Econometric analysis of potential outcomes time series: instruments, shocks, linearity and the causal response function

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Big picture

- The three great tasks of statistics are:
  - description (Y);
  - prediction ($Y|W$);
  - causality (change in Y if W is moved).
- Most empirical economics research is about causality:
  - highly structured models (e.g. I.O.);
  - randomized experimental methods (e.g. development economics);
  - observational methods; mimicking experiments (everywhere, e.g. corporate finance).
- Here build dynamic causality methods for observational time series:
  - mimicking potential outcome time series for randomized dynamic experiments;
  - core observational methods: causal response function, shocks, linear potential outcome time series, instrumented potential outcome time series;
  - relate to: impulse response function, local projection, local projection-IV.

Bojinov, Rambachan and Shephard (2020) extend to panel data experiments.

Time series & causality: what we are not doing

Researchers quantify causality in time series in a variety of ways

- Model-free "Granger causality" (Wiener(1956), Granger(1969)). Famously, but this is about forecasting, not causality.
- Model-based:
  - highly structured models such as DSGE models (Herbst & Schorfheide(2015)), game theory (Toulis & Parkes(2016));
  - state space modelling (Harvey and Durbin(1986), Harvey(1996), Bondersen et al (2015));
  - intervention analysis (Box & Tiao(75));
  - linear models, VMA, VAR, IRF (Sims (1980)), local projection (Jorda(2005)), IV-local projection (Jorda et al (2015)).

The potential outcome time series is distinct from each of those approaches. It is model-free. Closest to Angrist and Kurnsteiner(2011) and Angrist, Jorda, Kurnsteiner(2018). Much like cross-sectional causal studies.

Main contributions of this paper

- Define causal response function, useful for observational studies:
  - give model-free causal meaning to "generalized impulse response function";
  - give model-free causal meaning to "impulse response function".
- Define a shock, linearity and instruments for potential outcome time series.
- Understand what local projection & LP-IV estimate:
  - usually studied under VMA models (impose shocks, linearity and causality all at once);
  - unpick where identification comes from, seperating it out from precision;
  - shocks help precision, not identification.

Potential outcome time series (Bojinov & Shephard (2020, JASA))

Defn: Treatment path is the stochastic process $W_{1:T}$, where $W_t \in \mathcal{W}$. Potential outcome path, for any deterministic $w_{1:T} \in \mathcal{W}^T$, is the stochastic process

$$Y_{1:T}(w_{1:T}) = (Y_1(w_{1:T}), Y_2(w_{1:T}), ..., Y_T(w_{1:T})).$$

Assumptions

1. "Non-anticipating POs": $Y_t(w_{1:T}) = Y_t(w_{1:t}, w'_{t+1:T})$ for all $w_{1:T}, w'_{t+1:T}$. Write:

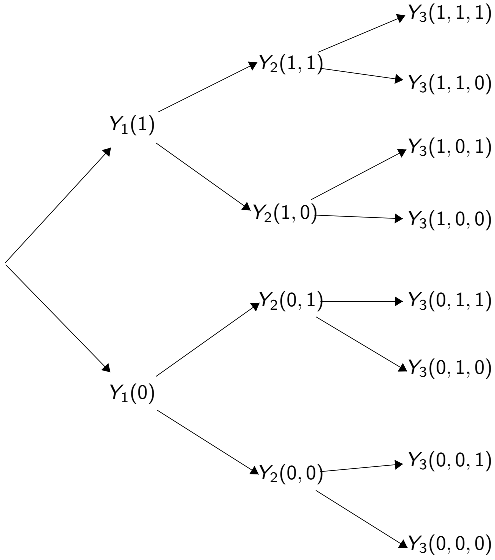
$$Y_{1:T}(w_{1:T}) = (Y_1(w_1), Y_2(w_{1:2}), ..., Y_T(w_{1:T})).$$

2. "Outcomes": See $W_{1:T}$ & $Y_{1:T} = Y_{1:T}(W_{1:T})$. Write $\mathcal{F}_t$ generated by $W_{1:t}, Y_{1:t}$.
3. "Non-anticipating treatments": For each t:

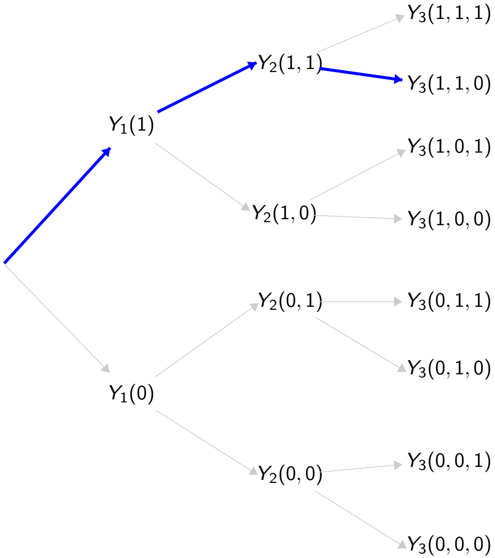
$$(\{Y_{t:T}(W_{1:t-1}, w_{t:T}), w_{t:T} \in \mathcal{W}^{T-t+1}\} \perp\!\!\!\perp W_t)|\mathcal{F}_{t-1}.$$

Defn: $W_{1:T}$ & $Y_{1:T}$ that satisfy Ass 1, 2 & 3 are "potential outcome time series".
Ass 1: time series non-interference (Cox(1958)). Ass 3: time series unconfoundedness.

$\{Y_{1:3}(w_{1:3}), w_{1:3} \in \{0,1\}^3\}.$



$W_{1:3} = (1, 1, 0)$, $Y_{1:3} = Y_{1:3}(W_{1:3}) = Y_{1:3}(1, 1, 0)$.



Example (Linear process)

Let $Y_t = Y_t(W_{1:t})$ where for all $w_{1:t} \in \mathcal{W}^t$,

$$\begin{pmatrix} Y_t(w_{1:t}) \\ W_t \end{pmatrix} = \begin{pmatrix} \mu + \phi Y_{t-1}(w_{1:t-1}) + \beta_0 w_t \\ \gamma + \theta W_{t-1} + \delta Y_{t-1}(W_{1:t-1}) \end{pmatrix} + \begin{pmatrix} \epsilon_t \\ \eta_t \end{pmatrix}, \quad \begin{pmatrix} \epsilon_t \\ \eta_t \end{pmatrix} \overset{iid}{\sim} N\left(0, \begin{pmatrix} \sigma_\epsilon^2 & \rho\sigma_\epsilon\sigma_\eta \\ \rho\sigma_\epsilon\sigma_\eta & \sigma_\eta^2 \end{pmatrix}\right). \tag{1}$$

- Not a potential outcome time series, as ϵ_t and η_t are contemporaneously correlated — disallows Assumption 3.
- If $\rho = 0$ then this is a potential outcome time series.

Example (Forward looking expectations)

$$W_t = \arg\max_{w_t} \left(\max_{w_{t+1:T}} E[U^*(Y_{t:T}(W_{1:t-1}, w_{t:T}), w_{t:T}) \,|\, \mathcal{F}_{t-1}] \right), \tag{2}$$

where U^* is a utility function of future outcomes and treatments. This decision rule delivers W_t and thus $Y_t(W_{1:t})$.

- Is a potential outcome time series.

Causal effect

Assume a potential outcome time series. Have seen $W_{1:t-p-1}$, $p \geq 0$. Bojinov and Shephard (2020) define causal effect of treatment $w_{t-p:t}$ compare to $w'_{t-p:t}$ as

$$\tau_t = Y_t(W_{1:t-p-1}, w_{t-p:t}) - Y_t(W_{1:t-p-1}, w'_{t-p:t}).$$

Usually place structure on $w_{t-p:t}$ and $w'_{t-p:t}$. We here introduce:

Definition

Assume a potential outcome times series and write $Y'_t := Y_t(W_{1:t-p-1}, W'_{t-p:t})$, recalling $Y_t := Y_t(W_{1:t})$. The **causal response function** is, if it exists,

$$CRF_{t,p}(w, w') := \mathbb{E}\left[\left(Y_t - Y'_t\right) \mid W_{t-p} = w, W'_{t-p} = w', \mathcal{F}_{t-p-1}\right], \tag{3}$$

where the expectation is generated by $\{Y_t, W_{t-p:t}, Y'_t, W'_{t-p:t}\}|\mathcal{F}_{t-p-1}$.

Then our causal estimand is: $\overline{CRF}_p(w, w') = \frac{1}{T-p}\sum_{t=p+1}^{T} CRF_{t,p}(w, w')$.

Generalized impulse response function

For a potential outcome time series assume additionally that
$\{Y'_t, W'_{t-p:t}\}|\mathcal{F}_{t-p-1} \overset{L}{=} \{Y_t, W_{t-p:t}\}|\mathcal{F}_{t-p-1}$, and

$$(\{Y_t, W_{t-p:t}\} \perp\!\!\!\perp W'_{t-p})|\mathcal{F}_{t-p-1}, \quad \text{and} \quad (\{Y'_t, W'_{t-p:t}\} \perp\!\!\!\perp W_{t-p})|\mathcal{F}_{t-p-1}.$$

Then

$$\boxed{CRF_{t,p}(w, w') := E\left[Y_t \middle| W_{t-p} = w, \mathcal{F}_{t-p-1}\right] - E\left[Y_t \middle| W_{t-p} = w', \mathcal{F}_{t-p-1}\right].}$$

- When $w' = 0$ this is the "generalized impulse response function" of Koop, Pesaran and Potter (1996). **Gives nonparametric causal meaning to GIRF.**
- $CRF_{t,p}(w, w')$ is our favorite summary measure for practical causal studies of observational time series.

Identifying dynamic causal effects

$$CRF_{t,p}(w,w') := E\left[Y_t \middle| W_{t-p}=w, \mathcal{F}_{t-p-1}\right] - E\left[Y_t \middle| W_{t-p}=w', \mathcal{F}_{t-p-1}\right].$$

For potential outcome time series, the central causal estimand is

$$\overline{CRF}_p(w,w') = \frac{1}{T-p}\sum_{t=p+1}^{T} CRF_{t,p}(w,w').$$

- A $T^{2/5}$-consistent kernel estimator of $\bar{\tau}_p(w,w')$ is given for continuous w, w'.
- Implications:
  - Average dynamic causal effects can be nonparametrically identified solely from assuming a potential outcome time series.
  - Identifying causal effects by "shocks", as econometricians often do through "Frisch-Slutsky paradigm", misses the point.
  - Later we will define a shock. It helps statistical precision, not identification!

Impulse response function

Definition (Impulse response function)

Assume $\{Y_t, W_t\}$ is strictly stationary and

$$IRF_p(w, w') := E[Y_t \mid W_{t-p} = w] - E[Y_t \mid W_{t-p} = w'],$$

exists, where $E[\cdot]$ is calculated from the law of Y_t, W_{t-p}. Then, $IRF_p(w, w')$ is called "impulse response function".

Classically, this has no causal content.

Thm: Assume $\{Y_t, W_t\}$ is stationary POTS. Then, if the expectations exist,

$$E[CRF_{t,p}(w, w')] = IRF_p(w, w'),$$

where the expectation is generated by the stationary distribution of treatments and outcomes. Again $T^{2/5}$ rate. **Gives nonparametric causal meaning to IRF.**

Special cases of POTS — searching for more precision

Definition (Linear potential outcome time series)

For a potential outcome time series, if, for every $w_{1:t}$,

$$Y_t(w_{1:t}) = U_t + \sum_{s=0}^{t-1} \beta_{t,s} w_{t-s},$$

where $\beta_{t,s}$ are non-stochastic, then $\{Y_t, W_t\}$ is **linear potential outcome time series**.

Note, this does not need that $\{Y_t, W_t\}$ is a linear process!

Definition (Shocked potential outcome time series)

For a potential outcome time series, if,

$$E[W_t \mid \mathcal{F}_{t-1}] = 0,$$

then W_t is called a **shock** and $\{Y_t, W_t\}$ is **shocked potential outcome time series**.

Linear, shocked POTS

Recall, linear POTS has $Y_t(w_{1:t}) = U_t + \sum_{s=0}^{t-1} \beta_{t,s} w_{t-s}$, so $Y_t(w_{1:t}) - Y_t(w'_{1:t}) = \sum_{s=0}^{t-1} \beta_{t,s}(w_{t-s} - w'_{t-s})$. Under linear, shocked POTS,

$$\boxed{\overline{CRF}_p(w, w') = (w - w')\bar{\beta}_p, \quad \bar{\beta}_p = \frac{1}{T-p} \sum_{t=p+1}^{T} \beta_{t,p}.}$$

Estimand is time-averaged of time-t causal effect! Local projection estimator:

$$\hat{\beta}_p^{OLS} = \frac{\sum_{t=p+1}^{T} Y_t W_{t-p}}{\sum_{t=p+1}^{T} W_{t-p}^2} \xrightarrow{p} \frac{\lim_{T\to\infty} \frac{1}{T-p} \sum_{t=p+1}^{T} \beta_{t,p} E(W_{t-p}^2)}{\lim_{T\to\infty} \frac{1}{T-p} \sum_{t=p+1}^{T} E(W_{t-p}^2)}.$$

Typically $T^{1/2}$ consistent. Linearity & shocks makes inference more precise. Under stationary, linear, shocked POTS:

$$\hat{\beta}_p^{OLS} \xrightarrow{p} \beta_p.$$

Special cases of POTS — instrumental variables

Definition (Instruments)

Assume $\{Y_t, V_t\}$ is a potential outcome time series, where $V_t = (W_t', Z_t')'$. Additionally:

1. Exclusion condition: $Y_t(w_1, z_1, ..., w_t, z_t) = Y_t(w_1, z_1', ..., w_t, z_t')$ for all $w_{1:t}, z_{1:t}, z_{1:t}'$.
2. Relevance condition: $Z_t \not\perp\!\!\!\perp W_t \mid \mathcal{F}_{t-1}$.

Then $\{Y_{1t}, V_t\}$ is "instrumented potential outcome time series", $Z_{1:t}$ is "instrument path."

Example

Y_t is a macro aggregate, W_t is a monetary policy shock, but cannot see it. $Z_t = \hat{W}_t$ is estimate of monetary policy shock! Here: assume actual shock impacts economy not estimated shock; estimate is not independent of shock.
Focus of the local projection-IV literature, e.g. Jorda et al (2015), Stock & Waton (2018) and Plagborg & Wolf (2018). Discussion there is in terms of linear models, based on shocks and stationarity.

Local projection-IV

Assume $\{Y_{1t}, (W_t, \hat{W}_t)\}$ is "instrumented potential outcome time series", $\hat{W}_{1:t}$ is "instrument path."
Define

$$\hat{\beta}_p^{IV} = \frac{\sum_{t=p+1}^{T} Y_t \hat{W}_{t-p}}{\sum_{t=p+1}^{T} Y_{t-p} \hat{W}_{t-p}}$$

Then, under a linear, shocked, instrumented potential outcome time series

$$\hat{\beta}_p^{IV} \xrightarrow{p} \frac{\lim_{T\to\infty} \frac{1}{T-p} \sum_{t=p+1}^{T} \beta_{t,p} E(W_{t-p} \hat{W}_{t-p})}{\lim_{T\to\infty} \frac{1}{T-p} \sum_{t=p+1}^{T} \beta_{t,0} E(W_{t-p} \hat{W}_{t-p})}. \tag{4}$$

In the LP-IV literature, conventional to assume $\beta_{t,0} = 1$ (Stock and Watson (2018)).

Linear projection

Shocked POTS where $E(Y_t^2) < \infty$, $0 < E(W_{t-p}^2) < \infty$, define "local" & "universal"

$$\beta_{t,p}^L := \arg\min_\beta \left[\min_\alpha E(Y_t - \alpha - \beta W_{t-p})^2\right],$$

$$\beta_p^U := \arg\min_\beta \min_\alpha \lim_{T\to\infty} \frac{1}{T-p} \sum_{t=p+1}^{T} E(Y_t - \alpha - \beta W_{t-p})^2.$$

Then, writing $\alpha_t = E(Y_t)$,

$$Y_t^L(w_{1:t}) := \alpha_t + \sum_{s=0}^{t-1} \beta_{t,s}^L w_{t-s}, \text{ and } Y_t^U(w_{1:t}) := \alpha + \sum_{s=0}^{t-1} \beta_s^U w_{t-s},$$

$$\beta_p^U = \frac{\lim_{T\to\infty} \frac{1}{T-p} \sum_{t=p+1}^{T} \beta_{t,p}^L E(W_{t-p}^2)}{\lim_{T\to\infty} \frac{1}{T-p} \sum_{t=p+1}^{T} E(W_{t-p}^2)}. \tag{5}$$

Conclusion

- Formal model-free definition of causality in times series: POTS and τ_t.
- Form favorite practical measure for observational time series:

$$CRF_{t,p}(w, w') = E\left[Y_t | W_{t-p} = w, \mathcal{F}_{t-p-1}\right] - E\left[Y_t | W_{t-p} = w', \mathcal{F}_{t-p-1}\right].$$

  Gives nonparametric causal meaning to GIRF.
- Can nonparametrically estimate $\overline{CRF}_p(w, w')$ at $T^{2/5}$.
- Stationarity + POTS implies:
  - $CRF_p(w, w') = IRF_p(w, w') = E\left[Y_t | W_{t-p} = w\right] - E\left[Y_t | W_{t-p} = w'\right].$
  - **Gives nonparametric causal meaning to IRF**.
  - Nonparametrically still $T^{2/5}$.
- Linearity + POTS, then estimate at $T^{1/2}$.
- Local projection: linearity + shocks + POTS: estimate weighted causal quantities.
- LP-IV: linearity + shocks + instrumented POTS: estimate weighted causal quantities.