Problem Set 3

EC2450A

Fall 2016

Problem 1

There are two types of individuals, $i = 1, 2$ with different abilities w_i. Let c_i be type i's consumption, l_i be his hours worked and income $y_i = w_i l_i$. Utility is increasing in consumption and decreasing in hours worked.

$$u_i = u_i(c_i, l_i)$$

Suppose there is a tax $T(y_i)$ on income.

1) Write the budget constraint of type i under this tax system.

2) Write the maximization problem of the individual under this tax system and derive the first-order conditions.

3) Express the marginal tax rate as a function of the primitives (marginal utilities and ability levels). This gives you a characterization of the marginal tax rate as a function of the allocations.

4) Define $V_i(c_i, y_i; w_i) = u_i\left(c_i, \frac{y_i}{w_i}\right)$ to be the utility expressed as a function of observable y_i. Express the derivatives: $\frac{\partial V_i}{\partial c_i}, \frac{\partial V_i}{\partial y_i}, \frac{\partial V_i}{\partial w_i}$ as functions of the derivatives of u_i. Express the marginal tax rate $1 - T'$ as a function of the derivatives of V_i.

5) What first-order condition would characterize the first-best allocation (in the first best allocation, there are lump-sum, type-specific taxes available since there is no private information).

6) Suppose that $w_2 > w_1$. Draw one indifference curve for each type in the (y, c) space. Which one is flatter? Explain intuitively which type experiences a higher reduction in utility from the same increase in income dy.

7) Types w_i are unobserved. Explain why choosing optimal taxes in this setting is equivalent to choosing a menu of (c_i, y_i) pairs. Is there a more general mechanism that we could come up with that would do better than setting this menu (please justify your answer).

8) Suppose that there are n_i individuals of type i and that there is a revenue requirement $\bar{R}$. Write the revenue constraint in terms of allocations (y_i, c_i).

9) Set up the Pareto problem: the Pareto problem maximizes the utility of type 2 subject to type 1 reaching some target utility $\bar{u}_1$ and subject to constraints. Think carefully what the constraints are (don't forget that types are unobservable).

10) Write the Lagrangian for this constrained maximization and provide the first-order conditions. Show that there are (only) three possible regimes, depending on which constraints bind.

11) Is there a case in which the first-best solution would apply? Characterize it in terms of the values of the multipliers and the allocation.

12) Suppose that the incentive constraint on type 2 is binding. Using your answer to 3) and 10), show that the marginal tax rate faced by type 2 is zero and explain this intuitively. Also show that the marginal tax rate faced by type 1 is positive and explain why. When is this regime likely to occur (as a function of the primitives of the problem)?

Problem 2

Consider an overlapping generations model. Suppose that there are two types of consumers, 1 and 2 in each generation. In period t there are $N_t^1 = N^1(1+n)^t$ young of type 1 whose productivity is $v < 1$ and $N_t^2 = N^2(1+n)^t$ young of type 2 with productivity 1. Everyone earns a wage equal to their productivity since the labor market is perfectly competitive.

There is an aggregate production function F such that:

$$Q_t = F(K_t, E_t) \quad \text{with:} \quad E_t = vN_t^1 L_t^1 + N_t^2 L_t^2$$

E_t are efficient units of labor. Denote by w_t the wage per unit of efficient labor. Type i's utility is $u_i(c_t^i, l_t^i)$ where $c_t^i = (c_{yt}^i, c_{o,t+1}^i)$ are his consumptions when he is young and when he is old.

1) Suppose that the government can freely choose K_t and K_{t+1} (effective saving every period) and the interest rate is $r = 0$ (so a unit saved just gives you one unit of consumption later). The government collects taxes to finance revenue requirements R_t. Write down the government budget constraint at time t as a function of consumption, capital, and revenue requirements (not taxes).

2) Argue why, if there is private information on the types, the government can restrict itself to choosing a menu of allocations for each type. What do these allocations specify for each type?

3) Argue that in general, type 2's incentive constraint will be binding and write it. You can assume for now that the incentive constraint on type 1 is slack.

4) The government objective is:

$$W = \sum_{t=1}^{\infty} W_t = \sum_{t=1}^{\infty} (\mu_t^1 N_t^1 u_t^1 + \mu_t^2 N_t^2 u_t^2)$$

Write the Lagrangian of this problem.

5) Suppose that the government can freely choose K_t as a control variable. Explain what this means and what would justify this assumption.

6) Take the first-order conditions of the government's maximization problem. Hint: If you need to take the FOC with respect to y_t, it may be easier to do a change of variables and to maximize with respect to $l_t^1 = \frac{y_t^1}{w_t v}$ and $l_t^2 = \frac{y_t^2}{w_t}$.

7) Using the FOC with respect to c^2_{yt} and $c^2_{0,t+1}$ and K_{t+1}, show that there should not be a marginal tax on the interest income paid to type 2 (you need to define what interest income means here in terms of primitives, i.e., marginal utility of consumption in different periods). What does this mean and why?

8) Take the FOCs with respect to c^1_{yt} and $c^1_{o,t+1}$. Impose a weak separability assumption, i.e.,

$$u_i(c^i_t, l^i_t) = \tilde{u}_i(h(c^i_t), l^i_t)$$

for some function $h()$.

Show that we can rewrite the FOC with respect to c^1_{yt} so as to make the FOC for $c^1_{o,t+1}$ appear in it. This will help you argue that the tax on type 1's capital income should also be zero.

9) What is driving the result in 8)? Think of several things and try to reason through them carefully. (Hints: you know what made the results go through in the FOCs mechanically. Stare at those FOCs and think what happens if you relaxed some assumptions. Question all FOCs you used!)