)u_t + \theta v_t \) is10

\[
(7) \quad u_{t+1|t-1}^+ = \gamma[(1 + \eta \theta)u_t + \theta v_t],
\]

\[
\gamma = \text{cov}[u_t, (1 + \eta \theta)u_t + \theta v_t]/\text{var}[(1 + \eta \theta)u_t + \theta v_t],
\]

9. In the text we implicitly assume that the monetary authorities set the money supply exactly. Suppose instead that the money supply process had a random component \( e_t \). Our analysis would be unaffected if \( m_t \) were regarded as the "systematic" part of monetary policy and \( e_t \) were incorporated into the money market disturbance \( v_t \) in equation (1c).

10. See Meditch [1969], Ch. 3.
\[ \gamma = (1 + \eta \theta) \sigma_u^2 / [(1 + \eta \theta)^2 \sigma_u^2 + \theta^2 \sigma_v^2]. \]

The price prediction error made by agents who use the interest rate is obtained by subtracting (5) from (4) to arrive at

\[ p_t - p_{t|t-1} = (1/\theta)(u_t - u_{t|t-1}). \]

Since the prediction of \( u_t \) given by (7) minimizes the variance of agents’ \( u_t \) prediction error, it also minimizes the agents’ price prediction error. Given (7), the price prediction error (8) can be rewritten as

\[ p_t - p_{t|t-1} = (1/\theta)[u_t - \gamma [(1 + \eta \theta) u_t + \theta u_{t|t}]]. \]

The variance of this price prediction error simplifies to

\[ \text{var}(p_t - p_{t|t-1}) = \sigma_u^2 \sigma_v^2 / [(1 + \eta \theta)^2 \sigma_u^2 + \theta^2 \sigma_v^2]. \]

Both the price prediction error and its variance are independent of the interest rate parameters \( \delta \) and \( \lambda \). This result is interesting because the interest rate is the current variable that agents observe. The parameters that do affect the price prediction error and its variance are the price level and output parameters \( \theta \) and \( \eta \); the price level and output are the variables that the agents do not directly observe. It should be noted that expressions (9) and (10) are valid whether or not wage setters or investors use the interest rate.

One further result is very important in what follows. Agents who use the interest rate never have a price prediction error variance greater than agents who use only information available at the end of period \( t - 1 \):

\[ \text{var}(p_t - p_{t|t-1}) \leq \text{var}(p_t - p_{t|t-1}). \]

Better informed agents can always achieve at least as low a price prediction error as less well-informed agents, since they are capable

11. The model of equations (1) does not subsume the case in which the aggregate demand function is given by

\[ y_t = -\delta [r_t - (p_{t|t-1} - p_t)] + u_t. \]

In (1b’) investors’ expected rate of inflation depends on \( p_t \) rather than \( p_{t|t-1} \). If (1b’) is used instead of (1b), the results are very similar to those reported throughout the text. The variance of the price prediction error of an agent who uses the interest rate would be somewhat lower with (1b’) than with (1b):

\[ \text{var}(p_t - p_{t|t-1}) = \sigma_u^2 \sigma_v^2 / [(1 + \eta \theta)^2 \sigma_u^2 + (\theta + \delta)^2 \sigma_v^2]. \]

This variance is lower because a given increase in \( p_t \) causes a larger reduction in the excess demand for goods, since it decreases demand as well as increasing supply. The variance of the price prediction error is lower even though \( \text{var}(u_t - u_{t|t-1}) \) is higher.
of calculating the price prediction of the less well-informed
agents.\textsuperscript{12}

IV. THE REDUNDANCY OF MONETARY POLICY WHEN
WAGE SETTERS USE THE INTEREST RATE

If wage setters use the interest rate, then equations (1) be-
come

\begin{align}
(12a) \quad y_t &= \theta(p_t - p^*_t|t-1), \\
(12b) \quad y_t &= -\delta[r_t - (p^*_t+1|t-1 - p^*_t|t-1)] + u_t, \\
(12c) \quad m_t - p_t &= \eta y_t - \lambda r_t + \nu_t.
\end{align}

When wage setters are agents who use the interest rate, any
monetary policy based on the interest rate is simply redundant.\textsuperscript{13} The
wage setters' price prediction error and its variance are given by
equations (9) and (10) in the preceding section. Both this price pre-
diction error and its variance are independent of monetary policy
because the information content of the interest rate, the linear com-
bination of $u_t$ and $\nu_t$ given by equation (6), is independent of mon-
etary policy based on the interest rate. Thus, we have shown that
monetary policy is redundant when wage setters and the monetary
authorities use the same incomplete current aggregate information.
This result is an extension of the Sargent and Wallace [1975] result
that monetary policy is redundant when all private agents and the
monetary authorities use the same lagged information.

Our result could be interpreted as implying that monetary policy
is "ineffective" when wage setters use the interest rate,\textsuperscript{14} but we prefer
to say that monetary policy is "redundant" here in order to emphasize
the symmetry between this case and cases in which wage setters do
not use the interest rate. For we shall demonstrate below that if the
monetary authorities exploit the information content of the interest
rate, then use of the interest rate by wage setters is redundant.

In the next two sections we consider two different monetary

\textsuperscript{12} A straightforward way to prove this result is to note that $p_t - p^*_t|t-1$ and $r_t$ 
- $r^*_t|t-1$ have a joint normal distribution. This is true because the underlying shocks
$u_t$ and $\nu_t$ are normally distributed, and the model is linear. The inequality (11) is a
property of joint normal distributions. See DeGroot [1975], p. 250.

\textsuperscript{13} It should be noted that this result holds even if investors use the interest rate.
In an otherwise very useful survey of rational expectations macroeconomic models,
McCallum [1980] states incorrectly that monetary policy can be effective in this
case.

\textsuperscript{14} McCallum [1980] and others have referred to the Sargent and Wallace [1975]
result as the "policy ineffectiveness result."
policies designed to take advantage of the information conveyed by the interest rate. Both of these policies succeed in making \( p_{t+1}^+ | t-1 \), the price prediction based on lagged information and the current interest rate, equal to \( p_t^+ | t-1 \), the prediction based only on lagged information. Both reduce \( \text{var}(p_t - p_{t+1}^+ | t-1) \) to \( \text{var}(p_t - p_{t+1}^+ | t-1) \), thereby making it unnecessary for wage setters to use the interest rate. In one case the monetary authorities can fulfill this role, even though they use information that is no more recent than the information used by wage setters and less recent than that used by investors.

V. OPTIMAL MONETARY POLICY WHEN THE MONETARY AUTHORITIES USE THE INTEREST RATE

In this section we show that if the monetary authorities use the interest rate, the optimal contemporaneous feedback rule is the same as the one derived by Poole [1970]. The monetary authorities’ optimal rule is the same whether or not investors use the interest rate. Monetary policy is not redundant because we assume that wage setters do not use the interest rate.

If wage setters do not use the interest rate, then equations (1) become

\[
\begin{align*}
(13a) & \quad y_t = \theta(p_t - p_{t+1}^+ | t-1), \\
(13b) & \quad y_t = -\delta[r_t - (p_{t+1}^+ | t-1 - p_{t+1}^+ | t-1)] + u_t, \\
(13c) & \quad m_t - p_t = \eta y_t - \lambda r_t + v_t.
\end{align*}
\]

The symbol (+) has been retained in equation (13b) to indicate that investors may or may not use the interest rate. Both cases can be analyzed, using equations (13).

Now, suppose that the monetary authorities adjust the current money supply in response to the incomplete current information conveyed by the current interest rate. We establish that they should

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15. In the presence of a productivity shock, the linear combination of disturbance terms observed by any agent who uses the interest rate would contain the productivity disturbance, and the minimum attainable variance for employment around its full information value would be different. However, monetary policy would still be redundant if wage setters used the interest rate and, under an optimal monetary policy, wage setters’ use of the interest rate would be redundant as long as either investors or the monetary authorities used the interest rate.

16. The case where investors do not use the interest rate is worth investigating mainly because it provides the closest analogy to Poole’s model, in which investors have static expectations. Sargent and Wallace [1975] assume that investors do not use the interest rate, but it is not clear that they would have retained this assumption had they allowed any other agents to use the interest rate.
use a rule of the form,

$$m_t = \beta (r_t - r_{t|t-1}),$$

and that $\beta$ should be chosen according to the principles developed by Poole [1970]. This monetary policy is optimal because it succeeds in lowering the price prediction error of wage setters down to the irreducible price prediction error of agents who use the interest rate.

It is convenient to begin by solving for some of the expectational variables. As a first step, it is shown that given a contemporaneous feedback rule of the form (14),

$$p_{t+1|t-1}^* = p_{t+1|t-1} = 0.$$  

To derive these results, it is necessary to obtain the difference equation that generates price predictions. Using (13a) and (13b) to eliminate $y_t$ and $r_t$ in (13c) leads to an expression for $p_t$:

$$p_t = \lambda (p_{t+1|t-1} - p_{t+1|t-1}) - (\eta + \lambda/\delta)\theta (p_t - p_{t|t-1})$$
$$+ m_t + (\lambda/\delta)u_t - v_t.$$  

Forwarding (16) by $j$ periods, taking expected values conditioned on $|_{t-1}$ information, and rearranging produces the required difference equation:

$$p_{t+j|t-1}^* = \left( \frac{\lambda}{1+\lambda} \right) p_{t+j+1|t-1}^* + \left( \frac{1}{1+\lambda} \right) m_{t+j|t-1}, \quad j \geq 1.$$  

Any monetary policy rule, such as (14), which sets $m_t$ in response to differences between period $t$ and period $t-1$ expectations, implies that

$$m_{t+j|t-1} = 0, \quad j \geq 1.$$  

After equation (18) is substituted into equation (17), the resulting difference equation can be solved by iterating forward starting at $j = 1$ to arrive at

$$p_{t+1|t-1} = \lim_{T \to \infty} \left( \frac{\lambda}{1+\lambda} \right)^{T-1} p_{t+j|t-1}^*.$$  

Ruling out “speculative bubbles” by assuming that $p_{t+j|t-1}$ is finite, yields (15).\(^{17}\)

\(^{17}\) In ruling out expected price (deviation) paths that increase or decrease without limit, we are following the suggestion of Sargent [1973]. Note that the arguments used in the text to establish the results in equations (15) and (20) are clearly valid if $\beta$ is finite. An expression for the optimal value of $\beta$, denoted by $\beta^*$, is given below equation (25). From that expression it is evident that the assumption that $\sigma_u^2 > 0$ is necessary and sufficient to ensure that $\beta^*$ is finite. In the Appendix we explain why the proofs of the results in equations (15) and (20) apply even in the extreme case in which $\beta^* \to \infty$ ($\sigma_u^2/\sigma_v^2 \to 0$).
As a second step, it is demonstrated that

$$p_t^{t-1} = r_t^{t-1} = 0.$$  

To derive these results, first take the expectations of equations (13) conditioned on lagged information. Then, simplify the three resulting equations by noting that equation (14) implies that $m_t^{t-1}$ equals zero and that $p_{t+1}^{t-1}$ equals zero from (15). Finally, solve the three simplified equations for $y_t^{t-1}$, $p_t^{t-1}$, and $r_t^{t-1}$. It turns out that all three of these variables must be equal to zero.

We are now ready to derive an optimal monetary policy. To do so, we use versions of the key equations (5) and (6) that reflect the assumptions of this section. These equations are obtained by taking the (+) symbol off of wage setters' predictions:

$$p_t^{t-1} = p_t^{t-1} - (\delta/\theta)[r_t - (p_{t+1}^{t-1} - p_t^{t-1})] + (1/\theta)u_t^{t-1},$$

$$(1 + \eta \theta)u_t + \theta v_t = \theta m_t + (\theta \lambda + \alpha)r_t$$

$$- \alpha(p_{t+1}^{t-1} - p_t^{t-1}) - \theta p_t^{t-1}.$$  

The question is this: can the monetary authorities choose a contemporaneous feedback rule of the form posited in (14) which insures that the optimality condition $p_{t+1}^{t-1} = p_t^{t-1}$ is satisfied?  
To see that they can, impose the optimality condition that $p_{t+1}^{t-1}$ be equal to $p_t^{t-1}$ which, in turn, equals zero from (20), and then eliminate $(1 + \eta \theta)u_t + \theta v_t$ using (7) and $p_{t+1}^{t-1}$ using (15). Equations (21) and (22) reduce to two optimality equations:

$$0 = -\delta r_t + u_t^{t-1},$$

$$(1/\gamma)u_t^{t-1} = \theta m_t + (\theta \lambda + \alpha)r_t.$$  

These optimality equations are satisfied if the monetary authorities set $m_t$ according to the rule,

$$m_t = [-\lambda + (\delta - \gamma \alpha)/\gamma \theta]r_t = \beta^* r_t,$$

$$\beta^* = -\lambda + [\delta \theta/(1 + \eta \theta)](\sigma_u^2/\sigma_x^2),$$

where use has been made of the definitions of $\alpha$ and $\gamma$ given below equations (6) and (7), respectively. The monetary rule of equation (25) is indeed of the form posited in equation (14), since $r_t^{t-1} = 0$ from (20).

Now we show that our results are consistent with those derived by Poole [1970] from a standard IS-LM model. To do this, we derive an optimal goods market equilibrium schedule and an optimal money
market equilibrium schedule that together can be employed to represent the equilibrium of our model in nominal interest rate-price level space. The optimal goods market equilibrium condition is obtained by equating the right-hand sides of equations (13a) and (13b), recognizing that \( p_{t+1}^{t+1} = 0 \) from (15), and noting that \( p_{t+1}^{t} = p_{t}^{t} = 0 \) from the optimality condition and (20):

(26) \[ \theta p_t = -\delta r_t + u_t. \]

The optimal goods market equilibrium schedule \( GG \) in Figure I is derived using equation (26) with \( u_t \) set equal to zero. The “modified” optimal money market equilibrium condition is obtained by substituting the right-hand side of equation (13a) for \( y_t \) in equation (13c), recognizing that \( m_t = \beta^* r_t \), and recalling that \( p_{t+1}^{t} = 0 \) from (20):

(27) \[ \beta^* r_t - p_t = \eta \theta p_t - \lambda r_t + v_t. \]

The optimal money market equilibrium schedule \( MM \) in Figure I is
derived using equation (27) with \( v_t \) set equal to zero. While the slope
of the optimal \( GG \) schedule is independent of \( \beta^* \), the slope of the optimal \( MM \) schedule,
\[
\frac{dr_t}{dp_t}_{MM} = \frac{1 + \eta \theta}{\beta^* + \lambda},
\]
depends on \( \beta^* \) and, therefore, on the relative sizes of the disturbances
to goods and money markets.\(^{19}\) It is easy to confirm that \( \beta^* \to \infty \) as
\( \sigma_o^2 / \sigma_u^2 \to \infty \) and that \( \beta^* \to - \lambda \) as \( \sigma_o^2 / \sigma_u^2 \to 0 \). Under our optimal
contemporaneous feedback rule—just as under Poole’s [1970] com-

When goods market disturbances are relatively small, it is opti-
mal to minimize the variance of the (investors’ expected) real interest
rate. Under the optimal contemporaneous feedback rule, the real
interest rate is equal to the nominal interest rate:
\[
\quad r_t = (p_{t+1|t-1}^+ - p_{t|t-1}^+) = r_t,
\]
as shown in the derivation of the optimal goods market equilibrium
(condition (26)). Thus, the optimal contemporaneous feedback rule
stabilizes the real interest rate by stabilizing the nominal interest rate.
However, when goods market disturbances are relatively large, the
optimal contemporaneous feedback rule induces offsetting changes
in the real interest rate even larger than those that would occur if the
money stock were held constant.

\(^{19}\) It might be thought that if investors use the interest rate, then the optimal
value of \( \beta \) would be different than if they did not, and the slope of the optimal goods
market equilibrium schedule would depend on this different optimal value. It is true
that for any value of \( \beta \) other than the \( \beta^* \) of equation (25), investors’ prediction of the
current price level \( p_{t|t-1}^+ \) is linked to the current interest rate, and the slope of the goods
market equilibrium schedule depends on \( \beta \). However, when the monetary authorities
set \( \beta \) equal to the \( \beta^* \) of equation (25), they fulfill the optimality condition that \( p_{t|t-1}^+ \)
always be equal to \( p_{t|t-1}^+ \) (which has the same constant value whether or not investors
use the interest rate). Therefore, they make \( p_{t|t-1}^+ \) independent of \( r_t \) and cause the slope
of the goods market equilibrium condition to be the same as it would be if investors
did not use the interest rate. It is worth noting that if the aggregate demand function
were given by equation (1b') in footnote 11, the slope of the goods market equilibrium
condition would be independent of \( \beta \) no matter what its value.

\(^{20}\) If there were a productivity disturbance \( \varepsilon \), only a minor change in the pro-
duced for finding the optimal contemporaneous feedback rule would be required. While
the resulting \( \beta^* \) would depend on the variance of the productivity disturbance \( \sigma_\varepsilon^2 \), as
well as on \( \sigma_v^2 \) and \( \sigma_o^2 \), and the parameters of the model, the results for the limiting cases
in which both elements of either \( \sigma_v^2 / \sigma_o^2 \) or \( \sigma_o^2 / \sigma_v^2 \) increase without limit
would be the same. It can also be shown that if money demand is unit elastic with re-
spect to real output, and if the labor supply curve is vertical and nonstochastic, then
\( \beta^* \) would have the same form as in (25) with \( \sigma_u^2 \) replaced by \( \sigma_u^2 + \sigma_v^2 \).
It is worth emphasizing that the optimal rule for monetary policy is the same whether or not investors use the interest rate in making their inflation predictions; the derivations in this section apply in either case. The explanation of this result is straightforward: the optimal contemporaneous feedback rule makes inflation predictions based on the current interest rate identical to those that are based only on lagged information. Under any contemporaneous feedback rule of the form of equation (14), $P^+_{t+1|t-1} = p_{t+1|t-1}$, and under the optimal one the monetary authorities, in satisfying their objective of minimizing wage setters’ price prediction errors, see to it that $P^+_{t|t-1} = p_{t|t-1}$ as well. So the optimal contemporaneous monetary feedback rule makes it unnecessary for investors as well as wage setters to use the interest rate; this is yet another redundancy result.

VI. OPTIMAL MONETARY POLICY WHEN ONLY INVESTORS USE THE INTEREST RATE

Here we assume that only investors use the interest rate and that the monetary authorities as well as wage setters base their decisions solely on lagged information. While the case in which the monetary authorities do not use the contemporaneous interest rate may seem improbable, it is quite relevant in institutional settings where the monetary authorities are committed to achieving medium-run monetary targets. A target range for a monetary aggregate limits, and a unique target value precludes, any contemporaneous adjustment to interest rate innovations. (Lagged adjustment is possible as long as the medium-run target is periodically reset.)

Here we show that even when the target value for the money supply is adjusted only periodically and one-period-ahead targets are hit exactly, monetary policy can still be effective as a stabilization tool. Indeed, in our model a lagged feedback rule for the monetary authorities works just as well as the contemporaneous feedback rule analyzed in the preceding section; it causes $P^+_{t|t-1}$ to equal $p_{t|t-1}$. Although this rule has the same implications for the expected real interest rate as the contemporaneous feedback rule, it has very different implications for the nominal interest rate.

21. In order to increase their credibility, the monetary authorities in several countries (for example, Canada, Germany, Switzerland, and the United States) have apparently made special efforts to achieve medium-run monetary targets at times during the last decade. The announcement of such targets and the periodic reporting on whether they are being attained are legal requirements in the United States. The reasons why several countries adopted monetary targets and descriptions of early experiences with monetary targeting are presented in a document by the Organization for Economic Cooperation and Development [1979].
If only investors use the interest rate, then equations (1) become

\[(30a)\]
\[y_t = \theta(p_t - p_{t|t-1}),\]

\[(30b)\]
\[y_t = -\delta[r_t - (p_{t+1|t-1} - p_{t|t-1})] + u_t,\]

\[(30c)\]
\[m_t - p_t = \eta y_t - \lambda r_t + v_t.\]

Now, suppose that the monetary authorities use a lagged feedback rule of the form,

\[(31)\]
\[m_t - m_{t-1} = \phi(r_{t-1} - r_{t-1|t-2}).\]

When equation (31) is forwarded by one period to obtain

\[(32)\]
\[m_{t+1} - m_t = \phi(r_t - r_{t|t-1}),\]

it becomes clear that this rule links the current growth rate of money directly to the current innovation in the interest rate. The lagged feedback rule can affect the variance of current output because it succeeds in relating investors' inflation prediction to the current interest rate. Under the optimal lagged feedback rule, it is the slope of the goods market equilibrium schedule rather than the slope of the money market equilibrium schedule that is made to depend on the relative sizes of the disturbances to the goods and money markets.

As in the preceding section we begin by solving for some of the expectational variables. As a first step, it is shown that

\[(33)\]
\[p_{t+1|t-1} = m_t + \phi(r_t - r_{t|t-1}) = m_{t+1}\]

for any value of \(\phi\). To do this, we follow the procedure employed to derive equation (15). Using (30a) and (30b) to eliminate \(y_t\) and \(r_t\) in (30c) and rearranging leads to an equation for \(p_t\):

\[(34)\]
\[p_t = \lambda(p_{t+1|t-1} - p_{t|t-1}) - (\eta + \lambda/\delta)\theta(p_t - p_{t|t-1})
+ m_t + (\lambda/\delta)u_t - v_t.\]

Forwarding equation (34) by \(j\) periods, taking expected values conditioned on \(p_{t-1}\) information, and rearranging yields

\[(35)\]
\[p_{t+j|t-1} = \left(\frac{\lambda}{1 + \lambda}\right) p_{t+1|t-1} + \left(\frac{1}{1 + \lambda}\right) m_{t+j|t-1}, \quad j \geq 1.\]

22. It is important to note that the lagged feedback rule analyzed here is a time-consistent policy. It is reasonable for investors to rely on the contingent promises of future money supply infusions because (in our model) the monetary authorities have no incentive to renege on these promises; the goals of the monetary authorities and the wage setters are the same. Fischer [1980] shows that some otherwise attractive policies may be time inconsistent when there is a conflict between the goals of the authorities and private agents.
The monetary policy rule (32) implies that

\[ m_{t+j|t-1} = m_t + \phi(r_t - r_{t|t-1}). \]  

After equation (36) is substituted into equation (35), the resulting difference equation can be solved by iterating forward beginning with \( j = 1 \) to arrive at

\[ p_{t+1|t-1} = \lim_{T \to \infty} \left( \frac{\lambda}{1 + \lambda} \right)^{T-1} p_{t+T|t-1} + m_t + \phi(r_t - r_{t|t-1}). \]  

Ruling out “speculative bubbles” yields the left-hand equality in (33). Of course, the right-hand equality follows from (32).

As a second step, it is noted that

\[ p_{t|t-1} = m_t \quad \text{and} \quad r_{t|t-1} = 0. \]  

To confirm these results, take expectations conditioned on \( |t - 1 \) information of equations (30). Then eliminate \( p_{t+1|t-1} \) and \( m_{t|t-1} \) using the relationships \( p_{t+1|t-1} = m_{t|t-1} = m_t \) implied by equations (33) and (31). The resulting equations can be solved for \( y_{t|t-1}, p_{t|t-1}, \) and \( r_{t|t-1} \) in terms of \( m_t \). The solutions for \( p_{t|t-1} \) and \( r_{t|t-1} \) are given by equations (38).

Now we are ready to prove that a lagged monetary feedback rule of the form (32) is optimal, since it causes \( p_{t|t-1} \) to equal \( p_{t|t-1} \). We follow the procedure employed in the preceding section to show that a contemporaneous feedback rule of the form we posited was optimal. Specializing the key equations (5) and (6) to the case in which only investors use the interest rate yields

\[ p_{t|t-1} = p_{t|t-1} - (\delta/\theta)[r_t - (p_{t+1|t-1} - p_{t|t-1})] + (1/\theta)u_{t|t-1}, \]  

\[ (1 + \eta \theta)u_t + \theta v_t = \theta m_t + (\theta \lambda + \alpha) r_t - \alpha (p_{t+1|t-1} - p_{t|t-1}) - \theta p_{t|t-1}. \]

Here the question is this: can the monetary authorities find a lagged feedback rule of the form posited in (32) which insures that the optimality condition \( p_{t|t-1} = p_{t|t-1} \) is satisfied? To see that they can, impose the optimality condition that \( p_{t|t-1} \) be equal to \( p_{t|t-1} \), which, in turn, equals \( m_t \) from (38) and then eliminate \( (1 + \eta \theta)u_t + \theta v_t \) using (7) and \( p_{t+1|t-1} \) using (33). Equations (39) and (40) reduce to two optimality equations:

\[ 0 = -\delta[r_t - (m_{t+1} - m_t)] + u_{t|t-1}, \]  

\[ (1/\gamma)u_{t|t-1} = (\theta \lambda + \alpha) r_t - \alpha (m_{t+1} - m_t). \]

These optimality equations are satisfied if the monetary authorities
set the money supply according to the lagged feedback rule,

\[ m_{t+1} - m_t = [1 + \theta \lambda \gamma / (\alpha \gamma - \delta)] r_t = \phi^* r_t, \]

\[ \phi^* = 1 - [\lambda (1 + \eta \theta) / \delta \theta] (\sigma_u^2 / \sigma_v^2), \]

where use has been made of the definitions of \( \alpha \) and \( \gamma \) given below equations (6) and (7), respectively. The monetary rule of equation (43) is indeed consistent with the rule posited in (32), since \( r_{t|t-1} = 0 \) from (38).

Now we show that under the optimal lagged feedback rule the slope of the goods market equilibrium schedule depends on the relative sizes of the disturbances to the goods and money markets while the slope of the money market equilibrium schedule is independent of the relative sizes of these disturbances. The optimal goods market equilibrium condition is obtained by equating the right-hand sides of equations (30a) and (30b) while recognizing that \( p^*_{t+1|t-1} = m_{t+1} \) from (33), that \( p^*_{t|t-1} = p_{t|t-1} = m_t \) from the optimality condition and (38), and that \( m_{t+1} - m_t = \phi^* r_t \):

\[ \theta (p_t - m_t) = -\delta (1 - \phi^*) r_t + u_t. \]

The optimal goods market equilibrium schedule \( GG \) in Figure II is derived using equation (44) with \( u_t = 0 \). The modified optimal money market equilibrium condition is obtained by substituting the right-hand side of (30a) for \( y_t \) in equation (30c) and recognizing that \( p_{t|t-1} = m_t \) from (38):

\[ m_t - p_t = \eta \theta (p_t - m_t) - \lambda r_t + u_t. \]

The optimal money market equilibrium schedule \( MM \) in Figure II is derived using equation (45) with \( u_t = 0 \). The slope of the optimal goods market equilibrium schedule is

\[ \left( \frac{dr_t}{dp_t} \right)_{GG} = -\frac{\theta}{\delta (1 - \phi^*)}, \]

and the slope of the optimal money market equilibrium schedule is independent of \( \phi^* \). It is easy to confirm that \( \phi^* \to 1 \) as \( \sigma_u^2 / \sigma_v^2 \to 0 \) and that \( \phi^* \to -\infty \) as \( \sigma_u^2 / \sigma_v^2 \to \infty \). Thus, if money market disturbances predominate, the optimal \( GG \) schedule is almost vertical, and if goods market disturbances predominate, the optimal \( GG \) schedule is almost horizontal.

The optimal lagged feedback rule has the same implications for the (investors’ expected) real interest rate as the optimal contemporaneous feedback rule. Under the optimal lagged feedback rule, the
real interest rate is given by
\begin{equation}
    r_t - (p_{t+1|t-1}^{\hat{t}} - p_{t|t-1}^{\hat{t}}) = (1 - \phi^*)r_t,
\end{equation}

as shown in the derivation of the optimal goods market equilibrium condition. When goods market disturbances are relatively small, the monetary authorities try to insure that the variance of real interest rates is small. They do this by setting $\phi^*$ near one so that nominal interest rate innovations induce nearly offsetting expected inflation rate innovations. When goods market disturbances are relatively large, the optimal lagged feedback rule generates large offsetting changes in the real interest rate by causing small nominal interest rate innovations to induce large reinforcing expected inflation rate innovations.

However, the optimal lagged feedback rule has implications for the nominal interest rate that are quite different from those of the optimal contemporaneous feedback rule. The reason for the difference
is that the lagged feedback rule operates to change the real interest rate by inducing changes in both the expected inflation rate and the nominal interest rate, while the contemporaneous feedback rule operates by inducing changes in the nominal interest rate alone. As a result, the lagged feedback rule yields a low (conditional) variance for the nominal interest rate when goods market disturbances predominate in contrast to the contemporaneous feedback rule that yields a low (conditional) variance for the nominal interest rate when money market shocks predominate.

An examination of the optimal modified money market equilibrium condition (45) suggests an explanation for why a low nominal interest rate variance makes it easier for wage setters to predict the price level when money market disturbances are small. Given a lagged money rule, wage setters know the current money supply before they set wages. Thus, if money market disturbances are small and if the monetary authorities can keep the variance of the nominal interest rate low, wage setters can use the money market equilibrium condition to make an accurate prediction of the current price level.

However, the optimal lagged feedback rule does not minimize nominal interest rate changes when money market disturbances are large. Under a lagged feedback rule the current nominal money supply is not allowed to change in response to a contemporaneous disturbance. Therefore, it is desirable to allow the nominal interest rate to change. Otherwise a large, unanticipated movement in the price level would be required to equilibrate the money market. The goods market remains insulated from the nominal interest rate change because the lagged rule induces an offsetting movement in the expected inflation rate. Of course, it is optimal to have the entire change in expected inflation take place through a change in investors' prediction of next period's price level, so that the investors' prediction of this period's price level coincides with the prediction of wage setters.

VII. CONCLUSIONS

Under rational expectations, private agents, as well as the monetary authorities, may make use of the incomplete current information conveyed by the current interest rate. We show how private agents solve the information filtering problem they face when forming their price expectations. In this environment the monetary authorities can insure that wage setters benefit from superior information used by other agents, either the authorities themselves or investors, thereby saving wage setters the costs of indexing.
Viewed one way, our results confirm those of Poole [1970]. The underlying mechanism through which monetary policy works is the same. Optimal monetary policy stabilizes the real interest rate when goods market disturbances are relatively small and amplifies offsetting movements in the real interest rate when goods market disturbances are relatively large.

Viewed another way, our results imply that Poole’s results must be qualified in an important way. If some private agents use more recent information than the monetary authorities, the practical policy prescription is just the opposite of Poole’s. When using a lagged feedback rule, the authorities should act so as to stabilize the nominal interest rate when goods market disturbances predominate. Only if no one uses more recent information than the monetary authorities is the practical policy prescription the same as Poole’s. When using a contemporaneous feedback rule, the authorities should stabilize the nominal interest rate when money market disturbances predominate.

**APPENDIX**

In the model of the text, we admit the possibility that the monetary authorities can fix \( r_t \) at \( r_{t|t-1}(\beta^* \to \infty) \), apparently ignoring the indeterminacy problem discussed by Sargent and Wallace [1975] and McCallum [1981]. Here we demonstrate that as long as nonstochastic trend monetary policy is sufficiently well specified so that the nonstochastic trend price level path is determinate, then the monetary authorities can fix \( r_t \) at \( r_{t|t-1} \).

The trend paths of the variables in the text must satisfy the following equations:

\[
\begin{align*}
(A1a) 
\hat{Y}_t &= k, \\
(A1b) 
\hat{Y}_t &= q - \delta [\hat{R}_t - (\bar{P}_{t+1} - \bar{P}_t)], \\
(A1c) 
\bar{M}_t - \bar{P}_t &= \eta \hat{Y}_t - \lambda \hat{R}_t,
\end{align*}
\]

where \( \hat{Y}_t, \bar{P}_t, \) and \( \bar{M}_t \) are the (logarithms of the) period \( t \) trend values of output, the price level, and the money supply, respectively; \( \hat{R}_t \) is the trend value of the nominal interest rate; \( k \) is the exogenous natural rate of output; and \( q \) is an intercept term in the aggregate demand function and \( q \geq k \). In order to simplify the analysis, we assume that \( k \) and \( q \) are constants, but similar conclusions could be derived if they varied over time.

It is easy to demonstrate the conventional indeterminacy result by analyzing the model of equations (A1) under the assumption that the monetary authorities announce only a constant trend nominal interest rate. Eliminating \( \hat{Y}_t \) from equations (A1b) and (A1c) using equation (A1a) and fixing the trend interest rate at \( \bar{R} \) reduces the
model of equations (A1) to

(A2a) \[ k = q - \delta \bar{R} - (\bar{P}_{t+1} - \bar{P}_t), \]

(A2b) \[ \bar{M}_t - \bar{P}_t = \eta k - \lambda \bar{R}. \]

Equation (A2a) is a first-order difference equation in the trend price level \( \bar{P}_t \) with a unit root. This equation is satisfied by any constant-inflation price path. Equation (A2b) does not imply any further restriction on the trend price path, since fixing the interest rate just sets trend real balances. Agents who try to calculate \( \bar{P}_t \) recognize that for any trend price level they choose, there is a corresponding trend money supply which is consistent with the fixed interest rate. Therefore, \( \bar{P}_t \) and \( \bar{M}_t \) are indeterminate for all \( t \).

To generate a determinate trend price path, the authorities must announce one value on a trend money supply path that is consistent with the constant trend nominal interest rate. Suppose that the authorities set the initial trend money supply \( \bar{M}_0 \) at a particular value. Then \( \bar{P}_0 \) is determined by (A2b). Suppose further that the authorities announce that the future path of the trend money supply will be the one implied by \( \bar{M}_0 \) and \( \bar{R} \). Agents can determine this path using equation (A2a):

(A3) \[ \bar{M}_{t+1} - \bar{M}_t = \bar{P}_{t+1} - \bar{P}_t = \bar{R} + (1/\delta)(k - q), \quad t \geq 0. \]

Thus, all the trend values of both the money supply and the price level are determined. Announcing a constant trend interest rate along with a consistent trend money supply path is a well-specified trend monetary policy, but simply announcing a constant trend interest rate is not.

As long as trend monetary policy is well specified, the model in Section V of the text, which is in terms of deviations from trend, is determinate even in the case where \( r_t \) is fixed at \( r_{t|t-1} = \beta^* \rightarrow \infty, \sigma_u/\sigma_v \rightarrow 0 \). To see this, note that the only steps in the proofs of equations (15) and (20) that are potentially affected as \( \beta \rightarrow \infty \) are where it is argued that \( m_{t+j|t-1}, j \geq 1 \), and \( m_{t|t-1} = 0 \). But these steps rely only on the fact that, for any contemporaneous feedback rule, all money supply innovations are temporary so that the money supply is always expected to return to its trend path. Therefore, as long as the trend path itself is determinate, as it will be under the well-specified monetary policy described in this Appendix, the proofs of Section V remain valid even as \( \beta^* \rightarrow \infty \).

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23. When \( r_t \) is fixed at \( r_{t|t-1} \), then the interest rate no longer embodies any current period information. A fixed interest rate policy is nevertheless optimal if the monetary authorities know that the money market is the sole source of disturbances \( \sigma_u^2/\sigma_r^2 = 0 \). Information on the magnitude of the money market disturbance would not affect the optimal policy of fixing \( r_t \) at \( r_{t|t-1} \).
REFERENCES


