

# New Dynamic Public Finance (NDPF)

**Stefanie Stantcheva**

Fall 2019

## GOALS OF THIS LECTURE

- (1) New Dynamic Public Finance: A toolbox.
- (2) Understand capital taxation from the point of view of NDPF
- (3) Application to human capital and how to finance investments in education

## SET UP: UNCERTAINTY ON EARNINGS

- So far: representative agents, ex ante heterogeneity, aggregate uncertainty
- We now consider idiosyncratic uncertainty that is not only ex ante, but unfolds over time
- Skill shocks or preference shocks
- Start with finite horizon:  $t = 0, 1$
- Preferences  $U(c_0, c_1(s), y(s)/s)$
- interpretation: skill shock  $s$  realized in period 1. Consumption decision  $c_0$  in period 0 is made before the shock is realized

## SET UP: FAILURE OF A-S and RESOURCE CONSTRAINT

- Note difference to time-0 shock (ex ante heterogeneity) as considered so far. Preferences would be  $U(c_0(s), c_1(s), y(s)/s)$
- Under separability + homogeneity, the Atkinson-Stiglitz (1976) theorem would rule out the optimality of a capital tax.
- With the period-1 shock, we will find a downward distortion of saving to be optimal (positive capital tax).
- Technology: linear storage with rate of return  $R^* = 1/q$ , so that the aggregate resource constraint is

$$c_0 + q \sum_s c_1(s)p(s) \leq q \sum_s y(s)p(s) \quad (1)$$

## FIRST BEST

- 

$$\max_{c_0, c_1(s), y(s)} \sum_s U(c_0, c_1(s), y(s)/s) p(s)$$

s.t. (1)

- FOCs for  $[c_0]$

$$\mathbb{E}[U_{c_0}, c_1(s), y(s)/s] = \lambda$$

and for  $[c_1(s)]$

$$U_{c_1(s)}(c_0, c_1(s), y(s)/s) = \lambda q$$

## FIRST BEST – FULL INSURANCE

- Hence,

$$\mathbb{E}[U_{c_0}, c_1(s), y(s)/s] = R^* U_{c_1(s)}(c_0, c_1(s), y(s)/s) \quad \forall s$$

$\Rightarrow$  Full Insurance

- Taking expectations on both sides

$$\mathbb{E}[U_{c_0}, c_1(s), y(s)/s] = R^* \mathbb{E}[U_{c_1(s)}(c_0, c_1(s), y(s)/s)] \quad (2)$$

- For instance, if  $U(c_0, c_1(s), y(s)/s) = u(c_0) + \beta u(c_1(s)) - h(y(s)/s)$ , then we obtain the usual Euler equation

$$u'(c_0) = \beta R^* \mathbb{E}[u'(c_1(s))]$$

and  $c_1(s) = c_1$  for all  $s$ .

## FREE SAVING GIVEN INCOME TAX

- Free saving with non-linear income tax  $T(Y)$ :

$$\max_{c_0, c_1(s), y(s)} \sum_s U(c_0, c_1(s), y(s)/s) p(s)$$

s.t.

$$c_0 + k_1 \leq e$$

$$c_1(s) \leq y(s) - T(y(s)) + Rk_1 \quad \forall s$$

- FOCs and  $R = R^*$  yields the Euler equation

$$u'(c_0) = \beta R^* \mathbb{E}[u'(c_1(s))]$$

- If agents can freely decide how much to save in a risk-free asset with return  $R = R^*$ , we obtain the Euler equation as in the first best

# PRIVATE INFORMATION AND INCENTIVE CONSTRAINTS

- Suppose  $s$  is private information and agents make reports  $r = \sigma(s)$ , where  $\sigma$  denotes the reporting strategy
- Truth-telling:  $\sigma^*(s) = s \quad \forall s$
- Denote

$$c_1^\sigma(s) = c_1(\sigma(s))$$

and

$$y^\sigma(s) = y(\sigma(s))$$

- The set of incentive constraints is

$$\mathbb{E}[U(c_0, c_1(s), y(s)/s)] \geq \mathbb{E}[U(c_0, c_1^\sigma(s), y^\sigma(s)/s)] \quad \forall \sigma, s$$

- This is equivalent to

$$U(c_0, c_1(s), y(s)/s) \geq U(c_0, c_1(r), y(r)/s) \quad \forall r, s \tag{3}$$

## SECOND BEST DYNAMIC MIRRLEES PROBLEM

- The second best (dynamic Mirrlees) problem is

$$\max_{c_0, c_1(s), y(s)/s} \mathbb{E}[U(c_0, c_1(s), y(s)/s)] \quad (4)$$

s.t.

$$c_0 + q \sum_s c_1(s)p(s) \leq q \sum_s y(s)p(s) \quad (\text{RC})$$

and

$$U(c_0, c_1(s), y(s)/s) \geq U(c_0, c_1(r), y(r)/s) \quad \forall r, s \quad (\text{IC})$$

## FEASIBLE VARIATIONS

- (RC) and (IC) define the set  $F$  of *feasible* allocations, i.e.

$$F \equiv \{(c_0, c_1(s), y(s)) \mid (c_0, c_1(s), y(s)) \text{ satisfies (RC) and (IC)}$$

- Key question: Is free saving feasible? Formally, if  $(c_0, c_1(s), y(s)) \in F$ , does this imply that  $(c_0 - \Delta, c_1(s) + R^* \Delta, y(s)) \in F$  as well, for some  $\Delta \in \mathbb{R}$ ?
- In other words, if the agent saves a little in period 0 ( $\Delta$ ) is she still willing to supply the same output (i.e. not lie about  $s$ )?
- Depends on income effects in general
- For instance, suppose

$$U(c_0, c_1(s), y(s)/s) = \hat{U}(c_0, c_1(s) - h(y(s)/s))$$

Then, given  $c_0$ , just maximize  $c_1(s) - h(y(s)/s)$ . There are no income effects due to quasilinearity, and the above variation is feasible.

## FEASIBLE VARIATIONS

- Easier to see using (IC):

$$\hat{U}(c_0 - \Delta, c_1(s) + R^* \Delta - h(y(s)/s)) \geq$$

$$\hat{U}(c_0 - \Delta, c_1(r) + R^* \Delta - h(y(r)/s))$$

if and only if

$$c_1(s) + R^* \Delta - h(y(s)/s) \geq c_1(r) + R^* \Delta - h(y(r)/s)$$

which is implied by the original allocation being feasible, i.e.

$$(c_0, c_1(s), y(s)) \in F$$

- But in general, saving in period 0 has a negative income effect on labor supply in period 1 (if leisure is a normal good)
- e.g. consider preferences

$$U(c_0, c_1(s), y(s)/s) = u(c_0) + \beta u(c_1(s)) - h(y(s)/s)$$

Additive separability + concavity of  $u(\cdot)$  mean leisure is normal (output is "inferior") and so the variation above is no longer feasible.

## CAN WE FIND A FEASIBLE VARIATION?

- Free saving is not feasible with these preferences due to negative income effect on labor supply
- Consider

$$(c_0 - \Delta, c_1(s) + \delta(\Delta, s), y(s)) \quad (5)$$

with  $\delta(\Delta, s)$  chosen such that (IC) is satisfied:

$$u(c_0 - \Delta) + \beta u(c_1(s) + \delta(\Delta, s)) = u(c_0) + \beta u(c_1(s)) + A(\Delta) \quad \forall s, \Delta \quad (6)$$

for some  $A(\Delta)$ , and such that it is resource neutral:

$$-\Delta + q \sum_m \delta(\Delta, s) p(s) = 0 \quad \forall \Delta \quad (7)$$

- With the “free saving” variation, we had  $\delta(\Delta, s) = -R^* \Delta$ . What is key difference?

## VERIFY INCENTIVE COMPATIBILITY OF VARIATION

- Verify that the variation maintains incentive compatibility:

$$u(c_0 - \Delta) + \beta u(c_1(s) + \delta(\Delta, s)) - h(y(s)/s) \geq$$

$$u(c_0 - \Delta) + \beta u(c_1(r) + \delta(\Delta, r)) - h(y(r)/s)$$

if and only if (6)

$$u(c_0) + \beta u(c_1(s)) + A(\Delta) - h(y(s)/s) \geq$$

$$u(c_0) + \beta u(c_1(r)) + A(\Delta) - h(y(r)/s)$$

if and only if

$$u(c_0) + \beta u(c_1(s)) - h(y(s)/s) \geq u(c_0) + \beta u(c_1(r)) - h(y(r)/s)$$

- Is this true?
- Key: Given separability, all that matters for incentive compatibility is the total utility from consumption.

## INVERSE EULER EQUATION

- Suppose the original allocation  $(c_0, c_1(s), y(s))$  solves the second best problem. Then, since the variation  $\delta(\Delta, s)$  is feasible as just shown, it cannot improve the objective.
- Formally,

$$\begin{aligned} 0 &= \operatorname{argmax}_{\Delta} \sum_s p(s) [u(c_0 - \Delta) + \beta u(c_1(s) + \delta(\Delta, s)) - h(y(s)/s)] \\ &= \operatorname{argmax}_{\Delta} \sum_s p(s) [u(c_0) + \beta u(c_1(s) + A(\Delta)) - h(y(s)/s)] \\ &= \operatorname{argmax}_{\Delta} A(\Delta), \end{aligned}$$

where we used

$$u(c_0 - \Delta) + \beta u(c_1(s) + \delta(\Delta, s)) = u(c_0) + \beta u(c_1(s)) + A(\Delta) \quad \forall s, \Delta \quad (8)$$

- FOC  $A'(0) = 0$

## INVERSE EULER EQUATION

- $\delta(\Delta, s)$  satisfies (IC); differentiate w.r.t  $\Delta$ :

$$-u'(c_0) + \beta u'(c_1(s)) \frac{\partial \delta(\Delta, s)}{\partial \Delta} \Big|_{\Delta=0} = A'(0)$$

rearrange (at optimum):

$$\frac{\partial \delta(\Delta, s)}{\partial \Delta} \Big|_{\Delta=0} = \frac{u'(c_0) + A'(0)}{\beta u'(c_1(s))} = \frac{u'(c_0)}{\beta u'(c_1(s))} \quad \forall s \quad (9)$$

- Condition for resource neutrality of the variation implies:

$$-1 + q \sum_s p(s) \frac{\partial \delta(\Delta, s)}{\partial \Delta} \Big|_{\Delta=0} = 0$$

## INVERSE EULER EQUATION (II)

- Using expression for  $\frac{\partial \delta(\Delta, s)}{\partial \Delta} \Big|_{\Delta=0}$ :

$$\frac{1}{u'(c_0)} = \frac{1}{\beta R^*} \sum_s \frac{p(s)}{u'(c_1(s))} \quad (10)$$

- With separable preferences, optimal allocation has to satisfy this inverse Euler equation (Diamond/Mirrlees 1978, Rogerson 1985, Golosov et al. 2003)
- Is this necessary and sufficient? (Think of optimality of  $y(s)$ ).
- Implies that the Euler equation is violated.

$$u'(c_0) = \beta R^* \sum_s p(s) u'(c_1(s)) \quad (11)$$

Is it always violated?

## POSITIVE SAVINGS WEDGE

- Inverse Euler implies that, at the optimum,

$$u'(c_0) = \left[ \frac{1}{\beta R^*} \sum_s \frac{p(s)}{u'(c_1(s))} \right]^{-1} = \beta R^* \left( \mathbb{E} \left[ \frac{1}{u'(c_1(s))} \right] \right)^{-1}$$

By Jensen's inequality and convexity of the function  $f(x) = 1/x$ ,

$$u'(c_0) < \beta R^* \mathbb{E}[u'(c_1(s))]$$

- The optimal allocation is incompatible with free saving. Is saving is discouraged or encouraged?
- Intuition: saving in period 0 increases income in period 1 across all shocks  $s \rightarrow$  negative income effect on  $y \rightarrow$  is this good or bad for Planner?
- Implications for capital taxation, but study distinction between the wedge derived here and actual implementations later

## THE DUAL APPROACH: VARIATIONAL ARGUMENT

- Easier to work with in dynamic problems: Minimize resource cost s.t. reaching given level of utilities.
- Consider allocation in terms of utils

$$u_0 \equiv u(c_0), \quad u_1(s) \equiv u(c_1(s))$$

- Move from original allocation  $(u_0, u_1(s))$  to variation  $(\tilde{u}_0, \tilde{u}_1(s))$  such that

$$u_0 + \beta u_1(s) = \tilde{u}_0 + \beta \tilde{u}_1(s) \quad \forall s$$

- In particular, set

$$\tilde{u}_0 = u_0 - \beta \Delta$$

and

$$\tilde{u}_1(s) = u_1(s) + \Delta \quad \forall s$$

## THE DUAL APPROACH: : VARIATIONAL ARGUMENT (II)

- Are incentive constraints affected?

$$\tilde{u}_0 + \beta \tilde{u}_1(s) - h(y(s)/s) \geq \tilde{u}_1(r) + \beta \tilde{u}_1(r) - h(y(r)/s) \quad \forall r, s$$

if and only if

$$u_0 - \beta \Delta + \beta(u_1(s) + \Delta) - h(y(s)/s) \geq$$

$$u_1(r) - \beta \Delta + \beta(u_1(r) + \Delta) - h(y(r)/s) \quad \forall r, s$$

if and only if

$$u_0 + \beta u_1(s) - h(y(s)/s) \geq u_1(r) + \beta u_1(r) - h(y(r)/s) \quad \forall r, s$$

which is this true because original allocation was IC.

- Variation by construction keeps total expected utility unchanged
- Dual problem: minimize total resource cost of allocation

$$\min_{\Delta} \left\{ C(u_0 - \beta \Delta) + q \sum_s p(s) C(u_1(s)) + \Delta \right\}$$

where  $C(u)$  is the inverse function of  $u(c)$

## INVERSE EULER AGAIN

- If the original allocation  $(u_0, u_1(s))$  is optimal,  $\Delta = 0$  must solve this problem
- The FOC evaluated at  $\Delta = 0$  is

$$-C'(u_0)\beta + q \sum_s p(s)C'(u_1(s)) = 0$$

- Use  $C'(u) = 1/u'(c)$

$$\frac{1}{u'(c_0)} = \frac{q}{\beta} \sum_s \frac{p(s)}{u'(c_1(s))}$$

which is the inverse Euler equation again (recall  $q = 1/R^*$ )

- Alternative interpretation:  $1/u'(\mathbf{c})$  is resource cost of providing some given incentives
- IEE requires the equalization of the expected resource cost of providing incentives across both periods

## IMPLEMENTATION

- STEP BACK: What is implementation? Why was it not discussed before?!
- Back to 2 period model, 2 shocks  $s \in \{H, L\}$
- Suppose have found the optimal allocation with consumption  $\{c_0^*, c_1^*(L), c_1^*(H)\}$
- It satisfies the inverse Euler equation

$$\frac{1}{u'(c_0^*)} = \frac{1}{\beta R^*} \left[ \frac{\rho_L}{u'(c_1^*(L))} + \frac{\rho_H}{u'(c_1^*(H))} \right]$$

- Suppose we introduce a linear capital tax  $\tau^k$  such that the Euler equation is satisfied

$$u'(c_0^*) = \beta R^*(1 - \tau^k) [\rho_H u'(c_1^*(L)) + \rho_H u'(c_1^*(H))] \quad (12)$$

## IMPLEMENTATION: LINEAR CAPITAL TAX, NONLINEAR INCOME TAX

- Introduce a non-linear income tax system  $T_0, T_1(y)$  so that the individuals' budget constraints become

$$c_0 + k_1 \leq e_0 - T_0$$

in period 0 and

$$c_1(s) \leq y(s) - T_1(y(s)) + (1 - \tau^k)R^*k_1$$

in period 1

- Note: Very restrictive tax system where the capital tax is linear and separable from the labor income tax
- Can we find a tax system  $T_0, T_1, \tau^k$  such that  $\{c_0^*, c_1^*(L), c_1^*(H)\}$  is incentive compatible? [What do you think?]
- If we could force the agent to choose  $c_0^*$  and thus  $k_1^*$ ? We'd be back to a standard Mirrlees problem in period 1, so we can always find  $T_1(y)$  that implements  $c_1^*(s), y^*(s)$

## PROBLEM WITH LINEAR CAPITAL TAX

- Suppose  $H$ 's incentive constraint is the binding one at the optimum (which means?)

$$u(c_1^*(H)) - h(y^*(H)/H) = u(c_1^*(L)) - h(y^*(L)/H) \quad (13)$$

i.e. if the agent saves optimally  $k_1^*$ , truth-telling is optimum

- Moreover, given truth-telling in period 1, Euler equation holds, so the agent finds it optimal to choose optimal savings  $k_1^*$
- But: double-deviation  $\sigma_1(s) = L$  for all  $s \in \{H, L\}$  and  $\tilde{k}_1 = k_1^* + \epsilon$
- If  $\sigma_1(s)$  for all  $s$  and  $k_1 = k_1^*$ , then

$$u'(c_0^*) < \beta R^*(1 - \tau^k) u'(c_1^*(L)) \quad (14)$$

Why?

- It is optimal to deviate to  $\tilde{k}_1 = k_1^* + \epsilon$  with  $\epsilon > 0$
- What is agent tempted to do here? Explain in words.

## DOUBLE DEVIATION

- Profitable deviation: save more in period 0 and always claim to be low type in period 1 period

$$\begin{aligned}\tilde{U} &= u(c_0^* - \epsilon) + \beta \left[ u(c_1^*(L) + R^*(1 - \tau^k)\epsilon) - p_L h(y^*(L)/L) - p_H h(y^*(L)/H) \right] \\ &\approx \epsilon \left[ -u'(c_0^*) + \beta R^*(1 - \tau^k) u'(c_1^*(L)) \right] \\ &\quad + u(c_0^*) + \beta \left[ u(c_1^*(L)) - p_L h(y^*(L)/L) - p_H h(y^*(L)/H) \right] \\ &> u(c_0^*) + \beta \left[ p_H (u(c_1^*(H)) - h(y^*(H)/H)) + p_L (u(c_1^*(L)) - h(y^*(L)/L)) \right]\end{aligned}$$

- Where is first approximation coming from? Where did the second equality come from?
- Hence, the double deviation makes the agent better off than truth telling under the optimal allocation that we wanted to implement.

## LINEAR CAPITAL TAX DOES NOT WORK – SOLUTIONS?

- State-dependent linear capital tax  $\tau^k(s)$  so that

$$u'(c_0^*) = \beta R^*(1 - \tau^k(s)) u'(c_1^*(s)) \quad \forall s$$

state by state (Kocherlakota 2005). Prevents profitable double-deviations.

$$\tau^k(s) = 1 - \frac{u'(c_0^*)}{\beta R^* u'(c_1^*(s))}$$

is high whenever  $c_1^*(s)$  is low.

- What does this mean? Returns to saving are made risky so as to make savings unattractive.
- However:  $\tau^k(s)$  is zero in expectation so that the government does not raise revenue with the capital tax.
- In general, capital taxes must be contingent on the entire history of shocks.

## ASSET TESTS IN DISABILITY INSURANCE AND MEDICAID

- Asset tests plus income tests are used in social programs in the U.S.
- Disability insurance: have to run down your assets first before applying. Is exactly a “downward distortion” in savings to prevent “shirking” (pretending to be unable to work).
- Medicaid long-term care also has asset test to prevent you from “shirking” (i.e., not using other options) first.

## Applications: Every Dynamic Problem with Asymmetric Information

- Many problems in the real world are dynamic.
- R&D and Innovation Policies: Firms produce “quality” products

$$q_t = (1 - \delta)q_{t-1} + \lambda_t(\theta_t, e_t, r_{t-1})$$

where  $\lambda_t$  is called “step size”,  $\theta_t$  is firms’ research productivity (unobservable),  $e_t$  is research effort (unobservable),  $r_{t-1}$  is R&D investments.

- Human Capital investments for people: Income is  $y_t = w_t(\theta_t, s_t)l_t$  where the human capital stock is acquired over time  $s_t = s_{t-1}(1 - \delta) + e_t$  with either a “money cost”  $M_t(e_t)$  or a time cost (that naturally will have some interaction with labor supply)  $\phi_t(l_t, e_t)$ .
- Physician’s incentives? Hospital Cost minimization under asymmetric information on hospital’s productivity?

## REFERENCES

Principle can be understood in 2 period model:

Cremer, H. and F. Gahvari "Uncertainty and Optimal Taxation: In defense of Commodity Taxes", Journal of Public Economics, Vol. 56, 1995, 291–310. (web)

Diamond, P. and J. Mirrlees "A Model of Social Insurance with Variable Retirement", Journal of Public Economics, Vol. 10, 1978, 295–336. (web)

Generalized to many periods:

Golosov, M., N. Kocherlakota and A. Tsyvinski. "Optimal Indirect and Capital Taxation," Review of Economic Studies, July 2003, Vol.70(3), pp.569–587.

Kocherlakota, N. "Zero Expected Wealth Taxes: A Mirrlees Approach to Dynamic Optimal Taxation," Econometrica, 2005, Volume 73, Issue 5.

## **Simple exposition:**

Kocherlakota, N. "Wedges and Taxes," American Economic Review Papers and Proceedings 2004, 109-113.

## **Two comprehensive surveys:**

Golosov, M., A. Tsyvinski and I. Werning ?New Dynamic Public Finance: A User?s Guide? in NBER Macroeconomic Annual 2006, MIT press.

[Book] Kocherlakota, N. "The New Dynamic Public Finance." Princeton University Press.

## **Applications and further extensions:**

Golosov, M. and A. Tsyvinski ?Designing Optimal Disability Insurance: A Case for Asset Testing? Journal of Political Economy, 114(2), (2006): 257-279.

Stantcheva, S. "Optimal Taxation and Human Capital over the Life Cycle", Journal of Political Economy, 2017, 125 (6), (web)

Akcigit, U., D. Hanley and S. Stantcheva. "Optimal Taxation and R&D Policies", NBER Working Paper 22908 (web)