For your reference (section 13.1):
The *tip* of the vector is the end with the arrow, while the *tail* is the end without it.

We say that we are *resolving* a vector into components when we write a vector in 
\( \vec{v} = v_1 \vec{i} + v_2 \vec{j} + v_3 \vec{k} \) form. In this form, we are thinking about the vector as a sum of three components (along perpendicular directions). As an alternative, we could also provide the magnitude of the vector and indicate the direction using angles.

For a vector that is resolved into components, its magnitude is given by 
\[ \| \vec{v} \| = \sqrt{v_1^2 + v_2^2 + v_3^2} \]
(this is coming from the distance formula).

A vector drawn with its tail at the origin is called a *position vector*.

A vector of length (magnitude) 1, so a vector where \( \| \vec{v} \| = 1 \), is called a *unit vector*.

The *zero vector* is the vector \( \vec{0} = 0 \vec{i} + 0 \vec{j} + 0 \vec{k} = (0, 0, 0) \).

**Example.** Resolve the vector \( \vec{u} \) into components. (Write it as \( \vec{u} = u_1 \vec{i} + u_2 \vec{j} + u_3 \vec{k} \)). Be careful to include the lines above \( i, j, k \) that indicate they are vectors.

**Example.** Find the magnitude of \( \vec{u} \). This is the length of the vector.

**Example.** Find a unit vector in the direction of \( \vec{u} \).

**Example.** Find all of the vectors, \( \vec{v} \), in 2-space such that the \( \vec{i} \) component is \( 3 \vec{i} \) and \( \| \vec{v} \| = 5 \).
For your reference (section 13.2):

*Velocity, acceleration,* and *force* are each quantities that have a magnitude and a direction, so are well represented by vectors. For a velocity vector, we refer to its magnitude as the *speed.* For acceleration and force vectors we don’t have special words to denote the size of the acceleration/force.

Vector addition and subtraction have some important mathematical properties:

1. Vector addition is associative: $\vec{u} + (\vec{v} + \vec{w}) = (\vec{u} + \vec{v}) + \vec{w}$.
2. There is an additive identity: $\vec{u} + \vec{0} = \vec{u}$.
3. There is an additive inverse: $\vec{v} + -\vec{v} = \vec{0}$ where $-\vec{v} = (-1)\vec{v}$.
4. Vector addition is commutative: $\vec{v} + \vec{w} = \vec{w} + \vec{v}$.

Scalar multiplication with vectors has important properties as well:

1. Distributivity of scalar multiplication with respect to vector addition $(\alpha + \beta)\vec{v} = \alpha\vec{v} + \beta\vec{v}$.
2. Distributivity of scalar multiplication with respect to vector addition $\alpha(\vec{v} + \vec{w}) = \alpha\vec{v} + \alpha\vec{w}$.
3. Compatibility of scalar multiplication with usual multiplication $\alpha(\beta\vec{v}) = (\alpha\beta)\vec{v}$.
4. Identity element of scalar multiplication: $1\vec{v} = \vec{v}$.

These properties are the properties of a mathematical object called a *vector space.* You can study vector spaces as mathematical objects in 21b.

*Relative motion:* If an object is moving at velocity $\vec{v}$ relative to a river, and the river is moving at velocity $\vec{w}$ relative to the shore, then the object will be moving at velocity $\vec{v} + \vec{w}$ relative to the shore.

When we think geometrically, we will use vectors in 2-space ($\mathbb{R}^2$) or in 3-space ($\mathbb{R}^3$). It is sometimes convenient to use a vector to keep track of data, such as a list of $n$ problem set scores, or a list of $n$ prices. In that case the vector is in *n*-space, $\mathbb{R}^n$.

**Example.** A boat is heading due east relative to the water at 25 km/hr. The current in the water is moving southwest at 10 km/hr. We want to understand the motion of the boat relative to the ground.

- Draw this scenario out using vectors.
- Find the speed of the boat relative to the ground.
- Find the angle that the boat is traveling relative to due east.

**Matlab Example:**

```matlab
% Magnitude of a vector:
vecv = [1,3,2];
% I can use a loop to do the addition
sumsq = 0;
for k = 1:3
    sumsq = sumsq + vecv(k)^2;
end
sqrt(sumsq) %sqrt to find the magnitude
norm(vecv) %norm is a built-in %command to find it.
```

**Command list**

- `norm(vecv)`
- `sqrt(sumsq)`