Standard Welfarist Approach: Critiques and Puzzles

- Maximize concave function of individual utilities or weighted sum of utilities.

\[
\text{max } SWF = \max \int_{i} \omega_i \cdot u_i
\]

- Special case: utilitarianism, \( \omega_i = 1 \).

- Cannot capture elements important in tax practice:
  - Source of income: earned versus luck.
  - Counterfactuals: what agents would have done absent tax system.
  - Horizontal Equity concerns that go against “tagging.”


- Methodological and conceptual critique: Policy makers use reform-approach rather than posit and maximize objective.
A Novel Approach to Model Social Preferences

- **Tax reform approach**: weighs gains and losses from tax changes.

  Tax reform desirable iff: \[ \int_i g_i dT_i > 0 \text{ with } g_i \equiv G'(u_i)u'_i \]  \hspace{1cm} (1)

- Optimality: no budget neutral reform can increase welfare.

- Weights directly come from social welfare function, are restrictive.


**A Novel Approach to Model Social Preferences**

- **Tax reform approach**: weighs gains and losses from tax changes.

  \[
  \text{Change in welfare: } \int g_i dT_i \text{ with } g_i \equiv g(c_i; z_i; x_i^s, x_i^b). \quad (2)
  \]

- Replace restrictive social welfare weight by **generalized social marginal welfare weights**.
  - A “price” for $1$ consumption/ social value of $1$ transfer for each person.
  - Specified to directly capture fairness criteria.
  - Not necessarily derived from SWF (nest standard ones).
Generalized social welfare weights approach

\[ u_i = u(c_i - v(z_i; x_i^u, x_i^b)) \]
\[ g_i = g(c_i, z_i; x_i^s, x_i^b) \]
A Framework to Resolve Puzzles and Unify Alternative Approaches

- **Resolve puzzles**: Can depend on luck vs. deserved income, can capture counterfactuals ("Free Loaders"), can model horizontal equity concerns.

- Can avoid problem of utility cardinality or representation.


- **Pareto efficiency** guaranteed (locally) by non-negative weights.

- As long as weights depend on taxes paid (in addition to consumption): non-trivial theory of taxation even absent behavioral responses.

- **Positive tax theory**: Can estimate weights from revealed social choices.

- Approach can be applied to other policies.
Related Literature


Outline

1. Outline of the Approach
2. Resolving Puzzles of the Standard Approach
3. Link With Alternative Justice Principles
4. Empirical Testing and Estimation Using Survey Data
5. Conclusion
Outline

1. Outline of the Approach
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General Model

- Mass 1 of individuals indexed by $i$.

- Utility from consumption $c_i$ and income $z_i$:
  \[ u_i = u(c_i - v(z_i; x_i^u, x_i^b)) \]
  where $x_i^u$ and $x_i^b$ are vectors of characteristics, $u$ increasing, $v$ decreasing.

- Productivity per unit of effort: $w_i \equiv z_i / l_i$ where $l_i$ is labor supply, distribution $f(w)$ on $[w_{\text{min}}, w_{\text{max}}]$.

- Cost of work $\theta_i$, distribution $p(\theta)$ on $[\theta_{\text{min}}, \theta_{\text{max}}]$.
  - E.g.: $\theta_i \tilde{v}(z_i / w_i)$.

- Typical income tax: $T(z)$, hence $c_i = z_i - T(z_i)$.
  - More general tax systems, with conditioning variables possible, depending on what is observable and politically feasible.
Standard Approach is Tax Reform with Restricted Weights

\[ \max SWF = \max \int_i \omega_i \cdot u_i \]

s.t: (1) \( \int_i T(z_i) \geq E \) (Government’s revenue constraint),
    (2) \( z_i \) responds to taxes (Incentive compatibility constraint).

- Standard social marginal welfare weight: \( g_i = \omega_i \cdot u_{ci} \).
- By envelope theorem of individual optimization, small tax reform \( dT(z) \)
  has welfare effect \( -\int_i \omega_i u_{ci} \cdot dT(z_i) = -\int_i g_i dT(z_i) \).
- Change in tax paid per person is \( dT(z_i) + T'(z_i)dz_i \).

Definition

A reform \( dT(z) \) is budget neutral if and only if \( \int_i [dT(z_i) + T'(z_i)dz_i] = 0 \).

Definition

Tax reform desirability criterion (standard approach). Small budget
neutral tax reform \( dT(z) \) desirable \( \Leftrightarrow \int_i g_i dT(z_i) < 0 \), with \( g_i = \omega_i \cdot u_{ci} \).
Generalized social welfare weights approach

- Novel theory of taxation starting directly from the social welfare weights.

Definition

The generalized social marginal welfare weight on individual \( i \) is:

\[
g_i = g(c_i, z_i; x_i^s, x_i^b)
\]

\( g \) is a function, \( x_i^s \) is a vector of characteristics which only affect the social welfare weight, while \( x_i^b \) is a vector of characteristics which also affect utility.

- Recall utility is: \( u_i = u(c_i - v(z_i; x_i^s, x_i^b)) \)
- Characteristics \( x^s, x^u, x^b \) may be unobservable to the government.
- \( x^s \): dimensions across which fair to redistribute – health costs of work?
- \( x^u \): dimensions across which unfair to redistribute – laziness for work?
Optimality Criterion with Generalized Weights

Proposition

$T(z)$ optimal iff, for any small budget neutral reform $dT(z)$, $\int_i g_i dT(z_i) = 0$, with $g_i$ the generalized social marginal welfare weight on $i$ evaluated at $(z_i - T(z_i), z_i, x_i^s, x_i^b)$. 

- No budget neutral reform can improve welfare as evaluated using generalized weights.
Aggregating Standard Weights at Each Income Level

Taxes depend on \( z \) only.: express everything in terms of observable \( z \).

\( H(z) \): CDF of earnings

\( h(z) \): PDF of earnings

Definition

\( \bar{G}(z) \) is the (relative) average social marginal welfare weight for individuals earning more than \( z \):

\[
\bar{G}(z) \equiv \frac{\int \{i: z_i \geq z\} g_i}{\text{Prob}(z_i \geq z) \cdot \int g_i}
\]

\( \bar{g}(z) \) is the average social marginal welfare weight at \( z \):

\[
\bar{g}(z) = -\frac{1}{h(z)} \frac{d}{dz} \left( \bar{G}(z) \cdot [1 - H(z)] \right)
\]
Standard Tax Formula Expressed with Welfare Weights

Result

The optimal marginal tax at $z$:

$$T'(z) = \frac{1 - \tilde{G}(z)}{1 - \tilde{G}(z) + \alpha(z) \cdot e(z)}$$

$e(z)$: average elasticity of $z$ w.r.t $1 - T'$ at $z_i = z$

$\alpha(z)$: local Pareto parameter $zh(z)/[1 - H(z)]$.

Can invert tax formula to obtain the weights (Werning, 2007, Hendren, 2013).

Proposition

If $T'(z) < 1$ exists for all $z$, there is an unique $\tilde{G}(z) < 1 + \alpha(z) \cdot e(z)$ defined by $\tilde{G}(z) = \left[1 - T'(z)(1 + \alpha(z) \cdot e(z))\right]/\left[1 - T'(z)\right]$ satisfying the optimal tax formula.
Proof

- Reform $dT(z)$ increases marginal tax by $d\tau$ in small band $[z, z + dz]$.
- Mechanical revenue effect: extra taxes $dzd\tau$ from each taxpayer above $z$: $dzd\tau[1 - H(z)]$ is collected.
- Behavioral response: those in $[z, dz]$, reduce income by $\delta z = -ezd\tau / (1 - T'(z))$ where $e$ is the elasticity of earnings $z$ w.r.t $1 - T'$. Total tax loss $-d zd\tau \cdot h(z)e(z)zT'(z)/(1 - T'(z))$ with $e(z)$ the average elasticity in the small band.
- Net revenue collected by the reform and rebated lump sum is:
  $$dR = dzd\tau \cdot \left[ 1 - H(z) - h(z) \cdot e(z) \cdot z \cdot \frac{T'(z)}{1 - T'(z)} \right].$$
- Welfare effect of reform: $-\int g_i dT(z_i)$ with $dT(z_i) = -dR$ for $z_i \leq z$ and $dT(z_i) = d\tau dz - dR$ for $z_i > z$. Net effect on welfare is
  $$dR \cdot \int g_i - d\tau dz \int_{\{i: z_i \geq z\}} g_i.$$
- Setting net welfare effect to zero, using
  $$(1 - H(z))\tilde{G}(z) = \int_{\{i: z_i \geq z\}} g_i / \int g_i$$ and $\alpha(z) = zh(z)/(1 - H(z))$, we obtain the tax formula.
Standard Linear Tax Formula Expressed with Welfare Weights

The optimal linear tax rate, such that \( c_i = z_i(1 - \tau) + \tau \int_i z_i \) can also be expressed as a function of an income weighted average marginal welfare weight (Piketty and Saez, 2013).

**Result**

The optimal linear income tax is:

\[
\tau = \frac{1 - \bar{g}}{1 - \bar{g} + e} \quad \text{with} \quad \bar{g} \equiv \frac{\int_i g_i \cdot z_i}{\int_i g_i \cdot \int_i z_i}
\]

\( e \): elasticity of \( \int_i z_i \) w.r.t \( (1 - \tau) \).
Applying Standard Formulas with Generalized Weights

- Individual weights need to be “aggregated” up to characteristics that tax system can conditioned on.
  - E.g.: If $T(z, x^b)$ possible, aggregate weights at each $(z, x^b) \rightarrow \bar{g}(z, x^b)$.
  - If standard $T(z)$, aggregate at each $z$: $\bar{G}(z)$ and $\bar{g}(z)$.

- Then apply standard formulas. Nests standard approach.

- If $g_i \geq 0 \ \forall i$, (local) Pareto efficiency guaranteed.

- Can we back out weights? Optimum $\Leftrightarrow \max \ SWF = \int_i \omega_i \cdot u_i$ with Pareto weights $\omega_i = g_i / u_{ci} \geq 0$ where $g_i$ and $u_{ci}$ are evaluated at the optimum allocation. Impossible to posit correct weights $\omega_i$ without first solving for optimum.
Outline

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1. Optimal Tax Theory with Fixed Incomes

Modelling fixed incomes in our general model.

- Focus on redistributive issues.

- Specialize general framework to: \( v(z; z_i) = 0 \) if \( z \leq z_i \) and \( v(z; z_i) = \infty \) if \( z > z_i \). (Thus, \( z_i \in x_i^u \)).

- In equilibrium, \( u_i = u(c_i) \), fully inelastic labor supply.

**Standard utilitarian approach.**

- Optimum: \( c = z - T(z) \) is constant across \( z \), full redistribution.

- Is this acceptable?

- Very sensitive to utility specification.

- Heterogeneity in consumption utility? \( u_i = u(x_i^c \cdot c_i) \)
1. Tax Theory with Fixed Incomes: Generalized Weights

Definition
Let \( g_i = g(c_i, z_i) = \tilde{g}(c_i, z_i - c_i) \) with \( \tilde{g}_c \leq 0, \tilde{g}_{z-c} \geq 0 \).

i) Utilitarian weights: \( g_i = g(c_i, z_i) = \tilde{g}(c_i) \) for all \( z_i \), with \( \tilde{g}(\cdot) \) decreasing.

ii) Libertarian weights: \( g_i = g(c_i, z_i) = \tilde{g}(z_i - c_i) \) with \( \tilde{g}(\cdot) \) increasing.

- Weights depend negatively on \( c \) – “ability to pay” notion.
- Depend positively on tax paid – taxpayers contribute socially more.
- Optimal tax system: weights need to be equalized across all incomes \( z \).
1. Tax Theory with Fixed Incomes: Optimum

Proposition

The optimal tax schedule with no behavioral responses is:

\[ T'(z) = \frac{1}{1 - \tilde{g}_{z-c}/\tilde{g}_c} \quad \text{and} \quad 0 \leq T'(z) \leq 1. \]  

(3)

Corollary

Standard utilitarian case, \( T'(z) \equiv 1 \). Libertarian case, \( T'(z) \equiv 0 \).

- Empirical survey shows respondents indeed put weight on both disposable income and taxes paid.
- Between the two polar cases, 
  \[ g(c, z) = \tilde{g}(c - \alpha(z - c)) = \tilde{g}(z - (1 + \alpha)T(z)) \]  
  with \( \tilde{g} \) decreasing.
- Can be empirically calibrated and implied optimal tax derived.
2. Luck versus Deserved Income: Setting

- Widely perceived that fairer to tax luck income than earned income and to insure against luck shocks.

- Provides micro-foundation for weights increasing in taxes, decreasing in consumption.

- $y^d$: deserved income due to effort

- $y^l$: luck income, not due to effort, with average $Ey^l$.

- $z = y^d + y^l$: total income.

- Society believes earned income fully deserved, luck income not deserved. Captured by binary set of weights:

  $$g_i = 1(y^l_i - Ey^l_i \leq z_i - c_i)$$

  $g_i = 1$ if taxed more than excess luck income (relative to average).

- $x^s_i = (y^l_i, Ey^l_i)$, with $Ey^l$ aggregate characteristic.
2. No behavioral responses: Observable Luck Income

- If luck income observable, can condition taxes on it: \( T_i = T(z_i, y_i^l) \).

- Aggregate weights for each \((z, y^l)\) pair:
  \[ \bar{g}(z, y^l) = 1(z - T(z, y^l) \leq z - y^l + Ey^l) \].

- Optimum: everybody’s luck income must be \( Ey^l \) with
  \[ T(z, y^l) = y^l - Ey^l + T(z) \] and \( T(z) = 0 \).

- Example: Health care costs.
2. No behavioral responses: Unobservable Luck Income

- Can no longer condition taxes on luck income: \( T_i = T(z_i) \).

- Assumption: \( \forall d z_i, 0 < d y_i / d z_i < d z_i \) or \( 0 > d y_i / d z_i > d z_i \).

- Micro-foundation for weights \( \tilde{g}(c, z - c) \) decreasing in \( c \), increasing in \( z - c \).

- Aggregating weights:
  \( \tilde{g}(c, z - c) = \text{Prob}(y_i^l - E y_i^l \leq z_i - c_i | c_i = c, z_i = z) \).

- If \( \uparrow z - c \) at \( c \) constant, means \( \uparrow z \). Then, \( y_i^l \) increases by less than \( z \), hence \( \tilde{g}(c, z) \uparrow \).

- If \( \uparrow c \) at \( z - c \) constant, means \( d z = d c > 0 \). Hence, \( y_i^l \) increases as well, so that \( \tilde{g}(c, z) \downarrow \).

- Optimum should equalize \( \tilde{g}(z - T(z), z) \) across all \( z \).

- Non-trivial theory of optimal taxation, even without behavioral responses.
2. Luck versus Deserved Income: Add Behavioral Responses

- Utility is \( u_i = u(c_i - v(z_i - y_i^l, w_i)), \) \( w_i \) is productivity.
- Hence, \( x_i^u = w_i, \) \( x_i^b = y_i^l \) and \( x_i^s = Ey^l \).

Proposition

There are multiple equilibria if the elasticity \( e \) of deserved income with respect to \((1 - \tau)\) is **sufficiently elastic** at low tax rates and **sufficiently inelastic** at high tax rates
(Precisely: \( e > 1 \) at \( \tau = 0.1 \) and at \( \tau = 0.9, e < \bar{g}_0 / 9 \) where \( \bar{g}_0 \) is the average income weighted welfare weight evaluated at \( \tau = 0 \)).

- "Low tax equilibrium:" deserved income large, justifies lower taxes.
- "High tax equilibrium:" luck income larger part, justifies higher taxes.
- Each equilibrium locally Pareto efficient, but low tax can Pareto dominate.
3. Transfers and Free Loaders: Setting

- Behavioral responses closely tied to social weights: biggest complaint against redistribution is “free loaders.”
- Generalized welfare weights can capture “counterfactuals.”
- Consider linear tax model where $\tau$ funds demogrant transfer.

$$u_i = u(c_i - \nu(l; \theta_i)) = u(c_i - \theta \cdot l) \text{ with } l \in \{0, 1\}.$$  

- Individuals can choose to not work, $l = 0$, $c_i = c_0$.
- If they work ($l = 1$), earn uniform wage $w$, consume $c_1 = w \cdot (1 - \tau) + c_0$.
- Cost of work $\theta$, with cdf $P(\theta)$, is private information.
- Individual: work iff $\theta \leq c_1 - c_0 = (1 - \tau) \cdot w$.
- Fraction working: $P(w(1 - \tau))$.
- $e$: elasticity of aggregate earnings $w \cdot P(w(1 - \tau))$ w.r.t $(1 - \tau)$. 
3. Transfers and Free Loaders: Optimal Taxation

Apply linear tax formula:

- \( \tau = \frac{(1 - \bar{g})}{(1 - \bar{g} + e)} \)

- In this model, \( \bar{g} = \int g_i z_i / (\int g_i \cdot \int z_i) = \bar{g}_1 / [P \cdot \bar{g}_1 + (1 - P) \cdot \bar{g}_0] \) with: 
  - \( \bar{g}_1 \) the average \( g_i \) on workers, and \( \bar{g}_0 \) the average \( g_i \) on non-workers.

Standard Approach:

- \( g_i = u'(c_0) \) for all non-workers so that \( \bar{g}_0 = u'(c_0) \).

- Hence, approach does not allow to distinguish between the deserving poor and free loaders.

- We can only look at actual situation: work or not, not “why” one does not work.

- Contrasts with public debate and historical evolution.
3. Transfers and Free Loaders: Generalized Welfare Weights

- Distinguish people according to what would have done absent transfer.

- Welfare weights function $g_i = g(c, z/w; \theta_i, w)$, with $x_i^b = (\theta_i, w)$.

- **Workers**: Fraction $P(w(1 - \tau))$. $g_i = u'(c_1 - \theta_i)$.

- **Deserving poor**: would not work even absent any transfer: $\theta > w$. Fraction $1 - P(w)$. $g_i = u'(c_0)$.

- **Free Loaders**: do not work because of transfer: $w \geq \theta > w \cdot (1 - \tau)$. Fraction $P(w) - P(w(1 - \tau))$. $g_i = 0$.

- Cost of work enters weights – fair to compensate for (i.e., not laziness).

- Average weight on non-workers $\bar{g}_0 = u'(c_0) \cdot (1 - P_0)/(1 - P)$ scales by fraction of deserving non-workers – lower than in utilitarian case.

- Reduces optimal tax rate not just through $e$ but also through $\bar{g}_0$. 
3. Transfers and Free Loaders: Remarks and Applications

- Ex post, possible to find suitable Pareto weights that rationalize same tax.
  - E.g.: $\omega(\theta) = 1$ for $\theta \leq w \cdot (1 - \tau^*)$ (workers)
  - $\theta \geq w$ (deserving poor)
  - $\omega(\theta) = 0$ for $w \cdot (1 - \tau^*) < \theta < w$ (free loaders).
- But: these weights depend on equilibrium tax rate $\tau^*$.

Other applications:
- **Desirability of in-work benefits** if weight on non-workers becomes low enough relative to workers.
- **Transfers over the business cycle**: composition of those out of work depends on ease of finding job.
4. Horizontal Equity: Puzzle and the Standard Approach

- Standard theory strongly recommends use of “tags” – yet not used much.

- Illustrate in Ramsey problem, where need to raise revenue $E$.

- 2 groups of measure 1, differ according to inelastic attribute $m \in \{1, 2\}$ and income elasticities $e_1 < e_2$.

- Standard approach: apply Ramsey tax rule, generates horizontal inequity:

$$\tau_m = \frac{1 - \bar{g}_m}{1 - \bar{g}_m + e_m} \quad \text{with} \quad \bar{g}_m = \frac{\int_{i \in m} u_{ci} \cdot z_i}{p \cdot \int_{i \in m} z_i},$$

$p > 0$: multiplier on budget constrained, adjusts to raise revenue $E$.

- Horizontal equity concern: aversion to treating differently people with same income.

- Social marginal welfare weights concentrated on those suffering from horizontal inequity.
  - Horizontal inequity carry higher priority than vertical inequity.

- If no horizontal inequity, a reform that creates horizontal inequity needs to be penalized: weights need to depend on direction of reform.

\[ g_i = g(\tau_m, \tau_n, d\tau_m, d\tau_n) \]

i) \[ g(\tau_m, \tau_n, d\tau_m, d\tau_n) = 1 \] and \[ g(\tau_n, \tau_m, d\tau_n, d\tau_m) = 0 \] if \( \tau_m > \tau_n \).

ii) \[ g(\tau, \tau, d\tau_m, d\tau_n) = 1 \] and \[ g(\tau, \tau, d\tau_n, d\tau_m) = 0 \] if \( \tau_m = \tau_n = \tau \) and \( d\tau_m > d\tau_n \).

iii) \[ g(\tau, \tau, d\tau_m, d\tau_n) = g(\tau, \tau, d\tau_n, d\tau_m) = 1 \] if \( \tau_m = \tau_n = \tau \) and \( d\tau_m = d\tau_n \).
4. Horizontal Equity: Equilibrium with Generalized Weights

Regularity Assumptions.

- There is a uniform tax rate $\tau_1 = \tau_2 = \tau^*$ that can raise $E$.
- $\tau_1 \rightarrow \tau_1 \cdot \int_{i \in 1} z_i$, $\tau_2 \rightarrow \tau_2 \cdot \int_{i \in 2} z_i$, and $\tau \rightarrow \tau \cdot (\int_{i \in 1} z_i + \int_{i \in 2} z_i)$ are single peaked.

Proposition

Let $\tau^*$ be the smallest uniform rate that raises $E$: $\tau^*(\int_{i \in 1} z_i + \int_{i \in 2} z_i) = E$.

i) If $1/(1 + e_2) \geq \tau^*$ the only equilibrium has horizontal equity with $\tau_1 = \tau_2 = \tau^*$.

ii) If $1/(1 + e_2) < \tau^*$ the only equilibrium has horizontal inequity with $\tau_2 = 1/(1 + e_2) < \tau^*$ (revenue maximizing rate) and $\tau_1 < \tau^*$ the smallest tax rate s.t. $\tau_1 \cdot \int_{i \in 1} z_i + \tau_2 \cdot \int_{i \in 2} z_i = E$. This inequitable equilibrium Pareto dominates $\tau_1 = \tau_2 = \tau^*$. 
4. Horizontal Equity: Equilibrium with Generalized Weights

- Horizontal inequity can be equilibrium only if helps discriminated group.

- Tagging must be Pareto improving to be desirable, limits scope for use.

- New Rawlsian criterion: “Permissible to discriminate against a group based on tags, only if discrimination improves this group’s welfare.”
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1. Libertarianism and Rawlsianism

Libertarianism:
- Principle: “Individual fully entitled to his pre-tax income.”
- Morally defensible if no difference in productivity, but different preferences for work.
- \( g_i = g(c_i, z_i) = \bar{g}(c_i - z_i) \), increasing (\( x_i^s \) and \( x_i^b \) empty).
- Optimal formula yields: \( T'(z_i) \equiv 0 \).

Rawlsianism:
- Principle: “Care mostly about those with lowest earnings.”
- \( g_i = g(u_i - \min_j u_j) = 1(u_i - \min_j u_j = 0) \), with \( x_i^s = u_i - \min_j u_j \) and \( x_i^b \) is empty.
- If least advantaged people have zero earnings independently of taxes, \( \bar{G}(z) = 0 \) for all \( z > 0 \).
- Optimal formula yields: \( T'(z) = 1/[1 + \alpha(z) \cdot e(z)] \) (maximize demogrant − \( T(0) \)).
2. Equality of Opportunity: Setting

- Ability to earn is result of i) family background $B_i \in \{0, 1\}$ (which individuals not responsible for) and to merit (which individuals are responsible for), $w_i$.
- Society is willing to redistribute across background, but not across income conditional on background.
- High family background gives “unfair” earnings advantage: $u_i = u(c_i - v(z_i/w_i, B_i))$ with $\partial v(z_i/w_i, 0)/\partial z_i > \partial v(z_i/w_i, 1)/\partial z_i$, for all $(z_i, w_i)$.
- $r_i$: percentile of $i$ in earnings distribution conditional on background – measure of merit.
- Conditional on earnings, those coming from $B_i = 0$ are more meritorious.
- $\bar{c}(r) \equiv (\int_{(i : r_i = r)} c_i) / \text{Prob}(i : r_i = r)$: average consumption at rank $r$.
- $g_i = g(c_i; \bar{c}(r_i)) = 1(c_i \leq \bar{c}(r_i))$, with $x_i^{s} = \bar{c}(r_i)$, $x_i^{u} = B_i$ and $x_i^{b}$ empty.
2. Equality of Opportunity: Results

- Suppose government cannot condition taxes on background.

- $\bar{G}(z)$: **Representation index**: % from disadvantaged background earning $\geq z$ relative to % in population.

- Implied Social Welfare function as in Roemer et al. (2003).

- $\bar{G}(z)$ decreasing since harder for those from disadvantaged background to reach upper incomes.

- If at top incomes, representation is zero, revenue maximizing top tax rate.

- Justification for social welfare weights decreasing with income not due to decreasing marginal utility (utilitarianism).
### 2. Equality of Opportunity vs. Utilitarian Tax Rates

<table>
<thead>
<tr>
<th>Income percentile</th>
<th>Equality of Opportunity</th>
<th>Utilitarian (log-utility)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fraction from low background (=parents below median) above each percentile</td>
<td>Implied social welfare weight G(z) above each percentile</td>
</tr>
<tr>
<td>z= 25th percentile</td>
<td>44.3%</td>
<td>0.886</td>
</tr>
<tr>
<td>z= 50th percentile</td>
<td>37.3%</td>
<td>0.746</td>
</tr>
<tr>
<td>z= 75th percentile</td>
<td>30.3%</td>
<td>0.606</td>
</tr>
<tr>
<td>z= 90th percentile</td>
<td>23.6%</td>
<td>0.472</td>
</tr>
<tr>
<td>z= 99th percentile</td>
<td>17.0%</td>
<td>0.340</td>
</tr>
<tr>
<td>z= 99.9th percentile</td>
<td>16.5%</td>
<td>0.330</td>
</tr>
</tbody>
</table>

3. Poverty Alleviation: Setting

- Poverty gets substantial attention in public debate.
- Poverty alleviation objectives can lead to Pareto dominated outcomes:
  - Intuition: disregard people’s disutility from work.
- Non-negative generalized welfare weights can avoid pitfall of Pareto inefficiency.
- $\bar{c}$: poverty threshold. "Poor": $c < \bar{c}$.
- $u_i = u(c_i - v(z_i/w_i))$.
- $\bar{z}$: (endogenous) pre-tax poverty threshold: $\bar{c} = \bar{z} - T(\bar{z})$.
- Poverty gap alleviation: care about shortfall in consumption.
  - $g_i = g(c_i, z_i; \bar{c}) = 1 > 0$ if $c_i < \bar{c}$ and $g_i = g(c_i, z_i; \bar{c}) = 0$ if $c_i \geq \bar{c}$.
  - Aggregating the weights: $\bar{g}(z) = 0$ for $z \geq \bar{z}$ and $\bar{g}(z) = 1/H(\bar{z})$ for $z < \bar{z}$.
  - $\bar{G}(z) = 0$ for $z \geq \bar{z}$ and $\bar{G}(z) = [1 - H(z)/H(\bar{z})]/[1 - H(z)]$ for $z < \bar{z}$.
3. Optimal Tax Schedule that Minimizes Poverty Gap

Proposition

\[ T'(z) = \frac{1}{1 + \alpha(z) \cdot e(z)} \quad \text{if} \quad z > \bar{z} \]

\[ T'(z) = \frac{(1/H(\bar{z}) - 1)H(z)}{(1/H(\bar{z}) - 1)H(z) + \alpha(z)[1 - H(z)] \cdot e(z)} \quad \text{if} \quad z \leq \bar{z} \]

\( c = z - T(z) \)

(a) Direct poverty gap minimization

(b) Generalized weights approach
4. Fair Income Taxation: Principle

- Agents differ in preference for work (laziness) and skill.


- Want to favor hard working low skilled but cannot tell them apart from the lazy high skilled.

- Show how their $w_{min}$-equivalent leximin criterion translates into social marginal welfare weights.

- We purely reverse engineer here to show usefulness of formula and generalized weights.
4. Fair Income Taxation: Setting and Optimal tax rates

- \( u_i = c_i - v(z_i/w_i, \theta_i) \), \( w_i \): skill, \( \theta_i \): preference for work.

- Labor supply: \( l_i = z_i/w_i \in [0, 1] \) (full time work \( l = 1 \)).

- Criterion: full weight on those with \( w = w_{\text{min}} \) with smallest net transfer.

- Start from optimal tax system: \( T'(z) = 0 \) for \( z \in [0, w_{\text{min}}] \),
  \( T'(z) = 1/(1 + a(z) \cdot e(z)) > 0 \) for \( z > w_{\text{min}} \).

- Implies \( \bar{G}(z) = 1 \) for \( 0 \leq z \leq w_{\text{min}} \).

- Hence, \( \int_{z}^{\infty} [1 - g(z')] dH(z') = 0 \).

- Differentiating w.r.t \( z \): \( \bar{g}(z) = 1 \) for \( 0 \leq z \leq w_{\text{min}} \).

- For \( z > w_{\text{min}} \), \( \bar{G}(z) = 0, \bar{g}(z) = 0 \).

- Let $T_{\text{max}} \equiv \max_{i: w_i = w_{\text{min}}}(z_i - c_i)$.

- $g_i = g(c_i, z_i; w_i, w_{\text{min}}, T_{\text{max}})$ where $x_i^b = w_i$, $x_i^u = \theta_i$, and $x_i^s = (w_{\text{min}}, T_{\text{max}})$, ($w_{\text{min}}$ exogenous aggregate characteristic, $T_{\text{max}}$ is endogenous aggregate characteristic.)

- $g(c_i, z_i; w_i, w_{\text{min}}, T_{\text{max}}) = \tilde{g}(z_i - c_i; w_i, w_{\text{min}}, T_{\text{max}})$ with:
  - $\tilde{g}(z_i - c_i; w_i, w_{\text{min}}, T_{\text{max}}) = 0$ for $w_i > w_{\text{min}}$, for any $(z_i - c_i)$ (no weight on those above $w_{\text{min}}$).
  - $\tilde{g}(.; w_{\text{min}}, w_{\text{min}}, T_{\text{max}})$ is an (endogenous) Dirac distribution concentrated on $z - c = T_{\text{max}}$

- Forces government to provide same transfer to all with $w_{\text{min}}$.

- If at every $z < w_{\text{min}}$ can find $w_{\text{min}}$ agents, forces equal transfer at all $z < w_{\text{min}}$.

- Zero transfer above $w_{\text{min}}$ since no $w_{\text{min}}$ agents found there.
Outline

1. Outline of the Approach
2. Resolving Puzzles of the Standard Approach
3. Link With Alternative Justice Principles
4. Empirical Testing and Estimation Using Survey Data
5. Conclusion
Online Survey: Goals and Setup

Two goals of empirical application:

1. Discover notions of fairness people use to judge tax and transfer systems.
   - Focus themes addressed in theoretical part.

2. Quantitatively calibrate simple weights

Online Platform:

- Amazon mTurk (Kuziemko, Norton, Saez, Stantcheva, 2013).
- 1100 respondents with background information.
Evidence against utilitarianism

- Respondents asked to compare families w/ different combinations of $z$, $z - T(z)$, $T(z)$.
- Who is more deserving of a $1000 tax break?
- **Both disposable income and taxes paid matter** for $sww$
  - Family earning $50K, paying $15K in taxes judged more deserving than family earning $40K, paying $5K in taxes
  - Family earning $40K, paying $10K in taxes judged more deserving than family earning $50K, paying $10K in taxes
- **Frugal vs. Consumption-loving** person with same net income

<table>
<thead>
<tr>
<th>Consumption-lover more deserving</th>
<th>Frugal more deserving</th>
<th>Taste for consumption irrelevant</th>
</tr>
</thead>
<tbody>
<tr>
<td>4%</td>
<td>22%</td>
<td>74%</td>
</tr>
</tbody>
</table>
Does society care about effort to earn income?

- **Hard-working vs. Easy-going person with same net income**
  
  “A earns $30,000 per year, by working in two different jobs, 60 hours per week at $10/hour. She pays $6,000 in taxes and nets out $24,000. She is very hard-working but she does not have high-paying jobs so that her wage is low.”

  “B also earns the same amount, $30,000 per year, by working part-time for 20 hours per week at $30/hour. She also pays $6,000 in taxes and hence nets out $24,000. She has a good wage rate per hour, but she prefers working less and earning less to enjoy other, non-work activities.”

<table>
<thead>
<tr>
<th>Hardworking</th>
<th>Easy-going</th>
<th>Hours of work irrelevant conditional on total earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>more deserving</td>
<td>more deserving</td>
<td>54%</td>
</tr>
<tr>
<td>43%</td>
<td>3%</td>
<td></td>
</tr>
</tbody>
</table>

43% more deserving
Do people care about “Free Loaders” and Behavioral Responses to Taxation?

Starting from same benefit level, which group most deserving of more benefits?

<table>
<thead>
<tr>
<th></th>
<th>Disabled unable to work</th>
<th>Unemployed looking for work</th>
<th>Unemployed not looking for work</th>
<th>On welfare not looking for work</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average rank (1-4)</td>
<td>1.4</td>
<td>1.6</td>
<td>3.0</td>
<td>3.5</td>
</tr>
<tr>
<td>% assigned 1st rank</td>
<td>57.5%</td>
<td>37.3%</td>
<td>2.7%</td>
<td>2.5%</td>
</tr>
<tr>
<td>% assigned last rank</td>
<td>2.3%</td>
<td>2.9%</td>
<td>25%</td>
<td>70.8%</td>
</tr>
</tbody>
</table>
Calibrating Social Welfare Weights

- Calibrate $g(c, T) = g(c - \alpha T)$
- 35 fictitious families, w/ different net incomes and taxes
- Respondents rank them pair-wise (5 random pairs each)

Which of these two families is most deserving of the $1,000 tax break?

- Family earns $100,000 per year, pays $50,000 in taxes, and hence nets out $50,000
- Family earns $25,000 per year, pays $1,250 in taxes, and hence nets out $23,750

Which of these two families is most deserving of the $1,000 tax break?

- Family earns $50,000 per year, pays $2,500 in taxes, and hence nets out $47,500
- Family earns $500,000 per year, pays $170,000 in taxes, and hence nets out $330,000
Eliciting Social Preferences

Is A or B more deserving of a $1,000 tax break?

\( T_1 \)

\( A \)

\( T_2 \)

\( B \)

\( T_3 \)

\( c \)

\( c_1 \)

\( c_2 \)

\( c_3 \)
Eliciting Social Preferences

Is A or B more deserving of a $1,000 tax break?
Eliciting Social Preferences

\( S_{ijt} = 1 \) if \( i \) ranked 1st in display \( t \) for respondent \( j \), \( dT_{ijt} \) (\( dc_{ijt} \)) is difference in taxes (net income) for families in pair shown.

\[
S_{ijt} = \beta_0 + \beta_T dT_{ijt} + \beta_c dc_{ijt} \quad \alpha = \frac{dc}{dT} \Big|_{S} = -\frac{\beta_T}{\beta_c} = -\text{slope}
\]

\[ g(c,T) = g(c - \alpha T) \]

indifference curves
## Eliciting Social Preferences

### Table 5: Calibrating Social Welfare Weights

<table>
<thead>
<tr>
<th>Sample</th>
<th>Full</th>
<th>Excludes cases with income of $1m</th>
<th>Excludes cases with income of $500K+</th>
<th>Excludes cases with income of $500K+ and $10K or less</th>
<th>Liberal subjects only</th>
<th>Conservative subjects only</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
</tr>
<tr>
<td>d(Tax)</td>
<td>0.0017***</td>
<td>0.0052***</td>
<td>0.016***</td>
<td>0.015***</td>
<td>0.00082***</td>
<td>0.0032***</td>
</tr>
<tr>
<td></td>
<td>(0.0003)</td>
<td>(0.0019)</td>
<td>(0.0019)</td>
<td>(0.0022)</td>
<td>(0.00046)</td>
<td>(0.00068)</td>
</tr>
<tr>
<td>d(Net Income)</td>
<td>-0.0046***</td>
<td>-0.0091***</td>
<td>-0.024***</td>
<td>-0.024***</td>
<td>-0.0048***</td>
<td>-0.0042***</td>
</tr>
<tr>
<td></td>
<td>(0.00012)</td>
<td>(0.00028)</td>
<td>(0.00078)</td>
<td>(0.00094)</td>
<td>(0.00018)</td>
<td>(0.00027)</td>
</tr>
<tr>
<td>Number of observations</td>
<td>11,450</td>
<td>8,368</td>
<td>5,816</td>
<td>3,702</td>
<td>5,250</td>
<td>2,540</td>
</tr>
<tr>
<td>Implied $\alpha$</td>
<td>0.37</td>
<td>0.58</td>
<td>0.65</td>
<td>0.64</td>
<td>0.17</td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td>(0.06)</td>
<td>(0.06)</td>
<td>(0.07)</td>
<td>(0.09)</td>
<td>(0.12)</td>
<td>(0.16)</td>
</tr>
<tr>
<td>Implied marginal tax rate</td>
<td>73%</td>
<td>63%</td>
<td>61%</td>
<td>61%</td>
<td>85%</td>
<td>57%</td>
</tr>
</tbody>
</table>
Outline

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Conclusion

- Generalized marginal social welfare weights are fruitful way to extend standard welfarist theory of optimal taxation.
  - Allow to dissociate individual characteristics from social criteria.
  - Which characteristics are fair to compensate for?

- Helps resolve puzzles of traditional welfarist approach.

- Unifies existing alternatives to welfarism.

- Weights can prioritize social justice principles in lexicographic form.
  1. Injustices created by tax system itself (horizontal equity)
  2. Compensation principle (health, background)
  3. Luck vs. effort or preferences for work.
  4. Utilitarian concept of decreasing marginal utility of consumption.