[Online Appendix]

Banking, Trade, and the Making of a Dominant Currency

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A Appendix

A.1 Proof of Proposition 2

From the risk averse importers problem we have,

$$D_\$ = M - \frac{1}{\psi\delta\sigma^2}\left(Q_\$ - Q_h\right)$$
$$Q_h = \delta$$

Market clearing requires that $D_\$ = B_\$ + X_\$$. There are two possible scenarios. One where the demand for dollar safe assets is high enough, that is $M \geq \bar{M}$, which incentivizes domestic banks to also produce them, that is $B_\$ > 0$. In this case it follows from Proposition 1 that,

$$\frac{Q_\$ - \beta}{Q_h - \beta} = \bar{\mathcal{E}}$$

Since $\delta \equiv \theta + \beta$, it follows that $Q_\$ - Q_h = \theta(\bar{\mathcal{E}} - 1)$ and $D_\$ = M - \frac{\theta(\bar{\mathcal{E}}-1)}{\psi\delta\sigma^2}$. A second scenario is one where the demand for dollar safe assets is so low that it is not worthwhile for domestic banks to produce them, that is $B_\$ = 0$. In this case, $D_\$ = X_\$$, to give the solution for $(Q_\$ - Q_h) = (M - X_\$)\psi\delta\sigma^2$. For this to be an equilibrium it must be that $(M - X_\$)\psi\delta\sigma^2 < \theta(\bar{\mathcal{E}} - 1)$. $\bar{M}$ is the cut-off level of imports M at which this equation holds with strict equality, that is $\bar{M} = X_\$ + \frac{\theta(\bar{\mathcal{E}}-1)}{\psi\delta\sigma^2}$.

A.2 Proof of Proposition 3

The model in Section 3 is characterized by the following system of equations:

$$D_\$ = M - \frac{1}{\psi\delta\sigma^2}(Q_\$ - Q_h) \tag{1}$$

$$D_\$ = B_\$ + X_\$ \tag{2}$$

$$\eta = \begin{cases} 0 & \text{if} \quad Q_\$ - Q_h < 0 \\ \in [0,1] & \text{if} \quad Q_\$ - Q_h = 0 \\ 1 & \text{if} \quad Q_\$ - Q_h > 0 \end{cases} \tag{3}$$

$$Q_\$ - Q_h = \theta(\bar{\mathcal{E}} - 1) \quad \text{if} \quad B_\$ > \gamma_L N \tag{4}$$

along with the condition, $B_\$ = \eta\gamma_L N$, when the return differential between the dollar and local currency is positive but not big enough i.e. $0 \leq Q_\$ - Q_h < \theta(\bar{\mathcal{E}} - 1)$. The first equation stems from the importer's demand for dollar deposits, the second comes from market clearing, the third and the forth equations are derived from from the bank's optimization problem. To ease exposition, we consider four different scenarios for M.

[Case 1: $M < X_\$$**]** In this region of the parameter space, $B_\$$ must be equal to zero because otherwise (i.e. if $B_\$ > 0$) both $D_\$ > X_\$$ and $Q_\$ - Q_h \geq 0$ should hold. The first inequality is a result of (2), while the second comes from the bank's funding decision. The two inequalities lead to a contradiction in view of (1). With $B_\$ = 0$ in place, it is now straightforward that $D_\$ = X_\$, \eta = 0$, and $Q_\$ - Q_h = (M - X_\$)\psi\delta\sigma^2$. Essentially, $\underline{M} \equiv X_\$$ acts as a cutoff for this type of solution.

[Case 2: $X_\$ \leq M < X_\$ + \gamma_L N$**]** Again, we prove $B_\$ = \eta\gamma_L N$ by contradiction. Suppose $B_\$ > \eta\gamma_L N$. To make it consistent with the bank's funding decision, $Q_\$ - Q_h = \theta(\bar{\mathcal{E}} - 1) > 0$ and $\eta = 1$ must hold. The first condition implies that $D_\$ - M < 0$ in light of (1), which contradicts $M < X_\$ + \gamma_L N < X_\$ + B_\$ = D_\$$. Suppose now that the opposite inequality is true i.e. $B_\$ < \eta\gamma_L N$. To make it consistent with the bank's funding decision, $Q_\$ - Q_h < 0$ has to be satisfied, which makes $\eta = 0$. This is again a contradiction since $X_\$ \leq M$ implies $D_\$ - M < 0$ when $B_\$ \leq 0$, which is not consistent with (1). Thus, $B_\$ = \eta\gamma_L N$. Given this result, consider the case $\eta > 0$,[1] which is equivalent to $Q_\$ - Q_h = 0$. Then we have $D_\$ = M$ in view of (1). η is also pinned down by $B_\$ = M - X_\$ = \eta\gamma_L N$. Again, using these results, we shall use $\hat{M} = \gamma_L N$ to denote the cutoff of M to be able to sustain this type of solutions.

[Case 3: $X_\$ + \gamma_L N \leq M < X_\$ + \gamma_L N + \frac{\theta(\bar{\mathcal{E}}-1)}{\psi\delta\sigma^2}$**]** In this region, there is no currency mismatch ($B_\$ = \gamma_L N$) because the resulting $Q_\$ - Q_h$ is positive but still smaller than $\theta(\bar{\mathcal{E}} - 1)$. This can be seen more clearly when you substitute $D_\$ = \gamma_L N + X_\$$ into (1). Thus, we have $\eta = 1$, $D_\$ = X_\$ + \gamma_L N$ and $Q_\$ - Q_h = (M - X_\$ - \gamma_L N)\psi\delta\sigma^2$. The cutoff for this region is $\bar{M}$.

[Case 4: $X_\$ + \gamma_L N + \frac{\theta(\bar{\mathcal{E}}-1)}{\psi\delta\sigma^2} \leq M$**]** This is where the currency mismatch kicks in. $Q_\$ - Q_h = \theta(\bar{\mathcal{E}} - 1)$ holds. $D_\$$ is thus equal to $M - \frac{\theta(\bar{\mathcal{E}}-1)}{\psi\delta\sigma^2}$ in view of (1), and $\eta = 1$.

A.3 Proof of Proposition 4

Consider a symmetric equilibrium in which $\eta_i \equiv \eta$ for all i. M in Proposition 3 is now replaced by $M = a + b\eta$. The cutoff of a, which corresponds to $\underline{M}$ previously, is then given by $\bar{a} = \underline{M}$. A Case-1 equilibrium (as in Proposition 3) is sustainable only if $a \leq \underline{a}$. The cutoff corresponding to $\bar{M}$ is $\underline{a} = X_\$ + \gamma_L N + \frac{\theta(\mathcal{E}-1)}{\psi\delta\sigma^2}$, so a Case-4 equilibrium is sustainable only if $a > \underline{a}$. Furthermore, if b is large enough, $b > \gamma_L N + \frac{\theta(\mathcal{E}-1)}{\psi\delta\sigma^2}$, the size of these two cutoffs is flipped so we have $\underline{a} < \bar{a}$.

[1] One can easily show that the case with $\eta = 0$ is not sustainable in this region

A.4 Proof of Proposition 5

We can rewrite the objective function of importers as

$$
\begin{aligned}
C_{0i} + \delta\left(\mathbb{E}_0[C_{1i}] - \frac{\psi}{2}\mathbb{V}_0[C_1]\right) =& W - Q_{hi}D_{hi} - Q_{\$}D_{\$i} - Q_{€}D_{€i} \\
&+ \delta(D_{hi} + D_{\$i} + D_{€i} - M) \\
&- \frac{\delta\psi\sigma^2}{2}[(D_{\$i} - M_{\$})^2 + (D_{€i} - M_{€})^2 + 2\rho(D_{€i} - M_{€})(D_{\$i} - M_{\$})]
\end{aligned}
$$

where $M \equiv M_{\$} + M_{€}$. The first order conditions yield

$$
\begin{aligned}
0 =& - Q_{\$} + \delta - \delta\psi\sigma^2\left((D_{\$i} - M_{\$}) + \rho(D_{€i} - M_{€})\right) \\
0 =& - Q_{€} + \delta - \delta\psi\sigma^2\left((D_{€i} - M_{€}) + \rho(D_{\$i} - M_{\$})\right) \\
0 =& - Q_h + \delta
\end{aligned}
$$

Solving this system of equations, it follows that

$$
\begin{aligned}
D_{\$i} =& M_{\$} - \frac{1}{\psi\delta\sigma^2(1-\rho)}\left((Q_{\$} - Q_{hi}) - \rho(Q_{€} - Q_{hi})\right) \\
D_{€i} =& M_{€} - \frac{1}{\psi\delta\sigma^2(1-\rho)}\left((Q_{€} - Q_{hi}) - \rho(Q_{\$} - Q_{hi})\right) \\
Q_h =& \delta
\end{aligned}
$$

From this point onwards, we consider three types of equilibria and provide conditions under which these equilibria exist.

[Case 1: $\eta_{\$i} = \eta_{\$i} = 0$] Suppose that all imported products are invoiced in local currency; we later verify that this is indeed an optimal behavior of firms since foreign currency interest rates are higher than local currency interest rate i.e. $Q_{\$} - Q_{hi} < 0$ and $Q_{€} - Q_{hi} < 0$ hold. Given this invoicing decision, importers face $M_{\$} = M_{€} = a$. The demands for foreign currency deposits are now characterized by

$$
\begin{aligned}
D_{\$i} =& a - \frac{1}{\psi\delta\sigma^2(1-\rho)}\left((Q_{\$} - Q_{hi}) - \rho(Q_{€} - Q_{hi})\right) \\
D_{€i} =& a - \frac{1}{\psi\delta\sigma^2(1-\rho)}\left((Q_{€} - Q_{hi}) - \rho(Q_{\$} - Q_{hi})\right) \\
Q_h =& \delta
\end{aligned}
$$

In view of the market clearing conditions, these equations can be converted to

$$\begin{aligned}X =& a - \frac{1}{\psi\delta\sigma^2(1-\rho)}\left((Q_\$ - \delta) - \rho(Q_€ - \delta)\right)\\X =& a - \frac{1}{\psi\delta\sigma^2(1-\rho)}\left((Q_€ - \delta) - \rho(Q_\$ - \delta)\right)\end{aligned}$$

So we have

$$Q_\$ = Q_€ = \delta - \delta\psi\sigma^2(1+\rho)\,(X-a)$$

A sufficient and neccesary condition to sustain this equilibrium is therefore $X \geq aM$. We shall use $\bar{a}^n = X$ to denote the corresponding cutoff.

[Case 2 $\eta_{\$i}, \eta_€ = (1,0)$ **or** $(0,1)$**]** Suppose now that exporters invoice all of their foreign revenues in dollars. That is, $\alpha_\$ = a+b$ and $\alpha_€ = a$. It follows from the demand functions that

$$Q_\$ = \delta - \delta\psi\sigma^2(D_{\$i} - (a+b)) + \rho\,(X-a)) \tag{5}$$

$$Q_€ = \delta - \delta\psi\sigma^2((X-a) + \rho\,(D_{\$i} - (a+b))) \tag{6}$$

On the other hand, the first order conditions for the bank's problem yield

$$Q_\$ - Q_{hi} = \theta(\bar{\mathcal{E}} - 1) \tag{7}$$

at an interior solution where $B_{\$i} > 0$ and $B_{hi} > 0$. Plugging (7) into (5) and (6), we can pin down the equilibrium value of dollar deposits:

$$D_{\$i} = (a+b) - \rho(X-a) - \frac{\theta(\bar{\mathcal{E}}-1)}{\delta\psi\sigma^2}$$

We can then compute interest rates on foreign currency deposits by plugging this back to (5) and (6). That is,

$$\begin{aligned}Q_\$ &= \theta(\bar{\mathcal{E}}-1) + \delta\\Q_€ &= \rho\theta(\bar{\mathcal{E}}-1) + \delta - \delta\psi\sigma^2(1-\rho^2)(X-a)\end{aligned}$$

To verify that this is indeed an equilibrium, we need to ensure that there is currency mismatch in the balance sheet and that the interest rate on dollar deposits is lower than the interest rate on euro deposits.

Put differently,

$$D_{\$i} \geq \gamma_L N + X \tag{8}$$
$$Q_{\$} \geq Q_{€} \tag{9}$$

Let $\underline{a}^s$ denote the threshold such that eq. (8) holds with equality and $\bar{a}^s$ denote the threshold such that eq. (9) holds with equality.

$$\underline{a}^s = X + \frac{\gamma_L N}{(1+\rho)} + \frac{\theta(\bar{\mathcal{E}}-1)}{(1+\rho)\delta\psi\sigma^2} - b$$
$$\bar{a}^s = X + \frac{\theta(\bar{\mathcal{E}}-1)}{(1+\rho)\delta\psi\sigma^2}$$

Therefore, it follows that a single dominant-currency equilibrium is sustainable if and only if $a \in [\underline{a}^s, \bar{a}^s]$

[Case 3: $\eta_{\$i} = \eta_{€} = 0.5$] Next, we turn to the case where both euros and dollars are used for international transactions. Consider a potential equilibrium where $\eta_{\$i} = 0.5$ and $\eta_{€i} = 0.5$ hold for all country i. From the bank's first order conditions, we have

$$Q_{\$} - Q_{hi} = Q_{€} - Q_{hi} = \theta(\bar{\mathcal{E}} - 1) \tag{10}$$

The demand functions, on the other hand, lead to

$$Q_{\$} = \delta - \delta\psi\sigma^2(D_{\$i} - (a + 0.5b)) + \rho\,(D_{€i} - (a + 0.5b))) \tag{11}$$
$$Q_{€} = \delta - \delta\psi\sigma^2\,(D_{€i} - (a + 0.5b)) + \rho(D_{\$i} - (a + 0.5b))) \tag{12}$$

Since $Q_{\$} - \delta = Q_{€} - \delta$ and $\rho < 1$, it is easy to show

$$D_{\$i} - (a + 0.5b) = D_{€i} - (a + 0.5b)$$

Let $D \equiv D_{\$i} = D_{€i}$. Combining (10), (11) and (12), we have

$$D - (a + b\eta_{\$}) = -\frac{\theta(\mathcal{E} - 1)}{\delta\psi\sigma^2(1+\rho)}$$

Let $\bar{a}^b$ denote the cutoff such that $D = X + 0.5\gamma_L N$. That is,

$$\bar{a}^b = X + \frac{\theta(\mathcal{E}-1)}{\delta\psi\sigma^2(1+\rho)} + \frac{\gamma_L N}{2} - \frac{b}{2}$$

Therefore, the proposed equilibrium is sustainable if and only if $a \geq \bar{a}^b$.

A.5 Asymmetric Case: Euro versus Dollar

In this section, we elaborate on computational details when the two currencies, the dollar and the euro, have different technological characteristics. We shed light on two factors: (i) the share of imports and (ii) the outside supply of foreign currency deposits. That is, the share of emerging markets' imports from the U.S. and European countries are different, $a_{\$} \neq a_{€}$, which constitutes $M_{\$} = a_{\$} + b\int_i \eta_{\$i} di$ and $M_{€} = a_{€} + b\int_i \eta_{€i} di$. Furthermore, the exogenous supply of foreign currency assets, $X_{\$}$ and $X_{€}$, may take on different values as opposed to the symmetric case developed in the main draft. Given this setup, this section aims to characterize equilibrium conditions under which the dollar or the euro emerges as a global dominant currency. We then present numerical examples that underscore the asymmetry. Path dependency, which we briefly discussed in Section 5, begins to play a key role.

[Case 1: $\eta_{\$i} = \eta_{\$i} = 0$**]** In this case, the demand fucntions for dollar- and euro-denominated deposits can be written as

$$
\begin{aligned}
Q_{\$} =& \delta - \delta\psi\sigma^2 \left((X_{\$} - a_{\$}) + \rho(X_{€} - a_{€})\right) \\
Q_{€} =& \delta - \delta\psi\sigma^2 \left((X_{€} - a_{€}) + \rho(X_{\$} - a_{\$})\right) \\
Q_h =& \delta
\end{aligned}
$$

To make this equilibrium sustainable, we should ensure that the parameter space, $(a_{\$}, a_{€}, X_{\$}, X_{€})$, satisfies $Q_{\$} < Q_{hi}$ and $Q_{€} < Q_{hi}$.

[Case 2 $\eta_{\$i}, \eta_{€} = (1, 0)$ **or** $(0, 1)$**]** Suppose now that exporters invoice all of their foreign revenues in dollars. As before, it follows from the demand functions that

$$Q_{\$} = \delta - \delta\psi\sigma^2(D_{\$i} - (a_{\$} + b)) + \rho\left(X_{€} - a_{€}\right)) \tag{13}$$

$$Q_{€} = \delta - \delta\psi\sigma^2(X_{€} - a_{€}) + \rho\left(D_{\$i} - (a_{\$} + b)\right)) \tag{14}$$

The first order conditions for the bank's problem yield

$$Q_{\$} - Q_{hi} = \theta(\bar{\mathcal{E}} - 1)$$

at an interior solution. In view of this condition, we can write the equilibrium value of dollar deposits as

$$D_{\$i} = (a_{\$} + b) - \rho(X_{€} - a_{€}) - \frac{\theta(\bar{\mathcal{E}} - 1)}{\delta\psi\sigma^2}$$

Substituting this equation into (13) and (14), we have

$$Q_\$ = \theta(\bar{\mathcal{E}} - 1) + \delta$$
$$Q_€ = \rho\theta(\bar{\mathcal{E}} - 1) + \delta - \delta\psi\sigma^2(1-\rho^2)(X_€ - a_€)$$

To make this equilibrium sustainable, we need to ensure that

$$D_{\$i} \geq \gamma_L N + X_\$$$
$$Q_\$ \geq Q_€$$

These conditions are used to draw a region of parameter space where a single dominant-currency equilibrium is sustainable.

[Case 3: $\eta_{\$i} = \eta_€ = 0.5$] Next, we turn to the case where both euros and dollars act as dominant currencies. Among possible equilibria, we focus on the symmetric invoicing case where $\eta_{\$i} = \eta_€ = 0.5$. It follows from the bank's first order conditions that

$$Q_\$ - Q_{hi} = Q_€ - Q_{hi} = \theta(\mathcal{E} - 1)$$

We can then write the demand functions as

$$\begin{bmatrix} Q_\$ - \delta \\ Q_€ - \delta \end{bmatrix} = \begin{bmatrix} -\delta\psi\sigma^2 & -\rho\delta\psi\sigma^2 \\ -\rho\delta\psi\sigma^2 & -\delta\psi\sigma^2 \end{bmatrix} \begin{bmatrix} D_{\$i} - (a_\$ + 0.5b) \\ D_{€i} - (a_€ + 0.5b) \end{bmatrix}$$

The above system of equations can be converted into

$$D_{\$i} = (a_\$ + 0.5b) - \frac{\theta(\bar{\mathcal{E}} - 1)}{\delta\psi\sigma^2(1+\rho)}$$
$$D_{€i} = (a_€ + 0.5b) - \frac{\theta(\bar{\mathcal{E}} - 1)}{\delta\psi\sigma^2(1+\rho)}$$

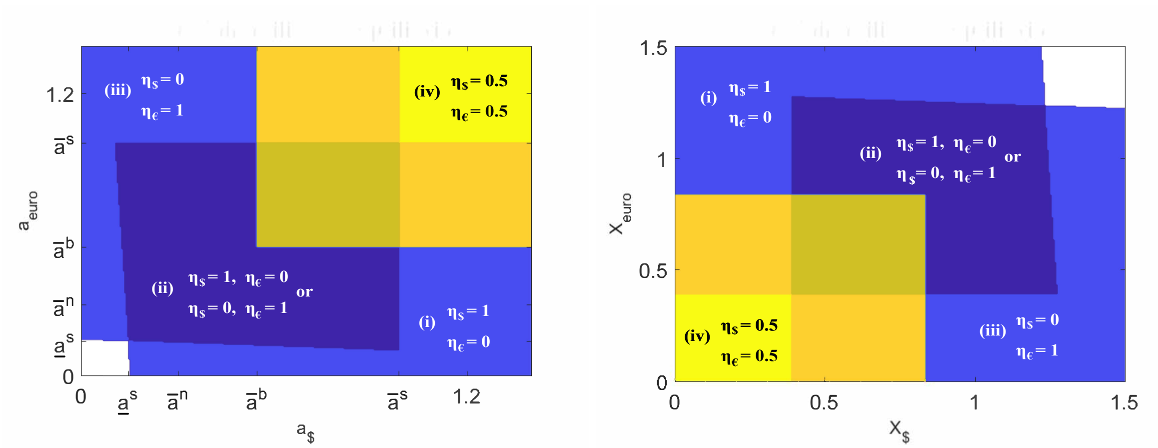
To make this equilibrium sustainable, we need to ensure that

$$(a_\$ + 0.5b) - \frac{\theta(\bar{\mathcal{E}} - 1)}{\delta\psi\sigma^2(1+\rho)} \geq X_\$ + 0.5\gamma_L N$$
$$(a_€ + 0.5b) - \frac{\theta(\bar{\mathcal{E}} - 1)}{\delta\psi\sigma^2(1+\rho)} \geq X_€ + 0.5\gamma_L N$$

We now turn to a numerical example.

A.5.1 Example 1: Asymmetric $a_\$$ and $a_€$

The equilibrium conditions we have derived in the preceding subsection can be used to illustrate various scenarios associated with the competition between the euro and the dollar to become a global dominant currency. As the first exercise, we revisit the formation of Eurozone in Section 5. Conceptually, the formation of the currency union can be thought of as a structural change which raises $a_{€i}$ relative to $a_{\$i}$ in emerging markets. We have claimed that the global economy prior to this change might as well be described by an equilibrium in which the dollar was the lone dominant currency. The diagram in Figure A.I illustrates this point when we allow for $a_€ \neq a_\$$. Assume, for the moment, that $X_€ = X_\$$. The set of no-dominant currency equilibria is ruled out here because this is not our current interest. One can easily compute the corresponding region by using conditions in Appendix A.5. In any case, the light blue area at the bottom of the figure displays the region of parametric space where the dollar is the only dominant currency. As $a_€$ rises, we can reach the space where the equilibrium selection is indeterminate. Multiple equilibria can survive in darker areas. We can consider two equilibrium pathways in association with the Eurozone example.



(a) Exogenous Trade Shares

(b) Exogenous Safe Asset Supply

Figure A.I: Sustainable Equilibria Regions

Path 1: (i) $\rightarrow$ (ii) One thought experiment we can conduct in this context is the rising importance of Euro as a medium of exchange in international trade. After the integration of European currencies, emerging markets have to bear a larger amount of euro-denominated revenue irrespective of their invoicing decisions. To describe such a trajectory, we investigate an equilibrium path starting from $(a_\$, a_€) = (0.5, 0)$ towards $(a_\$, a_€) = (0.5, 0.5)$. The diagram in Figure A.I shows that this equilibrium path passes through three different regions colored by light blue, dark blue and dark yellow respectively. We then computed how $\eta_\$$ and $\eta_€$ respond as the global economy moves along this trajectory. Our numerical example indeed illustrates the intuition that the dollar could still retain its sole dominance despite the fact that, in theory, multiple equilibria can arise. One caveat here is that we fixed $X_\$ = X_€$ in this example, while $a_€$ is increased. In the context of the Eurozone example, we can think of this assumption as implying that the supply of euro denominated assets has barely been changed relative to the trade invoicing.

Path 1: (i) $\rightarrow$ (iv) An alternative scenario we can consider is the rising importance of both central countries, U.S. and Eurozone, even with no catch-up of the later relative to the former. In other words, the two countries are so important as a share of global trade — that the only possible equilibrium is one where both the dollar and the euro are used by other countries to invoice their exports. To conduct this experiment, we investigate the equilibrium path starting from $(a_\$, a_€) = (0.5, 0)$ towards $(a_\$, a_€) = (1.2, 1.2)$. The diagram confirms our proposition that the global economy may end up in a situation where the only possible outcome is a dual-dominant equilibrium.

A.5.2 Example 2: Asymmetric $X_\$$ and $X_€$

In addition to $a_\$$ and $a_€$, our model can flexibly admit a difference between the foreign safe asset supplies $X_\$$ and $X_€$. Essentially, this setting reflects the fact that the market of the European bonds are more fragmented than that of U.S. treasury bonds. One can think of a situation in which the supply of dollar-denominated assets outside emerging markets is greater than the supply of Euro denominated assets. How much does this gap between $X_\$$ and $X_€$ matter in sustaining a single dominant currency equilibrium? To address this question, we revisit the numerical example and draw a region of the parameter space for sustaining each type of equilibria. We fix $a_\$ = a_€ = 1.08$ and, again, rule out no-dominant currency equilibria to focus on the main question. The diagram in Figure A.I(b) shows that a dollar dominant equilibrium is more likely to emerge when $X_\$$ is low relative to $X_€$. Yet, path dependency still matters as we have seen in the previous subsection