For your reference (single-variable calculus):
The average rate of change of a function $f(x)$ over the interval from $a$ to $a+h$ is given by 
\[
\frac{f(a+h)-f(a)}{h}.
\]

The instantaneous rate of change of a function $f(x)$ at the point $a$ is given by 
\[ \lim_{h \to 0} \frac{f(a+h)-f(a)}{h} \]
This is the derivative of $f$ at $a$, denoted $\frac{df}{dx}(a)$, $\frac{df}{dx}|_{a}$, or $f'(a)$.

When $h$ is sufficiently close to 0, the average rate of change of $f(x)$ at a point $a$ serves as an approximation to the derivative of $f$ at $a$. Example: $\frac{f(1+0.1)-f(1)}{0.1}$ is an approximation to $f'(1)$. $\frac{f(1+0.01)-f(1)}{0.01}$ is a better approximation to $f'(1)$.

Example. Let the point $P$ vary from $A$ to $B$. How does the sign of $\frac{df}{dx}(P)$ change?

Example. Let $P$ vary along the region of the $x$-axis shown in the figure below. How does $\frac{df}{dx}(P)$ change as you move from left to right?

For your reference (section 14.1):
To generalize the instantaneous rate of change to a multivariable context, we take partial derivatives along cross sections of the function. Given a function $f(x, y)$:

- The notation $\frac{\partial f}{\partial x}(a, b)$ means “Set $y = b$ and find the partial derivative with respect to $x$ at $x = a$.” $\frac{\partial f}{\partial x}(a, b) = \lim_{h \to 0} \frac{f(a+h, b) - f(a, b)}{h}$. This partial derivative is also denoted $f_x(a, b)$.
- We can also compute the partial with respect to $y$: $\frac{\partial f}{\partial y}(a, b) = \lim_{h \to 0} \frac{f(a, b+h) - f(a, b)}{h}$.

The symbol $\partial$ (see https://en.wikipedia.org/wiki/%E2%88%82) is used to distinguish partial derivatives from ordinary ones. (Fun fact: Legendre, inventor of the symbol, and Jacobi, key user of the symbol, each have a moon crater named for them.)

The units of a partial derivative are the units of the numerator divided by the units of the denominator: if $w$ is in kilograms and $x$ is in meters then $\frac{\partial w}{\partial x}$ will have units of kilograms per meter.
Example. For the figure on the left below, what is the sign of $f_x(A)$?

![Figure 14.7]

Example. For the figure on the right above, what is the sign of $f_y(Q)$? What is the sign of $f_x(P)$ and the sign of $f_y(P)$?

Example. Approximate the instantaneous rate of change $f_x(P)$ using an average rate of change.

Example. If possible, identify a point on the contour plot where $f_x < 0$ and $f_y < 0$. Explain your reasoning.

For your reference (section 14.2):
To compute a partial derivative with respect to $x$, treat other variables as constants. Compute an ordinary derivative with $x$ as the variable.

We can create a function $g_x(x, y)$ or $g_y(x, y)$. These functions allow us to evaluate the partial derivative of $g$ with respect to $x$ (or to $y$) at any point $(x, y)$.

Example.
- Let $f(x) = x^3$. Find $f_x(1)$.
- Let $g(x, y) = x^3 + y$. Find $g_x(1, 4)$.
- Let $g(x, y) = x^3y$. Find $g_x(1, 4)$.

Example. Let $h$ be the temperature of along a wall at point $x$ and time $t$. What does the function $h_x(x, t)$ measure? What does the function $h_t(x, t)$ measure?

Example. Find $f_x(x, y)$ if $f(x, y) = e^{xy} \cos x$.

Matlab Examples:
```matlab
syms x y
d1 = diff(x^3, x)
subs(d1, x, 1)
d2 = diff(x^3+y, x)
subs(d2, [x y], [1 4])
d3 = diff(x^3*y, x)
subs(d3, [x y], [1 4])
d4 = diff(exp(x*y)*cos(x), x)
```

Command list
diff    subs