The staffs of the Board of Governors of the Federal Reserve System and of the Federal Reserve Banks undertake studies that cover a wide range of economic and financial subjects. From time to time the results of studies that are of general interest to the professions and to others are published as Staff Studies.

Staff Studies 126–135 are a set of related papers on intervention in foreign exchange markets. Versions of parts of these papers were submitted to the Working Group on Exchange Market Intervention, which was established at the Versailles Summit in June 1982 to carry out an international study of experience with intervention. A summary of these papers will appear in the Federal Reserve Bulletin for November 1983.

The author of the following study has sole responsibility for its content, including interpretation and changes that have been made from the earlier submission to the Working Group. The analyses and conclusions set forth do not necessarily reflect the views of the Board of Governors of the Federal Reserve System, the Federal Reserve Banks, or other members of their staffs.
Nonstructural time-series techniques can capture statistical regularities in the strength and timing of relationships among exchange rates, intervention, and a small number of related variables. Moreover, they may be applied to data of any frequency. This versatility makes it worth considering whether nonstructural time-series analysis can be used to examine the impacts of intervention in the very short run. Of course, the term "nonstructural" is something of a misnomer. Although little economic structure is imposed in estimating time-series models, interpreting the results invariably requires making structural assumptions.

Despite the recent popularity of time-series analysis in empirical macroeconomics, only a few economists have applied this technique to exchange rates, and fewer still have used it to shed light on whether sterilized intervention is effective. There do exist two studies of the 1952–60 Canadian experience with floating rates, and one study of intervention by the United States, the United Kingdom, West Germany, and Japan during the period 1973:4 to 1980:4; these studies are discussed later in this paper. Both studies of the Canadian float are based on daily data; the authors reach opposite conclusions on whether intervention (represented by changes in the gold and U.S. dollar position of the Canadian Exchange Fund) helped stabilize the exchange rate of the Canadian dollar against the U.S. dollar. One possible problem with these two studies is that they do not include a variable to capture the separate effects of monetary policy. (Whenever monetary policy and intervention policy are correlated, failure to take account of the effects of the money supply on the exchange rate can bias results even when all intervention is sterilized.)

The third study, based on U.S., U.K., German, and Japanese data for the 1970s, does include the money supply as a separate variable. Although the framework of this study is potentially useful for examining the effects of intervention, the researcher chose instead to concentrate on confirming that central banks lean against the wind.

Because very little literature exists on the subject, much of this survey focuses on explaining the present popularity of nonstructural techniques, especially vector autoregressions, and on highlighting some of the uses and limitations of these techniques in studying the effects of intervention. For example, nonstructural analysis might be useful for testing whether intervention has any effect at all. The structural assumptions required to obtain such evidence are extremely weak. But obtaining evidence on the magnitude, timing, or even direction of the effects of intervention is much more difficult. The contemporaneous correlation of sterilized intervention, the money supply, and exchange rates confounds interpretation of the vector autoregression coefficients. In general, it does not make sense for a researcher to use vector autoregression techniques simply because he or she believes that structural modeling imposes assumptions (identifying restrictions) that are difficult to justify, as economic interpretation of the estimated coefficients of a vector autoregression requires making the same sort of assumptions. (See the appendixes for a more detailed discussion.)

**Popularity of Nonstructural Modeling**

Much of the current popularity of nonstructural time-series analysis may be viewed as a reaction to the poor forecasting performance of structural econometric models during the 1970s.¹ Proponents of nonstructural methods

¹ Given the enormous volatility of exchange rates during the floating-rate period, it is hardly surprising that 1970s-vintage structural models did not forecast exchange rates well. Meese and Rogoff find that a naive random-walk model, which predicts that all future exchange rates will be the same as today's exchange rate, forecasts as well as several popular, small structural exchange rate models at one-month to twelve-month horizons. Remarkably, the small structural models do no better than the random-walk model even when their forecasts are based on actual realized values of the explanatory variables (relative money supplies, interest rates, current accounts, and so forth). Meese and Rogoff find that time-series methods, such as the vector autoregressions discussed in the present paper, do just as badly. See Richard Meese and Kenneth Rogoff, "The Out-of-Sample Failure of Em-

NOTE: Appendix B was written by Robert Flood. The author wishes to thank Stephanie Sampson for editing the manuscript.
argue that while there has been much progress in solving some of the major problems with 1970s-vintage structural models—particularly in the modeling of private agents' expectations—1980s-vintage models have yet to prove themselves.² There are reasons to be skeptical, time-series advocates argue, especially in light of the "identification" problems that still plague structural modeling.³ A structural model of many equations is not identified if it is insufficiently articulated to sort out the simultaneous effects, which can confound and bias coefficient estimates.⁴

Although most empirical models are nominally identified, critics claim that identification is often based on questionable assumptions. For example, variables are excluded from some equations but not others without good theoretical justification; a priori knowledge of the serial-correlation properties of the unknown error terms is assumed; and policy variables are treated as exogenous (independent) when they are often endogenous. (Intervention is endogenous if the central bank intervenes in response to current developments in the exchange rate.) Sims argues that these identifying restrictions are so "incredible," and the estimates of the structural coefficients are therefore so badly biased, that structural and nonstructural modeling differ very little.

So instead of analyzing empirical coefficient estimates polluted by "incredible identifying restrictions," Sims recommends looking at unrestricted time-series correlations and using them to compare the relative plausibility of different theories.⁵ The next section examines Sims's preferred nonstructural technique, vector autoregressions.


Fair looks at macroeconomic variables, such as output and prices, and finds that the structural models often perform better than the time-series models. See Ray C. Fair, "Analysis of the Accuracy of Four Macroeconometric Models," Journal of Political Economy, vol. 87 (August 1979), pp. 701–18.

Litterman, however, finds that, by employing his Bayesian algorithm for reducing the number of parameters estimated in vector autoregressions, the time-series models forecast macroeconomic variables as well as the major commercial forecasting services do. See Robert B. Litterman, "Techniques of Forecasting Using Vector Autoregressions," Working Paper 115 (Federal Reserve Bank of Minneapolis, 1979; processed).

2. Cooley and LeRoy suggest that the poor forecasting performance of structural models might have been expected given the "specification search" algorithm employed by many econometricians. By reporting only the best results, econometricians sometimes exaggerate how closely their theory corresponds to the data. See Thomas F. Cooley and Stephen F. LeRoy, "Identification and Estimation of Money Demand," American Economic Review, vol. 71 (December 1981), pp. 825–44.


4. Identifying simultaneous-equation systems is necessary if, for example, the econometrician wants to be sure he is estimating a demand curve and not a supply curve. In early work in this area, researchers suggested that economic theory could provide the exclusion restrictions that are sufficient to identify systems of simultaneous equations. For example, while both the supply of and demand for coffee depend on price, the theorist might specify that the supply of coffee should depend on the weather in Brazil, but world demand for coffee should not. See Tjalling C. Koopmans, ed., Statistical Inference in Dynamic Econometric Models, Cowles Commission for Research in Economics, Monograph 10 (John Wiley, 1950); and Wm. C. Hood and Tjalling C. Koopmans, eds., Studies in Econometric Method, Cowles Commission for Research in Economics, Monograph 14 (John Wiley, 1953).

Modern expectations theory has slightly modified this view of identification. Because coffee can be stored, the demand for coffee in the world may depend on expectations about the temperature in Brazil. Identification is still obtainable by noting that the temperature in Brazil should only affect the demand for coffee through price expectations; see Sims, "Macroeconomics and Reality."


Vector Autoregressions

The vector autoregression methodology Sims uses is a representative and widely used multivariate, nonstructural time-series technique. In a vector autoregression, no variable is treated a priori as exogenous with respect to any other variable in the model. The list of possible explanatory variables is the same in every equation, and it includes lagged values of the left-hand variable together with lagged values of all the other variables in the model. For example, the equation for the exchange rate $S$ (dollars per units of foreign currency) might be specified as follows:

\[(1) \quad S_t = a_{11}S_{t-1} + a_{12}S_{t-2} + \ldots + a_{1n}S_{t-n} + a_{21}I_{t-1} + a_{22}I_{t-2} + \ldots + a_{2n}I_{t-n} + a_{31}M_{t-1} + a_{32}M_{t-2} + \ldots + a_{3n}M_{t-n} + a_{41}Z_{t-1} + a_{42}Z_{t-2} + \ldots + a_{4n}Z_{t-n} + \varepsilon_t,\]

where $I$ is intervention (for example, a simultaneous sale by the central bank of interest-bearing assets denominated in dollars and purchase of interest-bearing assets denominated in foreign currency); $M$ is the money supply of the foreign country; and $Z$ is a vector of other variables in the model such as interest rates, the U.S. money supply, and so forth. Subscripts on the variables denote calendar time; lowercase letters represent coefficients; $\varepsilon_t$ is an error term.

The vector autoregression model includes similar equations for $I$, $M$, and $Z$. The unconstrained vector autoregression system may be thought of as subsuming a wide class of structural models, each of which implies a different set of theoretical coefficient constraints. For example, a structural model in which intervention is ineffective would include among its constraints that the coefficients $a_{21}$, $a_{22}$, $\ldots$, $a_{2n}$ on intervention $I$ in equation 1 all be zero. Sims argues that by estimating a vector autoregression system with many parameters, it will often be possible to identify the most plausible set of restrictions on the coefficients of a structural model.

However, the vector autoregression implicitly imposes some constraints of its own. Given a finite data set, the maximum lag length cannot be set so long as to exhaust all degrees of freedom. Similarly, the number of variables included in the equation has to be limited to attain computational tractability. Nevertheless, one has to be careful in excluding variables. If important variables are omitted, the researcher will generally have difficulty drawing economic inferences from the vector autoregression coefficients. For example, if the researcher wants to study daily data but does not have daily observations on the monetary base, then he or she may find that the estimated intervention coefficients confound the effects of sterilized intervention and monetary policy.

Some further technical comments on the vector autoregression equation 1 are in order. The variables are assumed to be adjusted for trends and seasonality, when necessary, in a mutually consistent fashion; otherwise trend or seasonal terms may have to be included in equation 1. Other transformations of the data, such as taking logarithms or differencing, may be required as well. Some important problems may arise here: the researcher must take care not to alter causal relationships between the variables. (Similar problems arise in structural modeling.) Also, the choice of a linear specifi-

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7. There are many multivariate time-series techniques that a priori model some variables as exogenous. Such techniques are not ideal for studying the effects of intervention on exchange rates because intervention often responds to changes in exchange rates.


10. If data on the monetary base are unavailable on a daily basis, it may be possible to use daily interest rates as a coarse proxy.
cation for equation 1 is dictated by convenience rather than by economic theory. When theory strongly suggests that the relationships among the variables are nonlinear, equation 1 may provide only a poor approximation. 11

Although only the past values of the variables appear in equation 1, the vector autoregression can, in principle, subsume structural models in which forward-looking expectation mechanisms are very important. As long as there is no change in policy regime, expectations of the future values of each variable can be represented in terms of present and past values of some or all the variables in the system. Of course, in the face of a sharp change in policy regime, expectations of future events may have little relation to past events, and vector autoregression methods may not work very well. 12

One might ask why contemporaneous values of all the variables do not appear on the right-hand side of equation 1. The vector autoregression may be thought of as a convenient normalization, whereby contemporaneous correlations between the variables are captured in the covariance matrix of the disturbance terms. However, contemporaneous correlation can make economic interpretation of the vector autoregression extremely difficult (see appendix A). This problem is likely to hamper analysis of the effects of intervention: intervention often depends in part on contemporaneous changes in exchange rates, and these changes may depend on contemporaneous intervention. Appendix B demonstrates a robust method for testing whether sterilized intervention has a nonzero "portfolio balance" effect. But this methodology does not allow one to rule out the possibility that the effect is very small or to say anything about the timing of the effects.

### Bivariate Time-Series Studies of Exchange Rates

Most of the early international macroeconomic studies of nonstructural, multivariate time-series techniques examine only two variables at a time. Money and income, or money and prices, is the pair of variables most commonly analyzed. 13 The exchange rate regime enters these analyses only indirectly as a rationalization for differing results across countries or across exchange rate regimes. 14 A few other studies focus on testing aspects of the mone-

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12. Ibid. Sims makes the highly controversial argument that regimes seldom change as sharply or as permanently as they sometimes appear to and that a vector autoregression can adequately describe many fairly long time periods. See Sims, "Policy Analysis."


14. Some studies found that, under fixed exchange rates, the U.S. money supply was essentially independent of income, but in other countries money and income were mutually dependent. These results were rationalized on the basis of the dollar being the reserve currency. Other studies tested, with mixed results, the proposition that the money supply is more likely to be an independent variable under freely floating exchange rates than under fixed exchange rates.

tary approach to the balance of payments and on testing relationships among interest rates.\textsuperscript{15} But only two studies deal with the effectiveness of exchange market intervention.

One of these two is a study by Pippinger and Phillips in which the authors construct a model of the foreign exchange market and the operations of the Canadian Exchange Fund.\textsuperscript{16} They use "cross-spectral" bivariate time-series analysis of daily data to examine whether official intervention contributed to the stability of Canadian exchange rates.\textsuperscript{17} They conclude that

the analysis implies that official intervention reduced the variance of daily changes in exchange rates by 40 to 45 percent. . . . [and] that the Canadian authorities were able to let market forces essentially determine intermediate and long-run movements in exchange rates while at the same time playing a very important role in the short-run stability of the Canadian dollar from 1952 to 1960.\textsuperscript{18}

However, Pippinger and Phillips are not concerned with the distinction between sterilized and nonsterilized intervention. Because they do not choose to control for current or expected changes in the money supply in their analysis, one cannot interpret their results as clear evidence that sterilized intervention is effective. Even if one somehow knew that all intervention over the period 1952–60 were sterilized, one would still face the problem that the exchange rate effects of the omitted monetary policy variable could be picked up by the sterilized intervention variable whenever the two were correlated. Such a correlation might arise if, for example, a government directed its monetary policy and intervention policy at related macroeconomic targets, or if it used monetary policy after intervention proved inadequate.

Another reason for caution in interpreting the Pippinger and Phillips estimates of the stabilizing effects of intervention is the authors' assumption that, in the absence of intervention, all shocks to the exchange rate are permanent (so that the exchange rate follows a "random walk"). They do not allow for the possibility that some of the damping in exchange rate movements over time might be attributable to the actions of private agents.\textsuperscript{19} Pippinger and Phillips claim that the random-walk model is consistent with the "efficient markets" hypothesis. But this claim is true only if the interest rate differential between Canada and the United States always compensates exactly for exchange rate risk and differences in default risk.

In a study based on the data Pippinger and Phillips use, Sweeney relaxes the random-walk assumption and admits the possibility that exchange rate changes could be damped by forces other than intervention.\textsuperscript{20} When Sweeney adds lagged exchange rates to the lagged intervention in his exchange rate equation, he finds that intervention no longer enters significantly. Unfortunately, Sweeney's results cannot be considered decisive, as the time-series techniques he uses are open to the criticisms raised in appendix A.

\begin{footnotesize}
\begin{itemize}
\item[17.] Granger criticizes cross-spectral techniques as providing only simple correlations which have no causal interpretation; see C.W.J. Granger, "Investigating Causal Relations by Econometric Models and Cross-Spectral Methods," \textit{Econometrica}, vol. 37 (July 1969), pp. 424–38.
\item[18.] Hause also points out that the use of phase statistics and cross-spectral techniques to make inferences about lead-lag relationships can be misleading; see John C. Hause, "Spectral Analysis and the Detection of Lead-Lag Relations," \textit{American Economic Review}, vol. 61 (March 1971), pp. 213–17.
\end{itemize}
\end{footnotesize}
Multivariate Time-Series Studies of Exchange Rates

Recently, several economists have applied vector autoregression techniques to macroeconomic systems of three or more variables. Friedman studies the question of which credit aggregate the U.S. monetary authorities should attempt to stabilize.\(^{21}\) Johannes and Leiderman test certain assumptions about exogeneity that underlie applications of the monetary approach to the balance of payments.\(^{22}\) Fischer looks at relative price variability and inflation.\(^{23}\) Sims demonstrates how vector autoregressions might be helpful in determining the relative plausibility of different policy scenarios.\(^{24}\)

A small number of researchers have applied vector autoregression analysis to floating exchange rates.\(^{25}\) Genberg, Saidi, and Swoboda study systems involving interest and exchange rates of the United States, the United Kingdom, West Germany, and Canada.\(^{26}\) Meese and Rogoff present vector autoregression results as part of a study of small models of exchange rate determination.\(^{27}\)

Of all these analyses, the work of Branson is potentially the most informative if one is exclusively interested in intervention.\(^{28}\) He uses quarterly data from the International Monetary Fund on exchange rates, relative prices, home-country money (he tries both M1 and M3), short-term interest rates, reserves, and the current account for the period 1973–80. Branson estimates separate six-variable vector autoregressions for the United States, the United Kingdom, Germany, and Japan. This framework makes it possible to at least begin to analyze the relationship between exchange rates and intervention while holding the money supply constant. Yet Branson does not include every relevant variable in his framework. For example, he omits the foreign money supply and the intervention activities of the foreign monetary authorities.

Branson chooses to present only part of his empirical results—the contemporaneous correlation matrix of the error terms from the estimated vector autoregression equations. He presents neither the estimated vector autoregression lag coefficients nor any other information on the dynamic properties of his estimated model. He argues that the observed negative correlation between exchange rate movements and reserve levels (intervention), evident to varying degrees for all four countries, can be explained only by intervention policies of leaning against the wind. (In other words, the central bank attempts to damp movements in the exchange rate.) If intervention were not endogenous, and if it did influence the exchange rate, one might expect the correlation between exchange rate movements and reserve levels to be positive rather than negative. However, despite the fact that Branson finds a negative correlation, it is still possible that intervention affects the exchange rate as the theory predicts it will (see appendix A).

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\(^{23}\) Fischer's paper contains an objective exposition of the vector autoregression methodology; he highlights the difficulties in simulating shocks when contemporaneous correlation is important; see Fischer, "Relative Shocks." Taylor's comments on the costs and benefits of using non-structural analysis are worthwhile as well, see John B. Taylor, "Comment" on paper by Stanley Fischer, Brookings Papers on Economic Activity, vol. 2:1981, pp. 434–38.


\(^{25}\) Sims, "Policy Analysis."

\(^{26}\) Hans Genberg, Nassar Saidi, and Alexander K. Swoboda, "American and European Interest Rates and

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\(^{27}\) Meese and Rogoff, "The Out-of-Sample Failure of Empirical Exchange Rate Models."

\(^{28}\) William H. Branson, "Exchange Rate Policy after a Decade of 'Floating'" (Princeton University, 1982; processed).
Appendix A: Problems in Interpreting Vector Autoregressions

This appendix illustrates some of the problems in interpreting vector autoregressions when contemporaneous correlation is important. The structural model of equations A.1 and A.2 below is highly stylized:

(A.1) \[ S_t = b_1 S_{t-1} + c_{11} S_{t-1} + c_{12} I_{t-1} + \eta^1_t \]

(A.2) \[ I_t = b_2 S_t + c_{21} S_{t-1} + c_{22} I_{t-1} + \eta^2_t, \]

where \( S \) the rate of change of the spot exchange rate (positive \( S \) is an appreciation of the foreign currency against the dollar); \( I \) is intervention (positive \( I \) is an intervention in support of the foreign currency against the dollar); and \( \eta^1 \) and \( \eta^2 \) are uncorrelated white-noise disturbance terms. To make this bivariate example tractable, assume that the time period being examined is one in which (1) all intervention is sterilized; and (2) sterilized intervention is uncorrelated with money supply changes at all leads and lags. Also assume that \( S \) is adjusted for trends to remove the effects of stable monetary growth and that only the foreign monetary authorities intervene.

Equation A.1 determines the rate of change of the spot exchange rate. Assume for illustrative purposes that this behavioral equation for the private sector is structural in the sense that its form does not change when the intervention reaction function, equation A.2, changes. It would be useful for the foreign monetary authorities to know the coefficients of the exchange rate equation A.1, so that they can determine their optimal intervention reaction function. But, in this example, they cannot simply estimate A.1 because the coefficient estimates would be biased. Because intervention was endogenous over the historical sample period, identification of equations A.1 and A.2 is not possible.

Suppose, given the assumed lack of satisfactory identifying restrictions, the researcher decides to study the data by estimating a vector autoregression. The vector autoregression reduced-form representation of equations A.1 and A.2 is given by equations A.3 and A.4 below:

(A.3) \[ S_t = d_{11} S_{t-1} + d_{12} I_{t-1} + \epsilon^1_t \]

(A.4) \[ I_t = d_{21} S_{t-1} + d_{22} I_{t-1} + \epsilon^2_t, \]

where

(A.5) \[ d_{11} = (c_{11} + b_1 c_{21})/(1 - b_1 b_2) \]

(A.6) \[ d_{12} = (c_{12} + b_1 c_{22})/(1 - b_1 b_2) \]

(A.7) \[ d_{21} = (c_{21} + b_2 c_{11})/(1 - b_1 b_2) \]

(A.8) \[ d_{22} = (c_{22} + b_2 c_{12})/(1 - b_1 b_2) \]

(A.9) \[ \epsilon^1_t = (\eta^1_t + b_1 \eta^2_t)/(1 - b_1 b_2) \]

(A.10) \[ \epsilon^2_t = (\eta^2_t + b_2 \eta^1_t)/(1 - b_1 b_2). \]

If the contemporaneous coefficients \( b_1 \) and \( b_2 \) in the underlying structural model of equations A.1 and A.2 are nonzero, then, by A.9 and A.10, the error terms in the reduced-form vector autoregression equations A.3 and A.4 will in general be correlated. ²⁹

Suppose, having estimated equations A.3 and A.4, the researcher wants to determine the magnitude of the effects of contemporaneous and lagged intervention on the exchange rate (that is, the size of the coefficients \( b_1 \) and \( c_{12} \) in equation A.1). One piece of information may be available if the researcher is willing to assume that \( b_2 < 0 \). (This is simply the assumption that the foreign central bank adopts a policy of leaning against the wind.) Then, if the estimated correlation between the vector autoregression error terms \( \epsilon^1 \) and \( \epsilon^2 \) turns out to be positive, \( b_1 \) cannot be zero and must, in fact, also be positive.

One may verify this assertion by inspecting equations A.9 and A.10. The finding that \( b_1 \) is positive implies that intervention has an immediate (same-day) stabilizing effect on the exchange rate. Again this conclusion depends on two key assumptions: intervention is always sterilized, and monetary policy and intervention policy are uncorrelated.

The empirical finding that the correlation between \( \epsilon^1 \) and \( \epsilon^2 \) is positive provides some evidence that intervention is effective; however-

²⁹. Because equations A.3 and A.4 include identical right-hand side explanatory variables, ordinary least-squares estimation, equation by equation, is an efficient technique even though the error terms in the two equations are correlated.
er, it is not true that a negative correlation between \(e^1\) and \(e^2\) provides evidence that intervention is ineffective. Again inspecting equations A.9 and A.10, it is clear that, as long as the monetary authorities lean against the wind \((b_2 < 0)\), the correlation between \(e^1\) and \(e^2\) can be negative even if \(b_1 > 0\).

If one is willing to assume that \(b_1\) is nonnegative, then the estimated vector autoregression coefficients may contain another useful piece of information. In particular, if \(d_{12}\) (the coefficient on lagged intervention in the vector autoregression equation for the exchange rate) is significantly different from zero, then by equation A.6 either \(b_1\) or \(c_{12}\), or both, must be nonzero. Thus either contemporaneous or lagged intervention, or both, must affect the exchange rate in the underlying structural equation A.1.\(^{30}\) The explicit assumption that \(b_1\) and \(b_2\) are of opposite signs rules out the possibility that \((1 - b_1 b_2) \approx 0\). Otherwise, by equation A.6, \(d_{12}\) can be very large even when neither contemporaneous nor lagged intervention has a big effect on the exchange rate \((b_1\) and \(c_{12}\) are small).

Note also from equation A.6 that the assumptions made here on the signs of \(b_1\) and \(b_2\) do not rule out the possibility that \(d_{12}\) is small when \(b_1\) and \(c_{12}\) are large. For example, if the impact of intervention on the level of the exchange rate were partly reversed the next day, \(b_1\) and \(c_{12}\) would have opposite signs. When \(b_1\) and \(c_{12}\) are both positive, \(d_{12}\) can still be small if \(c_{22}\) is negative. A negative \(c_{22}\) means that, ceteris paribus (given today’s and yesterday’s rate of change in the exchange rate), the monetary authority gradually attempts to restore its optimal reserve position. So, although the finding that \(d_{12}\) is very small may cast doubt on whether intervention works, it cannot be decisive.

The above analysis is helpful in evaluating the potential usefulness of the other approaches to extracting economic information from the estimated vector autoregression coefficients in equations A.3 and A.4.\(^{31}\) For example, one standard methodology involves simulating a shock to one of the variables and studying the resulting dynamic effects. Contemporaneous correlation introduces subtle problems into this exercise. A true intervention shock is a one-time positive movement in \(\eta^2\), the error term in the underlying structural intervention equation A.2. Such a shock will typically move both of the error terms in equations A.3 and A.4, not just the error term in A.4, the vector autoregression equation for intervention. If, instead, one simulates a shock only to the error term in equation A.4, the resulting dynamic simulation has no direct correspondence with the effects of a shock to the true intervention equation A.2.

The solution, of course, must involve simulating simultaneous shocks to equations A.3 and A.4; the only question is what the relative magnitudes of the two shocks should be. Directly answering this question from estimates of A.3 and A.4 is virtually impossible unless it is known a priori that \(b_1\) or \(b_2\) is zero. Fischer addresses this problem by experimenting with different ways of simulating a shock and then testing the robustness of the dynamic response patterns.\(^{32}\)

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31. Geweke proposes extracting information from vector autoregressions by calculating measures of linear dependence among the variables at short and long horizons (or more precisely, at high and low “frequencies”). His measures capture how much lagged intervention helps forecast exchange rates, given lagged exchange rates. For the same reasons given in the appendix, interpreting Geweke’s measures may be difficult when contemporaneous correlation is important.

32. The technique for simulating shocks advocated by Sims and used by Fischer sets either \(b_1\) or \(b_2\) equal to zero. Neither of these two possibilities would provide the
The bivariate example presented here illustrates some of the potential difficulties in interpreting time-series results, even when no variables are omitted. Of course, daily data present a severe problem of omitted variables. The money supply is not available daily, and one can reasonably assume that the money supply is correlated with sterilized intervention and the exchange rate. Even if daily data on the money supply were available, the presence of contemporaneous correlation would continue to confound interpretation.

The vector autoregression methodology is therefore of limited use in studying the timing and quantitative effects of sterilized intervention. If "incredible" identifying assumptions are needed for structural estimation, then equivalent "incredible" identifying assumptions are necessary to interpret the results of a vector autoregression.
Appendix B: The Relationship between a Structural Model and a Vector Autoregression: An Example

The following is intended as an illustration and not as a general model of exchange rate determination. Suppose a two-country structural model consisting of the following equations:

\begin{align*}
\frac{M_t}{P_t} &= e^{-\lambda s} Y_t, \quad \lambda > 0 \\
\frac{M^*_t}{P^*_t} &= e^{-\lambda s^*} Y^*_t, \\
E_{t-1} \left[ \frac{M_t + B_t}{S_{t-1}(M^*_t + B^*_t)} \right] &= e^\theta \left[ E_{t-1}(i_t - i^*_t - [(S_t - S_{t-1})/S_{t-1}]) \right], \quad 0 \leq \theta \leq \infty \tag{B.3} \\
P_t &= P^*_t S_{t-1} e^{e^e}, \tag{B.4}
\end{align*}

where an asterisk (*) indicates a foreign variable and

\begin{align*}
E_{t-1} &= \text{expectation based on complete information about } t-1 \\
e &= \text{base of the natural logarithm} \\
M_t &= \text{level of domestic money supply} \\
P_t &= \text{domestic price level} \\
i_t &= \text{domestic interest rate} \\
Y_t &= \text{level of domestic output} \\
B_t &= \text{net supply to the private sector of interest-bearing government securities denominated in domestic currency} \\
S_t &= \text{exchange rate, defined as home-currency price of foreign currency (that is, of "dollars")}
\end{align*}

Equations B.1 and B.2 state domestic and foreign money market equilibrium. Equation B.3 states that the desired (which equals the expected) ratio of holdings of assets denominated in domestic currency \((M_t + B_t)\) to holdings of those denominated in foreign currency valued in units of domestic currency \([S_{t-1}(M^*_t + B^*_t)]\) depends on the expected difference in the yield between the assets denominated in different currencies, \(E_{t-1}(i_t - i^*_t - [(S_t - S_{t-1})/S_{t-1}])\).

Equation B.3 is a standard portfolio-balance equation, but it does reflect the market reality that agents must make their spot exchange market transactions before their investment transactions. Agents must contract today to make a net transaction tomorrow. (A one-day lag applies only to Canadian dollar-U.S. dollar exchange market transactions; the lag is two days for other currencies.) The exchange rate dating in the model reflects this institutional fact. The variable \(S_t\) is the rate quoted today, at time \(t\), for transactions tomorrow, at time \(t + 1\).

Equation B.4 states that purchasing power parity holds with a serially uncorrelated disturbance \(\epsilon_{pr}\). Implicit in equation B.4 is the assumption that, unlike foreign exchange contracts, goods market contracts are for same-day delivery. Relaxing this analytically convenient (if unrealistic) assumption would not affect the conclusions reached here.

Equations B.1-B.4 may be approximately represented in logarithms as

\begin{align*}
\tilde{m}_t - s_{t-1} &= \tilde{y}_t - \lambda \tilde{s}_t + \epsilon_{pr}, \tag{B.5} \\
(1 - \theta)E_{t-1} \tilde{m}_t - s_{t-1} &= \theta \left[ E_{t-1}\tilde{i}_t - E_{t-1}\tilde{s}_t + s_{t-1} \right], \tag{B.6}
\end{align*}

where lowercase letters denote logarithms of their uppercase counterparts, and a tilde (~) over a variable indicates the difference between domestic and foreign values (for example, \(m_t = m_t - m^*_t\)). Equation B.6 contains the simplification that \(M/(M + \tilde{B}) = M^*/(M^* + \tilde{B}) = \theta\), where the bar over a variable indicates a sample mean.

The effects of sterilized intervention on the exchange rate depend, in part, on the semi-elasticity of substitution parameter \(\eta\). As \(\eta\) approaches infinity, one obtains (from equation B.6) \(E_{t-1}\tilde{i}_t - E_{t-1}\tilde{s}_t + s_{t-1} = 0\). In this circumstance, equation B.5 alone determines the exchange rate, and relative bond supplies do...
not enter that equation. Thus as \( \eta \) approaches infinity, only unsterilized intervention will influence the exchange rate because such intervention will alter \( \tilde{m}_t \). One would consequently want to test the hypothesis \( \eta \to \infty \). In the context of this model, rejection of this hypothesis implies that sterilized intervention has at least some effect on exchange rates (and on prices and interest rates).

To close the model, some assumptions must be made about government actions in supplying \( \tilde{m}_t \) and \( \tilde{b}_t \) through domestic debt-management operations and foreign exchange intervention. For illustration, assume

\[
CIV_t = \alpha_0 + \alpha_1 CIV_{t-1} + \alpha_2 s_t + \varepsilon_{it}
\]

\[
\tilde{b}_t = \beta_0 + \beta_1 CIV_t + \varepsilon_{bt}
\]

\[
\tilde{m}_t = \gamma_0 + \gamma_1 CIV_t + \varepsilon_{mt},
\]

where \( \alpha_0, \alpha_1, \alpha_2, \beta_0, \beta_1, \gamma_0, \gamma_1 \) are known (to the investigator) parameters; \( \varepsilon_{it}, \varepsilon_{bt}, \varepsilon_{mt} \), are uncorrelated white-noise disturbances; and \( CIV_t \) is the cumulative intervention of the monetary authority, which is the monetary authority’s net dollar position. (Equations B.7–B.9 ignore many important policy reactions. However, they are meant only to be illustrative.) For instance, one might expect that \( \alpha_1 = 1 \) and \( \alpha_2 > 0 \) if intervention leans against the wind. One would also expect \( \beta_1 > 0 \) because an increase in \( CIV_t \) should signal an increase in the ratio of privately held domestic securities to privately held U.S. dollar securities. Further, \( \gamma_1 \) is greater than zero if some intervention is not sterilized.

Finally, assume that the countries examined have equal mean real growth rates so that

\[
\tilde{y}_t = \bar{y} + \varepsilon_{yt},
\]

where \( \bar{y} \) is a constant and \( \varepsilon_{yt} \) is a white-noise disturbance term.

Two of the variables of interest are the exchange rate, \( s_t \), and cumulative intervention, \( CIV_t \). In the model, solutions for these variables may be expressed as

\[
s_t = A_0 + A_1 CIV_{t-1} + u_{st}
\]

\[
\tilde{CIV}_t - B_0 + B_1 CIV_{t-1} + u_{it}.
\]

The constants \( A_0 \) and \( B_0 \) are ignored because they are functions of the choice of units. However, \( A_1 \) and \( B_1 \) are of interest in this investigation, and their values are

\[
A_1 = \frac{\alpha_1 \phi}{1 - \alpha_2 \phi}
\]

\[
B_1 = \frac{2 \alpha_1 \alpha_2}{1 - \alpha_2 \phi}
\]

\[
\phi = \frac{-k_4 + (k_2^2 + 4 \alpha_2 k_5 k_1)^{1/2}}{2 \alpha_2 k_2},
\]

where

\[
k_1 = \eta + \left( \frac{\theta + \eta}{\lambda} \right) \gamma_1 \alpha_2 - (1 - \theta) \beta_1 \alpha_2
\]

\[
k_2 = 1 + \frac{\eta}{\lambda} + \eta
\]

\[
k_3 = -\left( \frac{\theta + \eta}{\lambda} \right) \gamma_1 \alpha_1 + (1 - \theta) \beta_1 \alpha_1
\]

\[
k_4 = \alpha_2 k_3 + k_1 \alpha_1 - k_2.
\]

The disturbance terms \( u_{st} \) and \( u_{it} \) are both white noise, but they are correlated.

A key equation for determining whether sterilized intervention has any effect at all involves the expected rate of return differential, \( E_{t-1}(\tilde{i}_t - s_t + s_{t-1}) \). Manipulating equation B.6 results in

\[
E_{t-1}(\tilde{i}_t - s_t + s_{t-1}) = \frac{1}{\eta} \left[ \theta E_{t-1}(\tilde{m}_t - s_t - s_{t-1}) - (1 - \theta) E_{t-1}(\tilde{b}_t) \right].
\]

Substituting B.11 and B.12 and the assumptions about government behavior embodied in B.7–B.9 into equation B.13, one obtains

\[
E_{t-1}(\tilde{i}_t - s_t + s_{t-1}) = \frac{1}{\eta} \left[ \theta [\gamma_0 + \gamma_1 (\alpha_0 + \alpha_1 CIV_{t-1}) + \alpha_2 (A_0 + A_1 CIV_{t-1})] - s_{t-1} \right.
\]

\[
- (1 - \theta) [\beta_0 + \beta_1 (\alpha_0 + \alpha_1 CIV_{t-1}) + \alpha_2 (A_0 + A_1 CIV_{t-1})] \right].
\]

After consolidating terms, equation B.14 may be written as

\[
\tilde{i}_t - s_t + s_{t-1} = D_0 + D_1 CIV_{t-1} + D_2 s_{t-1} + u_{it},
\]
where

\[
D_0 = \frac{1}{\eta} \{ \theta [\gamma_0 + \gamma_1(\alpha_0 + \alpha_2A_0)] \\
+ (1 - \theta)[\beta_0 + \beta_1(\alpha_0 + \alpha_2A_0)] \}
\]

\[
D_1 = \frac{1}{\eta} \{ \theta(\gamma_1\alpha_1 + \gamma_1\alpha_2A_1) \\
- (1 - \theta)(\beta_1\alpha_1 + \beta_1\alpha_2A_1) \}
\]

\[
D_2 = \frac{1}{\eta}
\]

\[
u_{Rt} = (i_t - s_t + s_{t-1}) - E_{t-1}(i_t - s_t + s_{t-1}).
\]

The disturbance \(\nu_{Rt}\) is correlated with \(\nu_i\) and \(\nu_{it}\), but it is not correlated with either \(CIV_{t-1}\) or \(s_{t-1}\).

Interpreting the Complete System

The complete vector autoregression system consists of the following equations:

\[(B.16) \quad s_t = A_0 + A_1CIV_{t-1} + u_{st}\]

\[(B.17) \quad CIV_t = B_0 + B_1CIV_{t-1} + u_{it}\]

\[(B.18) \quad (i_t - s_t + s_{t-1}) = D_0 + D_1CIV_{t-1} \\
+ D_2s_{t-1} + u_{Rt}.
\]

Equations B.16–B.18 seem to be ideal for applying the vector autoregression method of Sims.\(^{33}\) The model is very simple and implies that only one lag of \(CIV\) should appear in B.16–B.18. It further implies that past values of \(s_t\) should enter only B.18. This gross simplification is due in part to the simplicity of the reaction functions chosen in B.7–B.9. But suppose estimates of the parameters are obtained and the model is not rejected on the grounds of having variables other than \(CIV_{t-1}\) and \(s_{t-1}\) enter equations B.16–B.18; what could one conclude about sterilized intervention from the estimates?

One needs first to look at the coefficients in B.18—\(D_0\), \(D_1\), and \(D_2\). All of these coefficients are proportional to \(1/\eta\). Consequently, if \(\eta\) approaches infinity, then \(D_0\), \(D_1\), and \(D_2\) all approach zero. Thus a test of the hypothesis \(\eta = \infty\) tests whether the systematic part of B.18 lacks explanatory power. As it turns out, this test is very robust to model specification: regardless of the choice of B.7–B.9, if equation B.18 has an explanatory power, one may conclude that \(\eta\) is finite. This conclusion implies that relative supplies of assets do influence the exchange rate, although the degree of influence remains unknown. Interestingly, if equation B.18 has no explanatory power, one cannot say that \(\eta\) approaches infinity because generally \(1/\eta\) will multiply complicated functions of the underlying structural coefficients, and these complicated functions could each be zero.

One could also ask whether the coefficients \(A_1\) and \(B_1\) also allow a test of \(1/\eta = 0\). For instance, if \(A_1 > 0\), can one conclude that sterilized intervention will affect the exchange rate? Stated another way, does \(A_1 > 0\) imply that \(\eta\) is finite? The answer is no. Even if \(\eta = \infty\), \(A_1\) will have a nonzero value. However, finding a nonzero \(A_1\) does imply that either \(\eta = \infty\) or \(\gamma_1 = 0\). (Recall that \(\gamma_1\) is the coefficient on \(CIV\) in the money supply equation. If \(\gamma_1 = 0\), then all intervention is sterilized.) The coefficient \(B_1\) is proportional to \(\alpha_1\) and is the sum of two terms. Only one of those terms involves the parameter \(\eta\). Hence \(B_1\) does not give identifying evidence concerning \(\eta\).

It is impossible to use the estimates of \(A_1\) and \(B_1\) to identify \(\eta\) even in the simple example at hand. In a more complex model, such as one involving lags of all variables in equations B.7–B.9, the relations of \(\eta\) to the estimated coefficients would become even more complex. However, estimates of the parameters \(D_0\), \(D_1\), and \(D_2\) do give evidence concerning \(\eta\), and that evidence is robust to a wide variety of specifications of equations B.7–B.9.

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33. Sims, "Macroeconomics and Reality."
References


