Today:

- ☐ Double integrals: Set up and evaluate an iterated integral for the integral of a function of two variables.
- ☐ Triple integrals: Set up and evaluate an iterated integral for the integral of a function of three variables.
- ☐ Mean value via integral: Find the mean value of a function using integration.

Example (iterated integrals).

Divide a rectangular region into $m * n$ boxes (m intervals along x and n intervals along y). Let (u_{ij}, v_{ij}) be a point in the box ij where $1 \le i \le m$ indicates the x-interval associated with the box and $1 \le j \le n$ indicates the y-interval associated with the box. In the example below, $m = 2$ and $n = 4$.

Our Riemann sum is $\sum\limits_{\text{all boxes}} f(u_{ij}, v_{ij})\Delta A = \sum\limits_{\text{all boxes}} f(u_{ij}, v_{ij})\Delta x \Delta y = \sum\limits_{j=1}^{j=n} \left(\sum\limits_{i=1}^{i=m} f(u_{ij}, v_{ij})\Delta x \right) \Delta y$

Taking limits to make Δx and Δy infinitesimal, we have

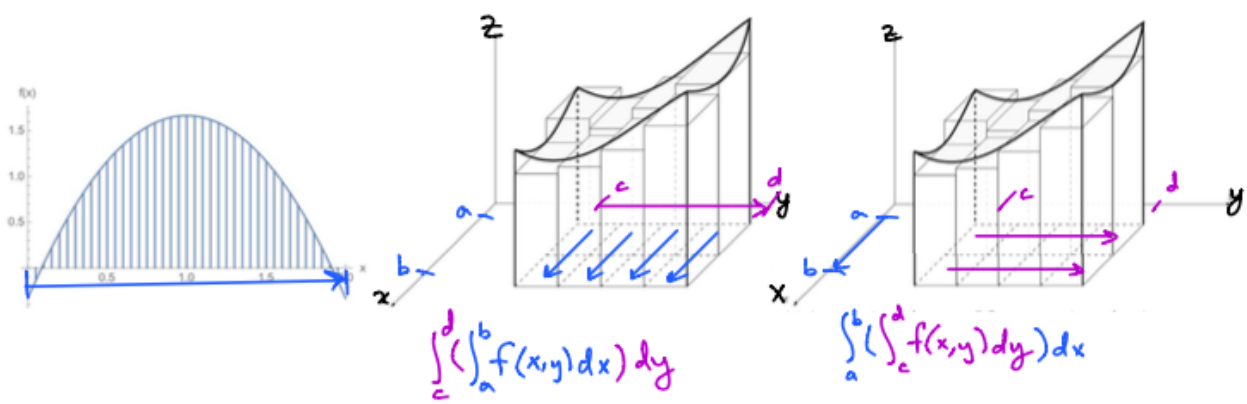
$$\int_R f \; dA = \int_{y=c}^{y=d} \left(\int_{x=a}^{x=b} f(x, y) \; dx \right) \; dy.$$

We usually write $\int_c^d \int_a^b f(x, y) \; dx \; dy$ for this.

We have two options for the order of integration:

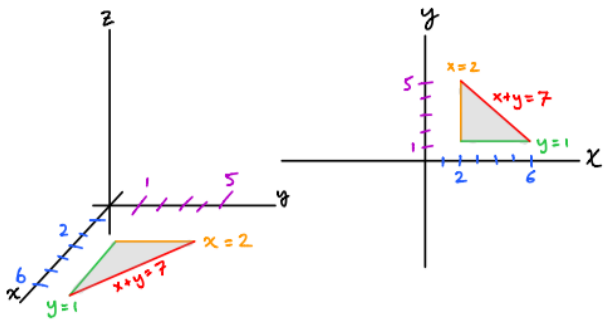
- (horizontal strips) integrating first in x and then integrating the resulting function in y. *See center figure*
- (vertical strips) integration first in y and then integrating the resulting function in x. *See right hand figure*

By convention, $\int_R f \; dA$ always denotes that the integral is taking from left to right in x and from bottom to top in y.

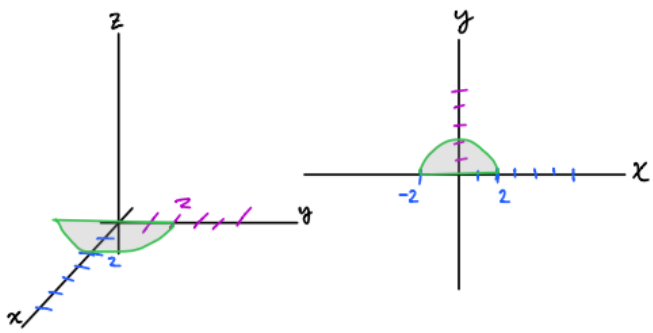


Example (rectangular region). Set up an iterated integral for $\int_R (x^2 + y^2) \; dA$ where R is the rectangular region $1 \le x \le 5$, $-2 \le y \le 2$.

Example (triangular region). Set up an iterated integral for $\int_R f\ dA$ where f is an unknown function of two variables and R is the triangular region shown below. *pollQ*

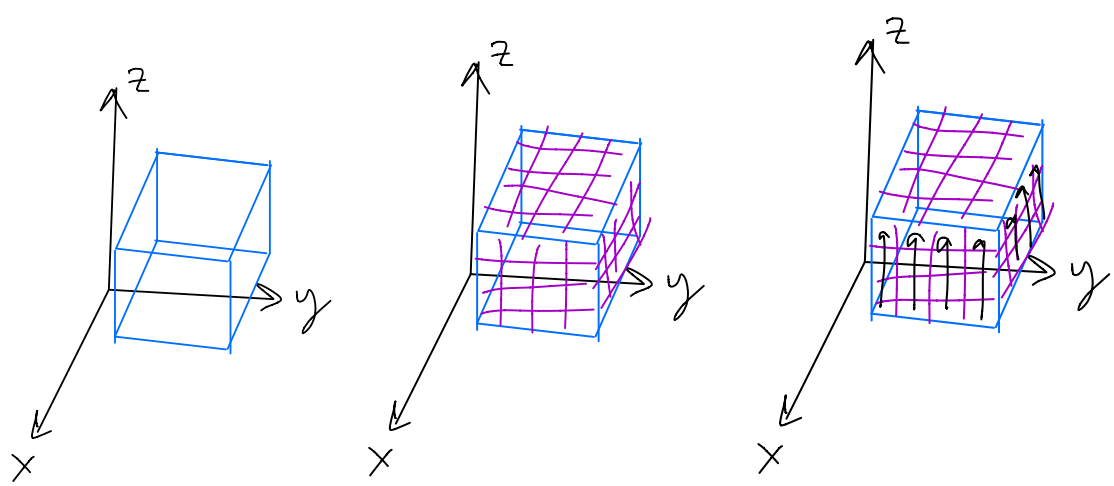


Example (half-disk region). Set up an iterated integral for $\int_R x\ dA$ where f is as given in the integral, and R is the half-disk shown below. *pollQ*



Example (function of three variables).

function of integration		
$f(x)$	$f(x,y)$	$f(x,y,z)$
region of integration		
line segment	region in 2-space (thin sheet)	region in 3 space (solid)
pieces of region		
dx, short line segments	$dA = dxdy$, small squares	$dV = dxdydz$, small cubes
possible orders of integration		
1 option	2 options	6 options



Example (tetrahedron). Consider the tetrahedron bounded by in the first octant and below the plane $3x + y + z = 1$. Set up an integral to find V, the volume of the tetrahedron. *pollQ*

Example (tetrahedron). Consider the tetrahedron bounded by in the first octant and below the plane $3x + y + z = 1$. Let the density of this tetrahedron be az g/cm^3 with position values x, y, z measured in cm. Set up an integral to find M, the mass of the tetrahedron. *pollQ*

Example (tetrahedron). For the tetrahedron above, find the mean density. *pollQ*

Text section	Initial practice (exercises)	PSet practice (problems)
16.2: Iterated integrals	3, 7, 15, 19, 25	31, 43, 61
16.3: Triple integrals	7, 11	15, 17, 37, 39, 53

Learning Objectives

These objectives are associated with Class 18 + Problem Set 7 questions + Quiz 5 + Workshop + Section + Office hours.

Students will be asked to

- define: triple integral
- sketch the region of integration for an iterated triple integral (§16.3 5-13)
- determine the sign of a triple integral by reasoning about the integrand and region of integration (§26.3 14-26)
- find bounds for a region of integration specified by the intersection of surfaces or by a plot of the region (§16.3 28-58)
- change the order of integration for a triple integral (§16.3 60-62, 64)
- identify when it would be appropriate to choose a single, double, or triple integral to compute a quantity
- use triple integrals to compute volume, mass, average value of a function, center of mass, or moment of inertia (§16.3 28-38, 43-53, 57, 59, 63, 65-69)