Selling Information Through Consulting

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We know a lot about selling items...
How do we sell a piece of information?
Market of Information

Consumer Information

Credit Reports

Recommendation

- acxiom
- EQUIFAX
- nielsen
- experian
- ORACLE
- TransUnion

- tripadvisor
- Waze
How is information different from items?

• Can be partially revealed

• e.g. FlightAware

<table>
<thead>
<tr>
<th>Flight</th>
<th>Airline</th>
<th>Flight Code</th>
<th>Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aberdeen</td>
<td>BD674</td>
<td></td>
<td>Delayed to 10:55</td>
</tr>
<tr>
<td>Newcastle</td>
<td>BA1326</td>
<td></td>
<td>Cancelled</td>
</tr>
<tr>
<td>Glasgow</td>
<td>BA1476</td>
<td></td>
<td>Cancelled</td>
</tr>
<tr>
<td>Durham Tees</td>
<td>GF5232</td>
<td></td>
<td>Cancelled</td>
</tr>
<tr>
<td>Cork</td>
<td>AA8025</td>
<td></td>
<td>Delayed to 11:10</td>
</tr>
<tr>
<td>Dublin</td>
<td>AA7991</td>
<td></td>
<td>Delayed to 11:35</td>
</tr>
<tr>
<td>Shannon</td>
<td>AA8017</td>
<td></td>
<td>Delayed to 10:55</td>
</tr>
<tr>
<td>Edinburgh</td>
<td>BA1442</td>
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</tr>
<tr>
<td>Manchester</td>
<td>BA1388</td>
<td></td>
<td>Cancelled</td>
</tr>
</tbody>
</table>

flight delay $\omega$

\[ \omega \leq 6 \]

Charge 20

\[ \omega > 6 \]

Charge 30

Charge 10

...
Outline

Selling information to **budget-constrained** buyers

1. **Motivation**
   - Why budget-constrained buyers?

2. **Our Results**
   - Model
   - Main theorem
   - Proof ideas
Motivation

• Babaioff et al. [2012]: buyers with unlimited budget

• Budget-constrained buyers
Motivation

• Suppose Haifeng @Boston is going to give a talk @NYC. Safe if the flight delay $\leq 4$ hrs
• Based on historical data & weather, $0 \leq \text{delay} \leq 10$
• Wait/take a train instead

<table>
<thead>
<tr>
<th>loss</th>
<th>delay $\leq 4$</th>
<th>delay $&gt; 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>wait</td>
<td>0</td>
<td>$+\infty$</td>
</tr>
<tr>
<td>take a train</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

• No information: will take a train
• Knowing the exact delay, take a train only when delay $> 4$
• Save $4/10 \times 100$ in expectation
• Willing to pay up to 40
Motivation

• Suppose FlightAware knows the exact delay and wants to sell it.
• 2 buyers:

<table>
<thead>
<tr>
<th></th>
<th>loss</th>
<th>prior belief</th>
</tr>
</thead>
<tbody>
<tr>
<td>buyer 1</td>
<td>400</td>
<td>[0, 7.9984]</td>
</tr>
<tr>
<td>buyer 2</td>
<td>600</td>
<td>[0, 8.0016]</td>
</tr>
</tbody>
</table>

• Babaioff et al. [2012]: when delay $> 4$ w.p. 0.5, the optimal mechanism is to
  • charge $\approx 250,000$ when delay $\leq 4$,
  • pay $\approx 249,600$ to the buyer when delay $> 4$.

• But the buyer’s expected gain $\approx 250$. 
Our Results

Consider *budget-constrained buyers*

- Simple optimal mechanism
- Compute via a *polynomial-size LP*
  - Improves the previous *exponential-size* LP by Babaioff et al. [2012] that solves one-round optimal mechanism when there is no budget constraint

Adding budget constraint makes the problem easier
Model: information buyer

• An unknown state of the world, random variable $\omega$
• The buyer needs to choose an action $a \in A$ that leads to utility $u(a, \omega)$
• The buyer has a private type $\theta$ that represents
  1. his belief about $\omega$: a probability distribution over $\Omega$
  2. his utility function $u_\theta(a, \omega)$
• The buyer has a private budget $b$ that can be used to purchase additional information about $\omega$
Model: information seller

- Fully observes $\omega$, has a budget $M$
- Knows the utility function $u_\theta(a, \omega)$ and the distribution $\mu(\theta, b, \omega)$
- Goal: sell $\omega$ to maximize the expected revenue

*What are the mechanisms that can possibly be used?*
Model: information as signals

- **Partial information**: a random variable (signal) $s$ that is correlated with the state of the world $\omega$
- Example: binary $\omega = \text{good}/\text{bad}$, reveals $\omega$ w.p. $2/3$

$$\theta = (0.5, 0.5)$$

- **Value of Information**: the gain from knowing $s$

$$V_\theta(s) = E_\theta[u(a_{\text{after}}, \omega)] - E_\theta[u(a_{\text{before}}, \omega)]$$
Model: mechanisms

- A menu of (partial information $s$, price $p_s$)
- The seller can interact with the buyer in multiple rounds Babaioff et al. [2012]

- Multiple-round mechanisms
- Recommend actions to the buyer and then charge some prices
Our Contribution

**Consulting Mechanism:**

1. Ask the buyer to report his type $\hat{\theta}$ and deposit his budget $\hat{b}$.

2. **For each reported $\hat{\theta}$ and $\hat{b}$**, according to $\omega$, (randomly) decides an action to recommend and an amount of refund, the amount of which is either 0 or $\hat{b} + M$.

• Buyers with different reported $\hat{\theta}$ and $\hat{b}$ will get different recommendations.
Our Contribution

**Consulting Mechanism:**

1. Ask the buyer to report his type $\hat{\theta}$ and deposit his budget $\hat{b}$.
2. For each reported $\hat{\theta}$ and $\hat{b}$, *according to $\omega$, (randomly) decides an action to recommend and an amount of refund*, the amount of which is either 0 or $\hat{b} + M$.

- For each $\hat{\theta}$ and $\hat{b}$:
  - **good**: $\frac{2}{3}$
    - $\frac{1}{3}$: wait, refund $t_1$
  - **bad**: $\frac{1}{3}$
    - $\frac{2}{3}$: take a train, refund $t_2$
Our Contribution

**Consulting Mechanism:**

1. Ask the buyer to report his type $\hat{\theta}$ and deposit his budget $\hat{b}$.
2. For each reported $\hat{\theta}$ and $\hat{b}$, according to $\omega$, (randomly) decides an action to recommend and an amount of refund, the amount of which is either 0 or $\hat{b} + M$.

- For each $\hat{\theta}$ and $\hat{b}$:
  
  ![Decision Tree Diagram]
  
  - good: 2/3
  - bad: 1/3

- wait, refund 0
- take a train, refund $\hat{b} + M$
Our Contribution

**Theorem:** There always exists an IC and IR Consulting Mechanism that achieves no less revenue than any (possibly multiple-round) mechanisms.
Proof ideas

- **Generic Protocol**: do not know how to compute
- **One-round Mechanism**: LP with exponentially many variables and exp. constraints
One-round Mechanism

1. Ask the buyer to report his type \( \hat{\theta} \) and deposit his budget \( \hat{b} \).
2. According to the reported \( \hat{\theta} \) and \( \hat{b} \), give the buyer a piece of partial information \( s \) and refund \( t(s) \).

- LP with exponentially many variables and exponentially many constraints
Proof ideas

Generic Protocol

do not know how to compute

One-round Mechanism

LP with exponentially many variables and exp. constraints

Consulting Mechanism

simple structure; LP with polynomial size
Proof ideas

\( \mathcal{P} \)
Exponential-size LP for computing optimal *one-round mechanism*

\( \mathcal{D} \)
Dual LP of \( \mathcal{P} \)

\( \mathcal{P}' \)
Exponential-size LP which provably admits a *consulting mechanism* as its solution

\( \mathcal{D}' \)
With the same optimal objective as \( \mathcal{D} \) but poly-size variables
Proof ideas

\[ P' \]

Exponential-size LP which provably admits a consulting mechanism as its solution

• Two possible payments
• Can be reduced to a consulting mechanism
Polynomial-size LP

• Variables: for each $\theta \in \Theta$ and $b \in B$, probability of recommending $a \in A$ and charge one of the two possible payments.

• Objective: expected revenue

• Constraints:
  • Individual Rationality
  • Incentive Compatibility
Summary

Selling information to budget-constrained buyers

• Simple one-round optimal mechanism: consulting mechanisms

• Compute the optimal mechanism via a polynomial-size LP
Thanks & Questions?