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A Model of English Demographic Change: 1573–1873*

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This paper documents an analysis of English demographic change over the period 1573–1873. A simultaneous equations model is developed in order to test four alternative demographic theories—the constant equilibrium wage theory, the constant fertility theory, Lee's original (1973) synthesis theory, and a new, composite theory. Each is a special case of a general demographic model which provides both for exogenous technological change and for endogenous migration behavior. Two-stage least-squares estimation yields parameter estimates and test statistics, which provide evidence of the superiority of the composite theory of demographic change. By statistically affirming the composite theory of demographic change, this paper confirms that the mortality level played a dominant role in English demographic change during the preindustrial period. The analysis also provides support, however, for the classical notion that shifts in the labor demand function were a dominant cause of long-run population changes, subsequent to the beginning of British industrialization.

I. INTRODUCTION

Since the time of Malthus, economists and historians have analyzed the links between changes in population levels and changes in various other demographic and economic variables. Two principal points of view have been brought to bear on the question of what caused population changes in preindustrial England: that of classical economists and that of historians and demographers.

The classical economists, in general, supported a Malthusian view in which the major driving mechanism behind changes in population was conceived to be one of shifts in the demand for labor, in the presence of a real wage which was stable, set by a social norm. The second approach, attributed principally to demographers and historians, has stressed the importance of exogenous changes in mortality.

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Some 14 years ago, Ronald Lee presented a third approach to examining the causes of population changes in preindustrial England. Lee (1973) developed a synthesis of the "classical" and "historic" approaches, couched all three approaches in simple simultaneous equations models, estimated econometrically those models' parameters, using aggregated English data for the period 1250–1750, and then evaluated the competing theories by conducting hypothesis tests on the relevant parameters.

Through this process, Lee rejected both the simple classical population theory (of shifting labor demand driving population changes) and the simple theory of the role of mortality. Instead, he affirmed his synthesis of the two models and found that population and wages were simultaneously determined by an equilibrating mechanism which was sensitive to mortality rates.

Previous Studies of English Demographic Change

Lee's original model of English demographic change was based upon aggregated data (Wrigley, 1969) covering the period 1250–1750. This period terminated prior to the onset of the British industrial revolution, an era which brought with it fundamental changes in the social and economic organization of society.[1] It is therefore of interest to ask whether Lee's original conclusions regarding the validity of the three alternative theories of demographic change hold up when examined in the light of population, mortality, and wage data which include the initiation of Britain's industrial revolution in the late 18th and early 19th century.[2] The model developed in this paper is sufficiently robust to account for demographic changes both prior to and subsequent to the beginnings of British industrialization, based upon an expanded data set (1573–1873), which was not available to Lee at the time of his original research (Wrigley and Schofield, 1981).[3]

One of the simplifying assumptions which Lee utilized in developing his generic model of population change in preindustrial England was that of a "closed population," i.e., one in which *net* migration from England to other countries (or vice-versa) did not occur. Drawing upon the newly available Wrigley and Schofield (1981) data, it becomes clear that net

[1] Lee stated that his "end point, 1700, was chosen because the relation between wages and population size began to shift unmistakably after this . . ." (1973, p. 583).

[2] Appendix 1 provides a tabular presentation of data for the period 1548–1873 on five variables—population, the crude death rate, the real wage, net migration, and degree of urbanization.

[3] After the research upon which this paper is based had been completed, the author became aware of Ronald Lee's recent study (1985b) of "Population Homeostasis and English Demographic History." For the most part, Lee's new analysis is complementary to that presented here, as is indicated in footnotes 6, 12, and 18. It should be noted that in his new paper, Lee states that the data set he utilized in his original (1973) work was "flawed," a conclusion which is borne out by the evidence presented in this study.

migration from England during the period was small but not negligible. Furthermore, numerous demographic and economic studies have indicated that migration behavior is often a function of economic conditions, as well as other factors.[4] Therefore, the present model provides for endogenous migration behavior.

Preview of the Analysis

In order to analyze the causes of English demographic change over the period 1573–1873, a simultaneous equations model is developed. The estimated model is used to test four alternative demographic theories—the constant equilibrium wage theory, the constant fertility theory, Lee's (1973) synthesis theory, and a new, composite theory.

The four theories are posited as special cases of a generalized formulation which provides both for exogenous technological improvement and endogenous migration behavior. Two-stage least-squares estimation yields parameter estimates and test statistics which provide evidence of the superiority of the new, composite theory of demographic change.

The dynamic performance of the model is compared with that of a closed-population model through a series of simulations, and it is found that the composite theory provides substantially more accurate tracking of the actual, historical changes in the English population level.

By statistically affirming the composite theory of demographic change, this paper supports the classical notion that shifts in labor demand were a dominant cause of long-run population changes, at least subsequent to the beginning of British industrialization. The analysis also indicates, however, that during the preindustrial period, the dominant role in demographic change was played by the level of mortality.

Thus, during the 16th and 17th centuries, changing rates of mortality constituted the principal driving force behind English population changes. With the initiation of industrialization in the 18th and 19th centuries, however, technological improvements led to structural changes whereby shifts in labor demand became the dominant force behind population increases. The composite model developed here is sufficiently robust to account for demographic changes during both periods.

II. THREE ALTERNATIVE THEORIES OF DEMOGRAPHIC CHANGE

In order to assess alternative theories of demographic change, Lee (1973) developed highly stylized versions of the two principal previous approaches to analyzing interactions between the population level, the real wage, and mortality. The first of these approaches, the constant

[4] Examples include: Easterlin (1961), Gallaway and Vedder (1971), Quigley (1972), Wilkinson (1970), and Williamson (1974).

equilibrium wage theory (CEW), associated with the classical tradition of Malthus, suggests that wages are a function of the labor supply, and that the long-term real wage is stable, being set by social norms. In Lee's simplified version of the CEW theory, fertility behavior automatically adjusts to changes in mortality. The rate of growth of population is a function of the real wage alone.

In the constant fertility (CF) model, fertility is relatively invariant, as the crude birth rate is a function of social institutions. Instead, fluctuations in the crude death rate drive population changes.

Lee pointed out that both of these (extreme) approaches have serious limitations. On the one hand, the CEW approach cannot account for the influence of changing mortality levels. On the other hand, the CF theory places the entire burden for explanation of population adjustments to resource changes on mortality.

To avoid the limitations of the two extreme viewpoints, Lee posited a synthesized (SYN) model in which fertility was responsive to the real wage level but was independent of mortality.

All three models may be viewed as special cases of a more general formulation

$$f = \mu + \alpha \ln (w) + \lambda d \tag{1}$$

$$w = \eta P^{-\beta} \tag{2}$$

$$P^*/P = f - d, \tag{3}$$

where

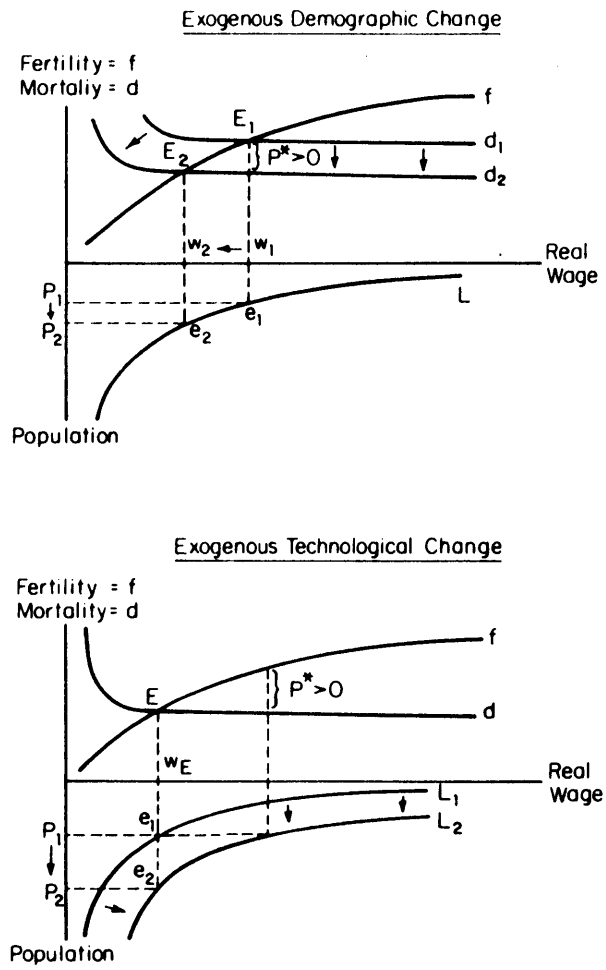
f = fertility rate;
w = real wage;
d = crude death rate;
P = population level;
P^* = derivative of population with respect to time; and μ, α, λ, and η are parameters to be estimated.[5]

Given this generic demographic model, the three previously described theories may be viewed as special cases as follows:

CEW Theory: $\lambda = 1$
CF Theory: $\lambda = 0$ and $\alpha = 0$
Synthesis Theory: $\lambda = 0$ and $\alpha \neq 0$.

Rather than limiting consideration of the alternative theories to a closed population context, an additional version of the model is developed, one which allows for endogenous migration.

[5] A pictorial representation of this model is provided in Fig. 1.



*Based upon Lee 1973.

FIG. 1 Simultaneous equations model of demographic transformation.

III. A MODEL OF DEMOGRAPHIC CHANGE

The major purpose of this part of the paper is to develop the structural equations (the parameters of which may be estimated econometrically) of two alternative versions of a model of demographic change in England during the period 1573–1873. Both versions are based upon 25-year averages of the relevant data series. This temporal aggregation is utilized, despite the availability of quinquennial data, because the hypotheses under consideration relate solely to long-term phenomena.

Model I: Closed Population

We begin with Lee's generic model of demographic change, in which it was assumed that net migration was zero.[6] The model, which consists of two equations plus one identity, was previously described in Part II.

[6] In a recent paper (1985b), Lee again assumes a closed population, but suggests that mortality be endogenized as a function of the real wage.

In Eq. (3), the midpoint between P_{t-1} and P_t is

$$P = \exp[1/2(\ln P_t + \ln P_{t-1})]. \tag{4}$$

So,

$$\ln P = 1/2(\ln P_t + \ln P_{t-1}). \tag{5}$$

And, from Eq. (2),

$$\ln (w) = \ln \eta - \beta \ln P. \tag{6}$$

Thus, from Eqs. (5) and (6),

$$\ln w_t = \ln \eta - \beta/2[\ln P_t + \ln P_{t-1}]. \tag{7}$$

Since

$$P^*/P = \ln [P_t/P_{t-1}] = [\ln P_t - \ln P_{t-1}] \tag{8}$$

from Eqs. (1) and (3), we have

$$[\ln P_t - \ln P_{t-1}] = \mu + \alpha \ln w_t + (\lambda - 1)d_t. \tag{9}$$

Thus, Eqs. (7) and (9) are the two structural equations of this simultaneous equations system[7]

$$\ln w_t = \ln \eta - \beta/2[\ln P_t + \ln P_{t-1}] \tag{7}$$

$$[\ln P_t - \ln P_{t-1}] = \mu + \alpha \ln w_t + (\lambda - 1)d_t. \tag{9}$$

The estimation of this two-equation model, via two-stage least squares, is described in Part V, below.

Model II: Endogenous Migration

As Wrigley and Schofield (1981, pp. 219–228) have pointed out, net out-migration from England was not insignificant during the period 1541–1871. Therefore, Model II provides for the inclusion of migration through a respecified identity

$$P^*/P = f - d - m, \tag{10}$$

where m = net out-migration.

From Eqs. (1), (8), and (10), we now have

$$[\ln P_t - \ln P_{t-1}] = \mu + \alpha \ln w_t + \lambda d_t - d_t - m_t \tag{11}$$

$$[(\ln P_t - \ln P_{t-1}) + m_t] = \mu + \alpha \ln w_t + (\lambda - 1)d_t. \tag{12}$$

A simple model of (push-type) migration would suggest that migration levels might be dependent upon the domestic standard of living, a reasonable

[7] A further improvement (not carried out in the present study) would be explicit provision for the age distribution of the population in the relationships posited in Eqs. (7) and (9).

proxy of which is the real wage (Wilkinson, 1970; Gallaway and Vedder, 1971; Quigley, 1972). In fact, empirical evidence indicates that English out-migration during the period of this analysis was indeed related to the domestic real wage (Wrigley and Schofield, 1981, p. 441). Therefore, migration is next endogenized through the addition of one more equation[8]

$$\exp\{m_t\} = \gamma[w_{t-1}]^{\rho} \tag{13}$$

or

$$m_t = \ln\gamma + \rho\ln w_{t-1}. \tag{14}$$

Thus, there are three structural equations in Model II:

$$\ln w_t = \ln\eta - \beta/2[\ln P_t + \ln P_{t-1}] \tag{7}$$

$$[(\ln P_t - \ln P_{t-1}) + m_t] = \mu + \alpha\ln w_t + (\lambda - 1)d_t \tag{12}$$

$$m_t = \ln\gamma + \rho\ln w_{t-1}. \tag{14}$$

IV. THE TEMPORAL RELATIONSHIP BETWEEN FERTILITY AND THE REAL WAGE

As is indicated by Eq. (1), Lee's original formulation specified that the relationship between the standard of living and fertility was a contemporaneous one. An inspection of the data, however, suggests otherwise; there is clearly a 25- to 50-year lag between the time pattern of fertility and that of the real wage. Smith (1981) noted the likely cause of this lagged relationship: during the period 1541–1871, English nuptiality appears to have responded to changes in real wages with a lag of about 30 years.[9] Because of this pattern, Wrigley and Schofield (1981) maintained that fertility is primarily a function of marriage age and incidence; little evidence exists of shifts in marital fertility during the period of interest. Thus, changes in the economic climate seem to have had a rather dramatic impact on the speed of entry into marriage.[10]

Equation (1) may be modified to allow for the lagged relationship:

$$f = \mu + \alpha\ln w + \pi\ln w_{t-1} + \lambda d. \tag{15}$$

Based upon this new formulation, Eqs. (9) and (12) may be expanded as follows:

[8] The semi-logarithmic functional form was selected because of its consistency with theoretical expectations regarding the migration–wage relationship: negative values of net out-migration are possible, while negative values of the real wage are not. Furthermore, this form of the equation provided the best fit with the empirical data.

[9] This evidence from nuptiality data was brought to my attention by Joseph Potter.

[10] Goldstone (1986) argues against Wrigley and Schofield's acceptance of a long (40-year) lag from wages to fertility, maintaining that wage shifts affected marriage incidence with a lag of 15 to 20 years.

$$[\ln P_t - \ln P_{t-1}] = \mu + \alpha \ln w_t + \pi \ln w_{t-1} + (\lambda - 1)d_t \quad (16)$$

$$[(\ln P_t - \ln P_{t-1}) + m_t] = \mu + \alpha \ln w_t + \pi \ln w_{t-1} + (\lambda - 1)d_t. \quad (17)$$

V. ADDING EXOGENOUS TECHNOLOGICAL CHANGE

Plotting (midpoint) population levels against the real wage, plus preliminary econometric analysis indicated that the English population–wage relationship underwent significant changes with the onset of the industrial revolution. These changes appear to be consistent with outward shifts in the labor demand schedule as a result of technological and other socioeconomic changes.[11]

Alternative methods of accounting for the effects of technological change on labor demand include the use of (1) dummy variables affecting the intercept and/or the slope of the labor–demand function; (2) trend variables which are some function of time; and (3) proxies for industrialization, such as the portion of the work-force employed in manufacturing or the portion of the population residing in urban areas.

On a purely theoretical basis, the last alternative—the degree of urbanization—appeared to represent the most attractive approach, and the requisite data on urbanization were available over the relevant period (de Vries, 1984). Moreover, preliminary econometric analysis indicated the clear superiority of this approach to any of the others.[12] Hence, the variable U was constructed, measuring the percentage of the English population which was resident in urban areas.

Equation (2) is now modified to produce a new relationship:

$$w = \eta P^{-\beta} e^{\delta U}. \quad (18)$$

Likewise, Eq. (7) becomes

$$\ln w_t = \ln \eta - \beta/2[\ln P_t + \ln P_{t-1}] + \delta U_t, \quad (19)$$

where U_t = the percentage of the English population residing in urban areas of population 10,000 or greater, on average during the 25-year period culminating in the year t. This modification, plus that associated with the lagged relationship presented above in Eqs. (16) and (17), yields the following simultaneous equations models to be estimated:[13]

[11] Assuming a relatively high degree of correlation between population and the total labor force, short- to medium-term labor supply will be highly inelastic. Thus, the population–wage relationship may be taken as providing an approximation to a true labor demand schedule. A more accurate estimate of the total labor force could be developed, based upon population data combined with evidence regarding the age distribution of the population (Weir, 1984).

[12] Lee (1985b) detected the shift in the labor–demand function, but accounted for it through the introduction of a single time-trend variable. For further discussion regarding shifts in labor demand, see Lee (1978, 1980).

[13] Whereas Eq. (9) is identified solely by the implicit constraint that the coefficient of

Model I

$$\ln w_t = \ln \eta - \beta/2[\ln P_t + \ln P_{t-1}] + \delta U_t \tag{19}$$

$$[\ln P_t - \ln P_{t-1}] = \mu + \alpha \ln w_t + \pi \ln w_{t-1} + (\lambda - 1)d_t. \tag{16}$$

Model II

$$\ln w_t = \ln \eta - \beta/2[\ln P_t + \ln P_{t-1}] + \delta U_t \tag{19}$$

$$[(\ln P_t - \ln P_{t-1}) + m_t] = \mu + \alpha \ln w_t + \pi \ln w_{t-1} + (\lambda - 1)d_t \tag{17}$$

$$m_t = \ln \gamma + \rho \ln w_{t-1}. \tag{14}$$

Given these expanded models, it is now necessary to reformulate the conditions under which the alternative theories of demographic change are to be evaluated.

Reformulation of Conditions on Alternative Theories of Demographic Change

Because of the modifications made in the previous section, it is now possible to view the expanded generic model as encompassing a fourth theory of demographic transformation, which will be denoted as the Composite Theory (COMP). This Composite Theory represents an additional step toward an empirical realization of what Wrigley and Schofield have proposed (on a theoretical level) to be a "full dynamic model of population and the environment" (1981, p. 465).[14]

The Composite Theory combines aspects of the CEW and CF theories, but in a manner distinct from Lee's SYN theory. Specifically, this fourth theory of demographic change shares with the classical (CEW) theory

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$\ln P_{t-1}$ is unity, note that Eqs. (16) and (17) are also identified by the urbanization variable, U_t. A further refinement in the present specification would make urbanization endogenous to the model through the inclusion of an additional equation in which U_t would be a function of the urban–rural wage differential.

[14] Actually, the empirical model developed here represents two steps toward the full Wrigley and Schofield theory, as we have provided for endogenous migration behavior, as well as exogenous technological change. Still omitted from our specification, but included in Wrigley and Schofield's theory, is an endogenous component for mortality. According to those authors, the endogenous component of mortality should be negatively related to the real wage and positively related to the degree of industrialization/urbanization ("dark, satanic mill towns"). Preliminary econometric analysis, however, indicated that both sources of hypothetical dependence are obscured (statistically) by exogenous changes in mortality over time. There is, however, a distinct inverse correlation of mortality with the real wage lagged 100 years, but there is no reason to expect this statistical relationship to be indicative of an actual association. An inverse relationship for short-run fluctuations was identified by Lee (1980), and in a recent paper, Lee (1985b) posits a (negative) dependence of mortality levels on the real wage, but no attempt at econometric estimation of the relationship was reported. Further research into endogenizing mortality in a more complete demographic model would be highly desirable.

the notion that shifts in the labor demand function played a major role in long-run population changes. On the other hand, the Composite Theory rejects the CEW premise of a major role for the real wage in determining levels of fertility.

This new theory is consistent with Lee's synthesis in that both recognize the important role played by exogenous movements in mortality. The Composite Theory allows for the possibility that exogenous mortality changes played a dominant role in the preindustrial period, while shifting labor demand due to technological changes played a more important role with the onset of industrialization.

The four alternative demographic theories may be viewed as special cases of an expanded generic model under the following conditions:

CEW Theory: $\lambda = 1$ and $\delta \neq 0$
CF Theory: $\lambda = 0, \alpha = 0, \pi = 0$, and $\delta = 0$
Synthesis Theory: $\lambda = 0, \alpha \neq 0, \pi \neq 0$, and $\delta = 0$
Composite Theory: $\lambda = 0, \alpha = 0, \pi = 0$, and $\delta \neq 0$.

These conditions are tested econometrically in the following part of the paper.

VI. PARAMETER ESTIMATION AND HYPOTHESIS TESTING

The parameters of Models I and II were estimated with data for the years 1573 through 1873 on five variables: population, the crude death rate, the real wage, net migration, and degree of urbanization.[15]

Parameter Estimation Procedure

Models I and II are simultaneous equations models, in which current and lagged endogenous variables appear on the right-hand side of various equations. Since ordinary least squares (OLS) estimation yields biased parameter estimates in such situations (Johnston, 1984), two-stage least squares (TSLS) estimation was utilized.[16]

Econometric Results: Parameter Estimates and Test Statistics

Parameter estimates and associated t-statistics are found in Table 1. On the basis of respective F-statistics, all equations are significant at the 1% level (at least), except for Eq. (14), which is significant at the 5% level.[17]

[15] Due to the use of lagged values, directly in the model and in the construction of the midpoint population and the rate of population growth (see Eqs. (4) and (8)), data are included in Appendix 1 for the year 1548 for population, the real wage, and urbanization, but not for other variables.

[16] The migration–real wage equation in Model II (Eq. 14) contains a single right-hand side variable, the real wage lagged one period, which is predetermined (assumed to be nonstochastic). Therefore, this equation was estimated using OLS.

[17] Because of the use of TSLS (an instrumental variable procedure), R^2 may not be used

TABLE 1
Two-Stage Least-Squares Results: Parameter Estimates and t Statistics

	$\ln\eta$	β	μ	α	π	λ	$\ln\gamma$	ρ	δ
Model I: closed population									
Parameters	20.5	1.13	−0.37	0.110	0.234	−0.30	—	—	0.069
t Statistics	5.90	4.80	−0.64	0.675	1.37	−0.91	—	—	5.96
Model II: endogenous migration									
Parameters	20.7	1.13	−0.12	0.087	0.214	−0.36	0.168	−0.03	0.069
t Statistics	5.98	4.88	−0.24	0.596	1.40	−1.3	2.88	−2.4	6.04

Before the parameter estimates are utilized for hypothesis testing, it is important to examine the estimated equations for the possible presence of serial correlation. The appropriate auxiliary regressions were estimated for all equations, and in no case was the null hypothesis of no serial correlation rejected (Appendix 2).

Hypothesis Testing: Evaluating the CEW, CF, Synthesis, and Composite theories

As can be seen in Table 1, the TSLS results indicate that $\lambda \neq 1$ in both versions of the demographic model. Additionally, it is not possible to reject the hypothesis that $\lambda = 0$. Thus, the CEW theory (which requires that $\lambda = 1$) may be eliminated from further consideration.

In order to discriminate among the CF, SYN, and COMP theories, it is next necessary to test whether α and π are equal to zero, given that λ is equal to zero. In keeping with Lee's (1973) approach, regressions were reestimated where λ was constrained to be equal to zero:

Model I

$$\left[[\ln P_t - \ln P_{t-1}] + d_t\right] = \mu + \alpha \ln w_t + \pi \ln w_{t-1}. \tag{20}$$

Model II

$$\left[[(\ln P_t - \ln P_{t-1}) + m_t] + d_t\right] = \mu + \alpha \ln w_t + \pi \ln w_{t-1}. \tag{21}$$

TSLS estimation yielded estimates of α, π, and associated t-statistics (Table 2). The calculated F-statistics for the two models are 3.31 and 3.15, respectively. Thus, the joint null hypothesis that α and π both equal zero is not rejected, even at the 5% level ($F_{(2,10)} = 4.10$ at 5% level, $F_{(2,10)} = 7.56$ at 1% level).

as a valid measure of goodness-of-fit. It should also be noted that since TSLS is devoid of desirable small-sample properties, the results reported here need to be compared with those derived from OLS estimation. Such estimation resulted in parameter and standard error estimates which were not substantially different from those obtained with TSLS.

TABLE 2
Constrained Equations Parameter Estimates and t Statistics

Model I		Model II	
α	π	α	π
0.1330	0.2358	0.1144	0.2160
(.808)	(1.36)	(.745)	(1.33)

Combining these results with those of the previous set of hypothesis tests, we find that $\lambda = 0$, $\alpha = 0$, and $\pi = 0$, which according to the previously specified set of conditions lends support both to the CF and the COMP theories of demographic change.

Last, in order to discriminate between the CF and the COMP theories, hypothesis tests are carried out on the urbanization parameter, δ. As may be seen in Table 1, the estimated parameter of this "technological-change" variable is significantly greater than zero in both versions of the model, indicating that the Composite Theory provides the best explanation of the observed pattern of English demographic change.

These findings for the period 1573–1873 differ from those of Lee's original (1973) analysis for the period 1250–1750 (based upon pre-Wrigley and Schofield data), in which it was found that $\lambda = 0$ but $\alpha \neq 0$, thereby supporting Lee's own Synthesis theory. Given the contrast between the findings reported here and those originally reported by Lee, it may be of interest to examine the degree to which the alternative versions of the model considered here are successful in simulating the actual demographic change which occurred during the relevant time period.

VII. DYNAMIC SIMULATION OF DEMOGRAPHIC CHANGE IN ENGLAND: 1573–1873

In this part of the paper, we compare the performance of Models I and II (using the Wrigley and Schofield database) in order to assess their relative merits as explanatory models of demographic change. One approach to testing the performance of a simultaneous equations model is to perform a dynamic, historical simulation and to examine how closely the endogenous variables of interest match their corresponding historical values through time.

Dynamic Simulation Results and Evaluation

Dynamic simulations of historical population levels were carried out using Models I (closed population) and II (endogenous migration). The results of these simulation exercises are provided in Fig. 2.

A number of methods were used for evaluating the alternative dynamic

simulations of historic population levels. First of all, we examine the estimated slope coefficients from regressing actual historical values on simulated values; the nearer the estimated slope is to 1.0, the better is the historical simulation. As can be seen in Table 3, Model I yields a regression coefficient of approximately 1.68, while Model II's estimated coefficient is 1.04, indicating substantially better performance by the latter model.

The most frequently utilized measure of dynamic performance is the root-mean-squared (RMS) error, which is defined for a variable Y_t as

$$\text{RMS error} = \left[\frac{1}{T}\sum_{t=1}^{T}[Y_t^s - Y_t^a]^2\right]^{1/2}, \tag{22}$$

where

Y_t^s = simulated value of Y_t;
Y_t^a = actual value of Y_t; and
T = number of time periods in the simulation.

The RMS error of the Model I simulation is more than five times as great as that of the Model II simulation (Table 3). While this measure is useful as an indication of the deviation of a simulated variable from its actual time path, it suffers from the limitation that its magnitude is not standardized (Pindyck and Rubinfeld, 1981).

To correct for this limitation, one last measure which may be employed is Theil's Inequality Coefficient (Theil, 1961):

$$U = \frac{\left[\frac{1}{T}\sum^{T}[Y_t^s - Y_t^a]^2\right]^{1/2}}{\left[\frac{1}{T}\sum_{t=1}^{T}[Y_t^s]^2\right]^{1/2} + \left[\frac{1}{T}\sum_{t=1}^{T}[Y_t^a]^2\right]^{1/2}}. \tag{23}$$

The numerator is the RMS error, and the scaling of the denominator insures that U has limits of 0 and 1, where $U = 0$ indicates a perfect

TABLE 3
Dynamic Simulation Evaluations of Models I and II

Alternative evaluation measures	Model I (closed population)	Model II (endogenous migration)
Regression coefficient of actual on simulated	1.67813	1.04334
Root-mean-squared error	2,823,732	529,844
Theil's inequality coefficient	0.08716	0.00307

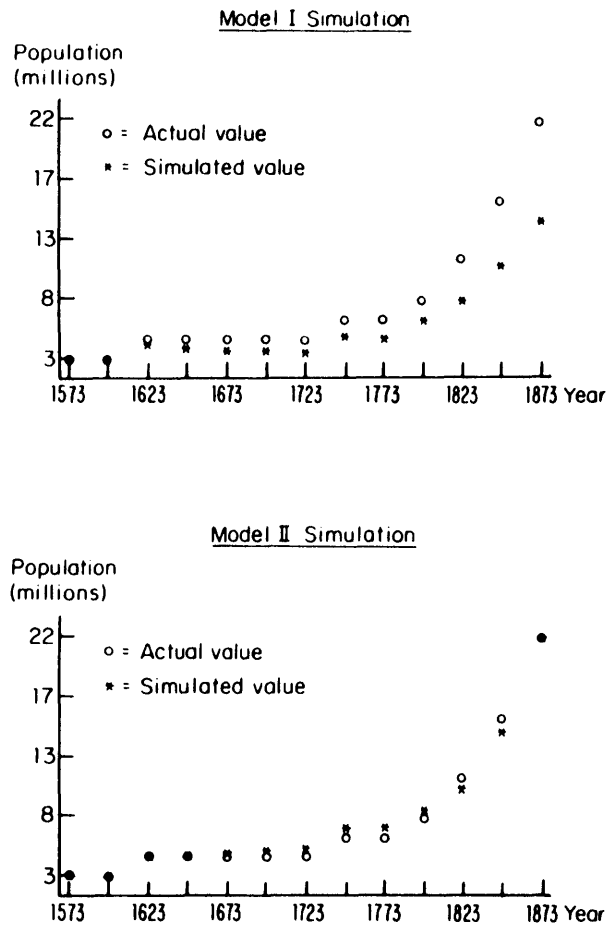

FIG 2. Dynamic simulation results, English population level, 1573–1873.

dynamic fit. In the present case, the simulation carried out using Model I has a U-statistic which is approximately 28 times the magnitude of the U-statistic associated with Model II (Table 3).

Thus, all three measures for evaluating dynamic simulations indicate the superiority of the demographic model which accounts for endogenous migration. If the dynamic simulation of Model II were to be compared with the dynamic performance of a model without migration *and* without provision for technological change or for the lagged relationship between fertility and the real wage, the difference would be even more dramatic.

It is particularly interesting, in the light of Lee's original (1973) analysis, to note that Model I (closed population) tracks the historical population level relatively well through the middle of the 18th century. But with the initiation of the British industrial revolution, the closed-population model produces increasingly poor results, particularly compared with the dynamic performance of the endogenous-migration demographic model (Fig. 2).[18]

[18] The data set used by Lee in his recent analysis (1985b) terminates in the year 1800, and hence does not include the dramatic demographic changes which occurred in the 19th century.

VIII. SUMMARY AND CONCLUSIONS

This paper has presented an analysis of the factors associated with English demographic change over the period 1573–1873, based upon the best available source of data on fertility, mortality, migration, and the real wage (Wrigley and Schofield, 1981). A simultaneous equations model was developed in order to test four alternative demographic theories—the constant equilibrium wage theory, the constant fertility theory, Lee's original synthesis theory, and a new, composite theory.

The analysis described in this paper includes three supplements to Lee's extensive work in this area. First, the model developed here takes into account endogenous migration behavior. Second, the fertility–real-wage relationship is modified to account for a lag which is empirically valid and is consistent with some theoretical expectations (Smith, 1981). Third, the final form of the model allows for shifts in labor–demand due to technological change which is proxied by degree of urbanization. The expanded formulation is sufficiently robust to accommodate the demographic transformation which occurred with the onset of the industrial revolution in Britain.

Two-stage least squares estimation yielded parameter estimates with which hypothesis tests were carried out, and these tests indicated the superiority of the composite theory of demographic change. The dynamic performance of this model was compared with a closed-population model through a series of simulations. On the basis of several alternative evaluation measures, it was found that the complete model with endogenous migration provides substantially better tracking of the actual, historical changes in the English population level.

Substantive Conclusions and Implications for Further Research

By statistically affirming the "composite theory" of demographic change, this paper supports the classical notion that shifts in labor demand were an important cause of long-run population changes, at least subsequent to the beginning of British industrialization.

Although there were dramatic shifts in labor demand late in the sample period which explain the increasingly rapid growth in population size,[19] the analysis (and the composite theory) affirms the dominant role played by mortality during the preindustrial period. The analysis also indicates that the direct effect of the real wage on fertility was, at best, weak.[20]

[19] This finding does not contradict Lee's original (1973) conclusion, since his empirical analysis focused exclusively on the preindustrial period. Indeed, Lee recognized that "the relationship between population and the real wage was stable *until* the beginning of the 18th century, at which time it began to change markedly" (1973, p. 604).

[20] Given the finding of an insignificant slope of the fertility–wage function, it is reasonable to speculate that the equilibrium population level is established by the intersection of a horizontal fertility schedule with a downward-sloping mortality–wage function. Such a

During the preindustrial period of relative "structural stability" in the 16th and 17th centuries, changing rates of mortality constituted the driving force behind English population changes. With the initiation of industrialization in the 18th and 19th centuries, however, technological improvements led to structural changes wherein shifts in the labor–demand function became the dominant force behind population increases.

Thus, it might be assumed that one model would be required for the preindustrial period and another one for the period commencing with the beginning of English industrialization. The composite model developed here, however, is sufficiently robust to account for demographic changes during both periods. In this sense, it provides a first-approximation of the changing demographic structure.

Such changing structure, while introducing additional statistical complexities,[21] does not necessarily indicate conceptual problems on a theoretical level. In fact, the major theme of the concluding chapter of Wrigley and Schofield's (1981) watershed study is that English demographic changes are best analyzed in the context of a complete model of potential interactions among population, fertility, mortality, migration, labor markets, and technological change; a model in which the numerous linkages between and among these factors themselves vary in importance over time.

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relationship has been suggested by Wrigley and Schofield (1981) and by Lee in his recent (1985b) study. For further discussion of this point, see footnote 14.

[21] One possible approach would be to model the structural changes (of the industrial revolution) as being endogenous to the system through the employment of "varying-parameter models" (Judge, Griffiths, Hill, Lutkepohl, and Lee, 1985, pp. 797–821).

APPENDIX 1: BASIC DATA FOR ECONOMETRIC MODELS

Year t	Population[a] P_t	Crude death rate[b] d_t	Real wage[c] w_t	Net out-migration[d] m_t	Urban portion[e] U_t
1548	2,928,476	—	64.40	—	3.4
1573	3,322,745	0.70700	57.76	0.0310	4.1
1598	3,985,129	0.60525	51.88	0.0398	5.3
1623	4,771,723	0.61525	40.50	0.0333	6.6
1648	5,226,311	0.65900	43.01	0.0398	8.1
1673	4,993,190	0.70175	49.57	0.0508	10.0
1698	4,997,611	0.75725	49.92	0.0210	12.2
1723	5,370,904	0.69475	62.15	0.0255	14.2
1748	5,668,980	0.76425	65.43	0.0235	15.9
1773	6,552,106	0.68200	65.82	0.0258	17.6
1798	8,398,737	0.67125	59.88	0.0128	19.4
1823	11,875,085	0.63500	53.23	0.0205	24.5
1848	16,183,051	0.56450	70.11	0.0348	31.0
1873	22,022,512	0.55850	81.33	0.0298	39.5

[a] Population is a point estimate for the indicated year, based on annual demographic data from Wrigley and Schofield (1981, Table A3.3, pp. 531–535).

[b] The crude death rate is a 25-year average, based on quinquennial data from Wrigley and Schofield (1981, Table A3.1, pp. 528–529). The respective 25-year periods for the crude death rate, the real wage, migration, and the urban portion all terminate in the indicated years.

[c] The real wage is a 25-year average, based on an annual real-wage index (divided by 10) developed by Wrigley and Schofield (1981, Table A9.2, pp. 642–644), from data assembled by Phelps Brown and Hopkins (1956).

[d] Migration is a 25-year average, based on quinquennial data from Wrigley and Schofield (1981, Table 7.11, p. 219, and Table A3.1, pp. 528–529).

[e] The urban portion is a 25-year average of the percentage of the population living in towns and cities with population of at least 10,000, linearly interpolated from 50-year data presented by de Vries (1984, pp. 39, 45, and 46).

APPENDIX 2: TESTING FOR SERIAL CORRELATION

Serial correlation of error terms is frequently present in time-series models, and ordinarily, Durbin–Watson statistics may be used to test for first-order serial correlation (AR1). When stochastic variables are present on the right-hand side of an equation, as is inevitably the case in a simultaneous-equation context, the Durbin–Watson statistic is biased toward a value of 2.0, and hence loses its validity as a test statistic (Judge, Griffiths, Hill, Lutkepohl, and Lee, 1985).

In this context, various other tests for serial correlation are available. Among these is the Breusch–Godfrey test (Breusch, 1978; Godfrey, 1978), wherein the estimated residuals, ε_t, from the original regression equation are regressed on a constant, α_0, the regressors from the original equation,

X, and lagged values of the estimated residuals, ε_{t-i}, where i equals 1 to p:

$$\varepsilon_t = \alpha_0 + \beta X + \varepsilon_{t-1} + \varepsilon_{t-2} + \cdots + \varepsilon_{t-p} + \mu_t. \tag{1}$$

Based upon this auxiliary regression, the null hypothesis of no serial correlation of degrees 1 through p is rejected if

$$(N)(R^2) > \chi_p^2 \tag{2}$$

where χ_p^2 is the critical value from a chi-square distribution with p degrees of freedom. The appropriate auxiliary regressions were estimated for all equations in the two models, and in no case was the null hypothesis of no serial correlation rejected.

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