New Dynamic Public Finance (NDPF)

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GOALS OF THIS LECTURE

(1) New Dynamic Public Finance: A toolbox.

(2) Understand capital taxation from the point of view of NDPF

SET UP: UNCERTAINTY ON EARNINGS

- So far: representative agents, ex ante heterogeneity, aggregate uncertainty
- We now consider idiosyncratic uncertainty that is not only ex ante, but unfolds over time
- Skill shocks or preference shocks
- Start with finite horizon: $t = 0, 1$
- Preferences $U(c_0, c_1(s), y(s)/s)$
- interpretation: skill shock s realized in period 1. Consumption decision c_0 in period 0 is made before the shock is realized

SET UP: FAILURE OF A-S and RESOURCE CONSTRAINT

- Note difference to time-0 shock (ex ante heterogeneity) as considered so far. Preferences would be $U(c_0(s), c_1(s), y(s)/s)$
- Under separability + homogeneity, the Atkinson-Stiglitz (1976) theorem would rule out the optimality of a capital tax.
- With the period-1 shock, we will find a downward distortion of saving to be optimal (positive capital tax).
- Technology: linear storage with rate of return $R^* = 1/q$, so that the aggregate resource constraint is

$$c_0 + q \sum_s c_1(s)p(s) \leq q \sum_s y(s)p(s) \tag{1}$$

FIRST BEST

- $$\max_{c_0, c_1(s), y(s)} \sum_s U(c_0, c_1(s), y(s)/s)p(s)$$

  s.t. (1)
- FOCs for $[c_0]$

  $$\mathbb{E}[U_{c_0}, c_1(s), y(s)/s] = \lambda$$

  and for $[c_1(s)]$

  $$U_{c_1(s)}(c_0, c_1(s), y(s)/s) = \lambda q$$

FIRST BEST – FULL INSURANCE

- Hence,

$$\mathbb{E}[U_{c_0}, c_1(s), y(s)/s] = R^* U_{c_1(s)}(c_0, c_1(s), y(s)/s) \quad \forall s$$

$\Rightarrow$ Full Insurance

- Taking expectations on both sides

$$\mathbb{E}[U_{c_0}, c_1(s), y(s)/s] = R^* \mathbb{E}[U_{c_1(s)}(c_0, c_1(s), y(s)/s)] \tag{2}$$

- For instance, if $U(c_0, c_1(s), y(s)/s) = u(c_0) + \beta u(c_1(s)) - h(y(s)/s)$, then we obtain the usual Euler equation

$$u'(c_0) = \beta R^* \mathbb{E}[u'(c_1(s))]$$

and $c_1(s) = c_1$ for all s.

FREE SAVING GIVEN INCOME TAX

- Free saving with non-linear income tax $T(Y)$:

$$\max_{c_0, c_1(s), y(s)} \sum_s U(c_0, c_1(s), y(s)/s)p(s)$$

s.t.

$$c_0 + k_1 \leq e$$

$$c_1(s) \leq y(s) - T(y(s)) + Rk_1 \quad \forall s$$

- FOCs and $R = R^*$ yields the Euler equation

$$u'(c_0) = \beta R^* \mathbb{E}[u'(c_1(s))]$$

- If agents can freely decide how much to save in a risk-free asset with return $R = R^*$, we obtain the Euler equation as in the first best

PRIVATE INFORMATION AND INCENTIVE CONSTRAINTS

- Suppose s is private information and agents make reports $r = \sigma(s)$, where σ denotes the reporting strategy
- Truth-telling: $\sigma^*(s) = s \quad \forall s$
- Denote
$$c_1^\sigma(s) = c_1(\sigma(s))$$
and
$$y^\sigma(s) = y(\sigma(s))$$
- The set of incentive constraints is
$$\mathbb{E}[U(c_0, c_1(s), y(s)/s)] \geq \mathbb{E}[U(c_0, c_1^\sigma(s), y^\sigma(s)/s] \quad \forall \sigma, s$$
- This is equivalent to
$$U(c_0, c_1(s), y(s)/s) \geq U(c_0, c_1(r), y(r)/s) \quad \forall r, s \tag{3}$$

SECOND BEST DYNAMIC MIRRLEES PROBLEM

- The second best (dynamic Mirrlees) problem is

$$\max_{c_0, c_1(s), y(s)/s} \mathbb{E}[U(c_0, c_1(s), y(s)/s)] \tag{4}$$

s.t.

$$c_0 + q \sum_s c_1(s) p(s) \leq q \sum_s y(s) p(s) \quad \text{(RC)}$$

and

$$U(c_0, c_1(s), y(s)/s) \geq U(c_0, c_1(r), y(r)/s) \quad \forall r, s \quad \text{(IC)}$$

FEASIBLE VARIATIONS

- (RC) and (IC) define the set F of *feasible* allocations, i.e.

$$F \equiv \{(c_0, c_1(s), y(s)) \mid (c_0, c_1(s), y(s)) \quad \text{satisfies (RC) and (IC)}$$

- Key question: Is free saving feasible? Formally, if $(c_0, c_1(s), y(s)) \in F$, does this imply that $(c_0 - \triangle, c_1(s) + R^*\triangle, y(s)) \in F$ as well, for some $\triangle \in \mathbb{R}$?
- In other words, if the agent saves a little in period 0 ($\triangle$) is she stil willing to supply the same output (i.e. not lie about s)?
- Depends on income effects in general
- For instance, suppose

$$U(c_0, c_1(s), y(s)/s) = \hat{U}(c_0, c_1(s) - h(y(s)/s))$$

Then, given c_0, just maximize $c_1(s) - h(y(s)/s)$. There are no income effects due to quasilinearity, and the above variation is feasible.

FEASIBLE VARIATIONS

- Easier to see using (IC):

$$\hat{U}(c_0 - \triangle, c_1(s) + R^* \triangle - h(y(s)/s)) \geq$$

$$\hat{U}(c_0 - \triangle, c_1(r) + R^* \triangle - h(y(r)/s))$$

  if and only if

$$c_1(s) + R^* \triangle - h(y(s)/s) \geq c_1(r) + R^* \triangle - h(y(r)/s)$$

  which is implied by the original allocation being feasible, i.e. $(c_0, c_1(s), y(s)) \in F$
- But in general, saving in period 0 has a negative income effect on labor supply in period 1 (if leisure is a normal good)
- e.g. consider preferences

$$U(c_0, c_1(s), y(s)/s) = u(c_0) + \beta u(c_1(s)) - h(y(s)/s)$$

  Additive separability + concavity of $u(.)$ mean leisure is normal (output is “inferior“) and so the variation above is no longer feasible.

CAN WE FIND A FEASIBLE VARIATION?

- Free saving is not feasible with these preferences due to negative income effect on labor supply
- Consider

$$(c_0 - \triangle, c_1(s) + \delta(\triangle, s), y(s)) \tag{5}$$

with $\delta(\triangle, s)$ chosen such that (IC) is satisfied:

$$u(c_0 - \triangle) + \beta u(c_1(s) + \delta(\triangle, s)) = u(c_0) + \beta u(c_1(s)) + A(\triangle) \quad \forall s, \triangle \tag{6}$$

for some $A(\triangle)$, and such that it is resource neutral:

$$-\triangle + q \sum_m \delta(\triangle, s) p(s) = 0 \quad \forall \triangle \tag{7}$$

- With the "free saving" variation, we had $\delta(\triangle, s) = -R^* \triangle$. What is key difference?

VERIFY INCENTIVE COMPATIBILITY OF VARIATION

- Verify that the variation maintains incentive compatibility:

$$u(c_0 - \triangle) + \beta u(c_1(s) + \delta(\triangle, s)) - h(y(s)/s) \geq$$

$$u(c_0 - \triangle) + \beta u(c_1(r) + \delta(\triangle, r)) - h(y(r)/s)$$

  if and only if (6)

$$u(c_0) + \beta u(c_1(s)) + A(\triangle) - h(y(s)/s) \geq$$

$$u(c_0) + \beta u(c_1(r)) + A(\triangle) - h(y(r)/s)$$

  if and only if

$$u(c_0) + \beta u(c_1(s)) - h(y(s)/s) \geq u(c_0) + \beta u(c_1(r)) - h(y(r)/s)$$

- Is this true?
- Key: Given separability, all that matters for incentive compatibility is the total utility from consumption.

INVERSE EULER EQUATION

- Suppose the original allocation $(c_0, c_1(s), y(s))$ solves the second best problem. Then, since the variation $\delta(\triangle, s)$ is feasible as just shown, it cannot improve the objective.
- Formally,

$$0 = argmax_{\triangle} \sum_s p(s)[u(c_0 - \triangle) + \beta u(c_1(s) + \delta(\triangle, s)) - h(y(s)/s)]$$

$$= argmax_{\triangle} \sum_s p(s)[u(c_0) + \beta u(c_1(s) + A(\triangle)) - h(y(s)/s)]$$

$$= argmax_{\triangle} A(\triangle),$$

where we used

$$u(c_0 - \triangle) + \beta u(c_1(s) + \delta(\triangle, s)) = u(c_0) + \beta u(c_1(s)) + A(\triangle) \quad \forall s, \triangle \tag{8}$$

- FOC $A'(0) = 0$

INVERSE EULER EQUATION

- $\delta(\triangle, s)$ satisfies (IC); differentiate w.r.t $\triangle$:

$$-u'(c_0) + \beta u'(c_1(s)) \frac{\partial \delta(\triangle, s)}{\partial \triangle}\Big|_{\triangle=0} = A'(0)$$

rearrange (at optimum):

$$\frac{\partial \delta(\triangle, s)}{\partial \triangle}\Big|_{\triangle=0} = \frac{u'(c_0) + A'(0)}{\beta u'(c_1(s))} = \frac{u'(c_0)}{\beta u'(c_1(s))} \quad \forall s \tag{9}$$

- Condition for resource neutrality of the variation implies:

$$-1 + q \sum_s p(s) \frac{\partial \delta(\triangle, s)}{\partial \triangle}\Big|_{\triangle=0} = 0$$

INVERSE EULER EQUATION (II)

- Using expression for $\left.\frac{\partial\delta(\triangle,s)}{\partial\triangle}\right|_{\triangle=0}$:

$$\frac{1}{u'(c_0)} = \frac{1}{\beta R^*}\sum_s \frac{p(s)}{u'(c_1(s))} \tag{10}$$

- With seperable preferences, optimal allocation has to satisfy this inverse Euler equatoin (Diamond/Mirrlees 1978, Rogerson 1985, Golosov et al. 2003)
- Is this necessary and sufficient? (Think of optimality of $y(s)$).
- Implies that the Euler equation is violated.

$$u'(c_0) = \beta R^* \sum_s p(s)u'(c_1(s)) \tag{11}$$

Is it always violated?

POSITIVE SAVINGS WEDGE

- Inverse Euler implies that, at the optimum,

$$u'(c_0) = \left[\frac{1}{\beta R^*} \sum_s \frac{p(s)}{u'(c_1(s))}\right]^{-1} = \beta R^* \left(\mathbb{E}\left[\frac{1}{u'(c_1(s))}\right]\right)^{-1}$$

By Jensen's inequality and convexity of the function $f(x) = 1/x$,

$$u'(c_0) < \beta R^* \mathbb{E}[u'(c_1(s))]$$

- The optimal allocation is incompatible with free saving. Is saving is discouraged or encouraged?
- Intuition: saving in period 0 increases income in period 1 across all shocks s $\rightarrow$ negative income effect on y $\rightarrow$ is this good or bad for Planner?
- Implications for capital taxation, but study distinction between the wedge derived here and actual implementations later

TECHNICAL POINT: DUAL APPROACH

- Consider allocation in terms of utils

$$u_0 \equiv u(c_0), \quad u_1(s) \equiv u(c_1(s))$$

- Move from original allocation $(u_0, u_1(s))$ to variation $(\tilde{u}_0, \tilde{u}_1(s))$ such that

$$u_0 + \beta u_1(s) = \tilde{u}_0 + \beta \tilde{u}_1(s) \quad \forall s$$

- In particular, set

$$\tilde{u}_0 = u_0 - \beta \triangle$$

and

$$\tilde{u}_1(s) = u_1(s) + \triangle \quad \forall s$$

TECHNICAL POINT: DUAL APPROACH (II)

- Are incentive constraints affected?

$$\tilde{u}_0 + \beta \tilde{u}_1(s) - h(y(s)/s) \geq \tilde{u}_1(r) + \beta \tilde{u}_1(r) - h(y(r)/s) \quad \forall r, s$$

if and only if

$$u_0 - \beta \triangle + \beta(u_1(s) + \triangle) - h(y(s)/s) \geq$$

$$u_1(r) - \beta \triangle + \beta(u_1(r) + \triangle) - h(y(r)/s) \quad \forall r, s$$

if and only if

$$u_0 + \beta u_1(s) - h(y(s)/s) \geq u_1(r) + \beta u_1(r) - h(y(r)/s) \quad \forall r, s$$

is this true?

- Variation by construction keeps total expected utility unchanged
- Dual problem: minimize total resource cost of allocation

$$min_{\triangle} \left\{ C(u_0 - \beta \triangle) + q \sum_s p(s) C(u_1(s)) + \triangle) \right\}$$

where $C(u)$ is the inverse function of $u(c)$

INVERSE EULER AGAIN

- If the original allocation $(u_0, u_1(s))$ is optimal, $\triangle = 0$ must solve this problem
- The FOC evaluated at $\triangle = 0$ is

$$-C'(u_0)\beta + q\sum_s p(s)C'(u_1(s)) = 0$$

- Use $C'(u) = 1/u'(c)$

$$\frac{1}{u'(c_0)} = \frac{q}{\beta}\sum_s \frac{p(s)}{u'(c_1(s))}$$

which is the inverse Euler equation again (recall $q = 1/R^*$)
- Alternative interpretation: $1/u'(c)$ is resource cost of providing some given incentives
- IEE requires the equalization of the expected resource cost of providing incentives across both periods

INFINITE HORIZON

- General model with separable preferences

$$\sum_{t,s^t} \beta^t [u(c(s^t)) - h(y(s^t)/s_t)] Pr(s^t)$$

  and $s^t = (s_0, s_1, ..., s_t)$
- Agents have reporting strategies such that (why does it depend on s^t not s_t?)

$$r_t = \sigma_t(s^t)$$

  where the truth telling strategy is such that

$$\sigma_t^*(s^t) = s_t \quad \forall s^t, t$$

- $\sigma^t(s^t)$ denotes the history of reports induced by the strategy $\sigma_t(s^t)$, i.e.

$$\sigma^t(s^t) = (r_0, r_1, ..., r_t) = (\sigma_0(s_0), \sigma_1(s^1), ..., \sigma_t(s^t))$$

DYNAMIC INCENTIVE CONSTRAINTS

- Dynamic incentive constraints

$$\sum_{t,s^t} \beta^t [u(c(s^t)) - h(y(s^t)/s_t)/Pr(s^t)$$

$$\geq \sum_{t,s^t} \beta^t [u(c(\sigma^t(s^t))) - h(y(\sigma^t(s^t))/s_t] Pr(s^t) \quad \forall \sigma$$

- Pick some node s^t. Then set

$$\tilde{u}(s^\tau) = u(s^\tau)$$

for any $s^\tau \neq s^t$ and $s^\tau \neq (s^t, s_{t+1})$. i.e. leave consumption utilities unchanged at any node that is not s^t or any of its direct successors

- At s^t, set

$$\tilde{u}(s^t) = u(s^t) - \beta \triangle$$

and

$$\tilde{u}(s^t, s_{t+1}) = u(s^t, s_{t+1}) + \triangle \quad \forall s_{t+1}$$

DYNAMIC INCENTIVE CONSTRAINTS

- Key: if initial allocation was incentive compatible, perturbed one is as well.
- Moreover, perturbed allocation does not change total expected utility (from any reporting strategy $\sigma_t(s^t)$, thus also from truth-telling)
- Minimize expected resource cost of the perturbed allocation by choosing $\triangle$

$$min_{\triangle}\Big\{C(u(s^t)-\beta\triangle)+q\sum_{s^{t+1}|s^t}Pr(s^{t+1}|s^t)C\big(u_1(s^{t+1})+\triangle\big)\Big\}$$

- If the initial allocation is optimal, this program must be solved at $\triangle = 0$ with FOC

$$\frac{1}{u'(c(s^t))}=\frac{1}{\beta R^*}\mathbb{E}\Big[\frac{1}{u'(c(s^{t+1}))}\Big|s^t\Big]$$

General inverse Euler equation has to hold for all nodes s^t

GENERAL INVERSE EULER EQUATION

- Implies
$$u'(c(s^t)) < \beta R^* \mathbb{E}\Big[u'(c(s^{t+1}))\Big| s^t\Big] \quad \forall s^t$$
i.e. savings need to be distorted downwards compared to the Euler equation from free saving
- May require individualized capital taxes that keep track of the entire history of skill shocks s^t such that
$$u'(c(s^t)) = \beta \mathbb{E}\Big[(1 + r^*(1 - \tau^k(s^{t+1})))u'(c(s^{t+1}))\Big| s^t\Big] \quad \forall s^t$$
where $r^* \equiv R^* - 1$
- However, simple linear capital tax may not work
- Farhi and Werning (2011) show how to use this framework to evaluate the welfare gains from optimal saving distortions starting from some baseline allocation, e.g. the free saving allocation (Aiyagari 1994)

IMPLEMENTATION

- STEP BACK: What is implementation? Why was it not discussed before?!
- Back to 2 period model, 2 shocks $s \in \{H, L\}$
- Suppose have found the optimal allocation with consumption $\{c_0^*, c_1^*(L), c_1^*(H)\}$
- It satisfies the inverse Euler equation

$$\frac{1}{u'(c_0^*)} = \frac{1}{\beta R^*}\left[\frac{p_L}{u'(c_1^*(L))} + \frac{p_H}{u'(c_1^*(H))}\right]$$

- Suppose we introduce a linear capital tax τ^k such that the Euler equation is satisfied

$$u'(c_0^*) = \beta R^*(1-\tau^k)[p_H u'(c_1^*(L)) + p_H u'(c_1^*(H))] \tag{12}$$

IMPLEMENTATION: LINEAR CAPITAL TAX, NONLINEAR INCOME TAX

- Introduce a non-linear income tax system $T_0, T_1(y)$ so that the individuals' budget constraints become

$$c_0 + k_1 \leq e_0 - T_0$$

  in period 0 and

$$c_1(s) \leq y(s) - T_1(y(s)) + (1 - \tau^k)R^* k_1$$

  in period 1
- Note: Very restrictive tax system where the capital tax is linear and separable from the labor income tax
- Can we find a tax system T_0, T_1, τ^k such that $\{c_0^*, c_1^*(L), c_1^*(H)\}$ is incentive compatible? [What do you think?]
- If we could force the agent to choose c_0^* and thus k_1^*? We'd be back to a standard Mirlees problem in period 1, so we can always find $T_1(y)$ that implements $c_1^*(s), y^*(s)$

PROBLEM WITH LINEAR CAPITAL TAX

- Suppose H's incentive constraint is the binding one at the optimum (which means?)

$$u(c_1^*(H)) - h(y^*(H)/H) = u(c_1^*(L)) - h(y^*(L)/H) \tag{13}$$

  i.e. *if* the agent saves optimally k_1^*, truth-telling is optimum
- Moreover, *given* truth-telling in period 1, Euler equation holds, so the agent finds it optimal to choose optimal savings k_1^*
- But: double-deviation $\sigma_1(s) = L$ for all $s \in \{H, L\}$ *and* $\tilde{k}_1 = k_1^* + \epsilon$
- If $\sigma_1(s)$ for all s and $k_1 = k_1^*$, then

$$u'(c_0^*) < \beta R^*(1 - \tau^k) u'(c_1^*(L)) \tag{14}$$

  Why?
- It is optimal to deviate to $\tilde{k}_1 = k_1^* + \epsilon$ with $\epsilon > 0$
- What is agent tempted to do here? Explain in words.

DOUBLE DEVIATION

- Profitable deviation: save more in period 0 and always claim to be low type in period 1 period

$$\tilde{U} = u(c_0^* - \epsilon) + \beta\Big[u(c_1^*(L) + R^*(1-\tau^k)\epsilon) - p_L h(y^*(L)/L) - p_H h(y^*(L)/H)\Big]$$

$$\approx \epsilon\Big[-u'(c_0^*) + \beta R^*(1-\tau^k)u'(c_1^*(L))\Big]$$

$$+u(c_0^*) + \beta\Big[u(c_1^*(L)) - p_L h(y^*(L)/L) - p_H h(y^*(L)/(H)\Big]$$

$$> u(c_0^*) + \beta\Big[p_H(u(c_1^*(H)) - h(y^*(H)/H)) + p_L(u(c_1^*(L)) - h(y^*(L)/L))\Big]$$

- Where is first approximation coming from? Where did the second equality come from?
- Hence, the double deviation makes the agent better off than truth telling under the optimal allocation that we wanted to implement.

LINEAR CAPITAL TAX DOES NOT WORK – SOLUTIONS?

- State-dependent linear capital tax $\tau^k(s)$ so that
$$u'(c_0^*) = \beta R^*(1 - \tau^k(s))u'(c_1^*(s)) \quad \forall s$$
state by state (Kocherlakota 2005). Prevents profitable double-deviations.
$$\tau^k(s) = 1 - \frac{u'(c_0^*)}{\beta R^* u'(c_1^*(s))}$$
is high whenever $c_1^*(s)$ is low.

- What does this mean? Returns to saving are made risky so as to make savings unattractive.

- However: $\tau^k(s)$ is zero in expectation so that the government does not raise revenue with the capital tax.

- In general, capital taxes must be contingent on the entire history of shocks.

Adding Human Capital Investments to the Model

- Interplay between HC policies and taxes.
- HC policies affect the income distribution - a key input for taxes.
- Taxes affect return and risk from HC investments.
- Calls for joint analysis of optimal taxation and HC policies.
- Optimal Taxation (Mirrlees) literature typically assumes exogenous ability
  - Mirlees 1971, Saez 2001...

Model: Risky investments in Human Capital

- **Wage**: $w_t = w_t(\theta_t, s_t, z_t)$
- **Ability** θ:
  stochastic, Markov $f^t(\theta_t|\theta_{t-1})$, private info, privately uninsurable.
- Two ways of acquiring HC:
  1. **Expenses** e_t at cost $M_t(e_t)$. Stock of HC expenses s_t :
  $$s_t = s_{t-1} + e_t$$
  2. **Training time** i_t at disutility cost $\phi_t(l_t, i_t)$. Accomplished training z_t :
  $$z_t = z_{t-1} + i_t$$
- Cost composition of College versus OJT?
- Income: $y_t = w_t l_t$

Hicksian complementarity

- **Hicksian coefficients of complementarity**:

$$\rho_{\theta s} = \frac{w_{\theta s} w}{w_s w_\theta} \qquad \rho_{\theta z} = \frac{w_{\theta z} w}{w_z w_\theta}$$

- $\rho_{\theta s} \geq 0$: Marginal wage gain from HC $\uparrow$ in ability.

- $\rho_{\theta s} \geq 1$: Elasticity of wage to HC $\uparrow$ in ability.

- If separable $w = \theta + h(s, z) \Rightarrow \rho_{\theta s} = \rho_{\theta z} = 0$

- If multiplicative $w = \theta h(s, z) \Rightarrow \rho_{\theta s} = \rho_{\theta z} = 1$

- If CES $w = \left[\alpha_1 \theta^{1-\rho_t} + \alpha_2 s^{1-\rho_t} + \alpha_3 z^{1-\rho_t}\right]^{\frac{1}{1-\rho_t}} \Rightarrow \rho_{\theta s} = \rho_{\theta z} = \rho_t$

Net Subsidy

Model: Preferences over Lifetime Allocations

- T periods of work, T_r periods of retirement.
- Per period utility: $u_t(c_t) - \phi_t(l_t, i_t)$.
- History $\theta^t = \{\theta_1, ..., \theta_t\} \in \Theta^t$, probability $P(\theta^t) = f^t(\theta_{t+1}|\theta_t)...f(\theta_1)$.
- Allocation: $\{c(\theta^t), y(\theta^t), s(\theta^t), z(\theta^t)\}_{\theta^t}$.
- Expected lifetime utility from allocation:

$$U\left(\left\{c\left(\theta^t\right), y\left(\theta^t\right), s\left(\theta^t\right), z\left(\theta^t\right)\right\}\right)$$
$$= \sum_{t=1}^{T+T_r} \int \beta^{t-1} \left[u_t\left(c\left(\theta^t\right)\right) - \phi_t\left(\frac{y\left(\theta^t\right)}{w_t\left(\theta^t\right)}, z\left(\theta^t\right) - z\left(\theta^{t-1}\right)\right)\right] P\left(\theta^t\right) d\theta^t$$

$$w_t\left(\theta^t\right) \equiv w_t\left(\theta_t, s_t\left(\theta^t\right), z_t\left(\theta^t\right)\right)$$

Government's/Planner's Goals: Insurance and Redistribution

- Govt's/Planner's goal: max expected social welfare given Pareto weights.
  - Insurance against earnings risk.
  - Redistribution across intrinsic ability heterogeneity (persistent).
  - Incentives for efficient work and HC investment.

- Asymmetric information about:

$$\begin{array}{c} \text{ability and its evolution} \qquad \text{labor supply} \\ \downarrow \qquad\qquad\qquad \downarrow \\ w_t(\underset{\uparrow}{\theta_t}, \underset{\uparrow}{s_t}, z_t) \times l_t = y_t \\ \text{2 cases: observable and unobservable HC.} \end{array}$$

$\rightarrow$ **"direct revelation mechanism"** with incentive compatibility.

Government's/Planner's Program: Dual Formulation

- Min expected resource cost s.t. utility targets and incentive compatibility $\rightarrow$ **constrained efficiency**.

$$\min_{\{c,y,s,z\}} \sum_{t=1}^{T} \frac{1}{R^{t-1}} \int \left(c\left(\theta^t\right) - y\left(\theta^t\right) + M_t\left(s\left(\theta^t\right) - s\left(\theta^{t-1}\right)\right)\right) P\left(\theta^t\right) d\theta^t$$

$$\text{s.t.: } U\left(\{c, y, s, z\}\right) \geq \underline{U}$$

$$\{c, y, s, z\} \text{ is incentive compatible.}$$

- If initial heterogeneity and non-utilitarian welfare function set any Pareto weights through $\underline{U} = \left(\underline{U}\left(\theta_1\right)\right)_{\Theta}$.

Incentive Compatibility Defined

- Reporting strategy: $r = \{r_t(\theta^t)\}_{t=1}^T$, with history $r^t \equiv \{r_1, .., r_t\}$.

- **Continuation utility** under reporting strategy r:

$$\omega^r(\theta^t) = u_t(c(r^t(\theta^t))) - \phi_t\left(\frac{y(r^t(\theta^t))}{w_t(\theta_t, s(r^t(\theta^t)), z(r^t(\theta^t)))}, i(r^t(\theta^t))\right) + \beta \int \omega^r\left(\theta^{t+1}\right) f^{t+1}(\theta_{t+1}|\theta_t)\, d\theta_{t+1}$$

- **Under truth-telling**: $\omega(\theta^t)$ with $r_t(\theta^t) = \theta_t$ for all θ^t.

- **Incentive Compatibility**

$$\omega(\theta^t) \geq \omega^r(\theta^t) \qquad \forall r, \forall \theta^t$$

Solving the Government's Program: Method

1. Solving the direct revelation mechanism:
   - Step 1: Relax program using first order approach (FOA).
   - Step 2: Formulate relaxed program recursively.
2. Characterize optimal allocations using "wedges" or implicit taxes.
3. Decentralize or "implement" optimum using policy instruments.

Step 1. Relaxing the Program: First-Order Approach

- Consider deviating strategy σ^r with report r:

$$\omega\left(\theta^t\right) = \max_r \Big(u_t\left(c\left(\theta^{t-1}, r\right)\right) - \phi_t\left(\frac{y\left(\theta^{t-1}, r\right)}{w_t\left(\theta_t, s\left(\theta^{t-1}, r\right), z\left(\theta^{t-1}, r\right)\right)}\right) + \beta \int \omega^{\sigma^r}\left(\theta^{t-1}, r, \theta_{t+1}\right) f^{t+1}\left(\theta_{t+1}|\theta_t\right) d\theta_{t+1}\Big)$$

- Replace by necessary **Envelope Condition**:

$$\frac{\partial\omega\left(\theta^t\right)}{\partial\theta_t} = \frac{\partial\phi_t}{\partial l_t}\frac{\partial w_t}{\partial\theta_t}\frac{l_t}{w_t} + \beta\int\omega\left(\theta^{t+1}\right)\frac{\partial f^{t+1}\left(\theta_{t+1}|\theta_t\right)}{\partial\theta_t}d\theta_{t+1}$$

- **Sufficiency**?
  a) Conditions on allocations (Pavan *et al.* 2013).
  b) Ex-post verification (Werning, 2007, Farhi and Werning, 2013).

▸ Verification

Step 2. Recursive Formulation: Define States

- Definition of continuation utility:

$$\omega\left(\theta^t\right) = u_t\left(c\left(\theta^t\right)\right) - \phi_t\left(\frac{y\left(\theta^t\right)}{w_t\left(\theta^t\right)}, i\left(\theta^t\right)\right) + \beta \int \omega\left(\theta^{t+1}\right) f^{t+1}\left(\theta_{t+1}|\theta_t\right) d\theta_{t+1}$$

- Envelope Condition:

$$\frac{\partial \omega\left(\theta^t\right)}{\partial \theta_t} = \frac{\partial \phi_t}{\partial l_t} \frac{\partial w_t}{\partial \theta_t} \frac{l\left(\theta^t\right)}{w\left(\theta^t\right)} + \beta \int \omega\left(\theta^{t+1}\right) \frac{\partial f^{t+1}\left(\theta_{t+1}|\theta_t\right)}{\partial \theta_t} d\theta_{t+1}$$

Step 2. Recursive Formulation: Define States

- Definition of continuation utility:

$$\omega\left(\theta^t\right) = u_t\left(c\left(\theta^t\right)\right) - \phi_t\left(\frac{y\left(\theta^t\right)}{w_t\left(\theta^t\right)}, i\left(\theta^t\right)\right) + \beta \int \omega\left(\theta^{t+1}\right) f^{t+1}\left(\theta_{t+1}|\theta_t\right) d\theta_{t+1}$$

- Envelope Condition:

$$\frac{\partial \omega\left(\theta^t\right)}{\partial \theta_t} = \frac{\partial \phi_t}{\partial l_t}\frac{\partial w_t}{\partial \theta_t}\frac{l\left(\theta^t\right)}{w\left(\theta^t\right)} + \beta \int \omega\left(\theta^{t+1}\right)\frac{\partial f^{t+1}\left(\theta_{t+1}|\theta_t\right)}{\partial \theta_t} d\theta_{t+1}$$

Step 2. Recursive Formulation: Define States

- Definition of continuation utility:

$$\omega\left(\theta^t\right)=u_t\left(c\left(\theta^t\right)\right)-\phi_t\left(\frac{y\left(\theta^t\right)}{w_t\left(\theta^t\right)}, i\left(\theta^t\right)\right)+\beta v_t\left(\theta^t\right)$$

- Envelope Condition:

$$\frac{\partial \omega\left(\theta^t\right)}{\partial \theta_t}=\frac{\partial \phi_t}{\partial l_t} \frac{\partial w_t}{\partial \theta_t} \frac{l\left(\theta^t\right)}{w\left(\theta^t\right)}+\beta \int \omega\left(\theta^{t+1}\right) \frac{\partial f^{t+1}\left(\theta_{t+1} | \theta_t\right)}{\partial \theta_t} d \theta_{t+1}$$

Step 2. Recursive Formulation: Define States

- Definition of continuation utility:

$$\omega\left(\theta^t\right) = u_t\left(c\left(\theta^t\right)\right) - \phi_t\left(\frac{y\left(\theta^t\right)}{w_t\left(\theta^t\right)}, i\left(\theta^t\right)\right) + \beta v_t\left(\theta^t\right)$$

- Envelope Condition:

$$\frac{\partial \omega\left(\theta^t\right)}{\partial \theta_t} = \frac{\partial \phi_t}{\partial l_t} \frac{\partial w_t}{\partial \theta_t} \frac{l\left(\theta^t\right)}{w\left(\theta^t\right)} + \beta \int \omega\left(\theta^{t+1}\right) \frac{\partial f^{t+1}\left(\theta_{t+1}|\theta_t\right)}{\partial \theta_t} d\theta_{t+1}$$

$$v_t\left(\theta^t\right) = \int \omega\left(\theta^{t+1}\right) f^{t+1}\left(\theta_{t+1}|\theta_t\right) d\theta_{t+1}$$

Step 2. Recursive Formulation: Define States

- Definition of continuation utility:

$$\omega\left(\theta^t\right) = u_t\left(c\left(\theta^t\right)\right) - \phi_t\left(\frac{y\left(\theta^t\right)}{w_t\left(\theta^t\right)}, i\left(\theta^t\right)\right) + \beta v_t\left(\theta^t\right)$$

- Envelope Condition:

$$\frac{\partial \omega\left(\theta^t\right)}{\partial \theta_t} = \frac{\partial \phi_t}{\partial l_t}\frac{\partial w_t}{\partial \theta_t}\frac{l\left(\theta^t\right)}{w\left(\theta^t\right)} + \beta \int \omega\left(\theta^{t+1}\right)\frac{\partial f^{t+1}\left(\theta_{t+1}|\theta_t\right)}{\partial \theta_t} d\theta_{t+1}$$

$$v_t\left(\theta^t\right) = \int \omega\left(\theta^{t+1}\right) f^{t+1}\left(\theta_{t+1}|\theta_t\right) d\theta_{t+1}$$

Step 2. Recursive Formulation: Define States

- Definition of continuation utility:

$$\omega\left(\theta^t\right) = u_t\left(c\left(\theta^t\right)\right) - \phi_t\left(\frac{y\left(\theta^t\right)}{w_t\left(\theta^t\right)}, i\left(\theta^t\right)\right) + \beta v_t\left(\theta^t\right)$$

- Envelope Condition:

$$\frac{\partial\omega\left(\theta^t\right)}{\partial\theta_t} = \frac{\partial\phi_t}{\partial l_t}\frac{\partial w_t}{\partial\theta_t}\frac{l\left(\theta^t\right)}{w\left(\theta^t\right)} + \beta\Delta_t\left(\theta^t\right)$$

$$v_t\left(\theta^t\right) = \int \omega\left(\theta^{t+1}\right) f^{t+1}\left(\theta_{t+1}|\theta_t\right) d\theta_{t+1}$$

Step 2. Recursive Formulation: Define States

- Definition of continuation utility:

$$\omega\left(\theta^t\right) = u_t\left(c\left(\theta^t\right)\right) - \phi_t\left(\frac{y\left(\theta^t\right)}{w_t\left(\theta^t\right)}, i\left(\theta^t\right)\right) + \beta v_t\left(\theta^t\right)$$

- Envelope Condition:

$$\frac{\partial \omega\left(\theta^t\right)}{\partial \theta_t} = \frac{\partial \phi_t}{\partial l_t}\frac{\partial w_t}{\partial \theta_t}\frac{l\left(\theta^t\right)}{w\left(\theta^t\right)} + \beta \Delta_t\left(\theta^t\right)$$

$$\begin{aligned} v_t\left(\theta^t\right) &= \int \omega\left(\theta^{t+1}\right) f^{t+1}\left(\theta_{t+1}|\theta_t\right) d\theta_{t+1} \\ \Delta_t\left(\theta^t\right) &= \int \omega\left(\theta^{t+1}\right) \frac{\partial f^{t+1}\left(\theta_{t+1}|\theta_t\right)}{\partial \theta_t} d\theta_{t+1} \end{aligned}$$

Step 2. Recursive Formulation: Rewrite Recursively

- Definition of continuation utility, using $\theta_-, \theta, \theta'$.

$$\omega(\theta) = u_t(c(\theta)) - \phi_t\left(\frac{y(\theta)}{w_t(\theta)}, z(\theta) - z_-\right) + \beta v(\theta)$$

- Envelope Condition:

$$\frac{\partial \omega(\theta)}{\partial \theta} = \frac{\partial \phi_t}{\partial l} \frac{\partial w_t(\theta)}{\partial \theta} \frac{l(\theta)}{w_t(\theta)} + \beta \Delta(\theta)$$

$$\begin{aligned} v(\theta) &= \int \omega(\theta') f^{t+1}(\theta'|\theta) \, d\theta' \\ \Delta(\theta) &= \int \omega(\theta') \frac{\partial f^{t+1}(\theta'|\theta)}{\partial \theta} d\theta' \end{aligned}$$

Recursive Formulation of Relaxed Program

$$K(v, \Delta, \theta_-, s_-, z_-, t) = \min \int \left(c(\theta) + M_t(s(\theta) - s_-) - w_t(\theta, s(\theta), z(\theta))\, l(\theta)\right.$$
$$\left. + \frac{1}{R} K(v(\theta), \Delta(\theta), \theta, s(\theta), z(\theta), t+1)\right) f^t(\theta|\theta_-)\, d\theta$$

$$\omega(\theta) = u_t(c(\theta)) - \phi_t(l(\theta), z(\theta) - z_-) + \beta v(\theta)$$
$$\dot{\omega}(\theta) = \frac{w_{\theta,t}}{w_t} l(\theta)\, \phi_{l,t}((\theta), z(\theta) - z_-) + \beta \Delta(\theta)$$
$$v = \int \omega(\theta)\, f^t(\theta|\theta_-)\, d\theta$$
$$\Delta = \int \omega(\theta) \frac{\partial f^t(\theta|\theta_-)}{\partial \theta_-} d\theta$$

over $(c(\theta), l(\theta), s(\theta), z(\theta), \omega(\theta), v(\theta), \Delta(\theta))$

Recursive Formulation of Relaxed Program

$$K(v, \Delta, \theta_-, s_-, z_-, t) = \min \int \Big(c(\theta) + M_t (s(\theta) - s_-) - w_t(\theta, s(\theta), z(\theta)) l(\theta) + \frac{1}{R} K(v(\theta), \Delta(\theta), \theta, s(\theta), z(\theta), t+1) \Big) f^t(\theta|\theta_-) \, d\theta$$

$$\omega(\theta) = u_t(c(\theta)) - \phi_t(l(\theta), z(\theta) - z_-) + \beta v(\theta)$$

$$\dot{\omega}(\theta) = \frac{w_{\theta,t}}{w_t} l(\theta) \phi_{l,t}((\theta), z(\theta) - z_-) + \beta \Delta(\theta)$$

$$v = \int \omega(\theta) f^t(\theta|\theta_-) \, d\theta$$

$$\Delta = \int \omega(\theta) \frac{\partial f^t(\theta|\theta_-)}{\partial \theta_-} d\theta$$

over $(c(\theta), l(\theta), s(\theta), z(\theta), \omega(\theta), v(\theta), \Delta(\theta))$

Recursive Formulation of Relaxed Program

$$
\begin{aligned}
K\left(v,\Delta,\theta_{-},s_{-},z_{-},t\right) &= \min \int \Big(c\left(\theta\right) + M_t\left(s\left(\theta\right)-s_{-}\right) - w_t\left(\theta, s\left(\theta\right), z\left(\theta\right)\right) l\left(\theta\right) \\
&\quad + \frac{1}{R} K\left(v\left(\theta\right), \Delta\left(\theta\right), \theta, s\left(\theta\right), z\left(\theta\right), t+1\right)\Big) f^t\left(\theta|\theta_{-}\right) d\theta
\end{aligned}
$$

$$
\begin{aligned}
\omega\left(\theta\right) &= u_t\left(c\left(\theta\right)\right) - \phi_t\left(l\left(\theta\right), z\left(\theta\right)-z_{-}\right) + \beta v\left(\theta\right) \\
\dot{\omega}\left(\theta\right) &= \frac{w_{\theta,t}}{w_t} l\left(\theta\right) \phi_{l,t}\left(\left(\theta\right), z\left(\theta\right)-z_{-}\right) + \beta \Delta\left(\theta\right) \\
v &= \int \omega\left(\theta\right) f^t\left(\theta|\theta_{-}\right) d\theta \\
\Delta &= \int \omega\left(\theta\right) \frac{\partial f^t\left(\theta|\theta_{-}\right)}{\partial \theta_{-}} d\theta
\end{aligned}
$$

over $\left(c\left(\theta\right), l\left(\theta\right), s\left(\theta\right), z\left(\theta\right), \omega\left(\theta\right), v\left(\theta\right), \Delta\left(\theta\right)\right)$

Recursive Formulation of Relaxed Program

$$K(v, \Delta, \theta_-, s_-, z_-, t) = \min \int \Big(c(\theta) + M_t(s(\theta) - s_-) - w_t(\theta, s(\theta), z(\theta)) \, l(\theta) + \frac{1}{R} K(v(\theta), \Delta(\theta), \theta, s(\theta), z(\theta), t+1) \Big) f^t(\theta|\theta_-) \, d\theta$$

$$\omega(\theta) = u_t(c(\theta)) - \phi_t(l(\theta), z(\theta) - z_-) + \beta v(\theta)$$

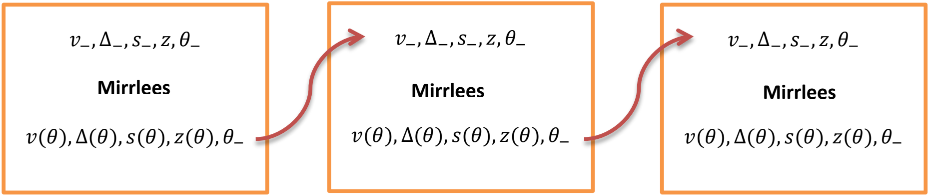
$$\dot{\omega}(\theta) = \frac{w_{\theta,t}}{w_t} l(\theta) \phi_{l,t}((\theta), z(\theta) - z_-) + \beta \Delta(\theta)$$

$$v = \int \omega(\theta) f^t(\theta|\theta_-) \, d\theta$$

$$\Delta = \int \omega(\theta) \frac{\partial f^t(\theta|\theta_-)}{\partial \theta_-} d\theta$$

over $(c(\theta), l(\theta), s(\theta), z(\theta), \omega(\theta), v(\theta), \Delta(\theta))$

Method summary:
Repeated Mirrlees Nested in Dynamic Programming
+ Endogenous Human Capital Formation



REFERENCES

Principle can be understood in 2 period model:

Cremer, H. and F. Gahvari "Uncertainty and Optimal Taxation: In defense of Commodity Taxes", Journal of Public Economics, Vol. 56, 1995, 291-310. (web)

Diamond, P. and J. Mirrlees "A Model of Social Insurance with Variable Retirement", Journal of Public Economics, Vol. 10, 1978, 295-336. (web)

Generalized to many periods:

Golosov, M., N. Kocherlakota and A. Tsyvinski. "Optimal Indirect and Capital Taxation," Review of Economic Studies, July 2003, Vol.70(3), pp.569-587.

Kocherlakota, N. "Zero Expected Wealth Taxes: A Mirrlees Approach to Dynamic Optimal Taxation," Econometrica, 2005, Volume 73, Issue 5.

Simple exposition:

Kocherlakota, N. "Wedges and Taxes," American Economic Review Papers and Proceedings 2004, 109-113.

Two comprehensive surveys:

Golosov, M., A. Tsyvinski and I. Werning ?New Dynamic Public Finance: A User?s Guide? in NBER Macroeconomic Annual 2006, MIT press.

[Book] Kocherlakota, N. "The New Dynamic Public Finance." Princeton University Press.

Applications and further extensions:

Golosov, M. and A. Tsyvinski ?Designing Optimal Disability Insurance: A Case for Asset Testing? Journal of Political Economy, 114(2), (2006): 257-279.

Stantcheva, S. "Optimal Taxation and Human Capital over the Life Cycle", Journal of Political Economy, 2017, 125 (6), (web)

Akcigit, U., D. Hanley and S. Stantcheva. "Optimal Taxation and R&D Policies", NBER Working Paper 22908 (web)