Topic: Contours

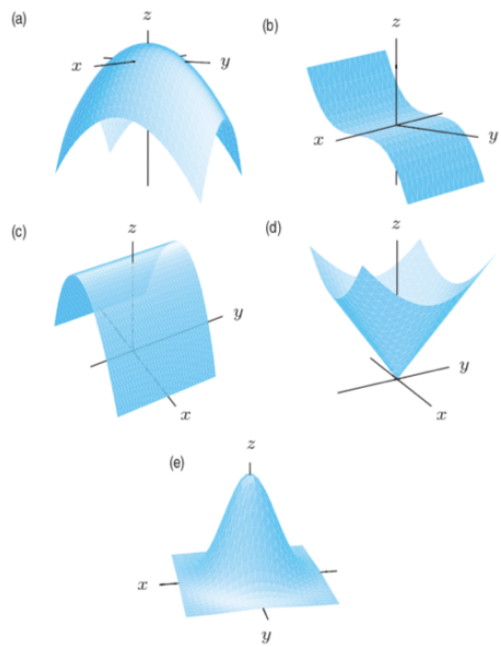
Examples. Identify the shape of the contour lines for each of the following functions:
$f(x,y) = y - x^2$, $f(x,y) = x^2$, $f(x,y) = x^2 + 2y^2$, $f(x,y) = y^2 - x^2$ (*pollQ*)

Topic: Contour diagrams

To draw a contour diagram for a function $f(x,y)$, choose four evenly spaced values of c (the spacing is the *contour interval*). Draw the contours corresponding to each value of c in the domain of the function (so in 2-space on an xy-plane).

Example. Construct a contour diagram for $f(x,y) = 2x^2 + y^2$. *pollQ*

Example. Match the contour diagrams to the appropriate graph. *pollQ*



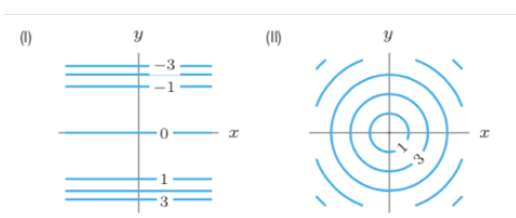


Figure 12.48

Matlab Example:

```
% Draw four contours on the xy-plane
syms x y
interval = 1;
c = 1;
f = @(x,y) x^2+y^2;
for contours = 1:4
    fimplicit(f-sym(c),[-4 4 -4 4])
    hold on
    c = c + interval;
end
axis equal
xlabel('x'); ylabel('y');
title('contour diagram for ...
   f(x,y) = x^2+y^2')
set(gca,'fontsize',16)
hAxis = gca;
hAxis.XRuler.FirstCrossoverValue  = 0;
% X crossover with Y axis
hAxis.YRuler.FirstCrossoverValue  = 0;
% Y crossover with X axis
```

Command list

```
for end       sym
fimplicit     gca
```

Topic: Linear functions

For your reference (section 12.4):
Linear functions are important because many nonlinear functions look linear when we zoom in on the function. This property is called *local linearity*. Locally approximating a function with a linear function (*linearizing* the function a point) is a key tool in applied mathematics.

A plane with slope m in the x-direction, slope n in the y-direction, and passing through the point (x_0, y_0, z_0) has equation $z = m(x - x_0) + n(y - y_0) + z_0$ (or $z = mx + ny + c$ with $c = z_0 - mx_0 - ny_0$). You can think of this as a *point-slope* form for the equation, analogous to $y = m(x - x_0) + y_0$ in 2-space.

To draw a plane in 3-space, it often works to draw a point at each of the axis intercepts and then to connect those with straight lines. The triangle that results can give a sense of the orientation of the plane.

Examples. Just as two points determine a line, three points determine a plane. Find the equation of the plane through the points $(0, 2, 4)$, $(3, 1, 4)$, $(0, 0, 2)$ (*pollQ*). Sketch the plane.

Examples. Construct a contour diagram for the plane you found.

Example. Find the equation of a plane corresponding to the function with values given in the table.

y \ x	2	3	4
1	3	5	7
5	2	4	6
9	1	3	5

Today:
- ☐ Construct a contour diagram given a function.
- ☐ Match graphs of functions to contour diagrams.
- ☐ Find the equation of a linear functions given information about points it passes through.
- ☐ Identify key features of the contour diagrams of linear functions.

Matlab Example:

```
% Make a contour plot (built-in)
syms x y
interval = 1;
c = 1;
f = @(x,y) 2*x+3*y+1;
domain = [-2 2 -2 2];
fcontour(f(x,y),domain,'levelstep',interval)
axis equal
xlabel('x'); ylabel('y');
title('contour diagram')
```

```
set(gca,'fontsize',16)
hAxis = gca;
hAxis.XRuler.FirstCrossoverValue  = 0;
% X crossover with Y axis
hAxis.YRuler.FirstCrossoverValue  = 0;
% Y crossover with X axis
```

Command list

```
fcontour
```