Problem Set 1

EC2450A

Fall 2016

Problem 1

An economy is populated by individuals with preferences over consumption and labor. They have utility $u_i(c, y)$ where y is income, $u_c(c, y) > 0$ and $u_y(c, y) < 0$. Suppose the tax schedule in place has a constant marginal tax rate τ above a fixed threshold y^*. The government wants to choose τ to maximize the tax revenue raised from top earners.

(a) As we saw in class, the tax rate that maximizes revenues depends on a Pareto parameter a and the elasticity of total income of the top earners who are in the top bracket, ε. Provide intuition about why ε is a mix of substitution and income effects.

(b) The individual solves the following utility maximization problem:

$$\max_{c,y} u_i(c, y)$$

subject to:

$$c = (1 - \tau)y + I$$

Denote by $y_i(1 - \tau, I)$ the Marshallian income supply. The uncompensated elasticity of labor supply with respect to $1-\tau$ is $\varepsilon_i^u = \big(\partial y_i/\partial(1-\tau)\big)\big((1-\tau)/y_i\big)$. We denote by $\eta_i = (1-\tau)\partial y_i/\partial I$ the income parameter.
Suppose a government advisor suggests to run an experiment where the top tax rate τ (above y^*) is raised by $d\tau$. The advisor claims that the response dy_i can be rewritten as a function of ε_i^u and η_i. Is the advisor right? If yes, show how dy_i depends on ε_i^u and η_i

(c) Using the expression derived in point b) write ε as a function of the Pareto parameter $a = y^m/(y^m - y^*)$ and a weighted average of the uncompensated elasticities and income effect parameters. Why are uncompensated elasticities weighted by incomes y_i, while the η_is are not?

(d) Now suppose the utility is logarithmic in consumption and exponential in income. It takes the following form:

$$u_i(c, y) = \log c - \phi_i y^{1+\frac{1}{\varepsilon}}$$

where ϕ_i can vary across individuals and captures heterogeneity in the disutility from labor. Derive the uncompensated elasticity, income, and compensated elasticity parameters (i.e., ε_i^u, η_i, ε_i^c) by solving the utility maximization problem of the individual under the linear tax and the same budget constraint as above.

(e) Study what happens to η_i and ε_i^u when ϕ_i becomes small (find their limits). Using the relation found previously, write the optimal top tax rate formula as a function of ε and the Pareto parameter a when ϕ_i is small.
The parameter ε is the Frisch elasticity of labor supply for this class of utility functions. Suppose we calibrate the parameters a and ε such that $a = 1.5$ and $\varepsilon = 1$. What is the optimal τ? What is the optimal τ when ε is very large? Discuss why the optimal tax rate is high even with a large Frisch elasticity.

Problem 2

Suppose that utility is quasi-linear and takes the form: $u(c,l) = c - \frac{l^{1+\epsilon}}{1+\epsilon}$ with $\epsilon > 0$. Each individual earns income $y = wl$ and consumes $c = y - T(y)$. The wage rate w can be interpreted as a measure of skills and is distributed with density $f(w) > 0$ over $[0, \infty)$. The total population is normalized to one so that $\int_0^\infty f(w)dw = 1$

(a) Suppose the tax schedule is linear with a flat tax rate τ. The tax is hence $T(y) = -S + \tau y$ where $S > 0$ is the transfer that the individual receives when labor supply is zero ($T(0) = -S$). Find the optimal labor supply choice as a function of the parameters S and $w(1-\tau)$. Also, derive the uncompensated and compensated elasticities of labor supply as a function of ϵ and find the income effect parameter.

(b) Assume that taxes are entirely rebated to the individuals in the economy. We have that $S = \tau Y$, where Y is average earnings in the economy. Find the optimal tax rate τ in the case where the government only cares about the worst-off individual (i.e. the government is Rawlsian) and in the case where the government maximizes the sum of utilities (i.e. the government is utilitarian). Always explain the intuition behind your results.

(c) Do points (a)-(b) again using utility function $u(c,l) = \log(c) - l$. If exact analytical expressions are not possible to derive, just provide implicit formulas with economic explanation. Is this utility function more or less realistic than the one used in questions (a)-(b)?

Go back to utility function $u(c,l) = c - \frac{l^{1+\epsilon}}{1+\epsilon}$. We now study an economy with two tax brakets such that:

$$T(y) = \begin{cases} -S + \tau_1 y & \textit{if } y \leq \hat{y} \\ -S + \tau_1 \hat{y} + \tau_2 (y - \hat{y}) & \textit{if } y > \hat{y} \end{cases}$$

$-S$ is the transfer to non-working individuals.

(d) Plot the budget constraint on a graph with axes (l, c).

(e) Suppose that $\tau_1 < \tau_2$. Find the optimal labor supply and earnings for an individual with wage w. Consider the three cases where the individual is in the bottom bracket, the top bracket, or exactly at $\hat{y}$.

Suppose that there are 3 types of individuals: disabled individuals unable to work $w_0 = 0$, low skilled individuals with wage rate w_1, and skilled individuals with wage rate w_2. We assume that $w_1 < w_2$. The fractions of disabled, low skilled, and high skilled in the population are respectively λ_0, λ_1 and λ_2 such that $\lambda_0 + \lambda_1 + \lambda_2 = 1$. Further assume that low skilled workers are always in the bottom bracket and that high skilled workers are always in the top bracket.

(f) Find the tax rate τ_2^* that maximizes taxes collected from the high skilled, assuming that S, τ_1, and $\hat{y}$ are given. Express it as a function of ϵ and $\hat{y}$.

(g) Compute the tax rate τ_1 that maximizes total taxes collected taking S and $\hat{y}$ as given and setting $\tau_2 = \tau_2^*$ (the optimal tax rate you found in the previous question). Explain why (intuitively) $\tau_2^* < \tau^* < \tau_1^*$, where τ^* is the one computed in question (b).

(h) (Bonus question:) Finally, the government introduces a third tax bracket with rate τ_3 above $\bar{y}$. Suppose $\tau_3 > \tau_2$, $\bar{y} > \hat{y}$ and that there is a continuous population with utility defined as in (a). You have access to 5 years of income data before the reform and 5 years of data after the reform. Suggest a time series graph that you would draw to visually test whether creating the 3rd tax bracket had an impact on reported incomes? How could you use the graph to estimate the elasticity of earnings with respect to $1 - \tau$ using this reform. Be precise about the assumptions needed for the estimate to be unbiased. Do you have ideas about how to test the robustness of your results with some alternative method of estimation?

Problem 3

In the standard Mirrlees model, the production function is implicitly additively linear: aggregate output is simply the sum of individual outputs, i.e., the substitution between different agents' outputs is infinite.

Consider instead the case where outputs are not perfectly substitutable.

We focus on a two-type model. There are n_1 individuals of type 1 (low productivity) and n_2 individuals of type 2 (high productivity). Let L_1 be the labor effort of individuals of type 1 and L_2 the labor effort of individuals of type 2. The production function has constant returns to scale:

$$Y = F(n_1 L_1, n_2 L_2)$$

Individuals have quasilinear preferences of the form

$$u_i = c_i - v(L_i)$$

where c_i is their consumption and L_i is their labor effort. If type i is paid a wage w_i and pays a tax T_i, his utility is:

$$u_i = w_i L_i - T_i - v(L_i)$$

The government's objective is to maximize social welfare equal to:

$$SWF = n_1 u_1 + \mu n_2 u_2$$

with $0 < \mu < 1$.

1) Show that aggregate output can be rewritten as:

$$Y = n_1 L_1 f(l)$$

for l to be defined and where f is i) increasing and ii) concave.

2) What is the social marginal welfare weight on agents of type 2? What is the social marginal welfare weight on agents of type 1? Are their absolute levels meaningful? Which type of agent is implicitly valued more by the government?

3) The government chooses a tax schedule $T(y)$ specifying for each income level the tax to be paid. Explain why the government can restrict itself to set a menu of contracts (T_1, Y_1) and (T_2, Y_2) that specifies for each type an output level and a tax level.

4) What are the two incentive compatibility constraints facing the government when setting this menu of contracts? What is the resource constraint (government budget constraint) if the government has an exogenous revenue requirement R?

5) Among the two incentives constraints, which one will be binding if μ is very low? Explain the intuition.

6) Suppose the wage is determined by the marginal product of labor given the aggregate production function. Write out an expression for w_1 and w_2.

7) Rather than solving the constrained program with respect to T_i and Y_i, replace Y_i by the auxiliary variable $L_i = Y_i / w_i$ (labor supplies of each type at the optimum). Eliminate T_1 and T_2 by using i) the government's budget constraint and ii) the incentive constraint of type 2. Give an expression for T_1 and T_2 as a function of the wages, labor supplies, and the government revenue requirement.

8) Use the expressions found for T_i to rewrite the objective of the government.

9) Suppose just for this question that, as in the Mirrlees model, wages are exogenous. Take the first-order conditions with respect to L_1 and L_2. Show that $T_1' > 0$ and $T_2' = 0$. Interpret this in light of the binding incentive constraint.

10) Now suppose that wages are as determined above through the aggregate production function. Give the derivatives of w_1 and w_2 with respect to L_1 and L_2 (note that both wages depend on the labor supplies of both types). When employment of the more productive type increases, what happens to w_1 and w_2?

11) Differentiate the government's objective function (into which you substituted the incentive and resource constraints as in 8)) with respect to L_2. Can you show that T_2', the marginal tax rate on type 2 has to be negative (if you need to make assumptions on some functions, do so)? What does this mean? Can you provide the intuition for why this is the case?