1 Problem set 08

1.1 Problem set

1. **Project proposal** It’s time to choose a project and to write an initial project proposal. You’ll work on this project throughout April as part of your out-of-class assignments, and we will also sometimes spend time on the project during class.

Milestones that will count towards the project grade:

- **Project proposal** (due by April 2nd) *1-2 pages long single-spaced. it can be longer if needed.*
  - Components are
    - brief background
    - question/goal - what do you want to know from doing your project?
    - references - these might be to existing work in the area, to a paper you plan to replicate, to a math source resource you plan to understand, or to other sources of background information.
    - if you’ll work with data, what your data sources will be
    - description of the mathematical tools from our course and/or from elsewhere in applied dynamical systems that you’ll make use of for the project
    - if you’ll use math tools from other areas, describe those as well.
    - what do you expect to be the main challenge of pursuing this project?
    - how would you measure success for your project?

- **Final project proposal** (due April 13th as part of your homework) *As you undertake the project your question and goals may evolve. This final project proposal will reflect the work you’ve done between March 30th and April 13th to make progress on the project. It will likely include some results as part of a revised proposal. This revised proposal will set out what you’ll work on during the final three weeks of the project.*

- **Short presentation** (April 25th in class, initially due April 23rd before class) *Length to be announced, likely 5-6 minutes.*

- **Feedback to another group** (April 23rd in class + due April 27th)

- **Final presentation** (May 3rd from 9am to noon, slides due May 2nd) *Length to be announced, likely 15-20 minutes.*
  - Components are
    - question/goal
    - background your classmates will need to understand your work
    - approach/model
    - data sources (if relevant)
    - results/conclusions
    - recommendations for next steps (what you would do if you had more time)
    - references

- **Annotated slides** (annotated slides due May 3rd by 9am) *More about annotations later - they are so that you can convey extra information about your work beyond what is directly on the slides you’ll use to present.*
  - Additional slide
    - attribution of individual effort

2. (8.4.4) Explore the phase portrait of

\[ \dot{\theta} + (1 - \mu \cos \theta) \dot{\theta} + \sin \theta = 0 \quad \text{for} \quad \mu \geq 0. \]

Classify the bifurcations that create and destroy a stable limit cycle as \( \mu \) increases from 0.

\( \dot{\theta} = f(\theta, y) \) equation is well-defined for \( \theta \) a 2\( \pi \)-periodic variable if \( f(\theta, y) = f(\theta + 2\pi, y) \). It looks like a Liénard transformation could be helpful here, but it actually is not - we need the \( \theta \) equation to be 2\( \pi \)-periodic and the Liénard transformation ruins this periodicity.

Extra practice: You can take a look at 8.3.2 if you’d like extra practice with Hopf bifurcations.