Economics 2450A: Public Economics
Section 5: Optimal Taxation with Income Effects and Bunching

Matteo Paradisi

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In today's section we will see the derivation of optimal income taxes proposed by Saez (2001). We will also introduce a way to estimate the elasticity of reported income that exploits the degree of bunching at the kinks of the existing tax schedule (Saez, 2010).[1]

1 Optimal Taxes with Income Effects

In this paragraph we derive optimal income taxes with income effects following the experiment in Saez (2001). Suppose individuals in the economy are heterogeneous in ability n and work l_n hours earning income z_n. We can write the labor supply as a Walrasian (uncompensated) demand such that $l_n(w_n, R_n)$ where $w_n = n(1-T')$ is the net of tax wage that the agent earns in equilibrium and R_n is the virtual (non-labor) income. Define R_n under the assumption that the tax is linear and tangent to the tax schedule at z_n such that $c = (1-T')\,nl_n + R_n$. Using the fact that $c = nl_n - T(nl_n)$ we can write $R_n = nl_n - T(nl_n) - nl_n(1-T')$. Using the Walrasian demand we can define uncompensated elasticity and the income parameter as follows:

$$\zeta_n^u = \frac{\partial l_n}{\partial(1-T')}\frac{(1-T')}{l_n} \tag{1}$$

$$\eta_n = \frac{\partial l_n}{\partial R_n}(1-T') \tag{2}$$

Using the definitions above and the Slutsky equation, we can write the compensated elasticity as:

$$\zeta_n^c = \zeta_n^u - \eta \tag{3}$$

We now derive a result that will be useful later. Totally differentiating l_n we get:

$$\dot{l}_n = \frac{\partial l_n}{\partial w_n}\left[1 - T' - n\left(n\dot{l}_n + l_n\right)T''\right] + \frac{\partial l_n}{\partial R_n}\left(n\dot{l}_n + l_n\right)(nl_nT'')$$

rearranging

$$\dot{l}_n = \frac{w_n}{l_n}\frac{\partial l_n}{\partial w_n}\frac{l_n}{n} + \left[w_n\frac{\partial l_n}{\partial R_n} - \frac{w_n}{l_n}\frac{\partial l_n}{\partial w_n}\right]\frac{l_nT''}{(1-T')}\left[l_n + n\dot{l}_n\right]$$

Notice that $\dot{z}_n = l_n + n\dot{l}_n$ and apply the definitions in (1) and (2) to get:

$$\dot{l}_n = \zeta_n^u\frac{l_n}{n} - \dot{z}_n\frac{l_nT''}{1-T'}\zeta_n^c \tag{4}$$

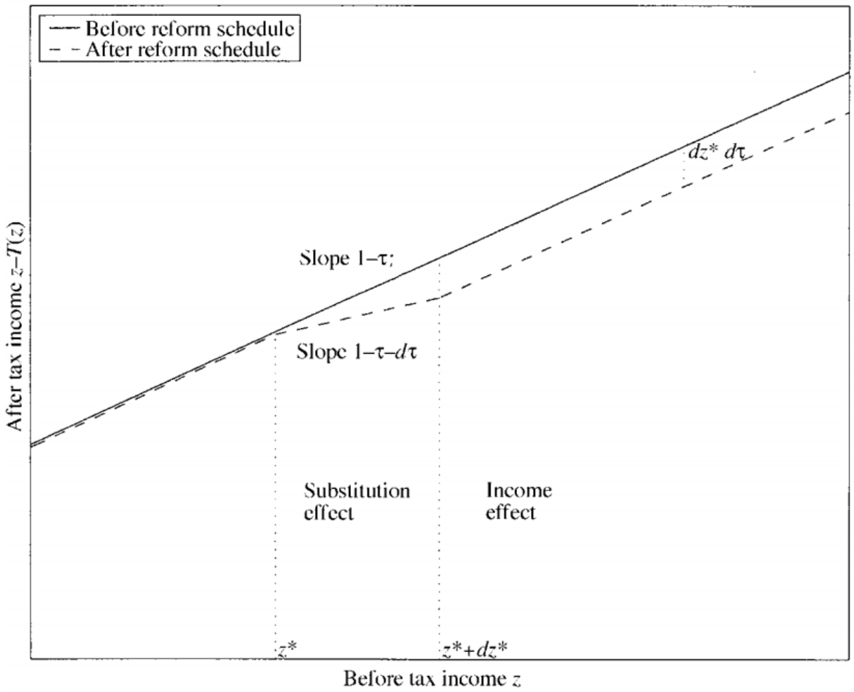
Using (4) we can write:

[1] The bunching paragraph is based on previous notes by Simon Jager.

$$\frac{\dot{z}_n}{z_n} = \frac{l_n + n\dot{l}_n}{nl_n} = \frac{1+\zeta^u}{n} - \dot{z}_n \frac{T''}{1-T'}\zeta^c \tag{5}$$

In order to derive the optimal tax, we follow the experiment in Saez (2001). Suppose we introduce a perturbation around the optimal tax schedule such that we raise the marginal tax rate by $d\tau$ in a small interval $[z^*, z^* + dz^*]$.[2] As we have already seen in previous Sections, the tax has three main effects: mechanical, welfare and behavioral. While the mechanical and welfare effects are similar to the ones we have previously studied, the behavioral effect now consists of two components: an elasticity effect for people in the interval $[z^*, z^* + dz^*]$ and an income effect for taxpayers above z^*.

Figure 1: Tax Reform Experiment



Mechanical and Welfare Effects Every taxpayer above z^* will pay an extra $dz^* d\tau$ of taxes, which for welfare purposes are valued according to the social marginal welfare weights $g(z)$. The net-of-welfare mechanical effect for pre-tax income z is $(1 - g(z))\, dz^* d\tau$. Summing up over all incomes above z^* we get:

$$M = dz^* d\tau \int_{z^*}^{\infty} (1 - g(z))\, h(z)\, dz$$

Social marginal welfare weights $g(z)$ represent the value for the government of giving one dollar to some level of income z. In particular, the government is indifferent between giving $1/g(z_1)$ dollars to individual 1 and $1/g(z_2)$ dollars to individual 2. The social marginal welfare weights are expressed in government money. Going back to the fully specified problem, we can interpret the weights as being normalized by λ, the multiplier on the resource constraint. The λ measures the value of transferring one dollar to every individual in the economy and captures the value of government funds. Higher λ means that the government can significantly raise welfare by transferring money to the individuals in the

[2] The experiment also assumes that $d\tau$ is second order relative to dz^* to avoid any bunching at kink.

economy, a low λ on the other hand implies that the gain from transfers is low and public funds are less valuable.

Elasticity Effect The increase in the marginal tax rate has an effect on individuals' labor supply that is denoted by dz. The effect consists of two parts. First, there is the direct consequence of the increase in taxes that depends on the compensated elasticity of labor supply. Second, since the taxpayer changes her labor supply by dz shifting on the tax schedule, she will face an additional change in the tax. We write the change in the marginal tax rate induced by the shift dz as $dT' = T''dz$. The behavioral response is proportional to the total tax change $d\tau + dT'$:

$$dz = \frac{dz^c}{d(1-T')}(d\tau + dT') = -\zeta^c z^* \frac{d\tau + dT'}{1-T'}$$

Rearranging:

$$dz = -\zeta^c z^* \frac{d\tau}{1 - T' + \zeta^c z^* T''} \tag{6}$$

Income Effect All individuals above z^* face a parallel shift of the tax schedule and pay additional taxes for $dz^* d\tau$. The mechanical increase in taxes paid has a direct income effect that depends on the income parameter $\eta = (dz/dR)(1-T')$. Moreover, since the individual shifts along the tax schedule we must take into account a further change in the tax rate. The two effects combined are:

$$dz = -\zeta^c z \frac{T'' dz}{1-T'} - \eta \frac{d\tau dz^*}{1-T'}$$

rearranging

$$dz = -\eta \frac{d\tau dz^*}{1 - T' - z\zeta^c T''}$$

In order to compute the total revenue effect we then need to sum over all taxpayers above z^* and account for the marginal tax rate T':

$$-\int_{z^*}^{\infty} \eta \frac{d\tau dz^*}{1 - T' - z\zeta^c T''} T' h(z)\, dz \tag{7}$$

Virtual and Actual Income Density Saez (2001) introduces the concept of virtual density in order to simplify the tax formulas. The virtual density is closely related to the virtual income in that it is the income density that would arise if the tax system was linear and tangent to the tax schedule $T(z)$ at every z. We denote the virtual density with $h^*(z)$. The mapping between virtual density and the type distribution $f(n)$ is given by the following:

$$h^*(z)\, \dot{z}^* = f(n)$$

where $\dot{z}^*$ is the derivative of earnings wrt $n$when the linear tax schedule is in place. A similar relation holds for $h(z)$ and we have $h(z)\,\dot{z} = f(n)$. Using the definition in (12) and the fact that $T'' = 0$ when a linear tax schedule is in place we can write:

$$h(z_n) \frac{1-T'}{1 - T' + z_n \zeta_n^c T''} \frac{1+\zeta_n^u}{n} z_n = h^*(z_n) \frac{1+\zeta_n^u}{n} z_n$$

It follows that:

$$\frac{h^*(z)}{1 - T'(z)} = \frac{h(z)}{1 - T'(z) + \zeta^c z T''(z)} \tag{8}$$

Optimal Income Taxes Starting from (6) and using equation (8) we can write the elasticity effect as follows:

$$E = -h(z)\,dz^* T' \zeta^c z^* \frac{d\tau}{1 - T' + \zeta^c z^* T''} = -\zeta^c z^* \frac{T'}{1 - T'} h^*(z)\,d\tau dz^* \tag{9}$$

Notice that in order to get the revenue effect that the elasticity effect should measure, we multiplied the expression in (6) by the marginal tax rate T' and by $h(z)\,dz^*$, which is the share of taxpayers affected by the tax reform. Using (8) we can write the income effect as follows:

$$I = -d\tau dz^* \int_{z^*}^{\infty} \eta \frac{T'}{1 - T'} h^*(z)\,dz \tag{10}$$

At the optimum, the sum of the three effects must be zero. We thus impose:

$$M + E + I = 0$$

and find

$$dz^* d\tau \int_{z^*}^{\infty} (1 - g(z))\,h(z)\,dz - \zeta^c z^* \frac{T'}{1 - T'} h^*(z)\,d\tau dz^* - d\tau dz^* \int_{z^*}^{\infty} \eta \frac{T'}{1 - T'} h^*(z)\,dz = 0$$

Rearranging:

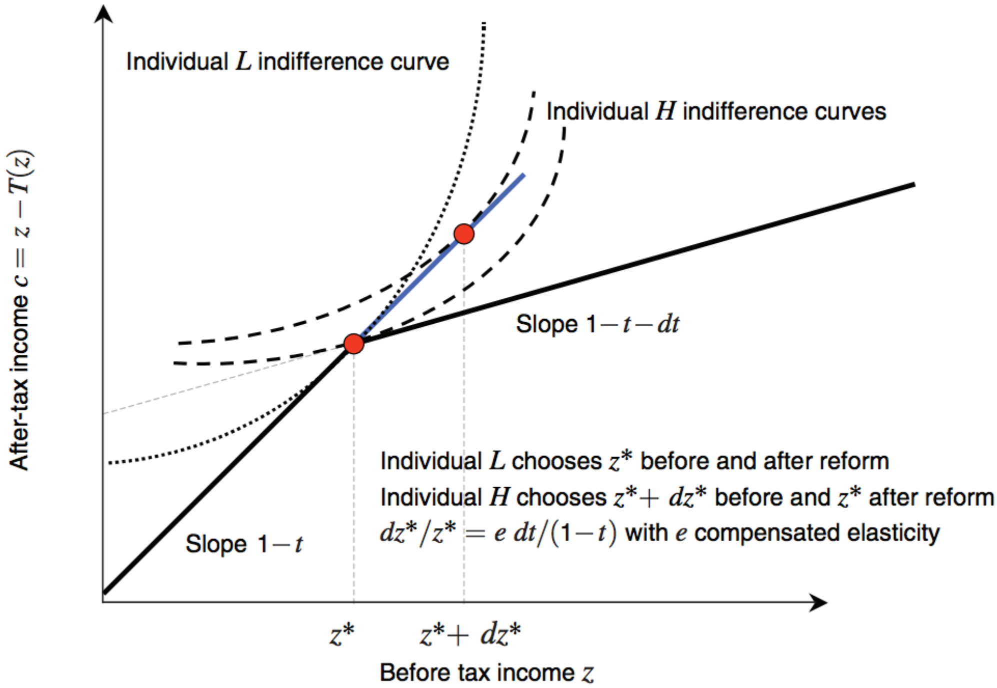
$$\begin{aligned}
\frac{T'}{1 - T'} &= \frac{1}{\zeta^c} \frac{1}{h^*(z)\,z^*} \left[\int_{z^*}^{\infty} (1 - g(z))\,h(z)\,dz - \int_{z^*}^{\infty} \eta \frac{T'}{1 - T'} h^*(z)\,dz \right] \\
&= \frac{1}{\zeta^c} \frac{1 - H(z^*)}{h^*(z)\,z^*} \left[\int_{z^*}^{\infty} \frac{h(z)}{1 - H(z^*)} (1 - g(z))\,dz - \int_{z^*}^{\infty} \eta \frac{T'}{1 - T'} \frac{h^*(z)}{1 - H(z^*)} dz \right] \qquad (11)
\end{aligned}$$

As we have already seen in the previous Sections the formula consists of different terms. $1 - H(z^*)/h^*(z)\,z^*$ captures the shape of the income distribution and measures how many people are above z^* relative to how much income is accumulated at z^* (i.e. $z^* h(z^*)$). The former is proportional to the mechanical increase in revenues, while the second measures the total income that is distorted by the tax. The marginal tax is also decreasing in the compensated elasticity ζ^c following a classical efficiency argument, and increases the larger the income effect is (in absolute value): a stronger income effect means that the negative fiscal externality from higher taxes is reduced.

2 Bunching Estimator

In this paragraph we will study a way to derive income elasticity that was introduced by Saez (2010). The methodology exploits the degree of bunching at the kinks that characterize the tax schedule. Suppose that individual incomes z are distributed according to a smooth density distribution $h(z)$. There is a constant marginal tax rate t at income z^* and a reform introduces an increase in marginal taxes such that it becomes $t + dt$ for all incomes above z^*. The kink will induce people that were falling in the interval $[z^*, z^* + dz^*]$ before the reform to bunch at z^*. Denote with L an individual who is exactly indifferent between the pre and post-reform tax schedule and does not change her income in equilibrium. This individual's indifference curve has slope $1 - t$ at z^*. There is also an individual H who represents the highest pre-reform income bunching at z^*. The indifference curve of H has slope $1 - t - dt$ at z^* and is tangent to the slope of the retention function above z^*.

Figure 2: Bunching



The response of income to the tax reform for individual H is:

$$dz^* = \frac{dz}{d(1-t)}\,|_{z=z^H}\; d(1-t) = e\frac{z^H}{1-t}d(1-t) = -e\left(z^* + dz^*\right)\frac{dt}{1-t}$$

Where e is the compensated elasticity of income. Notice that the dz^* is proportional to the ratio between the change in the tax rate and the net-of-tax rate $1-t$. It follows that, everything else being equal, a change in marginal tax rates from 0 percent to 10 percent should produce the same amount of bunching as a change from 90 percent to 91 percent. Rearranging:

$$dz^*\left(\frac{1-t+edt}{1-t}\right) = -e\frac{z^*}{1-t}dt$$

which implies:

$$dz^* = -e\frac{z^*}{1-t+edt}dt \tag{12}$$

It is not surprising that dz^* is increasing in the elasticity of income, implying that if income is more elastic more people will bunch.

Suppose the income distribution is locally continuous, the share of people bunching at the kink is:

$$s\left(z^*\right) = h\left(z^*\right)dz^*$$

Using the definition in (12):

$$\frac{s\left(z^*\right)}{h\left(z^*\right)} = -e\frac{z^*}{1-t+edt}dt$$

$$-e\left(\frac{s\left(z^{*}\right)}{h\left(z^{*}\right)}+z^{*}\right)dt=\frac{s\left(z^{*}\right)}{h\left(z^{*}\right)}\left(1-t\right)$$

$$e=-\frac{s\left(z^{*}\right)}{s\left(z^{*}\right)+z^{*}s\left(z^{*}\right)}\frac{1-t}{dt}\approx-\frac{s\left(z^{*}\right)}{h\left(z^{*}\right)}\frac{1-t}{z^{*}}\frac{1}{dt}$$

where we assumed that $s\left(z^{*}\right)/z^{*}\approx 0$. The formula shows that using the share of people bunching at kink and the income distribution that would arise under the no reform scenario we can estimate the elasticity of labor supply.

References

[1] Saez, E. "Using Elasticities to Derive Optimal Income Tax Rates", Review of Economics Studies, Vol. 68, 2001, 205-229.

[2] Saez, E. "Do Taxpayers Bunch at Kink Points?", AEJ: Economic Policy, Vol. 2, 2010, 180-212