

Revisiting Speculative Hyperinflations in Monetary Models: A Response to Cochrane*

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This paper responds to an appendix in Cochrane (2011) that characterizes as incorrect a central uniqueness result in Obstfeld and Rogoff (1983). Cochrane purports to show that, despite the existence of partial government backing of currency, there exist speculative price-bubble equilibria in which the price level goes to infinity and government money passes out of use, despite a money supply that itself is not increasing. Here we explain why our original result is, in fact, correct. Cochrane reaches a different conclusion only because he implicitly changes a critical assumption in the standard money-in-the-utility function model that we employed. We followed earlier literature in assuming that the individual utility derived from a given level of nominal balances depends on the behavior of others in the economy *only* through the aggregate price level. In particular, in that literature, utility from money services does *not* depend on the nominal money balances of others. In this note, we identify where Cochrane has changed the assumptions of our model, and explain why our original assumption is by far the more reasonable one. An interesting and broader point arises. The modern literature on the microfoundations of fiat money mostly fails to recognize that in practice, the government monopoly on currency issuance almost invariably rests on government-issued currency's being the unique legal tender.

We begin by putting our 1983 paper in context and then explaining the broader debate about speculative hyperinflations. We add that we are grateful to Cochrane for the challenge he poses to our original analysis, which helps bring out important aspects of the workings of the government currency monopoly.

Our 1983 paper, which built, in turn, on Brock (1974, 1975) and Calvo (1979), explored the conditions that could rule out speculative hyperinflations in micro-based models of money demand. Speculative hyperinflations have long been a subject of fascination for monetary economists: Is ultra-high inflation always and everywhere the result of excessive money growth, or can it happen even in a country that has stable monetary and fiscal policies?

We argued that a pure fiat regime, in which currency has absolutely no backing, is in fact vulnerable to the perversity of speculative hyperinflations. If the government provides a minimal exchange guarantee for the currency in terms of goods, however—in effect, a far-out-of-the-money put option—that is enough to rule out speculative hyperinflations. Indeed, as Obstfeld and Rogoff (1986) show, even a very small probability of very small contingent backstop is sufficient. Cochrane’s appendix argues, in sharp contrast, that such guarantees are *not* sufficient. We show here, however, that the candidate hyperinflationary equilibria Cochrane advances cannot survive the simple recognition that governments invariably decree their currencies to be legal tender, in which case a contingent fractional redemption guarantee assures it will always have transaction value.¹

Before proceeding, we should note that we agree with one central premise of Cochrane’s main text: Multiple-equilibrium problems are endemic to conventional monetary models of inflation. In fact, one cannot even exclude multiple *dynamically stable* equilibria without imposing arbitrary restrictions on individual utility functions or on the derived utility from real money balances (for example, see Obstfeld 1984). Thus, the appendix we are commenting on here is not, as such, really central to Cochrane’s main point.

Nevertheless, the question whether one can rule out dynamically *unstable* equilibria, while comparatively more narrow, is still a central one that has long fascinated economists. Conventional wisdom today is that explosive inflation (as in Weimar Germany or in Zimbabwe during this century) is *prima facie* evidence of wildly excessive government currency issuance. As far as we know, this has invariably been the case historically (see Reinhart and Rogoff 2009, chapter 12, for example). But because a fiat money’s value inherently de-

¹Generally, legal tender is a medium of payment that the legal system recognizes as final, in the sense that it cancels debt (including importantly taxes) when offered in repayment. Thus, every dollar issued by the United States promises that “This note is legal tender for all debts, public and private.” The legal tender assumption is implicit in virtually every model of government-monopoly currency, for example, the money-in-the-utility-function model that Obstfeld and Rogoff (1983) and the first part of this paper use. An even stronger version of the legal tender assumption is implicit in the widely-used cash-in-advance model, where the purchase of consumption goods requires the government-issued currency.

depends on the willingness of others to accept it, standard monetary models allow for a range of Pareto-dominated equilibria in which real money balances converge to zero—what Brock aptly referred to as “expectations pollution.”

In a canonical speculative hyperinflation, prices need to keep rising at an ever-increasing rate to maintain equilibrium when the currency supply itself is not growing explosively. Obstfeld and Rogoff (1983) argue that a modest fractional redemption guarantee makes this entire process unravel by placing an upper bound on the price level, while leaving no way for agents to replenish their nominal money balances should prices settle at that upper bound.² Once agents realize there is such a ceiling, no matter how high, the price level bubble can never get going in the first place. Our analysis demonstrating this result used a “money-in-the-utility-function” framework for modeling money demand, but analogous results hold in other models as, for example, Wallace (1981) showed independently in a “hybrid fiat-commodity money” model with overlapping generations.³ In sum, Obstfeld and Rogoff (1983) and Wallace (1981) both conclude that under apparently quite weak government interventions, purely speculative hyperinflations simply cannot be equilibria. Although Cochrane considers only the Obstfeld and Rogoff analysis, we will outline both approaches here to clarify the common underlying logic. For a comprehensive analysis of determinacy in modern monetary models, the reader can refer to Atkeson, Chari, and Kehoe (2010).⁴

So what, then, is there to debate? In an appendix to his 2011 paper, Cochrane suggests

²If the government promises to buy *and sell* money at the same limit price—equivalent to issuing a call option as well as a put option for each unit of currency at the same strike price—then price-level determinacy is lost. This result is well known from the literature on transitions from floating to fixed exchange rates, for example, Froot and Obstfeld (1991).

³Wallace’s term “hybrid” refers to the government’s posting of a low buying price for money in terms of goods and a high selling price for money in terms of goods—effectively attaching to currency a call as well as a put option, but with different strike prices. The presence of a ceiling as well as a floor on money’s value trivially precludes hyperdeflationary equilibria where $P \rightarrow 0$ as well as hyperinflationary equilibria where $P \rightarrow \infty$. In Obstfeld and Rogoff (1986) we concluded that reasonable preference restrictions suffice to rule out speculative hyperdeflationary equilibria (that is, those not driven by a falling money supply)—in sharp contrast to the case of hyperinflationary equilibria. Buiter and Sibert (2007) reaffirm this result.

⁴Atkeson, Chari, and Kehoe (2010) do not incorporate a legal tender assumption as we do here, but in a pure fiat money regime, a legal tender requirement does not *by itself* place any ceiling on the price level.

that the supposed barrier provided by a hybrid support scheme is in fact a Maginot line. He claims that in the Obstfeld-Rogoff model, there are still equilibria in which the economy drifts off into a speculative hyperinflation that ends abruptly with a demonetized economy. In the final period of the monetary economy, all individuals trade in their cash to the government for real output at the minimal guarantee price. Cochrane argues that, provided the demonetized situation is an equilibrium, the usual money Euler condition, which would otherwise be violated going forward (since the price level is capped) is not relevant in the period before demonetization. Instead, he constructs an alternative Euler condition that accounts for the consumption value of selling money to the government in the final monetized period of the equilibrium. Superficially, his analysis seems to parallel the argument of Obstfeld and Rogoff (1983) that, absent any backing, there are speculative equilibria where money simply becomes worthless on a finitely distant future date.

As we discuss here, however, there is an obvious, but nonetheless critical, difference between the cases of pure fiat money and hybrid fiat-commodity money: If people can always trade in money for some amount of output, no matter how minuscule, the price of money can never literally be zero, and hence, the price level cannot be infinite, as Cochrane's conjectured steady-state nonmonetary equilibrium supposes. If so, it is hard to contend that people will nonetheless be happy to do without money, which will always have some real trade-in value thanks to the government guarantee, and ongoing transactions value in *private markets* as well, should even a small number of individuals remain willing to accept money (at some nonzero price) in exchange for commodities. But as long as the government stands by currency as legal tender, this surely would be the case. Taking into account the two forms of government support (a fractional backstop and legal tender), it immediately follows that trading in all her currency will violate the individual's first-order condition, even if all other agents trade in their currency. In short, Cochrane's suggested outcome is not an equilibrium,

and thus our original result stands.⁵ It is important to note that in the simplistic money-in-the-utility-function approach of our model (which Cochrane also employs), an individual’s derived benefit from holding real balances depends on the behavior of others only through the aggregate price level and does not depend, say, on what percent of the population holds a positive level of money. We argue, however, that in any realistic model of money the same kind of hybrid scheme that we assume will still rule out speculative hyperinflationary equilibria in which the price level explodes even though the money supply does not.

This note proceeds by first reprising our argument that credible fractional backing of currency can foreclose the possibility of hyperinflationary equilibria. We then briefly review Cochrane’s critique, stressing the point at which we believe his analysis goes wrong. A final example, based on a simplified version of the setup in Wallace (1981), amplifies our point in a setting where the demand for money is motivated quite differently than in our heuristic money-in-the-utility-function model. We conclude with some reflections on the continuing inadequacy of theories of “money,” and how this long-standing gap in economic theory bedevils all attempts to pin down rigorously equilibria with pure fiat money (though we again emphasize that our analysis here focuses solely on conventional monetary models of inflation, and does not in any way address the fiscal theory of the price level that Cochrane advocates in the text of his paper).

1 Speculative Hyperinflations and Partial Backing: Reprise

We will use the slightly simpler version of the model in Obstfeld and Rogoff (1983, 1986), also employed in Cochrane (2011), which excludes physical capital. Importantly, in this setup the

⁵In private correspondence, V.V. Chari and Patrick Kehoe show that even without the (fairly compelling) legal-tender assumption, Cochrane’s hyperinflationary path still fails to describe an equilibrium as long as the government still injects some positive amount of money into the economy every period.

government can always make good any commitment to redeem money partially by levying lump-sum taxes. Here we also make explicit the assumption that the government-issued currency is legal tender.

Individuals receive y units of the perishable consumption good each period. Let c_t denote an individual's consumption rate at time t , M_t her nominal money holdings, and $\beta \in (0, 1)$ her subjective discount factor. The infinitely lived representative agent maximizes

$$U_0 = \sum_{t=0}^{\infty} \beta^t [u(c_t) + v(M_t/P_t)], \quad (1)$$

where P_t is the price level at time t , subject to

$$M_t - M_{t-1} = P_t(y - c_t) + H_t, \quad M_{-1} \text{ given,}$$

with H_t denoting transfers from the government that the individual takes to be exogenous. We assume that $u(\cdot)$ and $v(\cdot)$ are increasing and strictly concave, with the usual smoothness and Inada properties. The above assumption of separability between the utility from consumption and the derived utility from holding cash balances is quite important, as Obstfeld and Rogoff (1983, 1986) emphasize: Otherwise, even though there may not exist divergent speculative paths, there can still be multiple stable equilibrium paths converging to the same monetary equilibrium (as noted above). The Euler equation characterizing individual optimization is

$$\frac{u'(c_t)}{P_t} = \frac{v'(M_t/P_t)}{P_t} + \beta \frac{u'(c_{t+1})}{P_{t+1}}. \quad (2)$$

Defining real balances as $m_t = M_t/P_t$, and assuming purely for simplicity that the money supply is constant at M , the last equation above can be rewritten as the simple difference equation

$$\beta u'(y)m_{t+1} = m_t[u'(y) - v'(m_t)], \quad (3)$$

where we have imposed the market-clearing condition $c_t = y$. Following the diagrammatic technique of Brock (1974, 1975), we define $A(m) = m[u'(y) - v'(m)]$ and $B(m) = \beta u'(y)m$, in which case the equilibrium can be illustrated graphically as in Figure 1, replicated below from Obstfeld and Rogoff (1983, Figure 2).⁶

Figure 1 illustrates how, unless the price level starts out exactly at the unique stationary equilibrium level \bar{P} , then the price will either implode (if it starts out below the equilibrium level), or grow exponentially (if it starts out above the stationary equilibrium level). As we showed, in a pure fiat money system, it is not possible to rule out explosive bubbles where $m_{T+1} = 0$ (that is, where the price level reaches infinity at time $T + 1$) if the prior path of real balances $\{m_t\}_{t=0}^T$ ends at the point $m_T = \tilde{m}$ where

$$u'(y) = v'(\tilde{m}). \tag{4}$$

Even though money becomes worthless at $T + 1$, it still gives just enough marginal benefit at time T to compensate for the (small) loss of consumption the individual must forgo to hold on to it, which is why P_T can remain finite even when everyone knows $P_{T+1} = \infty$ is imminent. For future reference, note that in the case of pure fiat money, the assumption that currency is legal tender does not—in and of itself—prevent the public from ultimately valuing it at zero. In a pure fiat money regime, speculative hyperinflations cannot be ruled out just because agents are obligated to accept currency for debt repayment, if there is no simultaneous ceiling on the price level.

We went on to show, however, that speculative price bubbles can be ruled out if the government gives even a very small backing ϵ to the currency, sufficient to cap the price level

⁶Figure 1 assumes that

$$\lim_{m \rightarrow 0} mv'(m) = 0,$$

so that the utility level from money balances is bounded from below even as real balances become very small (Obstfeld and Rogoff 1983, Theorem 1). We argued that the case where $v(m)$ becomes unboundedly negative as $m \rightarrow 0$ is implausible.

at $\bar{P} = 1/\epsilon < \bar{P}$ through the simple arbitrage argument that money can never trade at a price below the value at which the government is willing to redeem it. (In fact, both the probability the government will repurchase the currency and the value of the backing can be uncertain.) It is immediately obvious from Figure 1 that such a price ceiling (a floor on the value of money) implies that all the aforementioned equilibrium hyperinflationary paths must unravel backward, because the price level cannot go to infinity as the equilibrium paths would require.

2 Cochrane's Critique

Cochrane (2011) argues that even if there is a ceiling \bar{P} on the price level (a floor $1/\bar{P}$ on the price of money), there will still be an equilibrium path leading to a steady state with $P = \infty$ and with $M = 0$. He contemplates the possibility of a date- T equilibrium in which all individuals trade their entire money balances to the government at the support price $1/\epsilon$, leaving the economy demonetized thereafter. He writes: “Here is how the hyperinflationary equilibrium actually ends, with the buyback guarantee in place: $P_{T+1} = \infty$ ” (Cochrane 2011, p. 613).

To support this ultimate equilibrium, Cochrane argues that an individual deciding money demand at the start of penultimate period T will balance the marginal utility of consuming another \$1 against the marginal flow utility from, instead, adding \$1 to money balances over period T plus consuming the \$1 at the end of T by trading it to the government at the redemption price $1/\bar{P}$. This possibility gives a terminal Euler equation for period T that

differs from the one set out above:⁷

$$\frac{u'(c_T)}{P_T} = \frac{v'(M/P_T)}{P_T} + \frac{u'(c_T)}{\bar{P}}. \quad (5)$$

Cochrane (p. 613) concludes that in the equilibrium with $c_T = y$, the price level just prior to the government's absorption of the money supply will be slightly below \bar{P} , and defined implicitly by

$$P_T = \left[1 - \frac{v'(M/P_T)}{u'(y)} \right] \bar{P} < \bar{P}.$$

Alas, this line of argument, albeit clever, falls afoul of one fundamental difficulty: The last Euler equation requires money to have no value on date $T + 1$, that is, $P_{T+1} = \infty$. But the government's promise to redeem money remains good on date $T + 1$. Any individual who deviates from the proposed equilibrium and instead carries \$1 into period $T + 1$, will be able to sell it on the market to other agents at any real price less than or equal to $1/\bar{P} = \epsilon$, because they, in turn, can then sell the \$1 to the government for ϵ in output. This simple arbitrage argument implies that the market price of money on date $T + 1$ will not be zero, it will be ϵ , and so the price level will be \bar{P} , not ∞ .

In this case, we need to ask if people would in fact all turn in their money at the end of T , and collectively revert thereafter to barter, when they know the price level will continue to be finite. More precisely, we need to ask if people will believe that $v(m) \equiv 0$, and that money will no longer have transaction value no matter *what* the price level is, making the first order condition involving $v'(m)$ collapse, so that individuals might as well turn in all their money in period T . One could perhaps argue that starting at time $T + 1$, everyone suddenly expects that no one will accept currency any longer, even though the government fully backs the currency and even though everyone was accepting currency a period before.

⁷Implicit in Cochrane's argument is that trade between the private sector and the government can take place at a different time than trade within the private sector, but we don't need to worry here about whether this assumption is reasonable, it is not a central issue.

Can't we then jump to a new equilibrium in which money is useless for transaction purposes simply because everyone shares the belief that it is? This strikes us as highly implausible, and for three reasons:

1. The modern theoretical literature on money seems to omit a basic point: Governments that invoke their right of a monopoly over paper currency issuance invariably make their money legal tender. Whereas this requirement can have different meanings in different countries (in the United States and the United Kingdom, legal tender carries the rather narrow meaning that currency must be accepted for repayment of any debt; see Rogoff, 2017), it is hard to conceive of a situation in which currency would not automatically inherit acceptability for goods transactions (for example, through settlement of credit card bills).
2. As Lerner (1947) and Brock and Scheinkman (1980) emphasize, governments can require that money be used to pay taxes (or more generally, simply accept it to discharge tax liabilities), always imparting transaction value as long as the price level is capped.
3. All the infrastructure for using cash is not going to disappear overnight, nor will the custom of using cash, even if the cash becomes convertible into a commodity rather than remaining as fiat currency. If one explores the history of currency (for example, the literature discussed in Rogoff 2017, chapter 2), one finds that the use of currency is surprisingly robust given its seeming fragility in our theoretical models.

So critically, currency will always retain transaction value, the standard first order condition will apply, and the candidate demonetized equilibrium fails; even a very weak government should be able to enforce such conditions, and has a huge incentive to do so.⁸

Is there a chance that once the government redeems money for output in period T , it has then exhausted its ability to redeem money in future periods—in the manner of a central

⁸If the price level were to hit \bar{P} and remain there, with people trading in some of their money for goods,

bank that exhausts its reserves defending a fixed exchange rate (Krugman 1979). That is not the case here—the government’s taxing capacity always allows it to redeem money at a sufficiently low price, so why would it withdraw its guarantee after a single period of testing?⁹

It is instructive to revisit the rhetorical argument Cochrane (2011, pp. 610–11) advances to support his analysis: “How could offering one kernel of corn for a billion dollars destroy an equilibrium? Given that people were holding money at T that they knew would be worthless at $T + 1$, why would a tiny residual value make any difference? It doesn’t.” The argument is seductive, but what the formal analysis is really saying is that equilibrium speculative hyperinflations are spectacularly fragile, and require very little effort to resist, which is why governments are invariably successful in doing so. Again, none of this contradicts the core argument of the text of Cochrane’s paper, because conventional models are vulnerable to multiple *stable* equilibria.

then the candidate Euler equation that would need to hold in the new steady state would be

$$(1 - \beta) \frac{u'(y)}{\bar{P}} = \frac{v'(M^*/\bar{P})}{\bar{P}}$$

where $M^* \leq M$. But the previously defined stationary price level \bar{P} satisfies

$$(1 - \beta) \frac{u'(y)}{\bar{P}} = \frac{v'(M^*/\bar{P})}{\bar{P}},$$

and so the candidate Euler equation can hold only if $\bar{\bar{P}} \leq \bar{P}$, contrary to our assumption about the redemption price for money.

⁹Cochrane (2011, p. 613) conjectures that his results may differ from ours because of a confusion (on our part) between discrete- and continuous-time modeling. He states: “The central problem is Obstfeld and Rogoff’s “arbitrage” condition (685) that $\bar{\bar{P}} = P_T$ in any period that people are tendering money. That argument is not valid in this discrete-time model because people can get $v(m)$ plus the redemption value. This arbitrage argument would be valid in a continuous-time version of the model, and perhaps the error comes from mixing correct continuous-time intuition with a discrete-time model.” As one can see from the analysis of this section and the next, the question of discrete versus continuous time, as usual, is irrelevant for the substantive economic conclusions.

3 An Alternative Model

The preceding money-in-the-utility-function framework is a crude shorthand for a much richer multi-good model in which the transactions value of money is derived from its ability to solve the problem of “double coincidence of wants” on the part of inherently heterogeneous market actors who might be unable fully to realize the available multilateral gains from trade without using a commonly accepted medium of exchange. Reasonably interpreting the model as capturing a richer underlying multi-good model with heterogeneous agents underscores the implausibility of a sudden rejection of a widely used—and backed—currency for no reason whatsoever.

This point is quite clear in Wallace’s (1981) overlapping-generations model of money. While arguably unrealistic as a complete model of money demand, it does illustrate rigorously how potential gains from monetary trade between heterogeneous agents can underpin the demand for money and lead some individuals to deviate from supposedly nonmonetary equilibria provided the government gives some backing to the currency.¹⁰ Here, we develop a simple example based on Gale (1973) and Brock and Scheinkman (1980).¹¹

In this example a generation lives for two periods, receiving an endowment w^y when young and $w^o < w^y$ when old and maximizing

$$U_t = u(c_t^y) + u(c_{t+1}^o) \tag{6}$$

subject to the constraints

$$M_t = P_t(w^y - c_t^y) = P_{t+1}(c_{t+1}^o - w^o), \quad M_t \geq 0, \tag{7}$$

¹⁰ As is well known, under perfect certainty, monetary equilibrium in this model might not survive the introduction of dominating assets such as capital.

¹¹ Sims (2013), also in an overlapping-generations framework, develops the related idea that monetary equilibrium becomes unique if the government each period levies taxes to repurchase some money (in contrast to the offer of a free put option that we model here).

where M_t is the money that a member of the generation born on date t (generation t) carries into its old age in period $t + 1$. The utility function $u(c)$ is, as usual, increasing and strictly concave. The assumption that $w^y > w^o$ creates an incentive for the young to acquire money balances so as to smooth their consumption over time—an incentive that can be offset by a sufficiently high expected rate of price-level inflation.

On the tentative assumption that the nonnegativity constraint on money balances will in fact never bind, the intertemporal Euler equation for an individual who is born on date t will be

$$\frac{1}{P_t}u'(c_t^y) = \frac{1}{P_{t+1}}u'(c_{t+1}^o). \quad (8)$$

On the assumption of a fixed aggregate money supply M and again defining aggregate real money balances on date t as $m_t \equiv M/P_t$, equilibrium paths satisfy the difference equation

$$A(m_t) \equiv m_t u'(w^y - m_t) = u'(w^o + m_{t+1}) m_{t+1} \equiv B(m_{t+1}). \quad (9)$$

There is a steady-state positive level of real balances \bar{m} that satisfies $A(\bar{m}) = B(\bar{m})$, and therefore a finite steady-state price level given by $\bar{P} = M/\bar{m}$. Figure 2 illustrates the determination of this Pareto-optimal steady state, but also shows there are other, inefficient equilibria (for example, the speculative hyperinflationary path starting at $m = m_0$) such that money asymptotically becomes worthless. The intuition is the same as in the Brock model.¹² However, in parallel to the Brock model, a government promise to redeem money

¹²For simplicity, the figure shows the simple special case in which $u(c) = \ln(c)$, but our main points carry over more generally. In the logarithmic case, $\bar{m} = \frac{1}{2}(w^y - w^o)$. The equilibrium of this economy when there is no money is inefficient. The government can raise every generation's welfare, however, by endowing the initial old with the stock of money M , which they immediately sell to the initial young for output, allowing the initial young to do the same next period when they are old and have lower income. The result will be efficient if the price level settles immediately at \bar{P} and stays there forever. But as our diagram shows, this need not happen if the market is left to find the equilibrium on its own—and this is where the government's partial backing of money is helpful. For a lucid discussion of the welfare economics of the overlapping-generations model, see Weil (2008). Figure 2 (the main features of which are valid more generally) makes clear that the nonnegativity constraint on money holdings will never bind in any equilibrium. The figure also shows that speculative hyperdeflations cannot arise in equilibrium (because the real money balances

for a small amount ϵ of goods effectively caps the price level at $\overline{P} = 1/\epsilon$, and this fact therefore rules out all paths but the steady-state path, because those paths are supported only by the self-fulfilling expectation of an ever-increasing path of prices.

An important nuance here is that absent any backing, there is a second steady-state equilibrium other than the monetary equilibrium \overline{m} , in which money is rejected instantly and entirely. In the nonmonetary equilibrium, people do not use money simply because no one else uses money—everyone thinks money is useless and will be forever, and so it is. Here, $P = \infty$ (permanently) and $m = 0$, so money is irrelevant: each generation is restricted to consuming its own current endowment. Instant rejection of money is a legitimate equilibrium, albeit an inefficient one. A date- t young person might *wish* for a way to transfer some savings into old age, but would never pay a positive price for money on date t if the money will be worthless on date $t + 1$. In other words, no individual young person would deviate from a date- t equilibrium with $P_t = \infty$ so long as she knows that $P_{t+1} = \infty$. And the date- $(t + 1)$ young will act the same, knowing that $P_{t+2} = \infty$, and so on, *ad infinitum*.

Things are different if the government guarantees for all dates a small redemption value for money, ϵ . To see why, let's ask if there is a Cochrane-like equilibrium where on some date t , the old suddenly tender their money to the government for backing at the price $1/\overline{P} = \epsilon$ and the price level then jumps permanently to $P = \infty$.

The easy answer is no. Any individual young person can dissuade an old person from exchanging their money with the government by offering to pay them instead the price $\epsilon + \eta$, where $\eta > 0$ is arbitrarily small. Because the autarky (nonmonetary) consumption levels over the young person's lifetime would satisfy $u'(w^o) > u'(w^y)$, she can raise her utility by purchasing money at $\epsilon + \eta$ from an old person and selling it at ϵ to the government later when she, herself, is old—provided η is small enough. The alleged equilibrium therefore collapses, just as the analogous one collapses in the Brock model.

preferred by the old can never exceed the endowment of the young).

4 Concluding Remarks

As Hahn (1965) argued over a half century ago, the absence of a rigorous and realistic theory of money opens up the possibility of multiplicities such as the nonmonetary equilibrium, and this is a continuing discomfort for macroeconomics. However, credible contingent redemption of the currency does rule out at least one possible type of indeterminacy: speculative hyperinflations unrelated to fundamentals, and it goes too far to say that this case suffers from the same indeterminacy as unbacked fiat money. Thus, if we observe a hyperinflation, we can be reasonably sure it has its roots in fundamentals and not in pure speculative frenzy. The results in our 1983 paper clearly still stand, Cochrane’s (2011) appendix comment notwithstanding. Our result is extremely robust and his result is extremely fragile, especially taking account of currency’s legal-tender status.

Yet again, we note that our result does not in any way contradict the main point in the text of Cochrane’s article, that conventional monetary models are replete with multiple equilibrium problems. Nonetheless, the truly striking thing about monetary equilibria with government-issued fiat money is that, contrary to the tenuousness predicted by our theoretical models, they seem to be remarkably stable and robust in reality. Undermining them seems to require extreme monetary malpractice. Monetary hyperinflations depend on large-scale government resort to monetary finance of deficits—a fiscal theory, not of the price level in general, but of its instability. And in such circumstances, there are always broader questions about more pervasive institutional breakdown—as Lerner (1947) put it, money is a “creature of the state.” While it is an exaggeration to argue that the literature on money has made no progress at all, there clearly remains a difficult puzzle to be solved.

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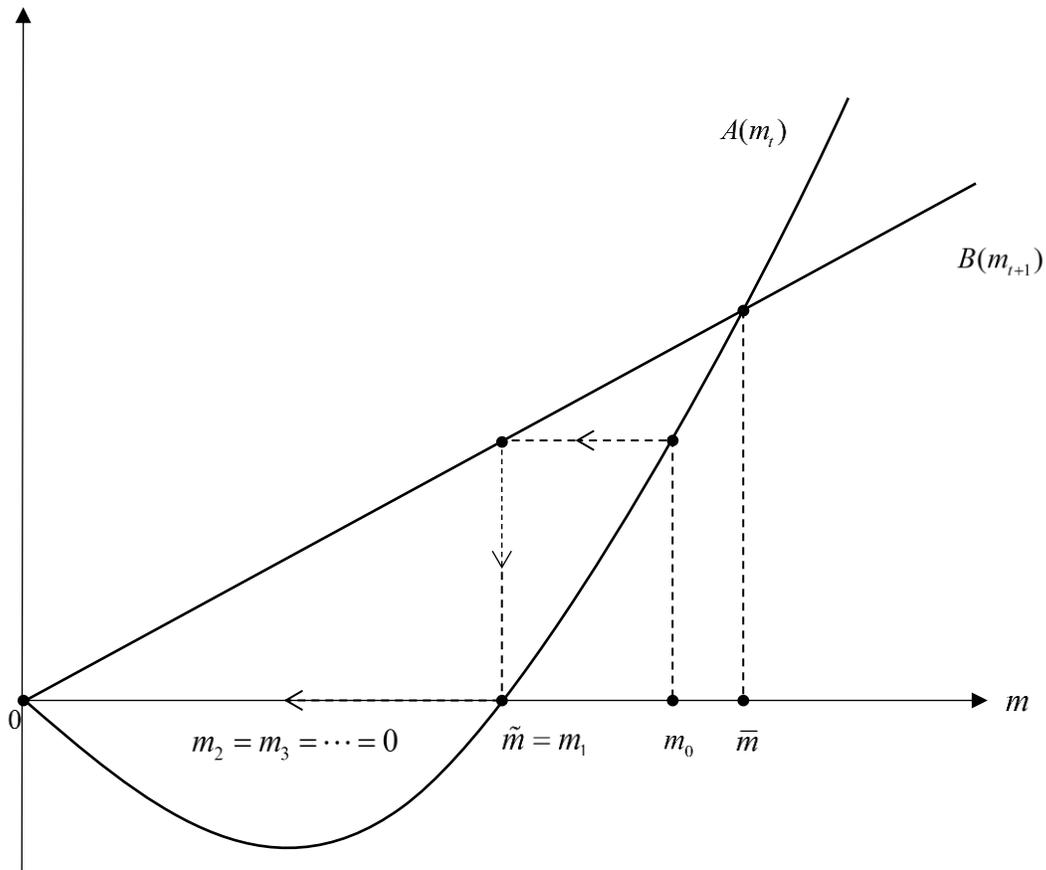


Figure 1: Speculative hyperinflation in the Brock representative-agent model

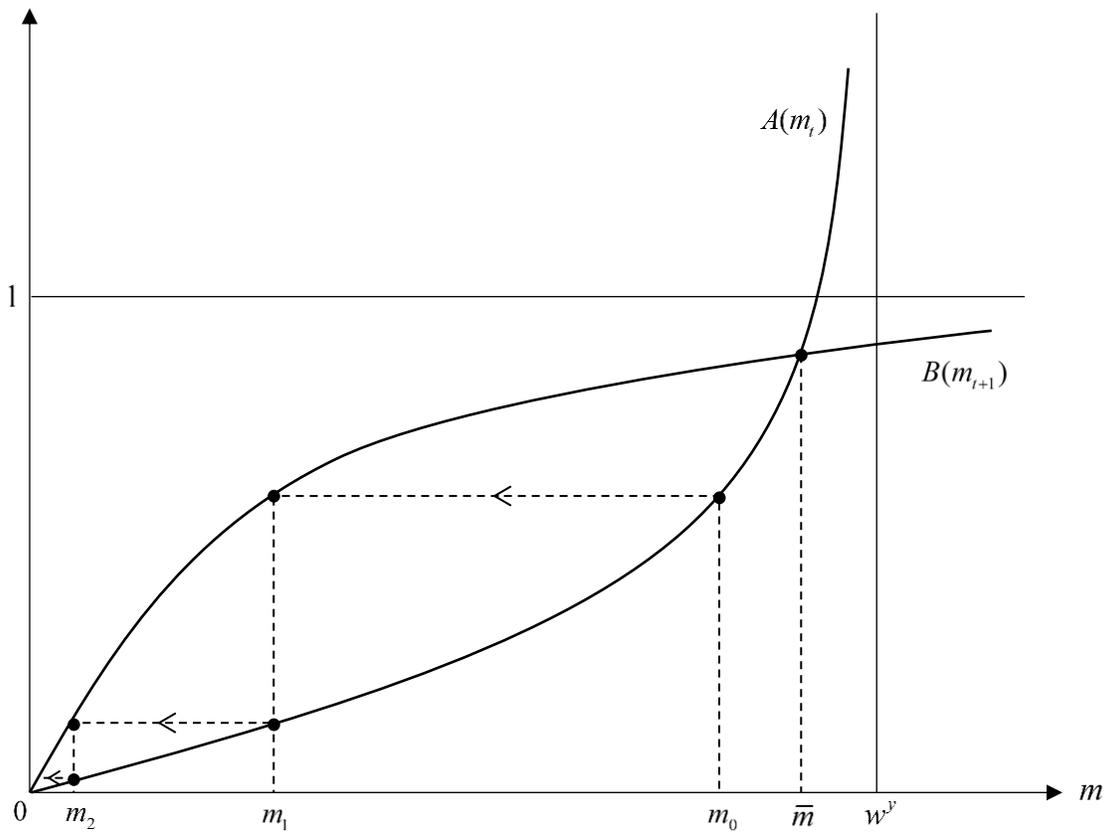


Figure 2: Speculative hyperinflation in an overlapping-generations model