1. Log in to WeBWorK and complete the problems assigned there under pset04.

Hughes-Hallett:
14.2: 27, 45,
14.3: 3, 14, 18, 21, 29
14.4: 30, 34, 63, 69,
14.5: 14
Stewart:
12.4: 26

2. The cardiac output, $c$, is the volume of blood flowing through a person’s heart per unit time. The systemic vascular resistance (SVR), $s$, is the resistance to blood flow in the veins and arteries. Let $p$ be a person’s blood pressure.

$$p = f(c, s).$$

(a) What does the instantaneous rate of change $\frac{\partial p}{\partial c}$ represent?

(b) Suppose that $p$ is proportional to $c$ and to $s$, so $p = kcs$. Create a contour plot in Matlab for the case $k = 1$.

```
syms c s
fcontour(function expression here,[domain info here])
```

Remember to include the * symbol when defining the function.

Use the 'LevelList' option to set the levels of the contours. For your plot, add labels, thicken your contour lines so that they are visible, add a title, and adjust the font size on the axes so that it is at least 14.

What do these level curves represent in the context of this problem?

(c) For a person with a weak heart, it is desirable to have the heart pump against less resistance while maintaining the same blood pressure. Such a person may be given the drug nitroglycerine to decrease the systemic vascular resistance, and the drug dopamine to increase the cardiac output. Add this scenario to your contour plot (within Matlab) by marking a point $A$ that represents the person’s state before drugs are administered and a point $B$ for their state after.

The command `plot(3,2,'k.','MarkerSize',10)` will place a black dot at the point $(3, 2)$. The command `text(3,2.2,'A','fontsize',14)` will place the letter “A” at the point $(3, 2.2)$ (to act as a label).

(d) After a heart attack a patient’s cardiac output drops, causing blood pressure to drop. The text says “a common mistake made by medical residents is to get the patient’s blood pressure back to normal by using drugs to increase the SVR, rather than by increasing the cardiac output”.

Using Matlab, add points $D$, $E$, and $F$ to your contour diagram to show this scenario. Choose $D$ for the patient before the heart attack, $E$ for the patient immediately after the heart attack, and $F$ after the patient has been given drugs to increase the SVR.

Make sure to choose a point $D$ where increasing SVR is the wrong thing to do.
For this problem, submit your matlab commands as well as your plots.

3. If it has been a while, or you don’t feel completely comfortable taking derivatives, complete §14.2: 3-39 odd, checking your answers in the back of the book. In addition, visit a member of the course staff to review differentiation. Compute the following partial derivatives, showing mathematical steps:

(a) \[
\frac{\partial}{\partial a} \left( \frac{1}{a} e^{-x^2/a^2} \right)
\]

(b) \[
\frac{\partial}{\partial \theta} \left( \sin(\pi \theta \phi) + \ln(\theta^2 + \phi) \right)
\]

The symbols are the variables \( \theta \) (said “theta”) and \( \phi \) (said “phi”). They are traditionally angular variables (\( \theta \) represents an angle in radians, and when people need a second angular variable they often use \( \phi \)).

If it makes it easier to think about the problem you can replace \( \theta \) with \( x \) and \( \phi \) with \( y \).

(c) Check your derivatives using the \texttt{diff()} command in matlab. Include the lines of matlab code you used as part of your submission.

4. Show that the Cobb-Douglas function \( Q = bK^\alpha L^{1-\alpha} \) where \( 0 < \alpha < 1 \) satisfies the partial differential equation \( K \frac{\partial Q}{\partial K} + L \frac{\partial Q}{\partial L} = Q \).

5. Find the equation of the tangent plane to the surface \( x^2 + y^2 - xyz = 7 \) at the point \((2, 3, 1)\). Do this in two ways:

(a) View the surface as a level set of a function of three variables, \( F(x, y, z) \).

(b) View the surface as the graph of a function of two variables, \( z = f(x, y) \).

6. Two surfaces can be said to be tangential at a point \((a, b, c)\) if they have the same tangent plane at that point. We want to find all points in 3-space where the two surfaces \( z = \sqrt{2x^2 + 2y^2 - 16} \) and \( z = \frac{1}{4}(x^2 + y^2) \) are tangential.

(a) Graph the two surfaces in matlab. Use transparency (the ‘FaceAlpha’ option in \texttt{fsurf}) to make the surfaces more visible.

```matlab
syms x y
f = @(x,y) fcn here;
g = @(x,y) fcn here;
domain = [domain here];
fsurf(x,y,f(x,y),domain,'facealpha','0.8','edgecolor','none')
hold on
fsurf(x,y,g(x,y),domain,'facealpha','0.8','edgecolor','none')
axis equal
```

Label your axes, adjust the font size, and give your plot a title. Use the rotation tool to explore the surfaces. I found the tangency was very visible in my plot.

Submit the graph as part of your problem set. You do not need to submit this code.

(b) Find the set of points where the surfaces are tangential.

\textit{It may be helpful to think of } \( z = \sqrt{2x^2 + 2y^2 - 16} \text{ as the surface } 2x^2 + 2y^2 - z^2 = 16.\)
(c) Check your work by showing (algebraically) that the each point you've identified satisfies the equation for each surface.

(d) In addition, confirm that the surfaces have the same tangent plane at each point in your set. Explicitly showing this mathematically is great. It is sufficient to explain how you know that you've found the tangent planes, and that they are identical.