Today:
□ Double integrals: Set up and evaluate an iterated integral for the integral of a function of two variables.
□ Triple integrals: Set up and evaluate an iterated integral for the integral of a function of three variables.
□ Mean value via integral: Find the mean value of a function using integration.

Example (iterated integrals).

Divide a rectangular region into \( m \times n \) boxes \((m\) intervals along \( x \) and \( n \) intervals along \( y \)). Let \((u_{ij}, v_{ij})\) be a point in the box \( ij \) where \( 1 \leq i \leq m \) indicates the \( x \)-interval associated with the box and \( 1 \leq j \leq n \) indicates the \( y \)-interval associated with the box. In the example below, \( m = 2 \) and \( n = 4 \).

Our Riemann sum is
\[
\sum_{\text{all boxes}} f(u_{ij}, v_{ij}) \Delta A = \sum_{\text{all boxes}} f(u_{ij}, v_{ij}) \Delta x \Delta y = \sum_{j=1}^{n} \left( \sum_{i=1}^{m} f(u_{ij}, v_{ij}) \Delta x \right) \Delta y
\]

Taking limits to make \( \Delta x \) and \( \Delta y \) infinitesimal, we have
\[
\int_{R} f \, dA = \int_{y=d}^{y=c} \left( \int_{x=b}^{x=a} f(x, y) \, dx \right) \, dy.
\]

We usually write \( \int_{c}^{d} \int_{a}^{b} f(x, y) \, dx \, dy \) for this.

We have two options for the order of integration:
- (horizontal strips) integrating first in \( x \) and then integrating the resulting function in \( y \). See center figure
- (vertical strips) integration first in \( y \) and then integrating the resulting function in \( x \). See right hand figure

By convention, \( \int_{R} f \, dA \) always denotes that the integral is taking from left to right in \( x \) and from bottom to top in \( y \).

Example (rectangular region). Set up an iterated integral for \( \int_{R} (x^2 + y^2) \, dA \) where \( R \) is the rectangular region \( 1 \leq x \leq 5, -2 \leq y \leq 2 \).
Example (triangular region). Set up an iterated integral for $\int_{R} f \, dA$ where $f$ is an unknown function of two variables and $R$ is the triangular region shown below. pollen

Example (half-disk region). Set up an iterated integral for $\int_{R} x \, dA$ where $f$ is as given in the integral, and $R$ is the half-disk shown below. pollen

Example (function of three variables).

<table>
<thead>
<tr>
<th>function of integration</th>
<th>region of integration</th>
<th>pieces of region</th>
<th>possible orders of integration</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>$f(x, y)$</td>
<td>$dx$, short line segments</td>
<td>1 option</td>
</tr>
<tr>
<td>$f(x, y)$</td>
<td>$f(x, y, z)$</td>
<td>$dA = dxdy$, small squares</td>
<td>2 options</td>
</tr>
<tr>
<td>$f(x, y, z)$</td>
<td></td>
<td>$dV = dxdydz$, small cubes</td>
<td>6 options</td>
</tr>
</tbody>
</table>
Example (tetrahedron). Consider the tetrahedron bounded by in the first octant and below the plane $3x + y + z = 1$. Set up an integral to find $V$, the volume of the tetrahedron. *poll*Q

Example (tetrahedron). Consider the tetrahedron bounded by in the first octant and below the plane $3x + y + z = 1$. Let the density of this tetrahedron be $\rho \text{ g/cm}^3$ with position values $x, y, z$ measured in cm. Set up an integral to find $M$, the mass of the tetrahedron. *poll*Q

Example (tetrahedron). For the tetrahedron above, find the mean density. *poll*Q
Learning Objectives

These objectives are associated with Class 18 + Problem Set 7 questions + Quiz 5 + Workshop + Section + Office hours.

Students will be asked to
• define: triple integral
• sketch the region of integration for an iterated triple integral (§16.3 5-13)
• determine the sign of a triple integral by reasoning about the integrand and region of integration (§26.3 14-26)
• find bounds for a region of integration specified by the intersection of surfaces or by a plot of the region (§16.3 28-58)
• change the order of integration for a triple integral (§16.3 60-62, 64)
• identify when it would be appropriate to choose a single, double, or triple integral to compute a quantity
• use triple integrals to compute volume, mass, average value of a function, center of mass, or moment of inertia (§16.3 28-38, 43-53, 57, 59, 63, 65-69)