Hopenhayn et al. (2019).

From Population Growth to Firm Demographics: Implications for Concentration, Entrepreneurship and the Labor Share

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Macro Reading Group

Introduction

- Aggregate trends in the economy: increase in concentration, decrease in entrepreneurship rate, decrease in labor share
- Hypothesis: driven by aging firm distribution, which can be explained by change in labor force growth
- Construct a model of firm dynamics in which changes in labor force growth lead to changes in firm demographics
- Show that labor force growth provides a unified explanation of the aggregate trends

Roadmap

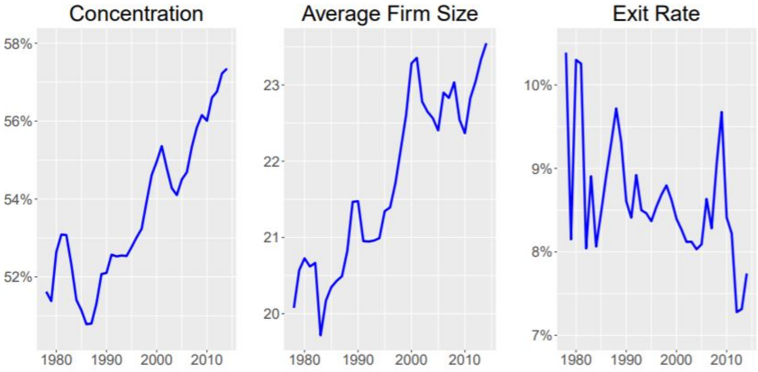
1. Data and Aggregate Trends
2. Model
3. Calibration
4. Discussion and Comments

Data

- Business Dynamics Statistics (BDS) data from Census; publicly available
- Covers almost all private sector US firms from 1977-2014
- Aggregate statistics by group (age, size, sector, etc.) but no firm-level data
- Interested in three aggregate trends:
  - Concentration: share of employment by firms with 250+ employees
  - Average firm size: number of employees per firm
  - Aggregate exit rate

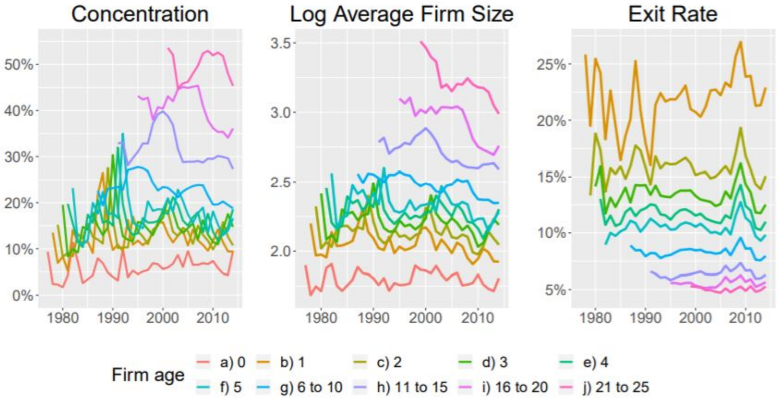
Aggregate Trends

Figure: Concentration and average firm size on a steady rise; exit rate falling in recent decade



Breaking Down by Firm Age

Figure: Trends by age are generally flat



Breaking Down by Firm Age

Figure: Regression of concentration on year, with and without age controls

	(1)	(2)
Year	0.003***	-0.000
	(0.001)	(0.000)
AGE:		
Age 0		0.666
		(0.439)
Age 1		0.730*
		(0.439)
Age 2		0.740*
		(0.440)
Age 3		0.756*
		(0.440)
Age 4		0.772*
		(0.440)
Age 5		0.786*
		(0.440)
Age 6 to 10		0.839*
		(0.440)
Age 11 to 15		0.928**
		(0.441)
Age 16 to 20		1.017**
		(0.441)
Age 21 to 25		1.115**
		(0.442)
R^2	0.080	0.976
Observations	301	301

What is driving aggregate trends, then?

Figure: Firm aging could be an explanation

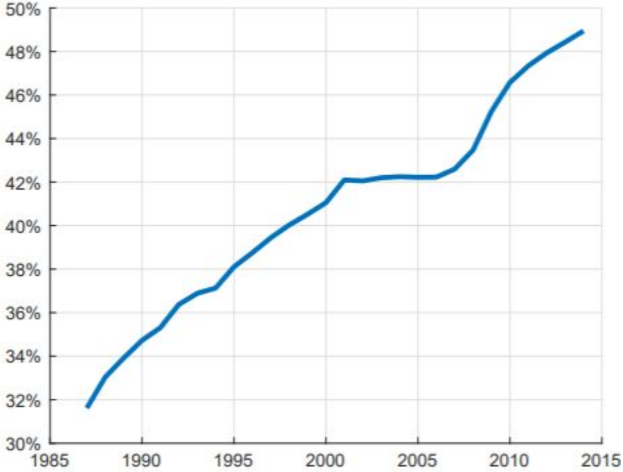
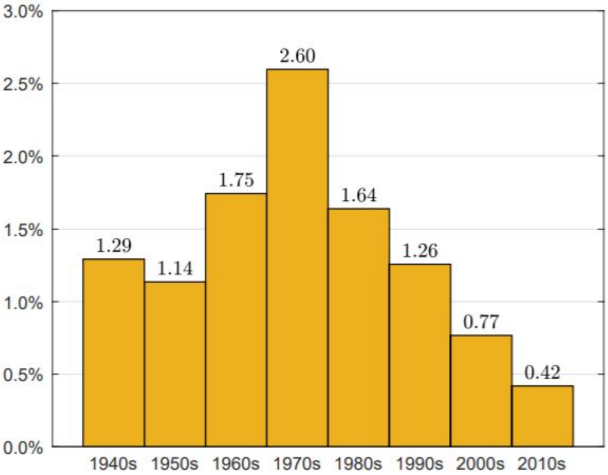


Figure 3: Share of Firms of Age 11+

Labor Force Growth

Figure: Growth rate of civilian labor force; in decline since the 1980s



Model Overview and Problems

- Hopenhayn-style tradition in Macro-IO – Hopenhayn-Rogerson
- ***Claims.*** Can match dynamism/concentration/labor share trends as GE outcomes from a L supply transition...
- ... in a perfect competition, friction-less, inelastic labor world
- ***Problems.*** More PE mechanics than GE economics.
- Limited interaction between aggregates and heterogeneity
- Neither real wages nor selection affected at all
- L movements just affects the scale, not shape of size dist.
- They hold interest rates fixed along the transition!
- Wages equalized along production and investment!
- ***Illustration***. My own notation and simulations.
- Endogenous competition induces interactions between firm heterogeneity and GE macro variables, which they miss...
- Accounting for GE properly fits many more facts! Channels?

Heterogeneous Firm GE Model Setup

- Homogeneous good produced by heterogeneous s firms

$$C = Y = \left[\int_s y(s) \cdot \mathbf{M}(s) \right], \quad y(s) = s \cdot f(l)$$

- Inelastic labor supply for production and investment

$$\bar{L} = L_{\mathrm{P}} + L_{\mathrm{X}}$$

- s_{it} not only heterogeneous, but stochastic
- Draw from common $G_0(s)$ on entry, i.i.d. Markov after

$$\pi(s, p, w) = \max_l p \cdot s \cdot f(l) - w \cdot l(\varphi) - w \cdot c_f, \quad s_{it} \sim F(s_{t+1} | s_t)$$

- Firms are p takers – key aggregate $\frac{w}{p}$, $w = 1$
- Go over steps to solve for recursive competitive equilibrium

Option Value and Investment

- $\pi(s,p)$ flow profits, $v(s,p) = V(s,p,w)/w$ value function

$$\pi(s,p,w) = \max_{l} p \cdot s \cdot f(l) - w \cdot l(\varphi) - w \cdot c_f)$$

$$v(s,p_t) = \max\left\{0, \pi(s,p_t) + \beta E_{s'} v\left(s', p_{t+1} | s\right)\right\}$$

- Survival investment follows an optimal cutoff rule

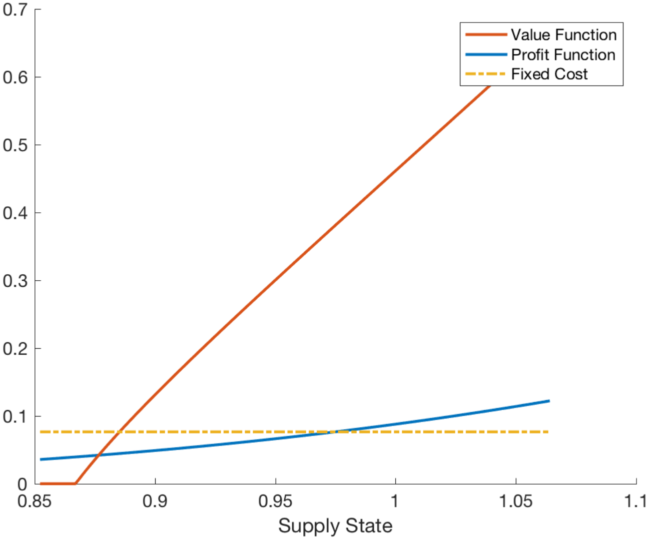
$$\chi(s \geq s_t^*(p_t)) = 1 \iff 0 \leq \pi(s,p_t) + \beta E_{s'} v\left(s', p_{t+1} | s\right)$$

- Entry investment yields ex-ante net profits

$$v^e(p_t) = \int v(s,p_t) \cdot G(s) - c_e$$

- Step 1. Solve value and policy functions for any p.
- Step 2. Solve for p that satisfies Free Entry.

Option Value and Selection (Threshold Rule)



Law of Motion of the Firm-size Distribution

$$M_{t+1}(s') = \underbrace{\int_s \chi\left(s' \geq s^*_{t+1}\left(p_{t+1}\right)\right) \cdot F(s'|s) \cdot M_t(s)}_{\text{Surviving Incumbents } \forall s \rightarrow\, s' \geq s'^*}$$

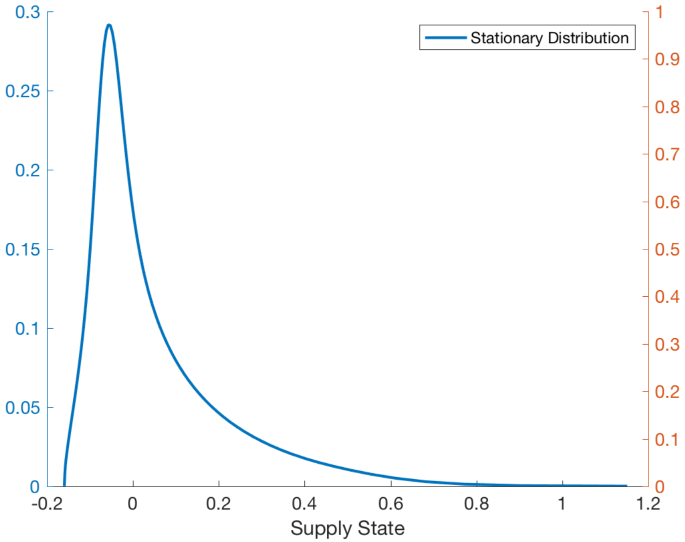
$$+ \quad \underbrace{\chi\left(s' \geq s^*_{t+1}\left(p_{t+1}\right)\right) \cdot \left[M^e_{t+1} G(s')\right]}_{\text{Surviving Entrants } \rightarrow\, s' \geq s'^*}$$

- M_e entrants automatically produce (no time-to-build)
- $M_t = \int_s M_t(s)$ total mass of firms' in equilibrium
- Iterate on it to find the fixed point where time-invariant

$$\mathbf{M} = M^e \bar{\boldsymbol{M}} = M^e \left[I - \mathbf{T}_M\right]^{-1} \chi \mathbf{G}$$

- Step 3. Given policies and shocks, solve for $\bar{\boldsymbol{M}}$.

Equilibrium Stationary Distribution of Firms.



Recursive Competitive Equilibrium

- All firms optimize (investment and production) given p

$$L_p(p_t) \equiv \int_s l\,(s, p_t)\, M_t(s)$$

- Labor market clears (by Walras law, so does goods market)

$$\underbrace{L_p(p_t) + M_t c_f}_{\text{Producing Firms}} + \underbrace{M_t^e c_e}_{\text{Entrants}} = \underbrace{L_t}_{\text{Supply}}$$

- Free entry yields complementary slackness

$$v^e\,(p_t) \cdot M_t^e = 0, \quad M_t^e > 0 \iff \int v\,(s, p_t) \cdot G(s) - w \cdot c_e = 0$$

- $\boldsymbol{M_t}$ distribution of firms defined recursively

$$\boldsymbol{M}_{t+1} = \boldsymbol{T}_{t,t+1} \boldsymbol{M}_t + M_{t+1}^e \chi_{t+1} \boldsymbol{G}$$

Central Result: L Affects Nothing but the Scale

- Stationary $\frac{w}{p}$ pinned down by Free Entry alone

$$\int v\left(s, \frac{p}{w}\right) \cdot G(s) = c_e \implies \frac{p^*}{w^*}$$

- Stationary $p = p^*$ imply a stationary threshold $s^*(p^*) = s^*$

$$\pi\left(s^*(p), p\right) + \beta E_{s'} v\left(s', p|s\left(p\right)\right) = 0$$

- M_t^e, and thus mass of firms M_t, will be the only thing growing with labor - no effects on the shape nor factor prices

$$M_t^e \cdot \left(\int_s \left[l\left(s, p\right) + c_f\right] \bar{\boldsymbol{M}}(s) + c_e\right) = L_t$$

- Their key result from a slowdown in population growth leading to slowdown in entry follows immediately.
- No economics, just mechanics

Why? Macro and Micro Separability.

- Firm-level variables (log)-separable from aggregates. HR:

$$\pi(\underbrace{s,\tau}_{\text{Firm}} \mid \underbrace{p,w}_{\text{Macro}}) = \underbrace{\left[\eta \cdot w^{-\eta} \cdot p\right]^{\frac{1}{1-\eta}} \left(\frac{1}{\eta} - 1\right)}_{\text{Macro Factors} \quad \Pi(p,w)} \cdot \underbrace{(1-\tau)^{\frac{1}{1-\eta}} \cdot s^{\frac{1}{1-\eta}}}_{\text{Firm-level Factors}}$$

- Pervasive feature of heterogeneous firm dynamics lit.
- Carries over to CES monopolistic competition. AB:

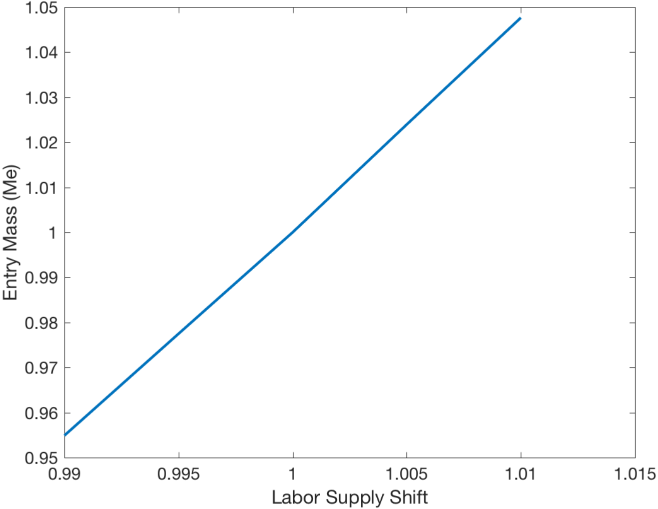
$$\pi(s|Y,w,P) = \underbrace{\left(1 - \frac{1}{\mu}\right)\left(\frac{1}{\mu}\right)^{\sigma-1}\left(\frac{w}{P}\right)^{1-\sigma} PY}_{\text{Macro Factors} \quad \Pi(Y,w,P)} \cdot \underbrace{s^{\sigma-1}}_{\text{Firm-Level}}$$

- Breaks down when you go beyond CES

$$\pi(s| \underbrace{Y,w,P,\Theta}_{\text{Macro States}}) = PY\Theta\left(1 - \frac{\eta}{\mu(s|S)}\right)\Upsilon'\left(\frac{s \cdot l(s)^{\eta}}{Y}\right)\frac{s \cdot l(s)^{\eta}}{Y}$$

Only Scale of Firm Size Distribution Changes.

Figure: Limits interactions between firm heterogeneity & macro



Sloppy Approach to GE Transition?

- In a GE transition, the interest is dynamic

$$(1+r_t) = R_{t,t+1} = \frac{1}{\beta}\left(\frac{C_{t+1}}{C_t}\right) \implies R_{t,t+T} = \prod_{\tau=t}^{T} R_{\tau,\tau+1}$$

- Firms discount at the real interest rate. Loose stationarity

$$v_t^+\left(s, \frac{w_t}{p_t}, r_t\right) = \pi_t\left(s, \frac{w_t}{p_t}\right) + \frac{1}{R_{t,t+1}} E_{s'|s} v_{t+1}\left(s', \frac{w_{t+1}}{p_{t+1}}, r_{t+1}\right)$$

- Entry capital pre-determined at steady state value initially
- Better approach to transitions (Atkeson-Burstein, 2012)

$$\frac{1}{R_{t,t+T}} \cdot \int_s v\left(s, \frac{w_t}{p_t}, r_t\right) \cdot G(s) - w_t \cdot c_e \to 0, \quad t \geq T > 0$$

- Compare evolution of relevant time series with data to evaluate model performance

Calibration

- Assume that US economy is in stationary equilibrium before 1940
- Match civilian labor force growth rate in the data starting in 1940
- Calibrate model parameters to target moments in 1978
- Compare evolution of relevant time series with data to evaluate model performance

Assumptions

- Production function of a firm:

$$f(s,n) = sn^{\alpha}$$

- Firm Productivity follows AR(1) process

$$\log(s_{t+1}) = \mu_s + \rho \log(s_t) + \varepsilon_{t+1}; \quad \varepsilon_{t+1} \sim \mathcal{N}(0, \sigma_\varepsilon^2)$$

- Distribution of startup productivities G is lognormal with mean s_0 and $\sigma_\varepsilon^2/(1-\rho^2)$
- Overhead labor increases linearly with firm size

$$c_f(n) = c_{fa} + c_{fb} \cdot n$$

Calibration

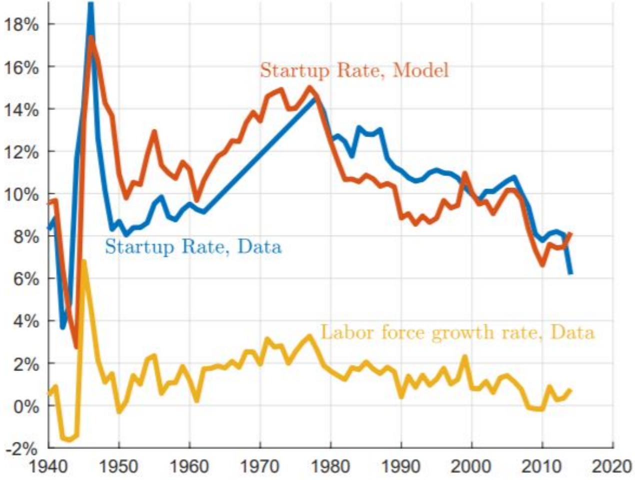
Figure: Calibrated parameters and targeted moments

	Assigned		
	Value	Definition	Basis
β	0.96	Discount factor	Annual real interest rate of 4%
α	0.64	Worker's share of output	Standard
g	0.01	Labor force growth rate (SS)	Standard

	Calibrated				
	Value	Definition	Target	Data	Model
c_e	0.0003	Entry cost	$p^* = 1$	—	—
c_{fa}	4.05	Operating cost intercept	Average firm size in 1978	19.50	19.58
c_{fb}	0.06	Operating cost slope	OP covariance 1993-2001	0.51	0.51
s_0	-3.28	Mean of G	Average startup size	6.05	6.01
μ_s	0.0006	Drift in AR(1)	Startup rate in 1978	14.52%	14.59%
ρ	0.97	Persistence of AR(1)	5-year growth rate	72.00%	74.78%
σ_ε^2	0.046	Variance of shocks	5-year exit rate	51.61 %	57.29%

Matching Startup Rate

Figure: Model matches startup rate in non-targeted years fairly well



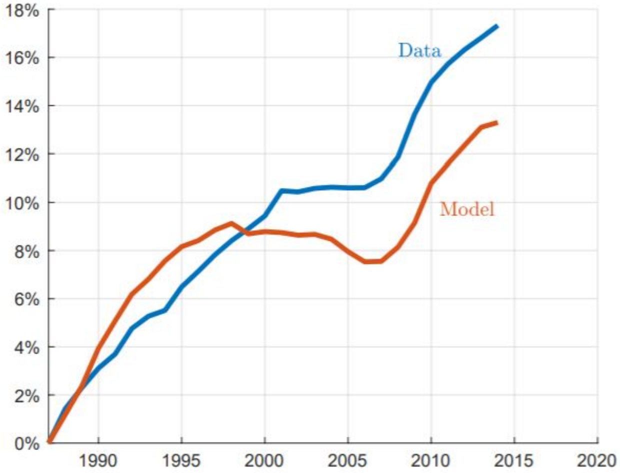
Matching Exit Rate and Average Firm Size

Figure: Exit rate and average firm size by age in the data and model

Age	Exit rate		Average firm size	
	Data(%)	Model(%)	Data	Model
0	–	–	6.05	6.01
1	21.85	25.57	7.73	6.81
2	15.86	17.20	8.46	7.63
3	13.43	13.43	9.14	8.51
4	11.68	11.24	9.77	9.47
5	10.48	9.79	10.36	10.51
6-10	8.30	7.58	11.98	13.92
11-15	6.40	5.68	15.08	21.06
16-20	5.56	4.83	18.81	29.04
21-25	4.99	4.36	24.03	37.04

Matching Firm Aging

Figure: Share of 11+ years old firms, data and model



Matching Concentration (Non-targeted)

Figure: Change in share of employment from 1987-2014, by age and by size

Category	Data (pp.)	Model (pp.)
Young		
Small	-11.06	-10.46
Large	-3.09	-0.29
Mature		
Small	4.51	3.33
Large	9.63	7.41

Aggregate Labor Share

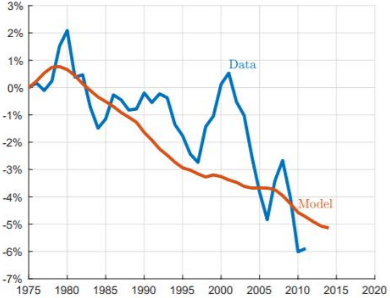
- Widely documented that aggregate labor share has declined since the 1980s
- In the model, share paid to production workers is fixed by α in equilibrium, so all differences come from the share paid to overhead labor
- Labor share of a firm i is given by

$$\text{labor share of firm } i = \alpha\left(1 + \frac{c_{fi}}{n_i}\right) = \alpha\left(1 + \frac{c_{fa} + c_{fb} \cdot n_i}{n_i}\right)$$

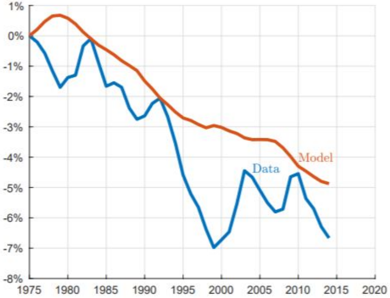
- Firm-level labor share declines with firm size, and thus with firm age
- Firm aging $\Rightarrow$ increase in weight of larger firms $\Rightarrow$ Decline in aggregate labor share

Decline in Aggregate Labor Share

Figure: Model matches decline in aggregate labor share, and in overhead to employment ratio



(a) Labor Share



(b) Nonproduction Workers to Employment

Counterfactuals

- Two channels driving the decline in startup rate
  - Firm demographics: older firms have higher size and lower exit rate $\Rightarrow$ feedback effect of aging $\Rightarrow$ lower exit and higher average size $\Rightarrow$ lower startup rate
  - Initial rise in labor force growth: initial rise $\Rightarrow$ expansion of pool of incumbents $\Rightarrow$ labor force growth slows down $\Rightarrow$ incumbent growth leaves less room for new firms
- Case 1: shut down i.i.d productivity process and assume that firms carry productivity at birth forever
- Case 2: shut down initial rise in labor force growth and assume that economy is at stationary equilibrium in 1977

Counterfactual: Shutting down channels

Figure: Firm demographics plays an important role for decline in entry rate