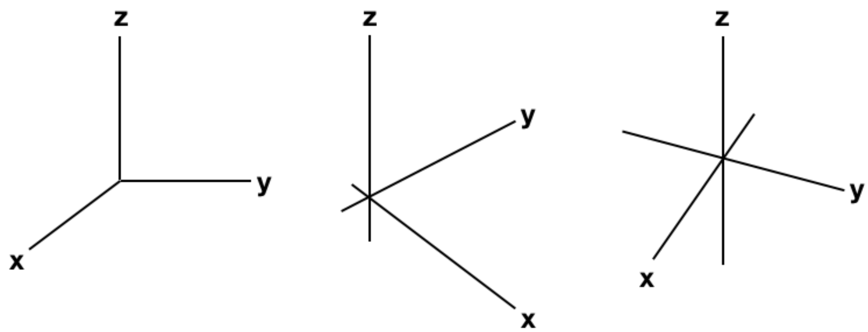
Topic: 3-space, distance and equations using distance; graphs of functions of two variables.

For your reference (section 12.1):
We plot points (x, y) in an xy-plane. This is 2-space. For x, y real numbers we write $\mathbb{R}^2$ for the space. We plot points (x, y, z) in an xyz-coordinate space. This is 3-space. For x, y, z real numbers we write $\mathbb{R}^3$ for the space.

In 3-space: coordinate axes meet at the *origin*. When sketching, place axis labels at the positive end of each axis. Axes are *right-handed*. Looking down the positive z-axis gives the standard view of the xy-plane.

Axes in 3-space with right-hand convention:



A *coordinate plane* is a plane in 3-space that contains two axes. There are three: the xy-, xz-, and yz-planes.

The *distance* between a point (a, b) and a point (x, y) is given by $\sqrt{(a-x)^2 + (b-y)^2}$. The distance between a point (a, b, c) and a point (x, y, z) is given by $\sqrt{(a-x)^2 + (b-y)^2 + (c-z)^2}$. We show this using the Pythagorean theorem.

A *sphere* is the set of all points equidistant to a reference point (a, b, c), so all solutions to the equation $(x-a)^2 + (y-b)^2 + (c-z)^2 = d^2$ where d, the distance to the reference point, is referred to as the *radius* of the sphere.

The graph of points (x, y, z) satisfying an equation is a *surface*. A surface is a deformed sheet. Note: the points on a sphere form a surface.

Examples:
- Plot $(1, 3, 4)$ in 3-space.
- Find the distance between $(1, 3, 4)$ and the xy-plane.
- Find the distance between $(1, 3, 4)$ and the plane $x = 7$.
- Find the distance from $(1, 3, 4)$ to the z-axis. *poll Q*
- Write an equation for the set of points distance 2 from the point $(1, 3, 4)$.
- Find the set of points in the intersection of the sphere of radius 3 centered around $(0, 0, 4)$ and the plane $z = 2$. *poll Q*

For your reference (section 12.2):
For a function of two variables, $f(x, y)$, the points (x, y, z) (in 3-space) that satisfy $z = f(x, y)$ form the *graph* of the function. These are the points $(x, y, f(x, y))$. The graph is a surface.

When plotted in 3-space, the equation $y = g(x)$ forms a vertical plane. The points in this plane are $(x, g(x), z)$. The intersection of a vertical plane with a surface is called a *cross-section* of the surface. For a function $z = f(x, y)$, this intersection is the set of points $(x, g(x), f(x, g(x)))$.

Examples:

- Graph function $f(x, y) = x^2 - y^2$.
- Plot a cross-section with $y = x$, a cross-section with $x = 0$, and a cross-section with $y = 0$. *poll Q*

Today:

- ☐ Find the distance between two points in 3-space, between a point and a plane parallel to a coordinate plane, or between a point and a line parallel to a coordinate axis.
- ☐ Set up an equation describing a set of points that satisfy a distance relationship.
- ☐ Find the set of points in the intersection of a sphere and a plane.
- ☐ Find the set of points in the cross-section of a surface.

Next time: cylinders, level sets, contour diagrams

Matlab Examples:

Find the **distance** between the points $(0, 4, 5)$ and $(3, 2, 4)$.

```
dist = sqrt((3-0)^2 + ...
(4-2)^2 + (5-4)^2)

p1 = [0, 4 5]; p2 = [3, 2, 4];
dist = sqrt((p1(1)-p2(1))^2 + ...
(p1(2)-p2(2))^2 + (p1(3)-p2(3))^2)

% The square of dist is an integer
distsq = distance^2

% The distance is exactly
sqrt(14)
```

Plot the **graph** of $z = x^2 - y^2$.

```
syms x y
f = @(x,y) x^2-y^2
fsurf(x, y, f(x,y))
xlabel('x'); ylabel('y'); zlabel('z')
title('surface: z = x^2 - y^2')
set(gca,'FontSize',14)
axis equal
axis([-3 3 -3 3 -5 5])
caxis([-5 5])
```

Add a **cross-section** with $x = 0$

```
hold on
fplot3(sym(0),y,f(0,y),'LineWidth',3)
```

And with $y = x$.

```
fplot3(x,x,f(x,x),'LineWidth',3)
```

Command list

(Use the syntax `doc command-name` to look up a command within Matlab).

title	[, ,]	zlabel
sqrt	;	caxis
-,^,+,()	fsurf	hold on