Topic: Contours

Examples. Identify the shape of the contour lines for each of the following functions:

\[ f(x, y) = y - x^2, \quad f(x, y) = x^2, \quad f(x, y) = x^2 + 2y^2, \quad f(x, y) = y^2 - x^2 \] (pollQ)

Topic: Contour diagrams

To draw a contour diagram for a function \( f(x, y) \), choose four evenly spaced values of \( c \) (the spacing is the \textit{contour interval}). Draw the contours corresponding to each value of \( c \) in the domain of the function (so in 2-space on an \( xy \)-plane).

Example. Construct a contour diagram for \( f(x, y) = 2x^2 + y^2 \). pollQ

Example. Match the contour diagrams to the appropriate graph. pollQ

Matlab Example:

```matlab
% Draw four contours on the xy-plane
syms x y
interval = 1;
c = 1;
f = @(x,y) x^2+y^2;
for contours = 1:4
    fimplicit(f-sym(c),[-4 4 -4 4])
    hold on
    c = c + interval;
end
axis equal
title('contour diagram for ... f(x,y) = x^2+y^2')
set(gca,'fontsize',16)
hAxis = gca;
hAxis.XRuler.FirstCrossoverValue = 0;
% X crossover with Y axis
hAxis.YRuler.FirstCrossoverValue = 0;
% Y crossover with X axis
Command list

for end     sym
fimplicit  gca
```
**Topic:** Linear functions

**For your reference (section 12.4):**

*Linear functions* are important because many nonlinear functions look linear when we zoom in on the function. This property is called *local linearity*. Locally approximating a function with a linear function (*linearizing* the function at a point) is a key tool in applied mathematics.

A plane with slope $m$ in the $x$-direction, slope $n$ in the $y$-direction, and passing through the point $(x_0, y_0, z_0)$ has equation $z = m(x - x_0) + n(y - y_0) + z_0$ (or $z = mx + ny + c$ with $c = z_0 - mx_0 - ny_0$). You can think of this as a *point-slope* form for the equation, analogous to $y = m(x - x_0) + y_0$ in 2-space.

To draw a plane in 3-space, it often works to draw a point at each of the axis intercepts and then to connect those with straight lines. The triangle that results can give a sense of the orientation of the plane.

**Examples.** Just as two points determine a line, three points determine a plane. Find the equation of the plane through the points $(0, 2, 4)$, $(3, 1, 4)$, $(0, 0, 2)$ (*pollQ*). Sketch the plane.

**Examples.** Construct a contour diagram for the plane you found.

**Example.** Find the equation of a plane corresponding to the function with values given in the table.

<table>
<thead>
<tr>
<th></th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>1</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>1</td>
<td>5</td>
</tr>
</tbody>
</table>

**Today:**

- Construct a contour diagram given a function.
- Match graphs of functions to contour diagrams.
- Find the equation of a linear functions given information about points it passes through.
- Identify key features of the contour diagrams of linear functions.

**Matlab Example:**

```matlab
% Make a contour plot (built-in)  set(gca,'fontsize',16)
syms x y
interval = 1;
c = 1;
f = @(x,y) 2*x+3*y+1;
domain = [-2 2 -2 2];
fcontour(f(x,y),domain,'levelstep',interval)
axis equal
xlabel('x'); ylabel('y');
title('contour diagram')
```

**Command list**

- `fcontour`
- `xlabel`
- `ylabel`
- `title`