I. Externalities

1. Consider a factory that emits pollution. The inverse demand for the good is \( P_d = 24 - Q \) and the inverse supply curve is \( P_s = 4 + Q \). The marginal cost of the pollution is given by \( MC = 0.5Q \).

(a) What are the equilibrium price and quantity when there is no government intervention?
(b) How much should the factory produce at the social optimum?
(c) How large is the deadweight loss from the externality?
(d) How large of a per-unit tax should the government impose to achieve the social optimum?

2. In Karro, Kansas, population 1,001, the only source of entertainment available is driving around in your car. The 1,001 Karraokers are all identical. They all like to drive, but hate congestion and pollution, resulting in the following utility function: \( U_i(f, d, t) = f + 16d - d^2 - 6t/1000 \), where \( f \) is consumption of all goods but driving, \( d \) is the number of hours of driving Karraoker \( i \) does per day, and \( t \) is the total number of hours of driving all other Karraokers do per day. Assume that driving is free, that the unit price of food is $1, and that daily income is $40.

(a) If an individual believes that the amount of driving he does won't affect the amount that others drive, how many hours per day will he choose to drive?
(b) If everybody chooses this number of hours, then what is the total amount \( t \) of driving by other persons?
(c) What will the utility of each resident be?
(d) If everybody drives 6 hours a day, what will the utility level of each Karraoker be?
(e) Suppose that the residents decided to pass a law restricting the total number of hours that anyone is allowed to drive. How much driving should be allowed if the objective is to maximize the utility of the typical resident?
(f) The same objective could be achieved with a tax on driving. How much would the tax have to be per hour of driving?
II. Public Goods

3. In Parkland County there are three types of people: Couch Potatoes, Joggers, and Bicyclists. There are 2 people of each type in the county. The different types of people exercise different amounts and have different marginal benefits for any given size of public park. Denoting the size of the Park as $A$ acres (to avoid confusion with the price), the benefits for each type of person are:

\[ B_{CP} = 0 \]
\[ B_{J}(A) = 10A - \frac{A^2}{4} \]
\[ B_{B}(A) = 20A - \frac{A^2}{2} \]

where subscripts denote the type of person. The marginal cost of increasing the size of the park has a fixed component and a component based on the size of the park. The cost of $A$ acres of park can be expressed as:

\[ C(A) = 10A + A^2. \]

Initially, the government of Parkland is committed to a policy of not charging different prices across different people and only having one park.

(a) What condition needs to hold for an efficient size of the park? Explain intuitively.

(b) Using the condition in part (a), calculate the efficient size of the park, $A$.

(c) Suppose the government decided to have a vote on the size of the park, $A$. The county government informs the voters that for each unit of $A$, each person will pay a tax of $10. What would be each person's preferred size of $A$? What would be the size of $A$ that would result from voting?

4. You’re an adviser to a governor who is deciding whether to provide low-income communities with state funds to increase their trash collection so that they don’t have to cut back on other services.\(^1\)

(a) Consider a community with preferences over trash collection ($T$) and all other goods ($C$) represented by $U(T, C) = 2T^{1/2} + 4C^{1/2}$. Trash collection costs $1, and the price of all other goods is $1. If the total income in the community is $5 million and there is no state grant, how much will the community spend on trash collection and how much on other goods? Draw the budget constraint.

(b) How much will the community spend on each good if it receives a $1 million block grant that can be spent only on trash collection? Draw the budget constraint.

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\(^1\)Note: this problem borrows from Emmanuel Saez’s public economics course
(c) How much will the community spend on each good if it receives a $2.5 million block grant that can be spent only on trash collection? Draw the budget constraint.

(d) How much will the community spend on each good if it receives a $1 for $1 matching grant on trash collection? Draw the budget constraint.

(e) Which of the grants gives the largest increase in town spending on trash collection per state dollar spent. Explain why.