Large Shareholders and Corporate Control

Andrei Shleifer
Massachusetts Institute of Technology

Robert W. Vishny
University of Chicago

In a corporation with many small owners, it may not pay any one of them to monitor the performance of the management. We explore a model in which the presence of a large minority shareholder provides a partial solution to this free-rider problem. The model sheds light on the following questions: Under what circumstances will we observe a tender offer as opposed to a proxy fight or an internal management shake-up? How strong are the forces pushing toward increasing concentration of ownership of a diffusely held firm? Why do corporate and personal investors commonly hold stock in the same firm, despite their disparate tax preferences?

I. Introduction

In an imperfect and evolving world, managers of some firms, though they may try hard, may just not be good enough. Sometimes they need to be persuaded and sometimes replaced. But who will monitor managers and look for ways to better the firm? In this paper this function is performed by large shareholders.

Grossman and Hart (1980) persuasively argue that outsiders without a share in a diffusely held firm would never take over in order to improve it. The reason is that if the outsiders' improvement plan is

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understood by atomistic incumbent shareholders, they will demand the value of the improvement in return for their shares or else they stay on. If the outsider can gain only on the shares he already owns (which are few if any) but has to pay all the monitoring and takeover costs, the deal may not be worth his while. For the same reason, small shareholders do not have a big enough stake in the firm to absorb the costs of watching the management.

The basic question then is: In a world in which search for improvements is a public good, how can its provision be ensured? Grossman and Hart permit the outsider to exclude incumbent shareholders from the gains his takeover produces. They call this exclusionary device “dilution.” It has the effect of lowering the acquisition price, possibly enough to ensure the efficient level of search for improvements by outsiders.1 Alternatively, improvements can be made by parties who already own a large share of the firm. As the largest consumers of the public good, they may pay for it themselves. In this paper we focus on the ways in which large shareholders bring about value-increasing changes in corporate policy.

Empirically, large shareholdings are extremely widespread and very substantial where present. In a sample of 456 of the Fortune 500 firms,2 354 have at least one shareholder owning at least 5 percent of the firm. In only 15 cases does the largest shareholder own less than 3 percent of the firm. The average holding of the largest shareholder among the 456 firms is 15.4 percent. The total average holding of the five largest shareholders is 28.8 percent. We suspect that for smaller firms the figures are even more dramatic.

Who are the large shareholders? In our sample, a large number of them are families represented on boards of directors (149 cases). Also prominent are pension and profit-sharing plans (90 cases), as well as financial firms such as banks, insurance companies, or investment funds (117 cases). The final category consists of firms and family holding companies with large stakes who do not have board seats (100 cases). We expect that financial managers and especially large individual and corporate investors would monitor the management and sometimes initiate a takeover or invite third parties to do so.3

1 Though a threat of dilution may indeed lower the acquisition price, the courts have taken measures to restrain it (see Easterbrook and Fischel 1982, p. 699). Evidence by Dodd and Ruback (1977) and Bradley (1980) suggests that the effect of dilution may be limited. This accords with the judgment of Grossman and Hart (1980, pp. 57–59) as well.

2 Our calculations are based on the data from CDE Stock Ownership Directory: Fortune 500, compiled for December 1980. The 44 excluded companies were subsidiaries, cooperatives, privately held firms, or firms that merged in 1980.

3 Even when a family with a large stake has seats on the board of directors, it may not have complete control (e.g., the Gettys in Getty Oil). Such seats in fact can facilitate monitoring and possible replacement of management.
Indeed, our preliminary evidence suggests that large shareholders play an important role in takeovers. Even when they cannot monitor the management themselves, large shareholders can facilitate third-party takeovers by splitting the large gains on their own shares with the bidder. Of the 456 firms in our sample, 52 merged, were taken private by management, or were taken over between January 1980 and December 1984. Of these transactions, 24 involved firms one of whose largest two shareholders was a nonmanaging large investor or a nonfinancial firm.4

To illustrate the role of large shareholders, we focus on a simple but realistic case. We describe a firm whose management acts to maximize profits but does so imperfectly. Accordingly, a monitor may have an opportunity to improve the firm’s operating strategy but needs to replace the incumbent management to produce the maximum increase in profits. Though the incumbents want to keep their jobs, they have no resistance options other than creating limited increases of takeover costs to the bidder. If the bidder completes a takeover despite these costs, he replaces the management.

Our firm is owned by one large shareholder, who does not participate in management, and a fringe of small ones.5 We do not consider strategic interactions between large shareholders. In our model, the large shareholder has a large enough stake that it pays for him to do some monitoring of the incumbent management. If higher profits justify a change, he attempts to implement it. All shareholders benefit since they enjoy gains on their own shares. The large shareholder’s return on his own shares suffices to cover his monitoring and takeover costs. However, because the large shareholder internalizes only the gains to his own shares, there is still too little monitoring and takeover activity. We do not model the factors such as wealth constraints and risk aversion that presumably are the impetus for a closely held concern to go public, despite the value loss associated with this free-rider problem.6

In Section II we specify the basic model and perform two important comparative statics exercises. First, we consider the implications of an

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4 In most of the rest of the cases, the sold company was family controlled.
5 In our sample of 456 firms, the modal number of shareholders with at least 5 percent of the firm is 1 (171 cases), and the mean is 1.4.
6 Even when the initial owners of the firm are unwilling or unable either to provide themselves or to borrow sufficient funds to invest, it might be argued that they could still raise sufficient funds and keep ownership of the firm concentrated by selling out to another large investor with more resources. However, initial shareholders (usually the management) might be reluctant to place control firmly in the hands of another party by selling its equity interest, while other large investors are unlikely to assume financial responsibility without a good degree of control attached. Harris and Raviv (1985) discuss related issues from a theoretical point of view, while Morck, Shleifer, and Vishny (1985) analyze empirically the relationship between management ownership and firm value.
increase in the holdings of the large shareholder. Not surprisingly, as the proportion of the firm’s shares held by the large shareholder rises, a takeover becomes more likely and the price of the firm’s shares increases. When a takeover does occur, the premium above the prevailing stock price paid to tendering shareholders (the takeover premium) is actually lower (proposition 1). The reason is that when the large shareholder owns more, he is willing to take over for a smaller increase in the firm’s profits. Hence bids not only signal smaller average posttakeover increases in profits but also become more likely and are more heavily reflected in the pretakeover market price. A second, related result says that an increase in the legal and administrative costs of takeovers reduces the welfare of small shareholders, despite an increase in takeover premia (proposition 2).

Though both theoretical and journalistic attention has focused on takeovers, they are not the only mechanism used to alter a firm’s operating strategy. When several ways to improve the firm are available, the large shareholder’s choice of mechanism informs the small shareholders about the likely value of his improvement plan. We illustrate this point in Section III. For example, when the large shareholder can use his information to improve the firm through informal negotiations with its current management, his resort to a takeover instead prompts small shareholders to demand a high premium. We further show why small shareholders might not tender if more than 50 percent of the firm’s shares are bid for in a takeover (proposition 3). Finally, we show that the share valuation results of Section II generalize to the case in which alternatives to the takeover mechanism are available (proposition 4).

One message of Section II is that a premonitoring purchase of shares by the large shareholder raises the firm’s expected profits. Section IV considers the scope for such value-increasing transactions. We show that, if an already large shareholder can commit to a ceiling on his pretakeover holdings, he will always make some purchase despite the insistence of small shareholders on an above-market price for their shares (proposition 6). On the other hand, we show that if the ownership structure of the firm is initially very diffuse and trading is public, it is not profitable to assemble a large block of shares (proposition 7). Corporation founders aside, this suggests that large positions must be either accumulated secretly or passed from one group of large shareholders to another. We discuss both of these possibilities in Section IV.

7 Particularly in the case of well-known takeover artists, pretakeover positions are very often the result of secret buying before the filing of the SEC’s 13-D disclosure form (Holderness and Sheehan, in press).
Our analysis indicates that large shareholders raise expected profits and the more so the greater their percentage of ownership. Yet they may be hard to keep. Section V considers the possibility of the differential valuation of shares by small and large shareholders in the presence of capital gains and dividend taxes. Small shareholders are usually individuals, so they prefer their returns as capital gains. Large shareholders usually have corporate tax attributes, so they prefer dividends to capital gains.\textsuperscript{8} Without the need to monitor the management, small shareholders are best served when no dividends are paid. In practice, though, dividends are widespread and large. This is the so-called dividend puzzle (see Black 1976). We argue that, if the large shareholder is valued, small shareholders may favor the payment of dividends to ensure that he stays with the firm. A simple model demonstrating this possibility is worked out in Section V.

II. The Value of a Large Shareholder

In this section we focus on corporate control transactions of a particular type, namely, cash tender offers made by large minority shareholders to replace inefficient management. Our objective is to establish the value of a large shareholder.

We assume that the firm's shares are initially held by a single risk-neutral large shareholder, unaffiliated with management and holding a fraction \( \alpha < .5 \) of the firm, and by a fringe of risk-neutral atomistic shareholders holding altogether a proportion \( 1 - \alpha \).\textsuperscript{9} For the purposes of this section, \( \alpha \) is taken as fixed. Although the management does its best to maximize the present value of profits, it faces possible replacement by insurgents led by the large shareholder, who can offer a more profitable operating strategy.

The large shareholder (hereafter \( L \)) is assumed to have exclusive access to a technology for finding valuable improvements of the incumbent's operating strategy through monitoring and independent research.\textsuperscript{10} This technology gives \( L \) a probability \( I \) of drawing an improvement of positive value \( Z \) from an atomless cumulative distribution function \( F(Z) \) for a cost \( c(I) \). The variable \( Z \) should be inter-

\textsuperscript{8} Even when large shareholders are individuals, they usually own their position indirectly through a holding company, not the least to avoid the dividend tax.

\textsuperscript{9} The fraction \( \alpha \) can also be thought of as the sum of the positions of a group of several large shareholders acting together.

\textsuperscript{10} We assume proprietary access to the monitoring technology in order to avoid modeling information spillovers and correlated research outcomes when several parties are watching the same firm. Even if many outsiders have access to the monitoring technology, the presence of a large shareholder is still a necessary condition for use of that technology to generate a positive profit.
interpreted as the increase in discounted profits resulting from replacement of inefficient management; \( I \) can be thought of as research intensity; \( F(Z) \) has a bounded support \((0, Z_{\text{max}}]\); and the cost function \( c(I) \) is assumed to satisfy \( c'(I) > 0 \) and \( c''(I) > 0 \). The expected value of profits under existing management is equal to \( q \).

In the event that \( L \) invests \( c(I) \) and finds an improvement of value \( Z \), he may attempt to gain control by making a cash tender offer for a proportion \( .5 - \alpha \) of the firm’s shares.\(^{11}\) But making a tender offer can involve substantial legal and administrative costs (in addition to any premium paid). We suppose that \( L \) must also incur a cost \( c_T \) if he decides to make a bid.\(^{12}\) He will move to make a tender offer if he can purchase \( .5 - \alpha \) of the shares from the small shareholders for any bid \( q + \pi \), with \( \pi \) satisfying

\[
.5Z - (.5 - \alpha)\pi - c_T \geq 0. \tag{1}
\]

It is important to note that \( \pi \) will not be equal to the difference between \( L \)'s bid and the share price prevailing before the takeover. This is so because the pretakeover share price will exceed \( q \) because of the prospect of a value-increasing takeover. Note also that, for a sufficiently large \( \alpha \), \( L \) need not bid less than the true value of the posttakeover firm in order to make a profit since he gains on the shares he already owns.

We assume that, if fewer than \( .5 - \alpha \) shares are tendered, the improvement is not made and \( L \) returns all shares tendered to their owners. Viewing the success of the takeover attempt as independent of his own tender decision, a rational atomistic shareholder will tender if and only if \( \pi \) exceeds his expectation of \( Z \), the rise in the firm’s profits after the takeover. Small shareholders form their expectations about \( Z \) using two pieces of information:\(^{13}\) (a) \( L \) has drawn an improvement from \( F(Z) \), and (b) \( L \) can cover takeover costs, pay \( (.5 - \alpha)\pi \) above \( q \), and still make a nonnegative profit. The small shareholders’ best forecast of \( Z \) is then given by

\[
E[Z|Z - (.5 - \alpha)\pi - c_T \geq 0],
\]

where the conditional expectation is taken with respect to \( F(Z) \).

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\(^{11}\) We show in Sec. III that tender offers for more than \( .5 - \alpha \) of the firm’s shares do not occur in equilibrium.

\(^{12}\) Easterbrook and Fischel (1981, pp. 1175–76) document the high cost of some recent takeover battles.

\(^{13}\) We do not consider the case in which \( L \) can credibly disclose the true value \( Z \) to small shareholders. We are also assuming that contracts that make shareholders’ takeover premia contingent on some tamper-proof future observation on \( Z \) are not available.
Tendering is the best strategy if and only if
\[ \pi - E[Z|Z \geq (1 - 2\alpha)\pi + 2c_T] \geq 0. \] (2)

We have assumed that, if small shareholders are indifferent between tendering and not tendering their shares, they choose to tender. Since \( L \) wants to obtain \((.5 - \alpha)\) shares for minimum cost, he will bid \( q + \pi^*(\alpha) \), where \( \pi^*(\alpha) \) is the minimum \( \pi \) that satisfies (2).

We should point out that, while we focus on the equilibrium in which \( L \) bids \( q + \pi^*(\alpha) \), there are in general many other pure strategy sequential equilibria. In these equilibria, \( L \) bids more than \( q + \pi^*(\alpha) \), but little enough to make a profit. Nevertheless, we believe that the case for the minimum bid equilibrium is compelling. In order to support any other sequential equilibrium, we would need to posit an out-of-equilibrium belief on the part of small shareholders that those bidding \( q + \pi^*(\alpha) \) had, on average, an improvement of value greater than the forecast based on \( a \) and \( b \) above. But there is no basis for such a belief since it is common knowledge that all \( L \) types would like to take over at the lowest possible price. It is also common knowledge that, if all those who could profit by taking over at \( q + \pi^*(\alpha) \) actually chose to deviate, then it would be rational to tender since \( \pi^*(\alpha) \) satisfies (2). Under these circumstances, it seems reasonable for small shareholders to believe that any \( L \) type able to profit by taking over at \( q + \pi^*(\alpha) \) would take the opportunity to pay a lower price, that \( L \) type recognizing that the bid \( q + \pi^*(\alpha) \) is acceptable when all his fellow \( L \) types are expected to do the same. This gives rise to beliefs based on \( a \) and \( b \) when \( \pi = \pi^*(\alpha) \) and to acceptance of the bid.

This type of argument is the basis for a refinement of the sequential equilibrium concept due to Grossman and Perry (1984). In our model, their requirement that out-of-equilibrium beliefs be “credible” can be interpreted as follows. Suppose that there is a group of potential bidders with improvement values drawn from a unique set \( K \subset (0, Z_{\max}] \) who wish to deviate from a proposed equilibrium strategy—such as no bid or a bid \( q + \pi' > q + \pi^*(\alpha) \)—and to bid \( q + \pi^*(\alpha) \) instead provided that small shareholders believe that \( L \) deviates if and only if \( Z \in K \). Suppose also that potential bidders with improvement values drawn from the complement of \( K \) do not wish to deviate when small shareholders believe that \( L \) deviates if and only if \( Z \in K \). Then, on seeing the bid \( q + \pi^*(\alpha) \), small shareholders must predict the value of the improvement using \( E(Z|Z \in K) \), where expectation is taken with respect to \( F(Z) \).\(^{14}\) When there is more than one set \( K \), any of the beliefs

\(^{14}\) One difference between the Grossman-Perry (1984) notion of credible beliefs and the criterion advocated by Kreps (1984) is that, for Grossman and Perry, the requirement of no deviation by types in the complement of \( K \) needs to hold only when it is
generated in this way can be credible. If there is no such set \( K \), any forecast of the form \( E(Z|Z \in J) \) for some \( J \subset (0, Z_{\text{max}}) \) is credible.

In the Appendix we prove the following theorem.\(^{15}\)

**Theorem.** When tender offers must be made for exactly \( 0.5 - \alpha \) shares and \( \pi^*(\alpha) < Z_{\text{max}} \), there is a unique pure strategy sequential equilibrium that is supported by out-of-equilibrium beliefs that are credible in the sense of Grossman and Perry. In that equilibrium, \( L \) bids \( q + \pi^*(\alpha) \) if \( 0.5Z - (0.5 - \alpha)\pi^*(\alpha) - c_T \geq 0 \) and does not bid otherwise.

In the later discussion (proposition 3), we show that, even when the strategy space is expanded to include offers for more than \( 0.5 - \alpha \) shares, such bids are never made in any pure strategy sequential equilibrium supported by credible beliefs.

We now examine several properties of the equilibrium. In particular, the characterization in (2) leads to the following result.

**Lemma 1.** \( \pi^*(\alpha) \) is decreasing in \( \alpha \).

One interpretation of lemma 1 is that, the more shares \( L \) owns, the easier it is to convince small shareholders that a low bid indicates a small posttakeover rise in price rather than an attempt to profit at their expense. Viewed another way, \( L \) is convincing because, as \( \alpha \) increases, it will be in its own interest to proceed with some lower-valued improvements at any given bid \( q + \pi \). As \( \alpha \) approaches \( 0.5 \), \( \pi^*(\alpha) \) is just \( E(Z).5Z \geq c_T \). At the other extreme, when \( \alpha \approx 0, L \) cannot take over for any \( \pi \) below \( Z_{\text{max}} \). This is essentially the case considered by Grossman and Hart (1980). With \( \alpha \approx 0 \), the raider can make a profit only if \( 0.5(Z - \pi) - c_T \geq 0 \); however, this implies \( Z > \pi \) so that no one will tender. Without differential valuations of the firm’s profits, the presence of a large shareholder is a necessary condition for the occurrence of value-increasing takeovers.\(^{16}\)

\(^{15}\) The proofs of all the subsequent results are also in the App.

\(^{16}\) This would not be true if an acquirer could secretly buy a very large number of shares on the open market. However, such trading is neither legal nor likely to stay secret. See our discussion in Sec. IV.
An immediate consequence of lemma 1 is that, given research intensity \( I \), the probability of a takeover increases with \( \alpha \). We define \( Z'(\alpha) \) as the cutoff value of \( Z \) that makes \( L \) just indifferent between taking over and not. The lower is \( Z'(\alpha) \), the more probable is a takeover for a given level of research intensity. Since as \( \alpha \) increases \( L \) can get \( .5Z \) by purchasing fewer shares and making a lower bid, we have the following lemma.

**Lemma 2.** \( Z'(\alpha) \) is strictly decreasing in \( \alpha \).

Having characterized the takeover process, we now consider \( L \)'s optimal choice of monitoring and research intensity. We let \( B(I, \alpha) \) be \( L \)'s expected benefit from research intensity \( I \):

\[
B(I, \alpha) = I \cdot E \{ \max[.5Z - (.5 - \alpha)\pi^*(\alpha) - c_T, 0] \}. \tag{3}
\]

Since \( \pi^*(\alpha) \) is just the expected value of an improvement condition on the takeover's being profitable, we have

\[
E \{ \max[.5Z - (.5 - \alpha)\pi^*(\alpha) - c_T, 0] \} = \{ \alpha E[Z | Z \geq Z'(\alpha)] - c_T \} \cdot \text{pr}\{Z \geq Z'(\alpha)\}.
\]

For all improvements with \( Z \geq Z'(\alpha) \), \( L \) proceeds with a takeover and, on average, receives \( \alpha \) of the value of the improvement less the takeover costs. Since small shareholders allow \( L \) to gain only on its own shares, the expected marginal benefit from an extra unit of research intensity \( I \) is an increasing function of \( \alpha \). An immediate consequence is lemma 3.

**Lemma 3.** \( L \)'s optimal choice of research intensity, \( I^*(\alpha) \), is increasing in \( \alpha \).

As \( \alpha \) increases, \( L \) is willing to pay for a higher probability of finding an improvement and is more likely to take over after finding an improvement of any given value \( Z \). Thus the probability of a value-increasing takeover rises unambiguously with \( \alpha \).

To explore the implications of this result, we write the market value of the firm as

\[
V(\alpha, q) = q + I^*(\alpha) \{ 1 - F[Z'(\alpha)] \} \cdot E[Z | Z \geq Z'(\alpha)]. \tag{4}
\]

The value of the firm is equal to the sum of the expected value of profits under existing management, \( q \), and the expected value of any future improvements in the firm's operating strategy. When \( L \) makes a takeover bid, he pays a premium \( (1 - I^*(\alpha) \cdot \{ 1 - F[Z'(\alpha)] \}) \cdot E[Z | Z \geq Z'(\alpha)] \) over the prevailing market price \( V(\alpha, q) \). A direct consequence of lemmas 1–3 is that this premium falls as \( \alpha \) rises. On the other hand, we have lemma 4.

**Lemma 4.** \( \{ 1 - F[Z'(\alpha)] \} \cdot E[Z | Z \geq Z'(\alpha)] \) is increasing in \( \alpha \).

Conditional on \( L \)'s having drawn an improvement, the expected increase in the firm's profits rises with \( \alpha \). While the larger range of
improvements acted on as \( \alpha \) rises leads to a lower takeover premium, the increased probability of an improvement’s being implemented more than compensates for this. Small shareholders receive a net gain equal to the expected value of the low-value improvements that would not have been made at a lower \( \alpha \).

We have proved the following proposition.

**Proposition 1.** An increase in the proportion of shares held by \( L \) results in a decrease in the takeover premium but an increase in the market value of the firm.

The presence of a large shareholder is a necessary condition for value-increasing takeovers to occur at all. Moreover, a large shareholder is more valuable the larger he is. Any transaction resulting in an increase in the proportion of the firm’s shares owned by \( L \) should therefore be reflected in a higher market price of the shares.

This result implies that a firm repurchasing a large number of its own shares should find it difficult to do so at the current market price if large shareholders do not tender any of their shares. Small shareholders holding on to their shares will enjoy a higher postrepurchase stock price, assuming that a significant number of shares are repurchased and \( \alpha \) rises. In fact, share prices almost always increase when a firm repurchases its own shares via a tender offer. In his 1962–76 sample, Dann (1981) finds that 50 out of 51 firms experienced increases in their estimated market values after repurchasing their own stock. We discuss the issues connected with \( L \)’s trading of shares in much greater detail in Section IV.

The model has another implication. This one concerns the consequences of a change in the administrative and legal costs of making a takeover bid, \( c_T \).

**Proposition 2.** An increase in the legal and administrative costs of a takeover will result in a rise in the takeover premium but a fall in the market value of the firm.

A decrease in \( c_T \) works just like an increase in \( \alpha \), facilitating a greater number of takeovers and causing a net increase in market value, despite a decrease in the takeover premium. This too has been observed in practice. Jarrell and Bradley (1980) analyze the effect of the 1968 Williams Act on takeover premia and the volume of takeover activity. That law is widely believed to have raised the legal and administrative costs of making a tender offer. Jarrell and Bradley find that takeover premia increased after the act while the volume of takeovers decreased.

So far we have focused on takeovers initiated by \( L \) based on his own monitoring efforts. However, the importance of the large shareholder is not limited to his role as monitor and bidder. He can also facilitate takeovers by well-informed outsiders who have no initial
position in the firm. The outsider first approaches the incumbent large shareholder and offers to split the gains from the rise in the price of the latter’s shares. Then the outsider takes over.

Because small shareholders decline all tender offers made by outsiders, \( L \) will recognize that his cooperation is necessary if any gains are to be realized at all. For example, he may sell some of his shares to the outsider for less than their expected value under new management. The outsider can thereby appropriate some of the gains from his superior information after making a tender offer and assuming control. While it is difficult to guess the precise outcome of this bargaining process, the presence of a large shareholder is likely to provide an incentive for outsiders to monitor and evaluate the performance of the incumbent management.

III. Other Mechanisms for Corporate Control

In Section II we assumed that \( L \) changes corporate policy via a cash tender offer for 50 percent of the firm's shares. This, however, is not the only way in which he may effect a change. In this section we discuss the properties of various mechanisms for influencing corporate policy. The focal point of the analysis is the dependence of takeover premia on the efficacy of alternative means of gaining control. When small shareholders have rational expectations, they realize not only that \( L \)'s tender offer yields him a nonnegative profit but also that it yields a higher profit than any other mechanism for gaining control he could have chosen. This will in general convey information about the likely value of the improvement \( L \) is trying to make.

For example, if 50 percent of the shares is enough for \( L \) to gain control and replace the incumbent management, an attempt by him to get more than that number of shares is a signal that he is trying to profit at their expense. We have the following proposition.

**Proposition 3.** Even when the strategy space is expanded to include bids for more than \( .5 - \alpha \) shares, there does not exist any pure strategy sequential equilibrium supported by credible beliefs in which bids for more than \( .5 - \alpha \) of the shares are made with positive probability. The equilibrium in which \( L \) bids \( q + \pi^*(\alpha) \) for \( .5 - \alpha \) shares if and only if \( Z \geq Z'(\alpha) \) remains supportable by credible beliefs.

Unless \( L \) has a good reason (other than the value of \( Z \)) to bid for more than 50 percent of the shares (e.g., he needs \( \frac{2}{3} \) of the shares to exercise effective control or there are tax advantages to owning 80 percent or more of the firm), he will never be better off making such a bid. Taking the analysis above further, we can consider the case in which \( L \) does not need to acquire *any* more shares to replace inefficient management. This would be so if proxy fights were an
effective means of ousting incumbents. A proxy contest is a voting mechanism by which shareholders can change the firm’s board of directors. Since the board of directors has the legal authority to replace the officers of the firm, gaining a majority of seats on the board is tantamount to gaining control of the operating decisions of the corporation.

Suppose that, for a cost \( c_p \), \( L \) can install efficient management via the proxy mechanism.\(^\text{17}\) Then use of this mechanism entails a profit for \( L \) of \( \alpha Z - c_p \). A decision to make a takeover bid \( q + \pi \) instead means that \( .5Z - (.5 - \alpha)\pi - c_T \geq \alpha Z - c_p \) or \( (.5 - \alpha)Z \geq (.5 - \alpha)\pi + (c_T - c_p) \).

If \( c_T > c_p \), then no \( L \) would ever do better than a proxy fight by bidding up to the true value of the posttakeover firm. If \( L \) can profit from \( Z \) on his own shares at a cost \( c_p \) less than \( c_T \), then his decision to buy more shares must reflect a positive profit at the expense of small shareholders. As a result there does not exist an equilibrium in which tender offers are made with positive probability. That we observe successful takeover bids implies either that the proxy mechanism is very costly to operate or that it is not an effective means of obtaining the full value of the improvement.

In fact, both seem to be true. Manne (1965, p. 114) argues that “the most dramatic and publicized of the take-over devices is the proxy fight; it is also the most expensive, the most uncertain, and the least used of the various techniques.” Dodd and Warner (1983) report that, in a 1962–78 sample of 71 proxy contests in which dissidents sought a majority of available board seats, only 18, or 25.4 percent, were successful.

Finally, we consider the possibility that informal negotiations with incumbent management can be used to institute changes. We refer to this means of influencing policy as the “jawboning” mechanism. Like proxy fights, jawboning does not involve the purchase of more shares by \( L \). Unlike proxy fights or tender offers, it is probably practically costless. If \( L \) is able to effect a change by getting incumbent management to go ahead with it, he can realize a gain \( \alpha Z \) at virtually no cost. He will choose to make a tender offer instead only if \( .5Z - (.5 - \alpha)\pi - c_T \geq \alpha Z \), so that \( (.5 - \alpha)(Z - \pi) \geq c_T \). But then \( Z \) must exceed \( \pi \), and tendering cannot be an equilibrium strategy. How then is the takeover mechanism used successfully at all?

While jawboning is less costly than making a tender offer, we suspect that it is also a much less effective means of improving the firm’s

\(^\text{17}\) We have assumed that \( c_p \) is independent of \( \alpha \). If \( c_p \) actually decreases with \( \alpha \), then \( L \) may want to buy more shares before attempting a proxy fight or even before doing any monitoring of the management. Related issues are addressed in Sec. IV.
operating strategy. Shareholder \( L \) may need to replace incumbent management by his own management team in order to get a significant part of the gains from his research. For example, if the competency of the incumbents is in question, they will have to be ousted. Furthermore, \( L \)'s ability to oversee his proposed changes may be limited if he does not own a controlling block of the firm’s shares.

To explore the choice between the takeover bid mechanism and the jawboning mechanism, we analyze the following simple case. All improvements made via informal negotiations are worth only a positive fraction \( \beta < 1 \) of the value of the same improvement when accompanied by a change of management. The proxy mechanism is either unavailable or prohibitively costly.

In this case, \( L \) would never want to bid \( q + \pi \) for \( .5 - \alpha \) shares unless \( .5Z - (.5 - \alpha)\pi - c_T \geq \alpha \beta Z > 0 \). Thus, for \( \beta \neq 0 \), the knowledge that he could have jawboned and received a gain of \( \alpha \beta Z \) conveys information not contained in the fact that the tender offer itself is profitable for \( L \). For \( \beta = 1 \), we are back in the case in which takeovers do not occur in equilibrium. With \( 0 < \beta < 1 \), the analog to condition (2) is

\[
\pi \geq E(Z|Z \geq \frac{.5 - \alpha}{.5 - \alpha \beta} \pi + \frac{1}{.5 - \alpha \beta} c_T).
\]  

(5)

Let \( \pi^*(\alpha, \beta) \) be the minimum \( \pi \) satisfying (5). If we insist on credible beliefs, then we can restrict our attention to the equilibrium in which \( L \) will use the takeover mechanism provided

\[
Z \geq \frac{.5 - \alpha}{.5 - \alpha \beta} \pi^*(\alpha, \beta) + \frac{1}{.5 - \alpha \beta} c_T,
\]

and the jawboning mechanism otherwise.\(^{18}\) For some \( \beta > 0 \) sufficiently less than one and \( c_T \) not too large, the takeover mechanism will be used with positive probability in equilibrium. If the decrease in the value of an improvement made through jawboning, \((1 - \beta)Z\), is sufficiently large, it will pay \( L \) to spend \( c_T \) and \((.5 - \alpha)\pi^*(\alpha, \beta) \) to take over and get half of the full value of a highly valued improvement. Rational small shareholders take this into account when forming their

\(^{18}\) The arguments in the proof of the theorem in Sec. II and the proof of proposition 3 can be used to show that bids for more than \( .5 - \alpha \) shares do not occur with positive probability in equilibrium and that bids for \( .5 - \alpha \) shares are made only at the price \( q + \pi^*(\alpha, \beta) \). To show that jawboning by all types is not an equilibrium when beliefs are credible is essentially the same. Consider the deviation by all types who can make a higher profit by paying \( q + \pi^*(\alpha, \beta) \) for \( .5 - \alpha \) shares than by jawboning. Given that the deviators consist of that set, small shareholders should accept the bid by definition of \( \pi^*(\alpha, \beta) \). Finally, the beliefs from the proof of proposition 3 can be used to show that our equilibrium can be supported by credible beliefs.
expectations of Z if a tender offer is made. Thus $\pi^*(\alpha, \beta)$ and $Z'(\alpha, \beta)$, the value of Z above which L takes over rather than jawbones, are simultaneously determined.

Note that the firm's market value rises with $\alpha$ since it is in L's own interest to take over and produce an extra $(1 - \beta)Z$ of gains more often; that is, for $\alpha_1 < \alpha_2$, $Z'(\alpha_2, \beta) \leq Z'(\alpha_1, \beta)$. The increased profitability of takeovers over jawboning at higher $\alpha$'s along with L's greater share of any improvement provide him with a greater incentive to do research. We prove the following proposition.

**Proposition 4.** When both the jawboning and the takeover mechanisms are available, the market value of the firm, $V(\alpha, q, \beta)$, rises with $\alpha$.

Our analysis suggests that the jawboning mechanism will be used to make less valuable improvements. As a result, resort to the takeover mechanism informs small shareholders that Z is high. This informational effect may help to explain the surprisingly large takeover premia paid to target shareholders. Jensen and Ruback (1983) report excess returns to targets of successful tender offers in the neighborhood of 30 percent.

Finally, it is interesting to note that the option to jawbone may actually be of negative value to the large shareholder. He may be worse off with the option to jawbone because the required bid on the takeovers he continues to execute rises. Small shareholders can also be worse off since, on average, takeovers benefit them more than jawboning does. Thus, in some cases, all shareholders can gain if L publicly commits to not dealing with the incumbent management. A simple example demonstrating this possibility is available from the authors.

**IV. Pretakeover Trading by the Large Shareholder**

So far we have taken L's share of the firm as exogenously fixed. We now discuss ex ante changes in $\alpha$, that is, trading by L before his investment in monitoring takes place. In practice, ex ante trading can be limited by L's inability to claim credibly that he has not already found an improvement.\textsuperscript{19} We examine such trades to illustrate the magnitude of the free-rider effect even in the absence of problems of credibility.

\textsuperscript{19} Our assumption that information is symmetric ex ante may be a valid approximation to situations in which L needs a seat on the board of directors to be privy to information necessary to evaluate the incumbent management. In that case, he can probably increase his stake at lower prices as long as he stays off the board. Moreover, once he has increased his stake, he is more likely to be successful in obtaining the necessary board seat.
As we showed earlier, the size of L’s share affects the value of the firm. Public trades by L change share prices. If he can buy anonymously at the current market price, he can deprive small shareholders of their gains from his larger holding. In fact, L always wants to engage in secret buying, eventually acquiring the whole firm. But the possibility of such trading seems very limited, not the least because he is required by law to disclose his trades.\textsuperscript{20}

Alternatively, L can announce his desired holding and try to trade at the price small shareholders demand. For these trades, he may be charged above the market price since small shareholders anticipate more monitoring and takeovers. Further, we show that if L raises his stake, he will want to do so again. If small shareholders anticipate this, they will hold out and not sell the first time L tries to buy more shares. To raise his stake, he needs to commit to not doing so again. The mechanics of such trades are examined below.

Suppose L starts with an initial position $\alpha_0$, corresponding to the valuation $V(\alpha_0, q)$ of the firm. We assume that he can commit to adjusting his position only once and therefore transacts at the price $V(\alpha, q)$, where $\alpha$ is the final position to which L commits himself. For simplicity we also assume that only the takeover mechanism is available. Shareholder L chooses his final position $\alpha^*$ to maximize his net gain, given by

$$G = \alpha_0[V(\alpha, q) - V(\alpha_0, q)]$$

$$- (I^*(\alpha)[1 - F[Z'(\alpha)]] - I^*(\alpha_0)[1 - F[Z'(\alpha_0)]]))c_T$$

$$- \{c[I^*(\alpha)] - c[I^*(\alpha_0)]\}.$$  

In moving from $\alpha_0$ to $\alpha > \alpha_0$, L enjoys a rise in the value of his initial holdings but must spend more on research and takeover costs to realize that gain. For the rest of this section we make the assumption that $F(Z)$ is atomless with density $dF > 0$ on $(0, Z_{\text{max}}]$. In order to characterize $\alpha^*$, we rewrite the net gain omitting all terms not depending on $\alpha$:

$$\tilde{G}(\alpha) = I^*(\alpha)[1 - F[Z'(\alpha)]])\alpha_0E[Z|Z \geq Z'(\alpha)] - c_T - c[I^*(\alpha)].$$

Given that larger $\alpha$’s bring us closer to the socially efficient level of research and takeover activity, we would hope at the least that L has no incentive to decrease his holdings. Intuition suggests that this is indeed the case. The only potential gains from a reduction in $\alpha$ are the savings on $c(I)$ from less monitoring and on $c_T$ from fewer takeovers. But at $\alpha_0$, L has the same options as he does at any $\alpha < \alpha_0$. By

\textsuperscript{20} This will be true if L holds more than 5 percent of the firm’s shares. We discuss 13D disclosure statements later in this section.
choosing $Z'(\alpha_0) = Z'(\alpha)$ and $I(\alpha_0) = I^*(\alpha)$ for any $\alpha < \alpha_0$, he can be as well off at $\alpha_0$ as at $\alpha < \alpha_0$. Moreover, if there is any surplus at all from the higher but optimally chosen levels of research intensity and takeover activity at $\alpha_0$, $L$ will be strictly better off if he does not sell. We have the following proposition.

**Proposition 5.** Even if $L$ can sell $(\alpha_0 - \alpha)$ shares at the price $V(\alpha, q)$ for any $\alpha < \alpha_0$, he will never gain by doing so. Moreover, if $L$ is taking over with positive probability at $\alpha_0$, he is strictly better off not selling.

The important asymmetry between the large shareholder's buying and selling in this model is that selling can serve only to decrease the opportunity set. It increases the required bid and reduces takeover activity at any level of research intensity. It does not follow, however, that $L$ can make a large net gain by increasing his holdings dramatically. Even if he can commit to adjusting his holdings only once, small shareholders will free ride on the gains from his increased expenditure on research and more frequent takeovers at a higher $\alpha$. But as long as $L$ already has substantial holdings, in the sense that he is taking over with a positive probability at $\alpha_0$, he always wants to acquire at least a little more.

**Proposition 6.** If $L$ is taking over with positive probability at $\alpha_0$ and can buy $(\alpha - \alpha_0)$ shares at the price $V(\alpha, q)$, then $\partial G/\partial \alpha(\alpha_0) > 0$. A small purchase always results in a net gain for $L$.

The idea behind proposition 6 is the following. If $L$ could just appropriate his share $\alpha$ of the improvements of value just under $Z'(\alpha)$, he could still make a strictly positive profit net of takeover costs by implementing those improvements. But because he pays $E[Z | Z \geq Z'(\alpha)]$ rather than $Z'(\alpha)$, he does not get his full share of the marginal improvements. Ex ante buying of shares is a means for him to lower his required takeover bid by making it in his own interest to take over for a wider range of improvements. This allows him effectively to get his original share $\alpha$ of the additional improvements he makes when he owns more of the firm. In a neighborhood of the original $Z'$ these gains exceed his cost of taking over, $c_T$.

We have shown that, if $L$ already has a substantial position and can commit to a ceiling on his holdings, he will increase his holdings. But what happens if ownership is diffuse and $L$ is not too large? Unfortunately, the answer is that, if trading is public, it is not in the interest of a relatively small party with access to the improvement technology [$c(I), F(Z)$] to acquire a substantial position in the firm. We make this precise in proposition 7.

**Proposition 7.** If $L$ initially holds $\alpha_0 < c_T/Z_{\text{max}}$ of the firm's shares and can make one purchase of $\alpha - \alpha_0$ shares at the price $V(\alpha, q)$, he will be made strictly worse off by moving to any position $\alpha$ from which he will take over with positive probability.
If ownership of the firm is sufficiently diffuse, a large block of shares will be assembled only if some degree of anonymous trading is present. It is therefore important to discuss the scope for secret trading as a means of creating large shareholdings.

By law, anyone holding more than 5 percent of a firm's shares is required to file a 13-D disclosure form with the Securities and Exchange Commission (SEC) within 10 days of passing the 5 percent mark. Among other items, this filing includes the name and background of each acquiring individual or of any individuals who control the acquiring corporation. The acquirer must also file an amended schedule 13-D if there is any "material change," such as purchase or sale of shares, in the information reported in an earlier filing.

While the 13-D must be filed within 10 days of passing the 5 percent holding, it is common for filers to have accumulated significantly more than 5 percent at the time of the filing.\(^{21}\) This suggests that 13-D's do not eliminate all secret buying after the 5 percent mark. At the same time, the 13-D filing may not be the market's first news of \(L\)'s accumulation of shares. For example, it seems likely that the increased volume of trading occurring as \(L\) is building up his position would signal to the market that something is happening. Such signals would restrict secret buying.

In addition to secret buying, there are other explanations for the prevalence of large shareholders. One simple explanation is that large shareholders are individuals who have always held a substantial portion of the firm. A corporate life-cycle story, in which firms are initially closely held and become less so as they grow and require more capital, is consistent with finding large shareholders at any point in time, despite the free-rider problem.

Our results also suggest that, once a large block of shares is assembled, the position is unlikely to be dissipated. Even after \(L\) (or his heir) has outgrown his usefulness as a monitor, it will probably be in his interest to wait until someone who can monitor effectively expresses interest in his shares. For if he sells his shares on the open market, he (along with everyone else) loses that part of the firm's value that comes from the possibility of a value-increasing takeover. If that possibility materializes, the incumbent large shareholder and the newcomer can profit by splitting the gains on the incumbent's shares. This suggests that large blocks of shares will tend to be passed on rather than dissipated.

\(^{21}\) See Mikkelsen and Ruback (1984). We think that the ownership of shares significantly in excess of 5 percent at the time of the 13-D filing indicates one of two things. Either the acquirer has bought out some incumbent shareholders with large blocks of stock or he has bought very quickly on the open market during the 10 days before he must file the 13-D.
V. Dividends as a Subsidy to Large Shareholders

We now introduce tax considerations and allow large and small shareholders to have different relative preferences for dividends and capital gains. While small shareholders are likely to prefer capital gains, large shareholders probably favor dividends. Many large shareholders are corporations themselves. As such, they are allowed to exclude 85 percent of dividends from income tax and so face an effective dividend tax rate of $0.46 \times 0.15 = 0.069$. This contrasts with the corporate capital gains tax rate of 28 percent. Even when the timing of capital gains realizations is taken into account, corporations seem to have a decided preference for dividends over capital gains (see Poterba and Summers 1984). On the other hand, personal investors pay their full marginal income tax rates on dividends but can exclude 60 percent of long-term capital gains from tax. Thus they usually prefer capital gains to dividends.

To explain why most corporations pay positive dividends though they are predominantly owned by personal investors is a long-standing puzzle in corporate finance. Some finance economists, while recognizing that corporate shareholders prefer dividends, do not think them to be important enough (see, e.g., Black 1976, p. 6). Since we believe that large shareholders can significantly influence share valuation, we explore the possibility that they are compensated for monitoring through dividends.

In our framework, if the firm’s dividend policy does not put sufficient weight on $L$’s preferences, it may be in his interest to sell his shares on the open market. Because this could significantly lower the value of the firm to small shareholders, they may favor a compromise dividend policy, which may be viewed as a side payment to $L$ for holding the firm’s shares. Though they cannot subsidize $L$’s monitoring directly, small shareholders can support a financial policy that favors the large shareholder. They rely on the fact that, if $L$ stays, it is in his own interest to monitor.

We illustrate this compensatory role of dividends in a simple intertemporal model. At time 0, a proportion $\alpha_0 < .5$ of the firm’s shares is held by $L$ and the rest by a fringe of atomistic personal investors. Also at time 0, the firm commits to the payment of a dividend of size $d$ at time 1 as long as $L$ is still holding the firm’s shares. Immediately after $d$ is set, trading of shares takes place. Between time 0 and time 1, $L$ monitors the management. For a cost $c(\tilde{I})$ paid at time 0, $L$ can purchase a probability $\tilde{I}$ of finding an improvement of value $\bar{Z}$ at time 1. To simplify matters, we take $\tilde{I}$ and $\bar{Z}$ as exogenous and fixed. Of course, $L$ can choose not to monitor, in which case $I = 0$, $c(0) = 0$, and no improvement is in sight. At time 1, the dividend is paid, and then $L$
takes over if he has found the improvement. The administrative and legal costs of a takeover are equal to c_T.

If management is found to be satisfactory, we assume that neither L nor anyone else is needed to monitor further. In that event, we assume that L sells his shares on the open market. We sidestep the modeling of the transfer of large blocks of shares from one large shareholder to the next.

We assume that, at time 0, the firm’s discounted after-profits-tax stream of income is equal to one. At time 0, L’s unrealized capital gains are assumed to be zero. Between time 0 and time 1, the value of the firm’s profit stream is expected to rise at the interest rate R. At time 1, this value falls by the amount of the dividend paid. Thus, if no improvement is made, the firm’s profits will be worth 1 + R − d at time 1.

But the value of the firm’s profit stream will not be equal to a shareholder’s valuation of the firm’s shares if dividends and capital gains are taxed. To capture personal investors’ tax preference for capital gains and corporate investors’ tax preference for dividends, we make two simple assumptions: corporate investors value a dollar of dividends at $1 but a dollar of realized capital gains at $\eta_1$; personal investors value a dollar of capital gains at $1$ but a dollar of dividends at $\eta_2$. The values of the parameters $\eta_1$ and $\eta_2$ may include considerations other than taxes but are less than one to incorporate the conventional wisdom.

Within the context of this model, we discuss small shareholders’ optimal choice of dividend policy. While a positive dividend will not be optimal for all values of the parameter vector $[\alpha_0, \bar{I}, \bar{Z}, \eta_1, \eta_2, R, c_T, c(\bar{I})]$, we give a range of numerical examples to illustrate the point that small shareholders are often better off when dividends are paid.

Before presenting the optimization problem for small shareholders, we need to determine the tender price they receive if L finds the improvement. Since it is in L’s interest to pay all profits in dividends after he takes over, there may be scope for dilution. That is, small shareholders might be paid $\eta_2(1 + R − d + \bar{Z})$ rather than $(1 + R − d + \bar{Z})$ per share.

We do not allow for such dilution in the model presented here. Indeed, there may be substantial legal obstacles to such dilution by L. Large increases in posttakeover dividend policies typically bring lawsuits from corporate bondholders as well as from minority sharehold-

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22 Once dividends are set to zero, it would probably be in L’s interest to sell his shares in order to avoid a future stream of capital gains. This assumes that L can get the after-tax return $R$ elsewhere.
ers (see Fox and Fox 1976). Arguably, such dilution is also limited by the fact that small shareholders could always sell their shares to corporations and tax-free institutions after a takeover. To keep the model simple, we have ignored corporations (except for $L$) and tax-free institutions. For the same reason, we do not consider ex ante buying by $L$.

If $L$ monitors the management at intensity $I$ and a dividend $d$ is paid, small shareholders' valuation of the firm as of time 0 is
\[
p_s(d, I) = \frac{\eta_s d + I[1 + (R - d) + \bar{Z}] + (1 - I)(1 + R - d)}{1 + R}.
\]

If $L$ does not draw an improvement, small shareholders retain control of the firm and pay no future dividend, so they value the profits at $1 + R - d$ at time 1.

Small shareholders favor the dividend policy that maximizes $p_s[d, I(d)]$. While they would like to set $d = 0$, this may cause $L$ to sell out at time 0, eliminating a chance of a takeover. Let $d_{\min}$ be the lowest dividend at which $L$ retains enough shares that it pays him to monitor with intensity $\bar{I}$. Small shareholders' optimal choice of dividends, then, will be either $d_{\min}$ or zero, depending on whether $p_s(d_{\min}, \bar{I}) \geq p_s(0, 0) = 1$. Note that $d_{\min}$ may be equal to zero. In that case, as well as the case in which $p_s(0, 0) = 1 > p_s(d_{\min}, \bar{I})$, no equilibrium with positive dividends exists. From now on, we consider the case in which $d_{\min}$ is positive.

We now characterize the minimum level of dividends required to make it in $L$'s interest to hold onto enough of his shares that it pays for him to monitor. In particular, it will turn out that, at $d = d_{\min}$, $L$ does not sell any shares. Assuming transactions are public, his net gain from selling his entire position is
\[
G(d, \alpha_0) = \alpha_0 \left\{ 1 - \frac{d}{1 + R} - \frac{\bar{I}(1 + R - d)}{1 + R} \right. \\
\left. - \frac{(1 - \bar{I})[1 + \eta_1(R - d)]}{1 + R} \right\} - \left[ \bar{I}\left(\frac{\alpha_0 \bar{Z} - c_T}{1 + R}\right) - c(\bar{I}) \right].
\]

The net gain from selling consists of two terms. The first term will be positive at any $d < R$. It reflects the fact that, if $L$ sells out to small shareholders, they will pay no dividends. Thus the shares are worth more to them than to $L$, who may have to pay taxes on capital gains. The second term is negative by assumption. It represents the lost surplus from monitoring and takeover activity that $L$ enjoyed on his original position.

A few points are worth noting here. First, $L$'s surplus from monitoring must be positive or else it is never in the interest of small
shareholders to bribe him to stay. Recall that small shareholders are not permitted to pay $L$ directly for his monitoring; they can only subsidize his stay with the firm. Second, since the first term in (6) is decreasing in $d$, a higher dividend decreases $L$'s net gain from selling. Finally, the optimal dividend will always satisfy $d^* < R$. When $d \geq R$, there is no capital gains tax penalty and $L$ will never wish to sell. Thus $d_{\min}$ must be less than $R$.

We now show that if $d_{\min}$ is greater than zero, it is equal to the dividend level $\bar{d}$ that solves $G(\bar{d}, \alpha_0) = 0$. First, we show that, for any $\bar{d} < \bar{d}$, $L$ would rather sell out completely at a price $p_i(0, 0) = 1$ than keep any portion of his initial position and monitor. If $G(\bar{d}, \alpha_0) = 0$ and $\bar{d} < \bar{d}$, then $G(\bar{d}, \alpha_0) > 0$ because $\partial G/\partial d < 0$. Furthermore, $G(\bar{d}, \bar{\alpha}) > 0$ for any $\bar{\alpha} < \alpha_0$. This can be seen as follows. We can rewrite $G(d, \alpha)$ as

$$G(d, \alpha) = \alpha \left\{ \frac{1 - d - \bar{I}(1 + R - d + \bar{Z}) - (1 - \bar{I})[1 + \eta_1(R - d)]}{1 + R} \right\} \frac{\bar{I} \cdot c^r}{1 + R} + c(\bar{I}).$$

If the term in braces is nonnegative, then $G(\bar{d}, \alpha)$ is positive for all $\alpha$. If that term is negative, then $G(\bar{d}, \bar{\alpha}) > G(\bar{d}, \alpha_0) > 0$. In either case, we have $G(\bar{d}, \bar{\alpha}) > 0$. So, for any $\bar{d} < \bar{d}$, $L$ prefers selling out completely to keeping any portion of his initial position.

Now suppose dividends are set equal to $\bar{d}$. Then the term in braces above is negative. So $G(\bar{d}, \bar{\alpha}) > G(\bar{d}, \alpha_0) = 0$ for any $\bar{\alpha} < \alpha_0$. That is, small shareholders realize that once $L$ sells anything he will sell everything. Hence he cannot sell any shares at any price above $p_i(0, 0) = 1$.

But then, since $G(\bar{d}, \alpha_0) = 0$, he will be just as well off to remain at $\alpha_0$ and monitor. This shows that $d_{\min} = \bar{d}$.

Small shareholders set $d^* = d_{\min}$ as long as $L$ is worth keeping as a monitor. He is worth keeping if $p_i(d_{\min}, \bar{I}) > p_i(0, 0) = 1$ or, equivalently, if $\bar{I} \cdot \bar{Z} > (1 - \eta_2)d_{\min}$ (the dividend tax small shareholders pay is smaller than the expected benefit from monitoring by the large shareholder).

To reiterate, the conditions on the parameter vector $[\alpha_0, \bar{I}, \bar{Z}, \eta_1, \eta_2, R, c^r, c(\bar{I})]$ required to get an equilibrium of the type we have described are (1) $G(0, \alpha_0) > 0$, that is, $d_{\min}$ is positive; (2) $\bar{I}(\alpha_0 \bar{Z} - c^r)/(1 + R) - c(\bar{I}) > 0$; and (3) $\bar{I} \cdot \bar{Z} > (1 - \eta_2)d_{\min}$, where $d_{\min}$ satisfies $G(d_{\min}, \alpha_0) = 0$.

Table 1 contains a range of numerical examples in which plausible parameter values give rise to an equilibrium of this type. In all cases, small shareholders favor strictly positive dividends.

Our view of dividends has the advantage of accurately predicting
TABLE 1
Numerical Examples: Optimal Dividends

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<th>Example Number</th>
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<th>$\bar{I}$</th>
<th>$\bar{Z}$</th>
<th>$c_T$</th>
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coownership of firms by corporate and personal shareholders. We argue that the analysis of a firm’s financial policy requires that attention be paid to externalities generated by coownership. For instance, unlike in the world with only personal investors, open market share repurchases are not equivalent to dividends since they may change the share of the firm held by the large shareholder.

Appendix

Proof of Theorem

Consider any pure strategy sequential equilibrium in which all $L$ types with $Z \geq Z'(\pi')$ bid $q + \pi'$ and those with $Z < Z'(\pi')$ do not bid. (This includes the case in which no $L$ bids, i.e., $Z' > Z_{\text{max}}$.) First, note that all pure strategy equilibria must be of this form since there can be only one equilibrium bid given that bidders will always be better off making the minimum acceptable bid. Second, notice that we must have $\pi' \geq \pi^*(\alpha)$ because it is rational for small shareholders to accept a bid only if equation (2) is satisfied. Finally, note that for any $\pi' > \pi^*(\alpha)$ or $Z' > Z_{\text{max}}$ there is a unique set $K$ of deviators—consisting of those potential bidders who can make a nonnegative profit by taking over with a bid of $q + \pi^*(\alpha)$—who would be better off bidding $q + \pi^*(\alpha)$ if that bid were accepted. Moreover, if small shareholders believe that the set of deviators is $K$, then they will accept the bid, by definition of $\pi^*(\alpha)$. But then a bid of $q + \pi'$ or $Z' > Z_{\text{max}}$ is inconsistent with equilibrium if we insist on credible out-of-equilibrium beliefs in the sense of Grossman and Perry (1984).

To see that our $q + \pi^*(\alpha)$ equilibrium can be supported by credible out-of-equilibrium beliefs, consider the following beliefs. For any bid $q + \pi \neq q + \pi^*(\alpha)$, let small shareholders believe that $Z$ is a random draw from the distribution $F(Z)$ restricted to the set of $L$ types who would be better off making that bid if it is accepted than they would be playing according to their equilibrium strategy, that is, bidding $q + \pi^*(\alpha)$ or not bidding. If there are no $L$ types who are better off deviating, then small shareholders just believe that $Z$ is a random draw from the entire distribution $F(Z)$. These beliefs are credible since deviation would be rational only if the beliefs lead to acceptance of the bid, and so we have assigned beliefs consistent with the only candidate for the set $K$. Also, the $\pi^*(\alpha)$ equilibrium is supported by these beliefs. Any bid $q + \pi$
$q + \pi^*(\alpha)$ will be rejected since, if it is accepted, all those making a non-negative profit bidding $q + \pi$ will deviate, but no $\pi$ less than $\pi^*(\alpha)$ satisfies (2). Further, no $L$ would ever bid $q + \pi > q + \pi^*(\alpha)$ as long as $q + \pi^*(\alpha)$ is accepted.

**Proof of Lemma 1**

$\alpha_2 > \alpha_1$ implies $(1 - 2\alpha_2)\pi + 2c_T < (1 - 2\alpha_1)\pi + 2c_T$ for any $\pi > 0$. Thus any $\pi$ satisfying (2) for $\alpha = \alpha_1$ will satisfy (2) for $\alpha = \alpha_2$. Since $\pi^*(\alpha)$ is the minimum $\pi$ satisfying (2), we must have $\pi^*(\alpha_2) \leq \pi^*(\alpha_1)$.

**Proof of Lemma 2**

$.5Z'(\alpha) - (.5 - \alpha)\pi^*(\alpha) - c_T = 0$ implies $Z'(\alpha) = (1 - 2\alpha)\pi^*(\alpha) + 2c_T$. Since $\pi^*(\alpha)$ decreases in $\alpha$ and $\alpha < .5$, $(1 - 2\alpha)\pi^*(\alpha)$ is strictly decreasing in $\alpha$.

**Proof of Lemma 3**

$I^*(\alpha) = \arg\max_{I \in [0, 1]} B(I, \alpha) - c(I)$. Since $\frac{\partial^2 B(I, \alpha)}{\partial I^2} = 0$ and $c''(I) > 0$, $I^*$ increases with $\frac{\partial B}{\partial I} = E \max[.5Z - (.5 - \alpha)\pi^*(\alpha) - c_T, 0]$. But $.5Z - (.5 - \alpha)\pi^*(\alpha) - c_T$ increases with $\alpha$ for each realization of $Z$. Thus $\frac{\partial B}{\partial I}$ increases with $\alpha$ and so does $I^*$.

**Proof of Lemma 4**

Suppose $\alpha_2 > \alpha_1$. Then $Z'(\alpha_2) < Z'(\alpha_1)$. Write

$$E[Z|Z \geq Z'(\alpha_2)] = E[Z|Z \geq Z'(\alpha_1)] \cdot \Pr[Z \geq Z'(\alpha_1)|Z \geq Z'(\alpha_2)]$$

$$+ E[Z|Z \in [Z'(\alpha_2), Z'(\alpha_1)] \cdot \Pr[Z \in [Z'(\alpha_2), Z'(\alpha_1)]|Z \geq Z'(\alpha_2)].$$

Then

$$\{1 - F[Z'(\alpha_2)]\} E[Z|Z \geq Z'(\alpha_2)] = E[Z|Z \geq Z'(\alpha_1)]\{1 - F[Z'(\alpha_1)]\}$$

$$+ E[Z|Z \in [Z'(\alpha_2), Z'(\alpha_1)] \cdot \Pr[Z \in [Z'(\alpha_2), Z'(\alpha_1)].$$

Since the second term in the last expression is nonnegative, we are done.

**Proof of Proposition 2**

Let $c_T < c_T^2$. $\pi^*(\alpha, c_T)$ is the minimum $\pi$ satisfying equation (2) for cost $c_T$. But if $\pi$ satisfies equation (2) for $c_T = c_T^2$, then $\pi$ must also satisfy equation (2) for $c_T = c_T^1$. Since $\pi^*(\alpha, c_T^1)$ is the minimum of all these, we must have $\pi^*(\alpha, c_T^1) \leq \pi^*(\alpha, c_T^2)$.

Now we show that $V(\alpha, q, c_T^1) \geq V(\alpha, q, c_T^2)$. Since $\pi^*(\alpha, c_T^1) \leq \pi^*(\alpha, c_T^2)$ and $c_T^1 < c_T^2$, we have that

$$\frac{\partial B}{\partial l}(c_T^1) = E \max[.5Z - (.5 - \alpha)\pi^*(\alpha, c_T^1) - c_T^1, 0]$$

$$\geq E \max[.5Z - (.5 - \alpha)\pi^*(\alpha, c_T^2) - c_T^2, 0] = \frac{\partial B}{\partial l}(c_T^2).$$
Thus $I^*(\alpha, \epsilon_T^1) \geq I^*(\alpha, \epsilon_T^2)$.

Finally,

$$Z'(\alpha, \epsilon_T^1) = (1 - 2\alpha)\pi^*(\alpha, \epsilon_T^1) + 2\epsilon_T^1 < (1 - 2\alpha)\pi^*(\alpha, \epsilon_T^2) + 2\epsilon_T^2 = Z'(\alpha, \epsilon_T^2).$$

The argument from the proof of lemma 4 can be applied directly to show that

$$\{1 - F[Z'(\alpha, \epsilon_T^1)|E[Z] \geq Z'(\alpha, \epsilon_T^1)] \geq \{1 - F[Z'(\alpha, \epsilon_T^2)|E[Z] \geq Z'(\alpha, \epsilon_T^2)] \}.$$

The takeover premium rises with $\epsilon_T$ since $Z'$ increases with $\epsilon_T$ and $I^*$ falls.

**Proof of Proposition 3**

Suppose that bids for more than $0.5 - \alpha$ shares occur with positive probability in equilibrium. Choose $\pi(\alpha, x)$ with $x > 0.5$ such that the bid $q + \pi(\alpha, x)$ for $x - \alpha$ shares is accepted and such that offers for more than $0.5 - \alpha$ shares at or above the price $q + \pi(\alpha, x)$ occur with positive probability.

First, we show that there must be a positive probability of bids for more than $0.5 - \alpha$ shares at $q + \pi(\alpha, x)$ or above for which $Z \neq \pi(Z)$. This follows because, if $Z = \pi(Z)$ for an interval of $Z$'s, then all $Z$'s in the interval would be better off bidding the lowest $Z$ (or any lower $Z$) in the interval, and hence $\pi(Z) = Z$ could not be an equilibrium. This is true regardless of how many shares these types are bidding for since moving from $\pi = Z$ for $x - \alpha$ shares to $\pi < Z$ for $y - \alpha$ shares is profitable for any $y$.

With a positive probability of $Z \neq \pi$ we must also have a positive probability of bids with $Z < \pi$ or else small shareholders would be better off rejecting bids of $q + \pi(\alpha, x)$ and above for more than $0.5 - \alpha$ shares. But we show that, if out-of-equilibrium beliefs are credible, then no $L$ would ever make a bid of $q + \pi(\alpha, x)$ or above for more than $0.5 - \alpha$ shares in which $\pi(Z) > Z$.

Consider the following deviation. Let all those bidding $q + \pi(\alpha, x)$ and above for more than $0.5 - \alpha$ shares, who are also bidding $\pi(Z) > Z$, instead bid for $0.5 - \alpha$ shares at the price $q + \pi(\alpha, x)$. In fact, all these $L$ types must also have $\pi(\alpha, x) \leq \pi(Z)$, or else they would have bid $q + \pi(\alpha, x)$ for $x - \alpha$ shares. These types are better off paying the same or less and buying fewer shares. In addition, there may be other $L$ types who are better off buying $0.5 - \alpha$ shares at the price $q + \pi(\alpha, x)$ than they are playing their proposed equilibrium strategy. Let $K$ be the set of the $Z$'s who wish to deviate if the bid $q + \pi(\alpha, x)$ for $0.5 - \alpha$ shares is accepted. For the reason above, any $L$ type in the set $K$ must have $\pi(\alpha, x) \leq \pi(\alpha, x)$. But then if small shareholders believe that the deviators are from the set $K$, they will accept the bid, thus making deviation profitable.

Hence the only credible beliefs lead to acceptance of the bid $q + \pi(\alpha, x)$ for $0.5 - \alpha$ shares. But then the proposed equilibrium cannot be an equilibrium since there will be a positive probability of bids at $q + \pi(\alpha, x)$ or above for more than $0.5 - \alpha$ shares in which $Z > \pi$ and a zero probability of such bids with $Z < \pi$. So no equilibrium exists in which bids for more than $0.5 - \alpha$ shares are made with positive probability.

To see that the equilibrium of Section II is still supportable by credible beliefs, consider the following beliefs. For any bid for $0.5 - \alpha$ or more shares, let small shareholders believe on seeing that bid that $Z$ is a random draw from the distribution $F(Z)$ restricted to the set of $Z$'s who would be better off making that bid if it is accepted than they would be playing according to their equilibrium strategy for the equilibrium of Section II, that is, bidding $q + \pi^*(\alpha)$ for $0.5 - \alpha$ shares or not bidding. If there are no $Z$'s who are better off deviating, then small shareholders just believe that $Z$ is a random draw from
the entire distribution $F(Z)$. These beliefs are credible since deviation would be rational only if the beliefs at the deviation lead to acceptance of the bid. Thus we have assigned beliefs consistent with the only candidate for the set $K$.

Now we show that these beliefs support the equilibrium of Section II by showing that any bid for $.5 - \alpha$ or more shares that would be more profitable for some $Z \in (0, Z_{\text{max}}]$ than the proposed equilibrium strategy would be rejected if small shareholders formed beliefs as above.

First, consider deviations to bids at $q + \pi* (\alpha)$ or above. Clearly, $L$ would never bid more than $q + \pi* (\alpha)$ for $.5 - \alpha$ shares, but he might wish to pay more if he could purchase more shares. But he would want to do this only if he could make a positive profit on the shares that he purchases (i.e., $Z > \pi$). But small shareholders will reject any bid for which they believe $Z > \pi$ with probability one. If $L$ bids exactly $q + \pi* (\alpha)$ for more than $.5 - \alpha$ shares, then small shareholders know that $Z = \pi* (\alpha)$ and reject the bid if $\pi* (\alpha) < Z_{\text{max}}$.

Second, consider deviations to bids $q + \pi < q + \pi* (\alpha)$. Since all those who could make a positive profit by making a successful bid at $q + \pi$ for $.5 - \alpha$ shares would be better off deviating and $\pi* (\alpha)$ is the minimum $\pi$ satisfying (2), any bid less than $q + \pi* (\alpha)$ for $.5 - \alpha$ shares will be rejected.

Finally, suppose $L$ bids $q + \pi < q + \pi* (\alpha)$ for $Y - \alpha > .5 - \alpha$ shares. A necessary condition for him to want to deviate is $YZ - (Y - \alpha)\pi - c_T \geq .5Z - (.5 - \alpha)\pi* (\alpha) - c_T$, which is equivalent to $Z \geq (Y - .5)^{-1}[(Y - \alpha)\pi - (.5 - \alpha)\pi* (\alpha)]$. It is also necessary that $L$ can do better bidding $q + \pi$ for $Y - \alpha$ shares than by not bidding at all. Further, these two conditions are jointly sufficient so that the set of deviators is the intersection of the two sets of $L$ types satisfying these conditions and is of the form $\{Z \in (0, Z_{\text{max}}] | Z \geq Z'(Y, \pi)\}$. But also, for any $Y > .5$ and any $\pi$ for which some $Z < \pi$ can make a positive profit bidding $q + \pi$ for $Y - \alpha$ shares, the set of $L$ types who can make a positive profit bidding $q + \pi$ for $Y - \alpha$ shares is a subset of the set of $L$ types who could make a positive profit bidding $q + \pi$ for $.5 - \alpha$ shares. This means that the set of deviators who would be better off bidding $q + \pi$ for $Y - \alpha$ shares than playing their strategy from the equilibrium in Section II is a subset of the form $\{Z \in (0, Z_{\text{max}}] | Z \geq Z'(Y, \pi)\}$. But this is the set of $L$ types who could make a positive profit bidding $q + \pi$ for $.5 - \alpha$ shares. We have $E(Z|Z)$ deviations to $q + \pi$ for $Y - \alpha$ shares) $\geq E[Z](.5Z - (.5 - \alpha)\pi - c_T \geq 0)$. But given that $\pi* (\alpha)$ is the minimum $\pi$ satisfying (2), we must have $\pi < E[Z](.5Z - (.5 - \alpha)\pi - c_T \geq 0) \leq E(Z|Z$ deviations to $q + \pi$ for $Y - \alpha$ shares). Hence any bid of less than $q + \pi* (\alpha)$ will be rejected when beliefs are as we specified them. This completes the proof.

**Proof of Proposition 4**

We have

$$V(\alpha, q, \beta) = q + I*(\alpha, \beta)(F[Z'(\alpha, \beta)] \cdot \beta \cdot E[Z|Z \leq Z'(\alpha, \beta)]$$

$$+ \{1 - F[Z'(\alpha, \beta)]\}E[Z|Z \geq Z'(\alpha, \beta)].$$

First, we show that $Z'(\alpha, \beta)$ and $\pi* (\alpha, \beta)$ are decreasing in $\alpha$. $\pi* (\alpha, \beta)$ is the minimum $\pi$ satisfying (5). Suppose $\alpha_1 < \alpha_2$. We show that any $\pi$ satisfying (5) for $\alpha = \alpha_1$ also satisfies that condition for $\alpha = \alpha_2$. We have

$$\frac{\partial}{\partial \alpha} \left[ \frac{(0.5 - \alpha)^2}{.5 - \alpha \beta} + \frac{c_T}{.5 - \alpha \beta} \right] = \frac{.5(\beta - 1)\pi + \beta c_T}{.5 - \alpha \beta} < 0$$

$$\Leftrightarrow .5(\beta - 1)\pi + \beta c_T < 0.$$
But this last inequality must hold for any $\pi$ satisfying (5) for any $\alpha$. Any $\pi$ satisfying (5) must satisfy

$$\pi \geq \frac{(5 - \alpha\pi)}{.5 - \alpha\beta} + \frac{c_T}{.5 - \alpha\beta}$$

$$\Rightarrow (5 - \alpha\beta)\pi \geq (5 - \alpha)\pi + c_T$$

$$\Rightarrow \alpha(1 - \beta)\pi \geq c_T \Rightarrow \alpha(1 - \beta)\pi \geq \beta c_T \Rightarrow .5(1 - \beta)\pi > \beta c_T$$

for any $\alpha < .5$. Hence, $\pi^*(\alpha_2, \beta) \leq \pi^*(\alpha_1, \beta)$.

It follows that

$$Z'(\alpha_2, \beta) = \frac{5 - \alpha_2}{.5 - \alpha_2\beta} \pi^*(\alpha_2, \beta) + \frac{c_T}{.5 - \alpha_2\beta}$$

$$\leq \frac{5 - \alpha_2}{.5 - \alpha_2\beta} \pi^*(\alpha_1, \beta) + \frac{c_T}{.5 - \alpha_2\beta}$$

$$< \frac{5 - \alpha_1}{.5 - \alpha_1\beta} \pi^*(\alpha_1, \beta) + \frac{c_T}{.5 - \alpha_1\beta} = Z'(\alpha_1, \beta).$$

Next we show that $Z'(\alpha_2, \beta) < Z'(\alpha_1, \beta)$ implies that

$$(F[Z'(\alpha_2, \beta)] \cdot \beta \cdot E[Z|Z \leq Z'(\alpha_2, \beta)] + \{1 - F[Z'(\alpha_2, \beta)]\} \cdot E[Z|Z \geq Z'(\alpha_2, \beta)])$$

$-(F[Z'(\alpha_1, \beta)] \cdot \beta \cdot E[Z|Z \leq Z'(\alpha_1, \beta)] + \{1 - F[Z'(\alpha_1, \beta)]\} \cdot E[Z|Z \geq Z'(\alpha_1, \beta)]) \geq 0.$

For this difference is just equal to

$$\text{pr}[Z \in [Z'(\alpha_2, \beta), Z'(\alpha_1, \beta)]] \cdot (1 - \beta) \cdot E[Z|Z \in [Z'(\alpha_2, \beta), Z'(\alpha_1, \beta)]]$$

which is clearly nonnegative.

Finally, we show that $I^*(\alpha_1) \leq I^*(\alpha_2)$. The expected benefit from research intensity $I$ is

$$B(I, \alpha, \beta) = I \cdot E \max[.5Z - (.5 - \alpha)\pi^*(\alpha, \beta) - c_T, \alpha\beta Z, 0].$$

Since $(.5 - \alpha)\pi^*(\alpha, \beta)$ decreases in $\alpha$ and $\alpha\beta Z$ increases in $\alpha$, $\partial B/\partial I$ increases in $\alpha$. Hence $I^*(\alpha_1) \leq I^*(\alpha_2)$.

\textbf{Proof of Proposition 5}

Let $\alpha < \alpha_0$:

$$\tilde{G}(\alpha) - \tilde{G}(\alpha_0) = I^*(\alpha)(1 - F[Z'(\alpha)])\alpha_0 E[Z|Z \geq Z'(\alpha)] - c_T$$

$$- I^*(\alpha_0)(1 - F[Z'(\alpha)])\alpha_0 E[Z|Z \geq Z'(\alpha_0)] - c_T$$

$$+ \{c[I^*(\alpha)] - c[I^*(\alpha_0)]\}.$$

First, we have:

$$\{1 - F[Z'(\alpha)]\}\alpha_0 E[Z|Z \geq Z'(\alpha)] - c_T$$

$$- \{1 - F[Z'(\alpha_0)]\}\alpha_0 E[Z|Z \geq Z'(\alpha_0)] - c_T$$

$$= - \text{pr}[Z \in [Z'(\alpha_0), Z'(\alpha)]]\alpha_0 E[Z|Z \in [Z'(\alpha_0), Z'(\alpha)] - c_T] \leq 0.$$
This last inequality follows because if \( Z'(\alpha_0) \geq Z_{\text{max}} \), then \( \Pr\{Z \in [Z'(\alpha_0), Z'(\alpha)]\} = 0 \). If \( Z'(\alpha_0) < Z_{\text{max}} \), we have \( \pi^*(\alpha_0) > Z'(\alpha_0) \) and thus

\[
0.5Z'(\alpha_0) - (0.5 - \alpha_0)\pi^*(\alpha_0) - c_T = 0 \Rightarrow \alpha_0Z'(\alpha_0) - c_T > 0
\]

\[
\Rightarrow \alpha_0 E\{Z|Z \in [Z'(\alpha_0), Z'(\alpha)]\} - c_T > 0.
\]

Hence

\[
\hat{G}(\alpha) - \hat{G}(\alpha_0) \leq [I^*(\alpha) - I^*(\alpha_0)][1 - F[Z'(\alpha)]]\{\alpha_0 E\{Z|Z \geq Z'(\alpha)\} - c_T\}
\]

\[
+ \{c[I^*(\alpha_0)] - c[I^*(\alpha)]\}.
\]

But \( \{1 - F[Z'(\alpha_0)]\}\{\alpha_0 E\{Z|Z \geq Z'(\alpha_0)\} - c_T\} \) is just equal to \( \partial B/\partial l \) evaluated at \( \alpha_0 \).

Because \( I^*(\alpha_0) \) is optimal at \( \alpha_0 \), the extra \( c[I^*(\alpha_0)] - c[I^*(\alpha)] \) expenditure on research must be profitable. Moreover, since \( \partial B/\partial l \) is strictly increasing in \( \alpha \), the surplus will be strictly positive as long as \( L \) is already taking over with positive probability.

Thus \( \hat{G}(\alpha) - \hat{G}(\alpha_0) \leq 0 \) with strict inequality if \( I^*(\alpha_0) > 0 \).

Proof of Proposition 6

\[
\frac{\partial \hat{G}(\alpha)}{\partial \alpha} = \frac{\partial \hat{G}(\alpha)}{\partial \alpha} = \frac{\partial I^*}{\partial \alpha} \cdot (\{1 - F[Z'(\alpha)]\}\{\alpha_0 E\{Z|Z \geq Z'(\alpha)\} - c_T\} - c'[I^*(\alpha)]
\]

\[
+ I^*(\alpha) \frac{\partial}{\partial \alpha} (\{1 - F[Z'(\alpha)]\}\{\alpha_0 E\{Z|Z \geq Z'(\alpha)\} - c_T\} - c_T).
\]

For the second term,

\[
\frac{\partial}{\partial \alpha} (\_\_\_) = \frac{\partial}{\partial \alpha} \int_{Z'(\alpha)}^{Z_{\text{max}}} (\alpha_0 Z - c_T) dF(Z)
\]

\[
= - \frac{\partial Z'(\alpha)}{\partial \alpha} \cdot [\alpha_0 Z'(\alpha) - c_T] dF[Z'(\alpha)].
\]

Thus the whole expression evaluated at \( \alpha_0 \) is

\[
\frac{\partial \hat{G}}{\partial \alpha}(\alpha_0) = \frac{\partial I^*}{\partial \alpha}(\alpha_0)\{1 - F[Z'(\alpha_0)]\}\{\alpha_0 E\{Z|Z \geq Z'(\alpha_0)\} - c_T\} - c'[I^*(\alpha_0)]
\]

\[
- [I^*(\alpha_0)] \frac{\partial Z'}{\partial \alpha}(\alpha_0) \cdot [\alpha_0 Z'(\alpha_0) - c_T] dF[Z'(\alpha_0)].
\]

But the first term is zero since the second term in braces is zero because \( I^*(\alpha_0) \) is the optimal choice of research intensity at \( \alpha_0 \). As for the second term, since \( L \) takes over with positive probability at \( \alpha_0 \), \( I^*(\alpha_0) > 0 \) and \( Z'(\alpha_0) < Z_{\text{max}} \), which implies that \( dF[Z'(\alpha_0)] > 0 \). But also \( \partial Z'/\partial \alpha < 0 \) by lemma 2. Last, \( \alpha_0 Z'(\alpha_0) - c_T > 0 \) as we showed in the proof of proposition 5. Thus

\[
\frac{\partial \hat{G}}{\partial \alpha}(\alpha_0) = \frac{\partial \hat{G}}{\partial \alpha}(\alpha_0) > 0.
\]

Proof of Proposition 7

Since \( \alpha_0 Z_{\text{max}} - c_T < 0 \), \( L \) cannot profit from taking over unless he makes a profit at the expense of small shareholders. Thus \( \pi^*(\alpha_0) = Z_{\text{max}} \). Then .5Z -
\((.5 - \alpha_0)\pi^*(\alpha_0) - \epsilon_T \leq \alpha_0 Z_{\text{max}} - \epsilon_T < 0\) for any \(Z \in [0, Z_{\text{max}}]\). So \(I^*(\alpha_0) = \{1 - F[Z'(\alpha_0)]\} = 0\) and \(c(0) = 0 \Rightarrow \hat{G}(\alpha_0) = 0\).

On the other hand, if \(L\) is taking over with positive probability at \(\alpha\), then \(I^*(\alpha) > 0\) and \(\{1 - F[Z'(\alpha)]\} > 0\). But also, \(E[Z|Z \geq Z'(\alpha)] < Z_{\text{max}} \Rightarrow \alpha_0 E[Z|Z \geq Z'(\alpha)] - \epsilon_T < 0\), which implies that \(\hat{G}(\alpha) < 0\). Thus \(\hat{G}(\alpha) < G(\alpha_0)\).

References


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