Hopenhayn et al. (2019).
From Population Growth to Firm Demographics: Implications for Concentration, Entrepreneurship and the Labor Share

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Macro Reading Group
Introduction

- Aggregate trends in the economy: increase in concentration, decrease in entrepreneurship rate, decrease in labor share
- Hypothesis: driven by aging firm distribution, which can be explained by change in labor force growth
- Construct a model of firm dynamics in which changes in labor force growth lead to changes in firm demographics
- Show that labor force growth provides a unified explanation of the aggregate trends
Roadmap

1. Data and Aggregate Trends
2. Model
3. Calibration
4. Discussion and Comments
Data

- Business Dynamics Statistics (BDS) data from Census; publicly available
- Covers almost all private sector US firms from 1977-2014
- Aggregate statistics by group (age, size, sector, etc.) but no firm-level data
- Interested in three aggregate trends:
  - Concentration: share of employment by firms with 250+ employees
  - Average firm size: number of employees per firm
  - Aggregate exit rate
Aggregate Trends

**Figure:** Concentration and average firm size on a steady rise; exit rate falling in recent decade

![Graphs showing trends in concentration, average firm size, and exit rate from 1980 to 2010.](image_url)
Breaking Down by Firm Age

Figure: Trends by age are generally flat
### Figure: Regression of concentration on year, with and without age controls

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What is driving aggregate trends, then?

*Figure:* Firm aging could be an explanation

*Figure 3: Share of Firms of Age 11+*
Labor Force Growth

Figure: Growth rate of civilian labor force; in decline since the 1980s
Model Overview and Problems

- Hopenhayn-style tradition in Macro-IO – Hopenhayn-Rogerson

**Claims.** Can match dynamism/concentration/labor share trends as GE outcomes from a L supply transition...

... in a perfect competition, friction-less, inelastic labor world

**Problems.** More PE mechanics than GE economics.

- Limited interaction between aggregates and heterogeneity
- Neither real wages nor selection affected at all
- L movements just affects the scale, not shape of size dist.
- They hold interest rates fixed along the transition!
- Wages equalized along production and investment!

**Illustration.** My own notation and simulations.

- Endogenous competition induces interactions between firm heterogeneity and GE macro variables, which they miss...
- Accounting for GE properly fits many more facts! Channels?
Heterogeneous Firm GE Model Setup

- Homogeneous good produced by heterogeneous s firms

\[ C = Y = \left[ \int_s y(s) \cdot M(s) \right], \quad y(s) = s \cdot f(l) \]

- Inelastic labor supply for production and investment

\[ \bar{L} = L_P + L_X \]

- \( s_{it} \) not only heterogeneous, but stochastic

- Draw from common \( G_0(s) \) on entry, i.i.d. Markov after

\[ \pi(s, p, w) = \max_l p \cdot s \cdot f(l) - w \cdot l(\varphi) - w \cdot c_f, \quad s_{it} \sim F(s_{t+1} | s_t) \]

- Firms are \( p \) takers – key aggregate \( \frac{w}{p}, w = 1 \)

- Go over steps to solve for recursive competitive equilibrium
Option Value and Investment

- \( \pi(s, p) \) flow profits, \( \nu(s, p) = V(s, p, w)/w \) value function
  
  \[
  \pi(s, p, w) = \max_l p \cdot s \cdot f(l) - w \cdot l(\varphi) - w \cdot c_f
  \]

  \[
  \nu(s, p_t) = \max \{0, \pi(s, p_t) + \beta E_{s'} \nu(s', p_{t+1}|s)\}
  \]

- Survival investment follows an optimal cutoff rule

  \[
  \chi(s \geq s^*_t(p_t)) = 1 \iff 0 \leq \pi(s, p_t) + \beta E_{s'} \nu(s', p_{t+1}|s)
  \]

- Entry investment yields ex-ante net profits

  \[
  \nu^e(p_t) = \int \nu(s, p_t) \cdot G(s) - c_e
  \]

- Step 1. Solve value and policy functions for any \( p \).
- Step 2. Solve for \( p \) that satisfies Free Entry.
Option Value and Selection (Threshold Rule)
Law of Motion of the Firm-size Distribution

\[ M_{t+1}(s') = \int_s \chi \left( s' \geq s^*_{t+1}(p_{t+1}) \right) \cdot F(s'|s) \cdot M_t(s) \]

\[ + \chi \left( s' \geq s^*_{t+1}(p_{t+1}) \right) \cdot \left[ M^e_{t+1} G(s') \right] \]

- \( M_e \) entrants automatically produce (no time-to-build)
- \( M_t = \int_s M_t(s) \) total mass of firms’ in equilibrium
- Iterate on it to find the fixed point where time-invariant

\[ \mathbf{M} = M^e \tilde{\mathbf{M}} = M^e [I - \mathbf{T}_M]^{-1} \chi \mathbf{G} \]

- Step 3. Given policies and shocks, solve for \( \tilde{\mathbf{M}} \).
Equilibrium Stationary Distribution of Firms.
Recursive Competitive Equilibrium

- All firms optimize (investment and production) given $p$

$$L_p(p_t) \equiv \int_s l(s, p_t) M_t(s)$$

- Labor market clears (by Walras law, so does goods market)

$$L_p(p_t) + M_t c_f + M^e c_e = L_t$$

  - Producing Firms
  - Entrants
  - Supply

- Free entry yields complementary slackness

$$\nu^e(p_t) \cdot M^e_t = 0, \quad M^e_t > 0 \iff \int \nu(s, p_t) \cdot G(s) - w \cdot c_e = 0$$

- $M_t$ distribution of firms defined recursively

$$M_{t+1} = T_{t, t+1} M_t + M^e_{t+1} \chi_{t+1} G$$
Central Result: L Affects Nothing but the Scale

- Stationary \( \frac{w}{p} \) pinned down by Free Entry alone
  \[
  \int \nu \left( s, \frac{p}{w} \right) \cdot G(s) = c_e \implies \frac{p^*}{w^*}
  \]

- Stationary \( p = p^* \) imply a stationary threshold \( s^*(p^*) = s^* \)
  \[
  \pi (s^*(p), p) + \beta E_s' \nu (s', p|s(p)) = 0
  \]

- \( M_t^e \), and thus mass of firms \( M_t \), will be the only thing growing with labor - no effects on the shape nor factor prices
  \[
  M_t^e \cdot \left( \int_s \left[ l(s, p) + c_f \right] \tilde{M}(s) + c_e \right) = L_t
  \]

- Their key result from a slowdown in population growth leading to slowdown in entry follows immediately.

- No economics, just mechanics
Why? Macro and Micro Separability.

- Firm-level variables (log)-separable from aggregates. HR:

\[
\pi(s, \tau | p, w) = \left[ \eta \cdot w^{-\eta} \cdot p \right] \frac{1}{1-\eta} \left( \frac{1}{\eta} - 1 \right) \cdot (1 - \tau) \frac{1}{1-\eta} \cdot s^{\frac{1}{1-\eta}}
\]

- Macro Factors: \( \Pi(p, w) \) 
- Firm-level Factors: \( s^{\frac{1}{1-\eta}} \)

- Pervasive feature of heterogeneous firm dynamics lit.

- Carries over to CES monopolistic competition. AB:

\[
\pi(s | Y, w, P) = \left( 1 - \frac{1}{\mu} \right) \left( \frac{1}{\mu} \right)^{\sigma-1} \left( \frac{w}{P} \right)^{1-\sigma} PY \cdot s^{\sigma-1}
\]

- Macro Factors: \( \Pi(Y, w, P) \) 
- Firm-Level Factors: \( s^{\sigma-1} \)

- Breaks down when you go beyond CES

\[
\pi(s | Y, w, P, \Theta) = PY\Theta \left( 1 - \frac{\eta}{\mu (s | S)} \right) \gamma' \left( \frac{s \cdot I(s)^{\eta}}{Y} \right) \frac{s \cdot I(s)^{\eta}}{Y}
\]

- Macro States: \( \Pi(S) \) 
- Firm-level Factors: \( s \cdot I(s)^{\eta} \)
Only Scale of Firm Size Distribution Changes.

Figure: Limits interactions between firm heterogeneity & macro
Sloppy Approach to GE Transition?

- In a GE transition, the interest is dynamic

\[(1 + r_t) = R_{t,t+1} = \frac{1}{\beta} \left( \frac{C_{t+1}}{C_t} \right) \implies R_{t,t+T} = \prod_{\tau=t}^{T} R_{\tau,\tau+1}\]

- Firms discount at the real interest rate. Loose stationarity

\[v^+_t \left( s, \frac{w_t}{p_t}, r_t \right) = \pi_t \left( s, \frac{w_t}{p_t} \right) + \frac{1}{R_{t,t+1}} E_{s'|s} v_{t+1} \left( s', \frac{w_{t+1}}{p_{t+1}}, r_{t+1} \right)\]

- Entry capital pre-determined at steady state value initially

- Better approach to transitions (Atkeson-Burstein, 2012)

\[\frac{1}{R_{t,t+T}} \cdot \int_s v \left( s, \frac{w_t}{p_t}, r_t \right) \cdot G(s) - w_t \cdot c_e \to 0, \quad t \geq T > 0\]

- Compare evolution of relevant time series with data to evaluate model performance
Calibration

- Assume that US economy is in stationary equilibrium before 1940
- Match civilian labor force growth rate in the data starting in 1940
- Calibrate model parameters to target moments in 1978
- Compare evolution of relevant time series with data to evaluate model performance
Assumptions

- Production function of a firm:

\[ f(s, n) = sn^\alpha \]

- Firm Productivity follows AR(1) process

\[ \log(s_{t+1}) = \mu_s + \rho \log(s_t) + \varepsilon_{t+1}; \quad \varepsilon_{t+1} \sim \mathcal{N}(0, \sigma_\varepsilon^2) \]

- Distribution of startup productivities \( G \) is lognormal with mean \( s_0 \) and \( \sigma_\varepsilon^2/(1 - \rho^2) \)

- Overhead labor increases linearly with firm size

\[ c_f(n) = c_{fa} + c_{fb} \cdot n \]
Calibration

Figure: Calibrated parameters and targeted moments

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<th>Value</th>
<th>Definition</th>
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<td>Discount factor</td>
<td>Annual real interest rate of 4%</td>
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<tr>
<td>$\alpha$</td>
<td>0.64</td>
<td>Worker’s share of output</td>
<td>Standard</td>
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<td>$g$</td>
<td>0.01</td>
<td>Labor force growth rate (SS)</td>
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<td>Mean of G</td>
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<td>Startup rate in 1978</td>
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<td>14.59%</td>
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<td>$\rho$</td>
<td>0.97</td>
<td>Persistence of AR(1)</td>
<td>5-year growth rate</td>
<td>72.00 %</td>
<td>74.78%</td>
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<td>$\sigma^2_\varepsilon$</td>
<td>0.046</td>
<td>Variance of shocks</td>
<td>5-year exit rate</td>
<td>51.61 %</td>
<td>57.29%</td>
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Matching Startup Rate

**Figure:** Model matches startup rate in non-targeted years fairly well
Matching Exit Rate and Average Firm Size

Figure: Exit rate and average firm size by age in the data and model

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<th>Age</th>
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<th>Average firm size</th>
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<td>Model(%)</td>
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<td>16-20</td>
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<td>21-25</td>
<td>4.99</td>
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Matching Firm Aging

**Figure:** Share of 11+ years old firms, data and model
Matching Concentration (Non-targeted)

**Figure:** Change in share of employment from 1987-2014, by age and by size

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<td>Small</td>
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<td>Large</td>
<td>9.63</td>
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Aggregate Labor Share

- Widely documented that aggregate labor share has declined since the 1980s
- In the model, share paid to production workers is fixed by $\alpha$ in equilibrium, so all differences come from the share paid to overhead labor
- Labor share of a firm $i$ is given by

$$\text{labor share of firm } i = \alpha \left( 1 + \frac{c_f}{n_i} \right) = \alpha \left( 1 + \frac{c_{fa} + c_{fb} \cdot n_i}{n_i} \right)$$

- Firm-level labor share declines with firm size, and thus with firm age
- Firm aging $\Rightarrow$ increase in weight of larger firms $\Rightarrow$ Decline in aggregate labor share
Decline in Aggregate Labor Share

**Figure:** Model matches decline in aggregate labor share, and in overhead to employment ratio
Counterfactuals

- Two channels driving the decline in startup rate
  - Firm demographics: older firms have higher size and lower exit rate ⇒ feedback effect of aging ⇒ lower exit and higher average size ⇒ lower startup rate
  - Initial rise in labor force growth: initial rise ⇒ expansion of pool of incumbents ⇒ labor force growth slows down ⇒ incumbent growth leaves less room for new firms

- Case 1: shut down i.i.d productivity process and assume that firms carry productivity at birth forever
- Case 2: shut down initial rise in labor force growth and assume that economy is at stationary equilibrium in 1977
Counterfactual: Shutting down channels

Figure: Firm demographics plays an important role for decline in entry rate