GOLD RETURNS*

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From 1836 to 2011, gold’s average annual real rate of price change is 1.1%, with a standard deviation of 13.1% and a negligible covariance with consumption growth. Because gold does not serve as a hedge against macroeconomic declines, its expected real rate of return should be close to the risk-free rate of around 1%. These properties fit an asset-pricing model with rare disasters and a high elasticity of substitution between gold services and ordinary consumption. In this scenario, gold’s expected rate of return corresponds mostly to the unobserved dividend yield, with a small part comprising expected real price appreciation.

Gold has dominated monetary systems for centuries, and it plays a prominent role in transactions among financial institutions even in modern systems that rely on fiat money. Private holdings of gold are also important, and one source of private demand is often thought to be gold’s role as a hedge against macroeconomic declines. In the present research, we seek to understand these hedging properties. We find that changes in real gold prices co-vary negligibly with growth rates of consumption and, moreover, gold has not delivered high average real returns during macroeconomic disasters. Therefore, the expected real rate of return on gold should be close to the risk-free rate of around 1% per year. We show that this perspective accords with long-term data on real gold prices as assessed with an asset-pricing model applied to gold.

We analyse the returns on gold in a Lucas-tree model that incorporates rare disasters associated with ordinary consumption. The model is a two-tree version with reasonable restrictions that deliver tractability: ordinary consumption and gold services are imperfect substitutes in an effective consumption flow, the outlay on gold services is always negligible compared to that on ordinary consumption and disaster shocks apply directly to ordinary consumption but not to stocks of gold. In this setting, the expected real rate of return on gold ranges between the risk-free rate and the expected rate of return on equity if the elasticity of substitution between ordinary consumption and gold services is between infinity and one.

We relate the model’s predictions to empirical properties of US long-term returns on gold and other assets. From 1836 to 2011, the average real rate of price change for gold is 1.1% per year, the standard deviation is 13.1% and the correlation with consumption growth rates is small in magnitude and statistically insignificantly different from zero. Moreover, for 19 OECD countries, the average real rate of price change for gold during macroeconomic disasters is close to that in non-disaster periods. Thus, gold has not served as a hedge against macroeconomic declines.

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A problem is that the data reveal changes in real gold prices but not the dividend yields associated with gold’s service flows. We use the model to assess these omissions and find that the observed average real rate of price change substantially underestimates gold’s expected real rate of return. Nevertheless, the data on real rates of price change should provide good measures of the uncertainty in real gold returns, including the covariances between these returns and consumption growth rates or with other asset returns.

The model accords with long-term data on real rates of change of gold prices if the elasticity of substitution between ordinary consumption and gold services is high. Explaining the changing volatility of real gold prices over sub-periods requires that shocks to preferences for gold services – particularly those that reflect gold’s monetary services – be minor under a serious gold standard, notably 1880–1913, but large in other periods, such as 1975–2011.

1. A Model of Returns on Gold with Rare Disasters

The underlying demand for gold reflects a service flow proportional to the stock of gold. This perspective matches up with gold used for non-monetary purposes – jewellery, crafts, electronics, medicine – and also for monetary services. The latter function reflects a transactions and liquid store-of-value benefit of the sort usually considered in analyses of the demand for money.\(^1\)

From the non-monetary perspective, gold can be viewed as any durable commodity that provides consumption services to households. In contrast, Goldstein and Kestenbaum (2010) report the argument of the chemist Sanat Kumar that the commodities (specifically, the naturally occurring elements) that can readily provide monetary services are limited to a few precious metals, with gold emerging as the most attractive. That is, gold’s prominent monetary role is not an historical accident.

The representative household’s utility depends on an effective consumption flow, \(c^e_t\), which relates to ordinary consumption, \(c_t\) and the flow of services from the gold stock, \(g_0\), in a CES form:

\[
c^e_t = \left[ x_t \times c^e_t \right] \left[ 1 - x_t \right] \times \left[ g_0 \right]^{\frac{1 - x_t}{x_t}},
\]

where we assume \(\sigma > 0\) and \(0 < x_t < 1\).\(^2\) The variable \(x_t\) can be viewed as a preference shock for ordinary consumption compared to gold services. One source of shocks to \(x_t\) is a change in the monetary role of gold. In earlier periods – at least up to 1975 – these

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\(^1\) One difference between the two functions is that monetary services do not depend directly on the physical quantity of gold but rather on the real value of that quantity in units of consumption. This consideration implies that the service value from monetary gold depends on the relative price of consumables and gold. In this respect, the analysis falls into the ‘money-in-the-utility-function’ literature, originated by Sidrauski (1967), which captures the service value of money by entering the quantity of real money balances directly into the utility function. An analogous service flow – corresponding to liquidity benefits that may relate to maintenance of exchange rates – attaches to monetary gold held as reserves by central banks and governments.

\(^2\) The limit of the right-hand side as \(\sigma\) approaches one is the Cobb-Douglas form, \(c^e_t \times g_0^{1-x_t}\).

shifts sometimes reflected movements off or on aspects of the gold standard in the US or the rest of the world.

Given the functional form in (1), gold’s real rental price, \( \pi^g_t > 0 \), equals the ratio of the marginal utility of gold services to the marginal utility of ordinary consumption and is given by:

\[
\pi^g_t = \left( \frac{1 - z_t}{z_t} \right) \left( \frac{c_t}{g_t} \right)^{1/\sigma}.
\]

Hence, gold is relatively highly valued when \( c_t / g_t \) is high and \( z_t \) is low. \(^3\) This rental price determines the unobserved dividend flow accruing to holders of gold. \(^4\)

We use a ‘two-tree’ version of the Lucas model (Lucas, 1978) to consider the evolution of ordinary consumption, \( c_t \), and gold, \( g_t \). Our model is closest to that of Piazzesi et al. (2007), in which the second tree corresponds to residential housing. In their model, variations in the share of consumer expenditure directed to housing have important implications for asset pricing. In contrast, we assume (reasonably) that the fraction of total consumer outlay directed to gold services is always negligible and, therefore, do not get these types of effects. \(^5\)

The stochastic process for per capita consumption, \( c_t \), viewed as the fruit from a Lucas tree, takes the same form as in Barro (2006, 2009):

\[
\log(c_{t+1}) = \log(c_t) + h + u_{t+1} + v_{t+1},
\]

where \( h \) \( \geq \) 0 is exogenous productivity growth and \( u_{t+1} \) is an i.i.d. normal shock with mean 0 and variance \( \sigma_u^2 \). The number of trees is fixed, there is no possibility of loss of ownership, and the economy is closed.

The term \( v_{t+1} \) in (3) is a disaster shock, governed by a constant Poisson arrival probability \( p \geq 0 \) (expressed per unit of time) and a proportionate disaster size, \( b \geq 0 \), which is subject to a time-invariant probability distribution. Specifically, the disaster shock \( v_{t+1} \) equals \( \log(1-b) \), where \( b > 0 \) in a disaster state and \( b = 0 \) in a non-disaster state. The realisation of \( b > 0 \) can be thought of as a sharp loss in productivity or as sudden depreciation or loss of trees. The expected growth rate, \( h^* \), of \( c_t \) is given from (3) as the period length approaches zero by:

\[
h^* = h + \left( \frac{1}{2} \right) \times \sigma_u^2 - p \times E b.
\]

Let \( P_t \) be the price in units of consumables of an unlevered equity claim on a tree. The gross, one-period return on this equity is:

\(^3\) The dependence on \( (c_t)^{1/\sigma} \) is reminiscent of the treatment of leverage in Campbell (1986, p. 796) and Abel (1999, p. 15). In their representations, dividends on stocks are proportional to \( c_t \) raised to the power \( \lambda \), where \( \lambda > 1 \) represents leverage. In our model, the exponent is less than one in the cases that we emphasise, where \( \sigma > 1 \).

\(^4\) The analogous concept in Pindyck (1993) is the convenience yield from holding commodities such as gold.

\(^5\) Martin (2013) also emphasises cases in which the shares of the dividends from some assets in total consumption are negligible. However, he assumes that the dividends from all assets are perfect substitutes in consumption, corresponding to \( \sigma \) being infinite in our model. Cochrane et al. (2008) also construct a two-tree model in which the consumption flows are perfect substitutes.

We assume that utility is time-additive and depends on $c_t$ in the usual iso-elastic way with the curvature parameter (coefficient of relative risk aversion) $\gamma > 0$ and time-preference rate $\rho \geq 0$. To simplify the asset-pricing analysis, we make the reasonable assumption that the preference parameter, $q$, and the per capita quantities of gold, $g_t$, and consumption, $c_t$, are always such that the outlay on gold services, $p_t g_t$, is negligible compared to $c_t$. This condition implies that the marginal utility of $c_t$ can be approximated by the usual $c_t^{\gamma}$. In this case, the first-order condition for choosing $c_t$ over time and holding assets as equity claims on trees can be approximated using (5) as:

$$
R_t = \frac{c_{t+1} + P_{t+1}}{P_t}.
$$

The consumption flow, $c_t$, is the dividend accruing to an owner of tree equity. Because the shocks to $\log(c_t)$ in (3) are i.i.d., the ratio of the consumption dividend to the equity price, $c_t/P_t$, will be approximately constant in equilibrium at some value denoted by $d > 0$. (The approximation arises because we are neglecting effects from changing ratios of $c_t$ to $g_t$.) Equations (6) and (3) imply the condition for $d$:

$$
\frac{1}{1 + d} \approx e^{-\rho} \times E_t \left[ c_t^{\gamma} \times \left( \frac{c_{t+1} + P_{t+1}}{P_t} \right) \right].
$$

Define $r'$ to be the expectation of the rate of return on equity, $R_t - 1$. Using (4), (5), and (7), this expectation (constant in this model) is given, as the period length becomes negligible, by:

$$
r' \approx \rho + \gamma h^* - \frac{1}{2} \times \gamma \times (\gamma - 1) \times \sigma_n^2 - p \times [E(1 - b)^{1-\gamma} - 1 - (\gamma - 1) \times E b].
$$

We use the first-order condition for choosing $c_t$ over time and holding assets as risk-free claims to determine the (constant) risk-free rate, denoted $r^f$:

$$
r^f \approx \rho + \gamma h^* - \frac{1}{2} \times \gamma \times (\gamma + 1) \times \sigma_n^2 - p \times [E(1 - b)^{-\gamma} - 1 - \gamma \times E b].
$$

The equity premium follows from (8) and (9):

$$
r^e - r^f \approx \gamma \sigma_n^2 + p \times [E(1 - b)^{-\gamma} - E(1 - b)^{1-\gamma} - E b].
$$

As in previous applications of this result (Barro, 2006; Barro and Ursúa, 2008), we use calibrations where the disaster term involving $p$ on the far right is the main contributor.

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6 With i.i.d. shocks, the main results will hold with Epstein-Zin/Weil preferences, introduced by Epstein and Zin (1989) and Weil (1990).

7 The constancy of $d$ in (7) means that the model fails to account for the observed volatility of dividend-price ratios for stocks. Barro and Ursúa (2012, sections 7–9) suggest that this problem can be rectified by introducing shocks to the growth-rate parameter, $h$, or the disaster probability, $p$, as pursued respectively by Bansal and Yaron (2004) and Gabaix (2012).
to the equity premium. The term involving $\sigma_u^2$ is negligible, as in Mehra and Prescott (1985).\textsuperscript{8}

Consider now the pricing of gold. A unit of gold yields a dividend flow equal to the rental price, $\pi_t^g > 0$, in (2). We assume that gold does not depreciate in a physical sense and that there are no costs of storage or possibilities of loss of ownership. The stock of gold is also not subject directly to the kinds of disturbances that beset the consumption trees – the normal shock, $u_t$, and the disaster shock, $v_t$, in (3). Let $P_t^g$ be the price of gold in units of consumables. The gross, one-period return on holding gold is:

$$R_t^g = \frac{\pi_t^g + P_t^g}{P_t^g}.$$

The first-order condition for choosing $c_t$ over time and holding assets as gold can be approximated, using (11), as:

$$c_t^{-\gamma} \approx e^{-\rho} \times \mathbb{E}_t \left[ c_t^{-\gamma} \times \left( \frac{\pi_{t+1}^g + P_{t+1}^g}{P_t^g} \right) \right].$$

The assumption, as before, is that the outlay on gold is negligible compared to that on ordinary consumption, so that the marginal utility of consumption can be approximated by the usual $c_t^{-\gamma}$.

Recall that the rental price of gold, $\pi_t^g$, is given by (2):

$$\pi_t^g = \left( \frac{1 - \alpha_t}{\alpha_t} \right) \frac{c_t}{g_t}^{1/\sigma}.$$

Thus, for given $c_t$, $\pi_t^g$ varies because of shocks to preferences, $\alpha_t$, and the quantity of gold, $g_t$. If the shocks to $\alpha_t$ and $g_t$ are independent of $c_t$ and each other, as we assume, the dividend-price ratio for gold, $\pi_t^g / P_t^g$, will be approximately constant in this i.i.d. model. Specifically, using (2), (11), and (12), this dividend-price ratio, denoted by $\chi$, will satisfy:

$$1/(1 + \chi) \approx e^{-\rho} \times \mathbb{E}_t \left[ \left( \frac{(1 - \alpha_{t+1})/\alpha_{t+1}}{\alpha_t} \right) \times \left( \frac{g_{t+1}}{g_t} \right)^{-\sigma} \times \left( \frac{c_{t+1}}{c_t} \right)^{(1/\gamma)} \right].$$

We assume that the preference term, $[(1 - \alpha_{t+1})/\alpha_{t+1}]/[(1 - \alpha_t)/\alpha_t]$ in (13), moves proportionately in a random-walk fashion (with no drift), independently of $c_t$ and $g_t$. With respect to the quantity of gold, $g_t$, we assume that the mean growth rate of the per capita gold stock, $g_{\rho}$, is the constant $h_g$, which could be positive due to gold discoveries or negative due to population growth and depreciation or loss of gold.\textsuperscript{9} We suppose further that, relative to its drift, the term $(g_{t+1}/g_t)^{-\sigma}$ in (13) moves proportionately as a random walk, independently of $c_t$ and $\alpha_t$.\textsuperscript{10}

\textsuperscript{8} Barro (2009) shows that the form of the expression for the equity premium in (10) remains valid in this i.i.d. case with Epstein-Zin/Weil preferences, with $\gamma$ representing the coefficient of relative risk aversion.

\textsuperscript{9} In a later Section, we get a rough estimate of $h_g$ based on the long-run growth rate of the per capita world stock of gold. The results, summarised in Figure 2, imply that the average growth rate from 1875 to 2011 is between 0.4% and 0.9% per year if we neglect any loss or depreciation of the gold stock.

\textsuperscript{10} We are, therefore, neglecting the longer run endogenous response of gold mining to changes in gold prices, possibly reflecting influences from shifts in $c_t$ or $\alpha_t$ on the demand for gold. Barro (1979) considers this effect along with other features of the classical model of the gold standard, as developed by Thornton (1802), Ricardo (1819), Mill (1848), Fisher (1911) and Friedman (1951).
Our assumptions imply that shocks to preferences and the quantity of gold will not affect gold’s dividend-price ratio, \( \chi \), but will create volatility in gold’s dividend, \( \pi^g_t \) in (2), and, therefore, in the price of and rate of return on gold. In interpreting changes over time in the volatility of gold prices and returns, we assume that the main source of preference shocks involves changes in the monetary services from gold. This channel associates, in particular, with movements off or on the gold standard and includes changes in the demand for official gold reserves by central banks and governments.\(^{11}\)

The constancy of gold’s dividend-price ratio, \( \chi \), means, from (2), that the real gold price, \( P^g_t \), moves along with \( (c_t)^{1/\sigma} \). Thus, although the dividend from gold is not directly subject to disasters or other shocks, its price ultimately reflects the shocks that affect consumption trees, with the sensitivity of gold prices to these shocks depending inversely on the elasticity of substitution, \( \sigma \). Specifically, (3) and (13) imply that the condition for determining \( \chi \) is:\(^{12}\)

\[
1/(1 + \chi) \approx \exp \left[ \left( \frac{1}{\sigma} - \gamma \right) h - \frac{1}{2} \left( \frac{1}{\sigma} - \gamma \right)^2 \sigma_u^2 \right] \times \left[ 1 - p + p \times E(1 - b)^{(\frac{1}{\gamma} - \gamma)} \right].
\]

(14)

Define \( r^g \) to be the expectation of the rate of return on gold, \( R^g_t - 1 \). Using (2), (11) and (14), this expected return is:

\[
r^g \approx \rho + \gamma h^* - \frac{1}{2} \times \sigma_u^2 \times \left[ \gamma \times \left( \gamma + 1 - \frac{2}{\sigma} \right) \right] - p \times \left[ E(1 - b)^{(\frac{1}{\gamma} - \gamma)} - E(1 - b)^{\frac{1}{\gamma}} - \gamma \times E_0 b \right].
\]

(15)

The expected rate of return on gold, \( r^g \) in (15), can be compared with the expected rate of return on equity, \( r^f \) from (8) or the risk-free rate, \( r^f \) from (9). When compared to the return on equity, the result is:

\[
r^g - r^f \approx \gamma \sigma_u^2 \times \left( \frac{\sigma - 1}{\sigma} \right) + p \times \left[ 1 - Eb - E(1 - b)^{(1 - \gamma)} - E(1 - b)^{\frac{1}{\gamma}} + E(1 - b)^{(\frac{1}{\gamma} - \gamma)} \right].
\]

(16)

When compared to the risk-free rate, the result is:

\[
r^g - r^f \approx \left( \frac{\gamma}{\sigma} \right) \times \sigma_u^2 + p \times \left[ E(1 - b)^{\frac{1}{\gamma}} - E(1 - b)^{(\frac{1}{\gamma} - \gamma)} - 1 + E(1 - b)^{-\gamma} \right].
\]

(17)

The comparison of gold returns with other returns depends on \( \sigma \), the elasticity of substitution between gold services and ordinary consumption in the effective

\(^{11}\) Official gold reserves are quantitatively important compared to the total world stock of gold. Figure 1 shows that the share of the world’s gold stock held as official reserves by central banks and governments was less than 10% in 1877 but rose to a peak of 50–60% at the end of World War II in 1945. Then this share fell to about 20% in 2011. Our measure of official gold reserves is incomplete because holdings by some countries – notably China and the Soviet Union in some periods – are excluded in standard estimates. A more comprehensive measure of monetary gold would also include amounts held privately as minted coins and bullion. As an example, gold in monetary circulation was roughly equal to official gold reserves in 1903 and about half of these reserves in 1913 (using data from US Treasury Department, Bureau of the Mint, 1904, p. 324; 1915, p. 454). We lack a long-time series on a broad concept of monetary gold that includes amounts held privately as coins and bullion.

\(^{12}\) As discussed in footnote 7, the constancy of the dividend-price ratio, \( \theta \), for stocks in (7) means that the model fails to account for the observed volatility of this ratio. Similar issues may arise with respect to the constancy of gold’s dividend-price ratio, \( \chi \), in (14). However, because gold’s dividend is unobservable, we lack measures of the volatility of \( \chi \). In any event, as for stocks, gold’s dividend-price ratio might be volatile because of shocks to the growth-rate parameter, \( h \) and the disaster probability, \( p \).

consumption flow in (1). When $\sigma = 1$, returns on gold mimic the returns on equity, so that $\frac{r^f - r^e}{C_0} = 0$ in (16) and $\frac{r^f - r^e}{C_0} = 0$ in (17) equals the equity premium, $r^f - r^e$, in (10). As $\sigma$ approaches infinity, the rental price of gold, $\pi^e_f$, in (2), becomes unresponsive to $c_0/g$, and gold becomes risk-free. Therefore, $r^e - r^f = 0$ in (17), and $r^e - r^f = 0$ in (16) equals the equity premium, $r^f - r^f$, in (10). In other words, depending on the value of $\sigma$ in the range where $\sigma \geq 1$, the expected rate of return on gold ranges between the expected rate of return on (unlevered) equity, $r^e$, and the risk-free rate, $r^f$.\textsuperscript{13} To put it another way, given a measure of the expected rate of return on gold, $r^e$ – and assuming that this value lies between $r^f$ and $r^f$ there is a value of $\sigma > 1$ that makes that observation consistent with the model.

In the present model, there is no $\sigma > 0$ that generates an expected rate of return on gold, $r^e$, below the risk-free rate, $r^f$. That is, gold never serves as enough of a disaster hedge so that its risk premium would be negative. This conclusion might change if the shocks to preferences, $z$, or the quantity of gold, $g$, co-varied in a particular way with

\textsuperscript{13} If $\sigma < 1$, gold is riskier than equity, and $r^e$ exceeds $r^f$. 

consumption, $c_r$. However, since the empirically observed covariance between consumption growth and the growth rate of real gold prices is small, it is unclear that one wants to add these complications to generate $r^f < r^i$.

2. An Illustrative Calibration

We use a calibration of the model based on an updated version of the analysis of macroeconomic disasters in Barro and Ursúa (2008). Thus, we assume the following:

- $r^f = 0.011$ per year;
- $r^i = 0.059$ per year;
- $h = 0.025$ per year;
- $h_g = 0$;
- $\sigma_u = 0.02$ per year;
- $p = 0.037$ per year;
- $\gamma = 3.34$;
- $\rho = 0.027$ per year;
- $Eb = 0.208$;

\[ E(1-b)^{-\gamma} = 3.62; \text{ and} \]
\[ E(1-b)^{1-\gamma} = 2.16. \]

The values related to disaster sizes, \( b \), derive from the observed size distribution of macroeconomic disasters in the long-term history across countries based on short-term declines in real \( \text{per capita} \) GDP by at least 10%.\(^{14}\) (Barro and Ursua, 2008; Barro and Jin 2011.) The value for \( p \) comes from the observed probability per year of entering into these disaster states. Equation (4) and the assumed parameter values imply that the expected growth rate of real \( \text{per capita} \) GDP and consumption is \( h^* = 0.0175 \) per year.

The value \( \gamma = 3.34 \) was chosen to match the formula in (10) with the specified (unlevered) equity premium, \( r' - r^f = 0.048 \) per year. Matching \( r' = 0.011 \) in (9) turns out to require \( \rho = 0.027 \) per year. We assume this value for \( \rho \), although comparisons among the various rates of return do not depend on \( \rho \) (or \( h^* \)). We consider below the extension of this calibration exercise to returns on gold.

As mentioned before, we estimate the long-term \( \text{per capita} \) growth rate of the gold stock, \( h_g \), by using data on world gold production, as described in the notes to Figure 2. The data before 1875, including an estimate of cumulative world production from 1493 to 1875, are from Soetbeer (1887). The reported cumulative stock of world gold production (assuming no depreciation or loss) from 1493 to 2011 is 155,922 metric tons.

The calculations of stocks of gold require an initial stock in 1492, which is apparently subject to controversy, as noted in GoldMoney Foundation (2012). If we take this stock to be close to zero, we end up with the time series for world gold \( \text{per capita} \) since 1875 shown by the lower graph in Figure 2. To get a plausible range of possibilities, we assume as an alternative that the world gold stock in 1493 equals the reported cumulative production from 1493 to 1875 (9,528 metric tons). In this case, we get the upper graph in Figure 2.\(^{15}\)

The average growth rate of world gold \( \text{per capita} \) from 1875 to 2011 is 0.88% per year based on the lower graph in Figure 2 and 0.42% per year based on the upper graph. Hence, 0.4–0.9% per year provides a reasonable range for gold’s long-term \( \text{per capita} \) growth rate if we assume that stocks had zero depreciation and loss. However, even with small rates of depreciation and loss, the long-term \( \text{per capita} \) growth rate of the world gold stock could be zero or slightly negative. We take \( h_g \approx 0 \) in our main calculations.

### 3. Missing Data on Dividends

A problem in matching the model with data on asset returns is that dividend yields are missing for commodities such as gold, silver and copper. To assess this issue, consider first how the returns on equity divide up between a dividend yield and a price-

\[^{14}\] Results are similar using declines in real \( \text{per capita} \) consumer expenditure but the sample is smaller because of missing data on consumer expenditure.

\[^{15}\] The numbers on the \( \text{per capita} \) gold stock since 1875 were calculated using a time series for world population based on McEvedy and Jones (1978) and World Development Indicators. The WDI data were used for annual numbers since 1960, and the McEvedy-Jones data at 25-year intervals were used before 1960. The growth rate of world population from 1875 to 2011 is 1.17% per year.
The gross return on equity, given by (5), is

$$R_t = \frac{c_{t+1} + P_{t+1}}{P_t}.$$  (5)

Using the definition of the dividend-price ratio as $d = c_t/P_t$, the rate of return, $R_t - 1$, on equity can be expressed as:

$$R_t - 1 = d \times \left( \frac{c_{t+1}}{c_t} \right) + \left( \frac{c_{t+1}}{c_t} - 1 \right).$$  (18)

The first term on the right-hand side is approximately the dividend yield and the second term is the rate of price change (because $P_t$ always moves in the same proportion as $c_t$). As the length of the period approaches zero, the first term on the right-hand side of (17) approaches $d$ and the uncertainty about this term is negligible.\(^{16}\) Hence, with short periods, nearly all of the uncertainty about the rate of return is concentrated into the second term, which reflects price changes.

We know the expectation of the rate of return in (18) from the formula for $r^e$ in (8). The first term on the right-hand side comes from the formula for $d$ in (7). The expectation of the second term—the expected growth rate of consumption—equals $h^*$, given in (4).

In the calibration of the model, we had $r^e = 0.059$ per year and $h^* = 0.0175$ per year. Hence, the dividend yield was 0.0415 per year (a result that can be verified from the formula for $d$ in (7)). To put it another way, 30% of the overall expected rate of return on equity reflects expected real price appreciation and 70% represents the dividend yield. Hence, omitting the dividend yield is a major problem for matching the model’s expected rate of return to the data. In contrast, all of the model’s uncertainty is concentrated into the price-appreciation term, with none appearing in the dividend yield. Therefore, data on asset returns exclusive of dividend yields would be satisfactory for gauging standard deviations of returns and covariances of returns with consumption growth rates and among assets.

We can check this theoretical reasoning against US data on stock returns. Using a total-return stock-market index and a consumer price index (provided by Global Financial Data), the average (arithmetic) real rate of return from 1836 to 2011 was 7.4% per year, with a standard deviation of 16.1%.\(^{17}\) (Note that these data refer to levered returns, rather than the unlevered ones considered in the model.) In contrast, if one uses only stock-price-index data, thereby omitting dividends, the average rate of return was 2.5% with a standard deviation of 15.4%. Therefore, the omission of dividends has a major effect on the average rate of return—with only 2.5 of the total 7.4 percentage points or 34% captured by real price appreciation.\(^{18}\) In contrast, the standard deviation of the price-change series, 15.4%, is close to that for total returns.

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\(^{16}\) The dividend can jump in the model when a disaster occurs. However, the probability, $p$, of a jump occurring during a period becomes negligible as the length of the period approaches zero.

\(^{17}\) These results are based on stock-return indexes and CPI’s averaged over each year, as used in Table 1.

\(^{18}\) This kind of effect from the omission of dividends on measured mean real stock returns is well known; see, for example Jorion and Goetzmann (1999, table III).
Moreover, the covariance of the growth rate of per capita consumption (personal consumer expenditure) with the total-return series is 0.00224 (correlation of 0.36)\textsuperscript{19} compared to 0.00229 (correlation of 0.39) with the price-change series. Therefore, the covariance and correlation computed from the price-change series are close to those calculated from total returns. Hence, as found theoretically, the price-change series capture the uncertainty in stock returns well – and this result holds even though the dividend-price ratio is not constant in the data (unlike in the model) and the data are analysed annually, rather than at a higher frequency.

The model’s implications for dividend yield and price appreciation are analogous for gold. The gross return on gold, $R^g_t$, is defined in (11), the dividend on gold, $\pi^g_t$, is given by (2) and the dividend-price ratio, $\chi$, is determined from (14). Analogous to (18), the real rate of return on gold, $R^g_t - 1$, can be broken down into a dividend yield and a price-appreciation term:

$$R^g_t - 1 = \chi \left( \frac{\pi^g_{t+1}}{\pi^g_t} \right) + \left( \frac{\pi^g_{t+1}}{\pi^g_t} - 1 \right).$$

(19)

As before, the first term on the right-hand side approaches the dividend-price ratio, $\chi$, as the length of the period becomes negligible. The second term, which reflects real gold price appreciation, is more complicated than before because $\pi^g_t$ depends on $(e/\bar{g})^{1/\sigma}$.

Analogous to equity returns, when the length of the period is negligible, all of the uncertainty about gold returns is concentrated into the price-appreciation term (the second part of the right-hand side of (19)) and none appears in the dividend-yield term (the first term). Therefore, the available data on real gold price appreciation should provide good information about the standard deviation of gold returns and about the covariances of these returns with consumption growth rates and with other asset returns.

We know the expectation of the rate of return on gold in (19) from the formula for $r^g$ in (15). The first term on the right-hand side – approximately the dividend yield – comes from the formula for $\chi$ in (14). The expectation of the second term can be determined as:

$$E \left( \frac{\pi^g_{t+1}}{\pi^g_t} - 1 \right) = \left( \frac{1}{\sigma} \right) \times (h^\ast - h_g) - \left( \frac{1}{2} \right) \sigma^2 \times \frac{1}{\sigma} \times \left( \frac{\sigma - 1}{\sigma} \right) + p \times \left[ \frac{1}{\sigma} \times E b - 1 + E(1 - h)^{1/2} \right].$$

(20)

If $\sigma = 1$, this term equals $h^\ast - h_g$ whereas if $\sigma$ is infinite, this term equals zero.

Observations on the mean growth rate of real gold prices provide measures of the left-hand side of (20). Then, if we use our previous specification of parameters other than $\sigma$ on the right-hand side, we can derive the $\sigma$ needed to equate the right-hand side to the left-hand side. As discussed in the next Section, the mean growth rate of US real gold prices from 1836 to 2011 is 1.1% per year (based on the US dollar gold price and consumer price indexes). However, the annual standard deviation of the growth rate of real gold prices is 13.1%, which corresponds to a standard deviation for the mean over 176 years of 1.0% per year. Therefore, a one-standard-deviation confidence interval for

\textsuperscript{19} See Table 2(a).

the mean growth rate of US real gold prices is roughly (0.1%, 2.1%). This range for the left-hand side of (19) turns out to correspond on the right-hand side to a wide interval for the estimated \( \sigma \): from 16 to 0.84. The corresponding range for \( r^f \) (from (16)) is 1.6–6.4%, and the share of the unobserved dividend yield in this expected return (based on (15)) goes from 94% to 67%.

The bottom line is that this method fails, by itself, to provide a precise estimate of gold’s unobserved dividend yield and, thereby, the expected real rate of return on gold. In the next Section, we bring in additional information from the observed covariance between changes in real gold prices and consumption growth.

4. Empirical Regularities on Gold Returns

4.1. Means and Standard Deviations

We apply the model empirically by focusing on the historical patterns for real gold returns, gauged by rates of change in real gold prices. Tables 1–3 provide statistics from US data for 1836–2011. Apart from gold returns, the Tables consider returns on two other commodities (silver and copper), returns on three types of financial assets (stocks, 10-year bonds, and Treasury Bills), inflation rates, and growth rates of real per capita consumption and GDP. The starting date was chosen based on the available data on US consumption (personal consumer expenditure).

All real returns in Tables 1–3 are computed from a US perspective, including the deflation of US dollar commodity prices by US consumer price indexes. We think of the real US dollar returns on commodities as determined on world markets. In particular, we think about real gold returns as reflecting shifts to world supply and demand, some of which are captured in Figures 1 and 2.

Tables 1–3 cover four sub-periods, 1836–79, 1880–1913, 1914–74 and 1975–2011, chosen to reflect changes in the international regimes for gold and silver. The period 1880–1913 is the highpoint of the world gold standard, featuring a nearly constant US dollar price of gold (Figure 3). We think of this period as exhibiting comparatively low volatility in the demand for gold, as reflected in the preference term, \( \frac{1 - \alpha_i}{\alpha_p} \), which enters into the gold dividend in (2).

For 1836–79, many countries, including the UK and US, had monetary systems linked to gold or silver for much of the sample. However, a suspension of US currency convertibility occurred near the beginning of the Civil War in 1861, with full convertibility restored in 1879. We view this period as having greater volatility than 1880–1913 in the demand for monetary gold. However, even in 1880–1913, countries

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Pindyck (1993, figure 4 and table 4) estimates the dividend yield (percentage net basis) on gold and other commodities using spot and futures prices for periods when liquid futures markets exist. He infers that the average dividend yield for gold for 1975–89 was 0.4% per year and remained below 3% except for a spike in 1981.

Data on T-Bill returns are available for 1919–2011.

Erb and Harvey (2013) emphasise the period since 1975, following the lifting of US restrictions on holdings of monetary gold by US citizens. These authors focus on effects on the real price of gold from inflation, exchange-rate movements and tail risks.

Figure 4 shows the real gold price (US dollar price divided by the US consumer price index, with the real gold price in 1800 set to 1.0).
Table 1

US Real Asset Returns: Means and Standard Deviations

<table>
<thead>
<tr>
<th>Period</th>
<th>(1) Gold</th>
<th>(2) Silver</th>
<th>(3) Copper</th>
<th>(4) Stocks</th>
<th>(5) T-Bills</th>
<th>(6) 10-year bonds</th>
<th>(7) Inflation rate</th>
<th>(8) C growth</th>
<th>(9) GDP growth</th>
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</thead>
<tbody>
<tr>
<td>1836–2011</td>
<td>0.0112</td>
<td>0.0123</td>
<td>0.0091</td>
<td>0.0740</td>
<td>0.0097*</td>
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<tr>
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<td>-</td>
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<td>(0.1518)</td>
<td>(0.1251)</td>
<td>-</td>
<td>(0.0412)</td>
<td>(0.0263)</td>
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<td>1914–74</td>
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<td>(0.1933)</td>
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<td>(0.0723)</td>
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<td>1975–2011</td>
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<td>(0.0871)</td>
<td>(0.0291)</td>
<td>(0.0163)</td>
<td>(0.0201)</td>
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</table>

Notes. *1920–2011. †1920–74. The return on gold refers to the growth rate of real gold prices, calculated arithmetically for each year as: \(-1 + (\text{gold price}/\text{CPI})\)/\([\text{gold price}(-1)/\text{CPI}(-1)]\), where \((-1)\) indicates an annual lag. The US dollar gold price comes from Global Financial Data (GFD). The number up to 1933 is the official price set by the US government, except for 1861–78, when the data come from Commercial and Financial Chronicle. Data after 1933 are from Commodity Research Bureau, Commodity Yearbook. The CPI values reported by GFD derive from the BLS consumer price index for urban consumers since 1913. Data before 1913 are based on information from the Federal Reserve Bank of New York, including monthly data since 1875 described in Snyder (1924). The gold price used for each year is the average of daily or monthly values during the year. The CPI value used for each year since 1875 is the average of monthly values during the year. Only annual data on consumer prices are available before 1875. The real silver and copper returns are computed analogously, based on averages of daily or monthly US dollar silver and copper prices. The silver prices reported by GFD come from Officer (2008), Warren and Pearson (1937) and Commodity Research Bureau, Commodity Yearbook. Recent New York quotes are from Handy and Harman. The copper prices reported by GFD come from Bezanson (1956, 1954), Metal Statistics, The Financial Review, the National Bureau of Economic Research, American Metal Market and the New York Mercantile Exchange. The real stock return is computed analogously, based on averages of daily or monthly nominal total-return indexes computed by GFD for the S&P500. Values before 1971 are based on GFD estimates of total-return indexes comparable to the S&P500. The real T-Bill return is computed analogously, based on averages of monthly nominal total-return indexes for 90-day US Treasury Bills. The estimates of total returns from GFD since 1929 derive from yields on 90-day bills. Estimates from 1919 to 1928 are based on yields on short-term US Treasury bonds. The real return on 10-year US government bonds is computed analogously, based on averages of monthly nominal total-return indexes for US government bonds with roughly 10-year remaining maturity. Values from 1919–40 are based on the Federal Reserve’s 10–15 year Treasury bond index. Values before 1919 are based on various long-term US government bonds. Data for 1836 to 1841 are from Boston city bonds. The inflation rate is calculated as \(-1 + \text{CPI}/\text{CPI}(-1)\), using the CPI data described above. Consumption (C) and GDP growth are real per capita growth rates calculated arithmetically from the Barro-Ursúa annual data on real per capita personal consumer expenditure and GDP, available at www.rbarro.com/data-sets.
Table 2

<table>
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<tr>
<th>Period</th>
<th>Gold</th>
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<th>Copper</th>
<th>Stock</th>
<th>T-Bill</th>
<th>10-year bond</th>
<th>Inflation rate</th>
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<td>1836–2011</td>
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<td>0.0014*</td>
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<td>Silver</td>
<td>Copper</td>
<td>Stock</td>
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<td>(–0.67, 0.02)</td>
<td>(–0.46, 0.08)</td>
<td>(–0.10, 0.50)</td>
</tr>
<tr>
<td>1975–2011</td>
<td>–0.0008</td>
<td>–0.0001</td>
<td>0.0011</td>
<td>0.0012*</td>
<td>0.0001</td>
<td>–0.0001</td>
<td>–0.0001</td>
</tr>
<tr>
<td></td>
<td>(–0.0025, 0.0007)</td>
<td>(–0.0019, 0.0016)</td>
<td>(–0.0002, 0.0023)</td>
<td>(0.0002, 0.0022)</td>
<td>(–0.0001, 0.0003)</td>
<td>(–0.0006, 0.0005)</td>
<td>(–0.0003, 0.0002)</td>
</tr>
<tr>
<td></td>
<td>[–0.19]</td>
<td>[–0.01]</td>
<td>[0.23]</td>
<td>[0.43*]</td>
<td>[0.16]</td>
<td>[–0.05]</td>
<td>[–0.10]</td>
</tr>
<tr>
<td></td>
<td>(–0.53, 0.19)</td>
<td>(–0.34, 0.30)</td>
<td>(–0.05, 0.47)</td>
<td>(0.10, 0.68)</td>
<td>(–0.26, 0.55)</td>
<td>(–0.39, 0.28)</td>
<td>(–0.46, 0.32)</td>
</tr>
</tbody>
</table>

Notes. *Statistically significant at 5% level. †1920–2011. ‡1920–74. The data on real asset returns, inflation rates and growth rates of consumption and GDP are described in the notes to Table 1. Table 2(a) applies to covariances and correlations (shown in brackets) of changes in real gold prices with the growth rate of per capita consumption. Table 2(b) applies to the growth rate of per capita GDP. 95% confidence intervals are in parentheses below each sample value for covariance or correlation. These intervals were generated from percentile-method bootstraps with 100,000 iterations. Table 3 shows the covariances and correlations (shown in brackets) of changes in real gold prices with real asset returns and the inflation rate. 95% confidence intervals were constructed as described above.
Table 3

*Covariances (Correlations) of Real Gold Returns with Other Real Asset Returns*

<table>
<thead>
<tr>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period</td>
<td>Silver</td>
<td>Copper</td>
<td>Stock</td>
<td>T-Bill</td>
<td>10-year bond</td>
<td>Inflation rate</td>
</tr>
<tr>
<td>1836–2011</td>
<td>0.0167*</td>
<td>0.0062*</td>
<td>0.0001</td>
<td>–0.0001†</td>
<td>0.0007</td>
<td>–0.0009</td>
</tr>
<tr>
<td></td>
<td>(0.0089, 0.0260)</td>
<td>(0.0025, 0.0104)</td>
<td>(−0.0028, 0.0039)</td>
<td>(−0.0013, 0.0011)</td>
<td>(−0.0016, 0.0027)</td>
<td>(−0.0023, 0.0008)</td>
</tr>
<tr>
<td></td>
<td>[0.71*]</td>
<td>[0.28*]</td>
<td>[0.03]</td>
<td>[−0.01]†</td>
<td>[0.07]</td>
<td>[−0.13]</td>
</tr>
<tr>
<td></td>
<td>(0.58, 0.81)</td>
<td>(0.13, 0.44)</td>
<td>(−0.13, 0.20)</td>
<td>(−0.19, 0.16)</td>
<td>(−0.14, 0.31)</td>
<td>(−0.38, 0.09)</td>
</tr>
<tr>
<td>1836–79</td>
<td>0.0065*</td>
<td>0.0047*</td>
<td>0.0056*</td>
<td>–</td>
<td>0.0007</td>
<td>–0.0024</td>
</tr>
<tr>
<td></td>
<td>(0.0031, 0.0116)</td>
<td>(0.0014, 0.0084)</td>
<td>(0.0023, 0.0108)</td>
<td>–</td>
<td>(0.0011, 0.0063)</td>
<td>(−0.0052, 0.0004)</td>
</tr>
<tr>
<td></td>
<td>[0.98*]</td>
<td>[0.50*]</td>
<td>[0.44*]</td>
<td>[0.51*]</td>
<td>[−0.41]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.95, 0.995)</td>
<td>(0.22, 0.72)</td>
<td>(0.21, 0.63)</td>
<td>–</td>
<td>(0.21, 0.75)</td>
<td>(−0.83, 0.05)</td>
</tr>
<tr>
<td>1880–1913</td>
<td>−0.0001</td>
<td>−0.0012*</td>
<td>−0.0003</td>
<td>–</td>
<td>0.0006*</td>
<td>−0.0007*</td>
</tr>
<tr>
<td></td>
<td>(−0.0006, 0.0004)</td>
<td>(−0.0023, 0.0000)</td>
<td>(−0.0013, 0.0008)</td>
<td>–</td>
<td>(0.0003, 0.0009)</td>
<td>(−0.0010, −0.0004)</td>
</tr>
<tr>
<td></td>
<td>[−0.05]</td>
<td>[−0.30*]</td>
<td>[−0.09]</td>
<td>[0.60*]</td>
<td>[−0.999*]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(−0.36, 0.28)</td>
<td>(−0.55, −0.01)</td>
<td>(−0.41, 0.26)</td>
<td>(0.35, 0.80)</td>
<td>(−0.9997, −0.9992)</td>
<td></td>
</tr>
<tr>
<td>1914–74</td>
<td>0.0124*</td>
<td>0.0035</td>
<td>−0.0003</td>
<td>0.0016*‡</td>
<td>0.0026</td>
<td>−0.0020</td>
</tr>
<tr>
<td></td>
<td>(0.0012, 0.0259)</td>
<td>(−0.0018, 0.0004)</td>
<td>(−0.0089, 0.0078)</td>
<td>(0.0002, 0.0031)</td>
<td>(−0.0001, 0.0052)</td>
<td>(−0.0042, 0.0004)</td>
</tr>
<tr>
<td></td>
<td>[0.52*]</td>
<td>[0.15]</td>
<td>[−0.01]</td>
<td>[0.23*‡]</td>
<td>[0.27]</td>
<td>[−0.26]</td>
</tr>
<tr>
<td></td>
<td>(0.09, 0.74)</td>
<td>(−0.11, 0.38)</td>
<td>(−0.30, 0.35)</td>
<td>(0.02, 0.53)</td>
<td>(0.00, 0.66)</td>
<td>(−0.66, 0.05)</td>
</tr>
<tr>
<td>1975–2011</td>
<td>0.0505*</td>
<td>0.0187*</td>
<td>−0.0033</td>
<td>−0.0027*</td>
<td>−0.0061</td>
<td>0.0014</td>
</tr>
<tr>
<td></td>
<td>(0.0227, 0.0810)</td>
<td>(0.0055, 0.0340)</td>
<td>(−0.0096, 0.0029)</td>
<td>(−0.0045, −0.0011)</td>
<td>(−0.0153, 0.0017)</td>
<td>(−0.0021, 0.0058)</td>
</tr>
<tr>
<td></td>
<td>[0.83*]</td>
<td>[0.38*]</td>
<td>[−0.11]</td>
<td>[−0.55*]</td>
<td>[−0.34]</td>
<td>[0.24]</td>
</tr>
<tr>
<td></td>
<td>(0.72, 0.91)</td>
<td>(0.13, 0.65)</td>
<td>(−0.36, 0.12)</td>
<td>(−0.76, −0.30)</td>
<td>(−0.68, 0.12)</td>
<td>(−0.54, 0.66)</td>
</tr>
</tbody>
</table>

Notes. *Statistically significant at 5% level. †1920–2011. ‡1920–74. For a detailed description, see footnote to Table 2.
retained the option to move off gold under various circumstances, especially during wars. For example, the UK suspended convertibility of the currency in 1797 during the long period of wars with France and did not restore convertibility until 1821.

In the 1914–74 period, many countries moved off the gold standard during World War I, with resumptions and subsequent departures occurring in the 1920s and 1930s. The US maintained aspects of the gold standard throughout this period but a rise in the nominal gold price and a prohibition on large holdings of monetary gold by US citizens occurred in 1933. In 1971, the US formally dropped its commitment to foreign central banks to convert US dollars into gold at a fixed dollar price. Then at the beginning of 1975, the US lifted restrictions on private holdings of monetary gold. We view the partial prohibition on holdings of monetary gold by US citizens for 1933–74 as a shock to the preference parameter \( \alpha_t \) at \( t \), which relates to gold’s dividend in (2).

In the 1975–2011 period, gold retained a commodity-reserve role for central banks but one that was largely divorced from domestic monetary systems. We think that the elimination of the anchor provided by the gold standard led to an increase in the volatility of the demand for monetary gold, as reflected in the preference term \((1 - \alpha)/\alpha_t\) in (2).

Over the full sample, 1836–2011, the mean real rates of price change on gold, silver, and copper are similar – 1.1%, 1.2%, and 0.9% per year respectively in columns 2–4 of Table 1. The standard deviations of the real rates of price change for gold, silver and

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Notes. See the notes to Table 1 for the sources of data on annual average US dollar gold prices. After 1971, the US dollar gold price became highly variable, reaching an annual average of $613 for 1980 and $1,573 for 2011.

copper over the full sample – 13.1%, 17.9%, and 17.0% respectively – are also similar and close to that on stocks (16.1%).

For gold, the standard deviation of real rates of price change depends strongly on the sub-period. For example, the standard deviation for 1880–1913 is 2.6% per year, whereas that for 1975–2011 is 20.7%, higher than that for stocks (14.1%). We think that this pattern relates to the changing monetary role of gold; specifically, preference shocks related to gold’s monetary services seem to be much smaller under a serious gold standard (1880–1913) than at a time (1975–2011) when gold’s monetary role was much less clearly defined. The periods 1836–79 and 1914–74 then have intermediate volatility (SDs of 8.2% and 13.4% respectively) because the world monetary system involved an important role for gold but one that was less central than that for 1880–1913.

Note, further, that the time pattern for the standard deviation of gold’s real rate of price change differs sharply from that for copper. Copper’s standard deviation evolved much more smoothly over time – 11.4% for 1836–79, 15.2% for 1880–1913, 17.1% for 1914–74, and 23.5% for 1975-2011. In particular, the presence of a serious gold standard for 1880–1913 had no obvious impact on the standard deviation of copper’s real rate of price change. Our inference is that the changing volatility of gold’s real rate of price change derives mainly from the shifting monetary role of gold and not from other time-related changes in the economy (which would also likely have affected copper).

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25 Since the nominal gold price was essentially constant (Figure 3), this pattern reflects the behaviour of inflation (Table 1, column 8).

When compared with earlier times, the most recent sub-period, 1975–2011, shows a substantially higher mean and standard deviation of gold’s real rate of price change – 4.0% and 20.7% respectively. Although the increase in volatility is clear, the change in the mean is less sure because it is hard to pin down the expected value when the volatility is this high. If the annual standard deviation were known to be 20.7% (the sample value), the standard error of the mean over 37 years would be 3.4%. That is, the observed mean real rate of price change of 4.0% for 1975–2011 is not statistically significantly different from zero (or from the mean rates of change in earlier periods) at typical significance levels. Given this result, it seems best to focus our analysis on statistics for gold’s expected real returns over the full sample, 1836–2011.

4.2. Covariances of Gold Returns with Consumption and GDP Growth Rates

For matching the model to the data, a critical consideration is the covariance of real gold returns (real rates of price change) with consumption growth. In the model, gold’s expected rate of return, $r_g$, depends on $\sigma$, the elasticity of substitution between gold services and ordinary consumption. If $\sigma \geq 1$, $r_g$ varies between the expected return on equity, $r^e$ and the risk-free rate, $r^f$. Notably, $r_g$ cannot fall below $r^f$ because gold returns do not co-vary negatively with consumption growth. This property of the model would deviate from reality if it turned out that the covariance between gold’s real rate of price change and the consumption growth rate were substantially negative. In contrast, if this covariance is small in magnitude (as seems to be true), gold does not serve as a substantial hedge against macroeconomic declines, and its expected rate of return should be close to the risk-free rate.

Table 2 shows sample covariances and correlations of the various US real rates of return and inflation rates with respectively the growth rates of real per capita consumption (personal consumer expenditure) and GDP. Also included are 95% confidence intervals for these statistics, based on bootstrap methods.

For 1836–2011, the key finding in Table 2(a), column 2, is that the covariance of real gold returns with consumption growth is small, $-0.0002$ (with a correlation of $-0.05$), not statistically significantly different from zero. This negligible covariance implies that the expected real rate of return on gold, $r_g$, should be close to the risk-free rate, which is around 1.0%, based on the sample mean of real returns on US Treasury Bills (Table 1, column 6).

Another approach to gauging whether gold is empirically a hedge against macroeconomic declines is to focus on the behaviour of gold’s real rates of price change during the worst macroeconomic outcomes; that is, the rare disasters studied in previous research. Among the 185 disasters isolated over the long term for 40 countries, we were able to estimate real returns on gold for 56 of these cases, applying to 19 OECD countries. The calculated average annual real rate of price change for gold in each country during a disaster was based on the time paths of the world dollar price of gold, the nominal exchange rate between the home currency and the US dollar and the consumer price index for the home country. This calculation neglects restrictions on buying and selling gold (at the world dollar price) and also ignores controls on...

26 The sample covariance of real T-Bill returns with consumption growth for 1920–2011 is essentially zero (Table 2(a), column 6).

currency exchanges or the general price level. (These controls might mean that actual transactions typically could not be made at the stated prices.)

For 14 OECD countries with data on exchange rates and CPIs back to 1880, gold’s mean real rate of price change was 1.5% per year. For the 56 macro disasters with the necessary data (applying to 19 countries), gold’s mean real rate of price change during the disaster periods was 2.1% per year.\(^{27}\) Since the standard deviation of these gold returns was 22%, gold’s mean real rate of price change during the macroeconomic disasters was not statistically significantly different from the overall mean.\(^{28}\) Therefore, the bottom line from the disaster experience is consistent with the long-term US finding that gold does not serve regularly as a hedge against bad macroeconomic outcomes.\(^{29}\)

To match the model with this feature of the data, we need a value of \(\sigma\) (the elasticity of substitution between gold services and ordinary consumption) that is high enough to get \(\sigma^g\) close to \(\sigma^f\). Specifically, we found before that the long-term sample mean of gold’s real rate of price change \(-1.1\%\) per year, with a one-standard-deviation confidence interval of roughly \((0.1\%, 2.1\%)\) – required \(\sigma\) to fall into the range from 16 to 0.84. Low values of \(\sigma\) within this range are inconsistent with \(\sigma^g\) being close to \(\sigma^f\). For example, using (15), \(\sigma = 1\) implies \(\sigma^g = 5.9\%\) (the same as the expected return on unlevered equity) and \(\sigma = 2\) implies \(\sigma^g = 4.0\%\). Therefore, with a risk-free rate around 1.0%, the risk premia on gold implied by these low values of \(\sigma\) are inconsistent with the observed negligible covariance of gold’s real rate of price change with consumption growth.

The high end of the range for \(\sigma\) produces more satisfactory results. For example, \(\sigma = 10\) generates \(\sigma^g = 1.8\%\), with an expected rate of change in real gold prices of 0.2% and \(\sigma = 16\) generates \(\sigma^g = 1.6\%\), with an expected rate of change in real gold prices of 0.1%. Hence, specifications with high values of \(\sigma\) generate values of \(\sigma^g\) that are only small amounts above the estimated risk-free rate of 1.0%, while also producing expected rates of change in real gold prices within the one-standard-error band of \((0.1\%, 2.1\%)\). A notable feature of these results is that the bulk of gold’s expected return comes from the unobserved dividend yield, with only a small part reflecting the expected real rate of price change.

These results depend on the negligible observed covariance between gold’s real rate of price change and consumption growth in the long-term data. However, this pattern does not apply to all commodities; in particular, in the US data, the long-run covariance and correlation between copper’s real rate of price change and consumption growth are significantly positive – 0.0014 and 0.22 respectively for 1836–2011 in

\(^{27}\) A problem with this calculation is that, because of price controls and restrictions on currency exchanges, the reported CPIs and exchange rates are particularly likely not to reflect true transaction prices during disasters – especially war-related disasters.

\(^{28}\) The disaster sample has 21 war-related periods with a mean real rate of price change for gold of \(-0.8\%\) per year (standard deviation of 22%) and 35 non-war periods with a mean of 3.8% (standard deviation of 22%). The difference between the means in war and non-war disasters is not statistically significantly different from zero.

\(^{29}\) However, ‘T-Bills’ performed worse than gold as a hedge against the macroeconomic disasters. For the 49 OECD cases with data on returns on both gold and T-Bills, the mean real rate of return on T-Bills was \(-4.6\%\) per year, compared to \(1.3\%\) for gold. The mean of the differences in these returns has a t-statistic of 2.1, p-value = 0.044. The T-Bill returns had a mean of \(-6.6\%\) per year in wartime disasters (associated with high inflation) and \(-3.2\%\) in non-wars (heavily influenced by the value \(-97\%\) in 1923 during the German hyperinflation).

Table 2(a), column 4. This pattern likely arises because copper plays a more important role than gold in the production of overall goods and services.

4.3. Covariances Between Gold Returns and Other Asset Returns

In the model, with i.i.d. shocks to GDP and consumption, the covariance between real gold returns and the growth rates of GDP and consumption is positive but small if the elasticity of substitution, $\sigma$, is high. The same prediction applies to the covariance between real gold returns and real stock returns.

Table 3 shows covariances and correlations of growth rates of real gold prices with real asset returns and inflation rates for 1836–2011 and the various sub-periods. For stock returns (column 4), the covariance is negligible over the full sample. As discussed before, this result accords with the model if $\sigma$ is high.

In the model, the real T-bill return is constant and equals the short-term risk-free rate, and the real term structure is flat. Hence, the model predicts zero covariance between real gold returns and real returns on T-Bills or 10-year government bonds. This prediction would change if we bring in effects of inflation on assets such as conventional government bonds with pay-outs denominated in nominal units.

In Table 3, columns 5 and 6, the covariance between changes in real gold prices and real returns on the two forms of government securities is positive for sub-periods between 1836 and 1974 and negative for 1975–2011. These covariances are statistically significantly different from zero at the 5% level for 10-year bonds for 1836–79 and 1880–1913 and for T-Bills for 1920–74 and 1975–2011. The results are nearly statistically significant at the 5% level for 10-year bonds for 1914–74 and 1975–2011.

The empirical relation between changes in real gold prices and real returns on government securities is likely to derive from effects of inflation on the real value of assets – conventional Treasury bonds and T-Bills – that are denominated in nominal terms. The covariances between the real returns on the two forms of government securities and inflation are strongly negative for the overall sample, 1836–2011 and the various sub-periods. The covariances between gold’s real rate of price change and inflation are also negative in periods where the nominal gold price is virtually constant (1880–1913) or has some element of nominal pegging (1836–79 and 1914–74) – as shown in Table 3, column 7. This pattern for inflation creates positive correlation between the real returns on government securities and gold over the sub-periods from 1836 to 1974 (columns 5 and 6). From 1975 to 2011, there is no longer nominal pegging of gold prices and the covariance between real gold returns and inflation becomes positive, though not statistically significantly different from zero (column 7). This changed pattern between real gold returns and inflation is likely to explain why the covariances between real gold returns and real returns on the two forms of government securities are negative in this period.

The covariance between changes in real gold and silver prices (Table 3, column 2) is significantly positive except during the high point of the classical gold standard from 1880 to 1913. In this period, the nominal price of gold, but not silver, was essentially constant. For 1836–79, the correlation between changes in real gold and silver prices is close to one, because the ratio of gold to silver nominal prices changes

little, reflecting the bimetallic standard that was partially maintained in the UK and the US. The ratio of gold to silver prices from 1790 to 2011 is shown in Figure 5. For 1836–79, this ratio varies relatively little around the median for that sub-period of 16.30 Figure 5 also shows the ratio of world stocks of silver to world stocks of gold for 1790-2011. This ratio falls from 34 in 1840 to 19 in 1875 (partly due to gold discoveries), then trends downward further to 9 in 2011.31 This pattern in the quantity ratio does not align in an obvious way with the fluctuations in the price ratio. An interesting extension would relate the price and quantity ratios to exogenous changes in the relative supplies of and demands for these two precious metals.

5. Summary Observations

Our main objective was to match empirical regularities for real gold returns with the predictions from a simple asset-pricing model. As to regularities, we observe first that,

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30 Nevertheless, the changes in the US mint ratio in 1834 and 1853 and the full demonetisation of silver in 1873 (the ‘crime of 1873’) received a lot of attention – see Laughlin (1894, chs. IV, V and VII).

31 The estimated stock of silver in 2011 is 1,371,239 metric tons.
from 1836 to 2011, the US average of gold’s real rate of price change is 1.1% per year with a standard deviation of 13.1%, implying a one-standard-deviation confidence band for the mean of roughly (0.1%, 2.1%). Second, over the same period, the covariance of gold’s real rate of price change with consumption growth is small and statistically insignificantly different from zero. This negligible covariance implies that gold should carry a negligible risk premium; that is, gold’s expected real rate of return – which includes an unobserved dividend yield – should be close to the risk-free rate, estimated from real returns on Treasury Bills to be around 1.0%. Third, the volatility of the growth rate of real gold prices is small under the classical gold standard from 1880 to 1913 but high – comparable to that on stocks – in other periods, including 1975 to 2011.

Key features of our model are, first, ordinary consumption and gold services are imperfect substitutes for the representative household; second, outlays on gold services are always minor compared to ordinary consumption; and third, disaster and other shocks impinge directly on consumption and GDP but not on stocks of gold. With a high elasticity of substitution, \( \sigma \), between gold services and ordinary consumption, the model can generate a mean real rate of price change towards the lower end of the observed one-standard-deviation confidence band, (0.1%, 2.1%), along with a small risk premium for gold. In this scenario, the bulk of gold’s expected real rate of return reflects the unobserved dividend yield and only a small part comprises expected real price appreciation. Nevertheless, the uncertainty in gold returns is concentrated in the price-change component.

The model accords with the time-varying volatility of real gold prices if preference shocks associated with gold’s monetary services relate inversely to the extent that the gold standard was maintained. In particular, we need that these preference shocks were small during the high point of the world gold standard, 1880–1913, but much larger in 1975–2011, most probably because gold’s monetary role was less central than in earlier periods. A promising extension would explore more deeply the nature of the preference shocks that seem to generate a volatility of real gold prices that is inversely related to the extent to which the gold standard was followed and, relatedly, the degree to which nominal gold prices were pegged.

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References


